Equivalent Signal-Alignment-Based Frequency-Domain Equalization for MC-CDMA Two-Way Relay Systems

Tingting Liu, *Member, IEEE*, and Chenyang Yang, *Senior Member, IEEE*

***Abstract*—In multiuser relay systems, if the relay does not have sufficient degrees of freedom to eliminate multiuser interference, the spectral efficiency gain of two-way relay (TWR) transmission may vanish. Signal alignment (SA) signaling is possible to recover this gain but may suffer from a signal power loss. To improve the spectral efficiency of multicarrier code-division multiple-access (MC-CDMA) TWR systems, we develop an equivalent SA (ESA) signaling procedure. Simply by aligning the received symbols after despreading instead of aligning the received signals as in SA signaling, ESA signaling has more feasible solutions than SA signaling, and its performance and complexity depend on the spreading sequences. This allows us to improve the system performance by optimizing ESA signaling and to reduce its complexity by designing the spreading sequences. Specifically, we develop optimal ESA signaling to maximize the signal-to-noise-ratio (SNR) under the interference-free constraint. Then, we design the spreading sequence to obtain a low-complexity ESA-based frequency-domain equalizer (FDE), i.e., ESA-based one-tap FDE, and to provide the maximal diversity gain for the one-tap FDE. Both theoretical analysis and simulation results demonstrate that ESA signaling is more spectral efficient than SA signaling for MC-CDMA TWR systems.**

***Index Terms*—Equivalent signal alignment (ESA), frequency-domain equalizer (FDE), multiuser interference (MUI), multiuser two-way relay (TWR), spectral efficiency.**

I. Introduction

TWO-HOP communication systems using half-duplex relays suffer from a prelog factor 1/2 in the capacity expression [1], [2]. To mitigate such a loss in spectral efficiency, significant efforts have been devoted to develop spectral efficient relaying techniques [3]–[5]. Two-way relay (TWR) communication is one of the attractive techniques, which completes the data exchange between one or more pairs of users through two phases.

In multiuser TWR systems where multiple pairs of two users exchange their messages, the received signals of the users consist of both self-interference (SI) and multiuser interference (MUI). SI is the previously transmitted information of a user that returns to itself, which can be removed by SI cancellation (SIC) [6]. Various techniques have been developed to manage the MUI in multiuser TWR systems [7]–[12]. The TWR schemes in [7]–[9] treat the signals of K pairs of users as $2K$ independent signals. To remove the MUI thoroughly, the relay needs at least $2K$ degrees of freedom (DoFs), such as $2K$ codes, subcarriers, or antennas. As a result, these TWR systems achieve the same spectral efficiency as the two-hop relay systems, which use $2K$ DoFs to assist $2K$ pairs of users in four phases. When one pair of users employs *signal alignment (SA)* signaling [4] to compress their signals into a superimposed signal at the relay, the relay can forward the superimposed signal by *network coding* [13], [14]. Consequently, the TWR with SA signaling can use K DoFs to support K pairs of users in two phases, which recovers the spectral efficiency gain of the TWR in the high signal-to-noise ratio (SNR) region.

SA signaling was first proposed in [4] for multiple-input–multiple-output (MIMO) TWR systems. In [15] and [16], the precoder at the base station is developed and analyzed for aligning the signals from one pair of users in MIMO TWR cellular systems. It has been shown that the SA signaling procedures in [15] and [16] do not always outperform non-SA (NSA) signaling [17], because the transmit vectors lead to a signal power loss when encountering the alignment constraint. Fortunately, in multicarrier code-division multiple-access (MC-CDMA) TWR systems, there is more than one way to align the signals. Thus far, the existing studies [4], [15], [16] focus on how to find a feasible SA signaling procedure for TWR systems. However, how to find the optimal SA solution that recovers the spectral efficiency gain of the TWR for all SNR levels is still an open problem.

In this paper, we develop an *equivalent SA (ESA)* signaling procedure to improve the spectral efficiency of MC-CDMA TWR systems. By exploiting the feature of MC-CDMA signals, ESA signaling aligns the received symbols after despreading, whereas SA signaling [4] aligns the received signals directly. Although the idea is simple, such a difference brings various benefits that will become clear in the following sections. First, there will be more ways to align the symbols than to align the signals; therefore, it is possible to optimize ESA signaling that maximizes the SNR while ensuring MUI-free transmission. Second, ESA signaling ensures MUI-free transmission even when the spreading sequences are not orthogonal, but SA

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The authors are with the School of Electronics and Information Engineering, Beihang University, Beijing 100191, China (e-mail: ttliu@ee.buaa.edu.cn; cyyang@buaa.edu.cn).

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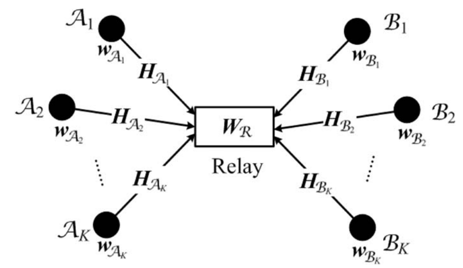

Fig. 1. Multiuser TWR system with K pairs of users assisted by a single relay.

signaling cannot ensure such transmission when the relay has no channel information. Finally, the complexity and performance of SA signaling only depend on the channel response, but those of ESA signaling also depend on the spreading sequences. This allows us to reduce the complexity and to improve the performance by judiciously designing the spreading sequence. Specifically, by decomposing and designing the spreading sequence, we develop an ESA-based one-tap frequency-domain equalizer (FDE) and provide the maximal diversity gain for the one-tap FDE. To show the performance gain and further optimize ESA signaling, we derive closed-form expressions of the asymptotic spectral efficiency for MC-CDMA TWR systems. Theoretical analysis and simulation results show that the proposed ESA signaling is spectral efficient for MC-CDMA TWR systems under various system settings.

The remainder of this paper is organized as follows: In Section II, we describe the system model. Then, we propose ESA signaling in Section III. In Section IV, we analyze the spectral efficiency of MC-CDMA TWR systems using our ESA signaling. Simulation results are provided in Section V, and finally, our conclusions are drawn in Section VI.

Notations: Conjugation, transpose, Hermitian transpose, and expectation are represented by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$, respectively. The trace of a square matrix is denoted by $\mathrm{tr}\{\cdot\}$. $\mathrm{diag}\{\cdots\}$ denotes the diagonal matrix, and $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^H\boldsymbol{x}}$ denotes the norm of vector $\boldsymbol{x}$.

II. System Description

We consider an MC-CDMA TWR system with M subcarriers, as shown in Fig. 1, where K $(K \leq M)$ pairs of users exchange their messages through a single half-duplex relay.

A. Transmitter at the Users

In a multiple-access channel (MAC) phase, the kth pair of users $\mathcal{A}_k$ and $\mathcal{B}_k$ uses two M-length frequency-domain transmit vectors $\boldsymbol{w}_{\mathcal{A}_k t}$ and $\boldsymbol{w}_{\mathcal{B}_k t}$ to convey their symbols $d_{\mathcal{A}_k}$ and $d_{\mathcal{B}_k}$, respectively. Transmit symbols $d_{\mathcal{A}_1}, \ldots, d_{\mathcal{A}_K}$ and $d_{\mathcal{B}_1}, \ldots, d_{\mathcal{B}_K}$ are independent and identically distributed (i.i.d.) random variables satisfying $\mathbb{E}\{d_{\mathcal{A}_k}\} = \mathbb{E}\{d_{\mathcal{B}_k}\} = 0$ and $\mathbb{E}\{|d_{\mathcal{A}_k}|^2\} = \mathbb{E}\{|d_{\mathcal{B}_k}|^2\} = P_{\mathcal{U}}$, where $P_{\mathcal{U}}$ is the maximal transmit power per symbol at each user. To meet the transmit power constraint, the transmit vectors satisfy $\boldsymbol{w}^H_{\mathcal{A}_k t}\boldsymbol{w}_{\mathcal{A}_k t} \leq 1$ and $\boldsymbol{w}^H_{\mathcal{B}_k t}\boldsymbol{w}_{\mathcal{B}_k t} \leq 1$. The frequency-domain received signal at the relay is expressed as

$$\boldsymbol{y}_{\mathcal{R}} = \sum_{k=1}^{K} (\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} d_{\mathcal{A}_k} + \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{w}_{\mathcal{B}_k t} d_{\mathcal{B}_k}) + \boldsymbol{n}_{\mathcal{R}} \quad (1)$$

where $\boldsymbol{H}_{\mathcal{A}_k}$=$\mathrm{diag}\{H_{\mathcal{A}_{k,1}}, \ldots, H_{\mathcal{A}_{k,M}}\}$, $\boldsymbol{H}_{\mathcal{B}_k}$=$\mathrm{diag}\{H_{\mathcal{B}_{k,1}}, \ldots, H_{\mathcal{B}_{k,M}}\}$, whose diagonal elements denote the frequency-domain channel responses over M subcarriers from the kth pair of users to the relay, $\boldsymbol{n}_{\mathcal{R}}$ is the M-length zero-mean Gaussian noise vector with a covariance matrix $\mathbb{E}\{\boldsymbol{n}_{\mathcal{R}}\boldsymbol{n}^H_{\mathcal{R}}\} = \sigma^2_{\mathcal{R}}\boldsymbol{I}_M$, and $\sigma^2_{\mathcal{R}}$ is the noise variance at the relay.

B. Transceiver at the Relay

If the SA signaling in [4] or our ESA signaling that will be introduced in the following is employed by the users, the symbols coming from one pair of users are aligned at the relay. In a broadcast channel (BC) phase, the relay can forward the superimposed symbols by various network-coding strategies in TWR systems. Two of the most popular strategies are physical-layer network coding (PNC) [13] and analog network coding (ANC) [14]. PNC can be used for estimate-and-forward (EF) [18], whereas ANC can be applied for amplify-and-forward (AF) [19].

1) PNC: The EF relay [18] first employs a linear receiver $\boldsymbol{w}_{\mathcal{R}_k r}$ to detect the superimposed symbol from the kth pair of users, i.e.,

$$\hat{d}_{\underline{\mathcal{A}+\mathcal{B}}_k} = \hat{d}_{\mathcal{A}_k} + \hat{d}_{\mathcal{B}_k} = \boldsymbol{w}^H_{\mathcal{R}_k r}\boldsymbol{y}_{\mathcal{R}} \quad (2)$$

then decodes the bitwise XORed message $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}$ from (2) by applying the PNC mapping principle in [13], where $b_{\underline{\mathcal{A}\oplus\mathcal{B}}_k} = b_{\mathcal{A}_k} \oplus b_{\mathcal{B}_k}$, and $b_{\mathcal{A}_k}$ and $b_{\mathcal{B}_k}$ are the messages coming from users $\mathcal{A}_k$ and $\mathcal{B}_k$, respectively. Then, the EF relay modulates $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}$ into $\hat{d}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}$ and employs a linear transmitter $\boldsymbol{w}_{\mathcal{R}_k t}$ to broadcast $\hat{d}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}$ to all users. The forwarded signal is

$$\boldsymbol{x}^{\mathrm{EF}}_{\mathcal{R}} = \sum_{k=1}^{K} \sqrt{\alpha^{\mathrm{EF}}_k}\boldsymbol{w}_{\mathcal{R}_k t}\hat{d}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k} \quad (3)$$

where $\alpha^{\mathrm{EF}}_k = 2P_{\mathcal{R}}/\mathrm{tr}\{\boldsymbol{w}_{\mathcal{R}_k t}\boldsymbol{w}^H_{\mathcal{R}_k t}\}$ is an amplification factor to meet the relay transmit power constraint, and $P_{\mathcal{R}}$ is the maximal transmit power per symbol at the relay.

2) ANC: When using ANC, the AF relay employs a linear processor to forward the received signal directly. The forwarded signal can be expressed as

$$\boldsymbol{x}^{\mathrm{AF}}_{\mathcal{R}} = \sum_{k=1}^{K} \sqrt{\alpha^{\mathrm{AF}}_k}\boldsymbol{w}_{\mathcal{R}_k}\boldsymbol{y}_{\mathcal{R}} \quad (4)$$

where $\boldsymbol{w}_{\mathcal{R}_k} = \boldsymbol{w}_{\mathcal{R}_k t}\boldsymbol{w}^H_{\mathcal{R}_k r}$ is the transceiver of the AF relay, which can be decoupled into a linear receiver for the MAC phase and a linear transmitter for the BC phase without performance loss [20], and $\alpha^{\mathrm{AF}}_k = 2P_{\mathcal{R}}/\mathrm{tr}\{\boldsymbol{w}_{\mathcal{R}_k}\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}}\boldsymbol{y}^H_{\mathcal{R}}\}\boldsymbol{w}^H_{\mathcal{R}_k}\}$ is the amplification factor.

C. Receiver at the Users

We assume that channel reciprocity holds for the TWR channel, i.e., the channel matrices from the relay to the kth pair of users are $\boldsymbol{H}_{\mathcal{A}_k}^T$ and $\boldsymbol{H}_{\mathcal{B}_k}^T$, respectively. Then, in the BC phase, the received signals at the kth pair of users are, respectively, expressed as

$$\boldsymbol{y}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{A}_k}^T \boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{A}_k} \quad \boldsymbol{y}_{\mathcal{B}_k} = \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{B}_k} \tag{5}$$

where $\boldsymbol{n}_{\mathcal{A}_k}$ and $\boldsymbol{n}_{\mathcal{B}_k}$ are the M-length zero-mean Gaussian noise vectors with $\mathbb{E}\{\boldsymbol{n}_{\mathcal{A}_k}\boldsymbol{n}_{\mathcal{A}_k}^H\} = \mathbb{E}\{\boldsymbol{n}_{\mathcal{B}_k}\boldsymbol{n}_{\mathcal{B}_k}^H\} = \sigma_{\mathcal{U}}^2 \boldsymbol{I}_M$, and $\sigma_{\mathcal{U}}^2$ is the noise variance at each user.

The received symbols are estimated as

$$z_{\mathcal{A}_k} = \boldsymbol{w}_{\mathcal{A}_k r}^H \boldsymbol{y}_{\mathcal{A}_k} \quad z_{\mathcal{B}_k} = \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{y}_{\mathcal{B}_k} \tag{6}$$

where $\boldsymbol{w}_{\mathcal{A}_k r}$ and $\boldsymbol{w}_{\mathcal{B}_k r}$ denote the receive vectors of users $\mathcal{A}_k$ and $\mathcal{B}_k$, respectively.

When different network coding strategies are used, the symbols from one pair of users are superimposed in different ways. Then, the users need to employ corresponding SIC methods to extract the desired signals or messages.

1) Bit-Level SIC: When PNC is applied, users $\mathcal{A}_k$ and $\mathcal{B}_k$ decode the bitwise XORed message $b_{\underline{\mathcal{A}\oplus\mathcal{B}}_k} = b_{\mathcal{A}_k} \oplus b_{\mathcal{B}_k}$ from (6), respectively. Let $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}^{\mathcal{A}_k}$ and $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}^{\mathcal{B}_k}$ represent the decoded messages, the users can recover their desired message by the bit-level SIC [5], i.e.,

$$\hat{b}_{\mathcal{B}_k} = \hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}^{\mathcal{A}_k} \oplus b_{\mathcal{A}_k} \quad \hat{b}_{\mathcal{A}_k} = \hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}_k}^{\mathcal{B}_k} \oplus b_{\mathcal{B}_k}. \tag{7}$$

2) Signal-Level SIC: When ANC is considered, users $\mathcal{A}_k$ and $\mathcal{B}_k$ demodulate the superimposed signal $d_{\underline{\mathcal{A}+\mathcal{B}}_k} = d_{\mathcal{A}_k} + d_{\mathcal{B}_k}$ from (6), respectively. Let $\hat{d}_{\underline{\mathcal{A}+\mathcal{B}}_k}^{\mathcal{A}_k}$ and $\hat{d}_{\underline{\mathcal{A}+\mathcal{B}}_k}^{\mathcal{B}_k}$ denote their demodulated signals, the users can estimate their desired signals by the signal-level SIC [5], i.e.,

$$\hat{d}_{\mathcal{B}_k} = \hat{d}_{\underline{\mathcal{A}+\mathcal{B}}_k}^{\mathcal{A}_k} - d_{\mathcal{A}_k} \quad \hat{d}_{\mathcal{A}_k} = \hat{d}_{\underline{\mathcal{A}+\mathcal{B}}_k}^{\mathcal{B}_k} - d_{\mathcal{B}_k}. \tag{8}$$

III. Signal Alignment Design

Here, we will introduce an ESA signaling procedure for MC-CDMA TWR systems. We first investigate the transmission in the MAC phase and then extend to the BC phase considering the duality.

We assume that each user only has the channel state information (CSI) between the relay and itself, whereas the relay has no CSI about any user. Note that the required CSI is much less than those in many existing TWR systems, where each user has the CSI of one pair of users and the relay has all the CSI [5]. This can reduce information exchange between the relay and the users since the CSI is obtained by broadcasting training sequences to all users during the procedure of initialization.

A. SA-Based FDE

To introduce ESA signaling, we first investigate the SA signaling in [4] as a starting point, where the transmit vectors of one pair of users in the MAC phase satisfy

$$\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} = \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{w}_{\mathcal{B}_k t} = \boldsymbol{h}_{\mathcal{A}\mathcal{B}_k} \tag{9}$$

where $\boldsymbol{h}_{\mathcal{A}\mathcal{B}_k}$ is an aligned signal vector.

In MC-CDMA systems, $M \times M$ channel matrices $\boldsymbol{H}_{\mathcal{A}_k}$ and $\boldsymbol{H}_{\mathcal{B}_k}$ have rank of M with a probability of 1; therefore, they are both invertible. Consequently, we can align the signals to an arbitrary vector $\boldsymbol{h}_{\mathcal{A}\mathcal{B}_k}$ by simply designing the transmit vectors as

$$\boldsymbol{w}_{\mathcal{A}_k t} = \boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{h}_{\mathcal{A}\mathcal{B}_k} \quad \boldsymbol{w}_{\mathcal{B}_k t} = \boldsymbol{H}_{\mathcal{B}_k}^{-1}\boldsymbol{h}_{\mathcal{A}\mathcal{B}_k}. \tag{10}$$

Considering that the relay has no CSI, it is impossible to apply a zero-forcing (ZF) receiver at the relay to eliminate the interference among multiple user pairs. To provide MUI-free transmission, each pair of users can align their signals to an orthogonal spreading sequence as in [10], i.e., $\boldsymbol{h}_{\mathcal{A}\mathcal{B}_k} = \sqrt{\alpha_k}\boldsymbol{c}_k$. Then, we have

$$\boldsymbol{w}_{\mathcal{A}_k t} = \sqrt{\alpha_k}\boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{c}_k \quad \boldsymbol{w}_{\mathcal{B}_k t} = \sqrt{\alpha_k}\boldsymbol{H}_{\mathcal{B}_k}^{-1}\boldsymbol{c}_k \tag{11}$$

where $\boldsymbol{C}^H\boldsymbol{C}=\boldsymbol{I}_K$, $\boldsymbol{C}=[\boldsymbol{c}_1,\ldots,\boldsymbol{c}_K]$, and $\alpha_k = \min\{\alpha_{\mathcal{A}_k}^{\mathrm{SA}}, \alpha_{\mathcal{B}_k}^{\mathrm{SA}}\}$ is a scaling factor to satisfy the transmit power constraint of users,[1] i.e.,

$$\alpha_{u_k}^{\mathrm{SA}} = \frac{1}{\left\|\boldsymbol{H}_{u_k}^{-1}\boldsymbol{c}_k\right\|^2} = \frac{1}{\boldsymbol{c}_k^H\left(\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\right)^{-1}\boldsymbol{c}_k}, \quad u_k = \mathcal{A}_k, \mathcal{B}_k. \tag{12}$$

Since $\boldsymbol{H}_{\mathcal{A}_k}$ and $\boldsymbol{H}_{\mathcal{B}_k}$ are diagonal matrices, the transmit vectors shown in (11) are in fact one-tap pre-FDEs. In the sequel, we call them *SA signaling* or *SA-based FDE*.

Now, the relay can simply employ a matched filter (MF) to eliminate MUI completely, i.e., $\boldsymbol{W}_{\mathcal{R}r} = \boldsymbol{C}$. Then, we have

$$\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} = \boldsymbol{C}^H\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{w}_{\mathcal{B}_k t} = \sqrt{\alpha_k}\boldsymbol{e}_k \tag{13}$$

where $\boldsymbol{e}_k$ is a basis vector with its kth entry being one and the other entries being zeros.

Unfortunately, SA signaling does not always outperform NSA signaling. This is because it suffers from a severe signal power loss due to the channel inversion operation in (11). In the following, we will introduce the ESA signaling procedure that can recover the power loss by exploiting the feature of MC-CDMA signals.

B. ESA-Based FDE

In MC-CDMA systems, the MF at the relay plays a role to despread the received signals from M signals to K symbols.

[1] Because each user only has the CSI between itself to the relay, user $\mathcal{A}_k$ only has $\alpha_{\mathcal{A}_k}$. To provide $\alpha_{\mathcal{B}_k}$ for user $\mathcal{A}_k$, the relay needs to estimate $\alpha_{\mathcal{B}_k}$ from the received signal power of user $\mathcal{B}_k$ and forward it to user $\mathcal{A}_k$, and *vice versa*.

In fact, (13) can be viewed as another kind of SA signaling, which aligns the symbols after despreading instead of aligning the signals before despreading as in (9). We refer to (13) as *ESA signaling*. Since transmit vectors $\boldsymbol{w}_{\mathcal{A}_k t}$ and $\boldsymbol{w}_{\mathcal{B}_k t}$ are performed in the frequency domain, we also call them *ESA-based FDEs*. Although the idea is simple, ESA signaling brings many benefits.

First, ESA signaling has more feasible solutions than the SA signaling procedure in (11). Take the transmit vector of user $\mathcal{A}_k$ as an example, to align the signals before despreading, i.e., $\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} = \sqrt{\alpha_k}\boldsymbol{c}_k$, there are M equations and M variables, which results in only one SA solution. By contrast, to align the symbols after despreading, i.e., $\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} = \sqrt{\alpha_k}\boldsymbol{e}_k$, there are K equations and M variables, which leads to $(M-K+1)^+$ ESA solutions, where $x^+ = \max\{x, 0\}$. This implies that we can find an optimal $\boldsymbol{w}_{\mathcal{A}_k t}$ to improve system performance.

Second, it is necessary for SA signaling to employ orthogonal spreading sequences to provide MUI-free transmission. By contrast, since $\boldsymbol{c}_j^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k t} = 0$, $\forall j \neq k$, as shown in (13), ESA signaling always ensures MUI-free transmission no matter if the spreading sequences are orthogonal or not.

Finally, the complexity and performance of SA signaling depend on the channel characteristic, whereas those of ESA signaling rely on both the channels and the spreading sequences. This indicates that we can design the spreading sequences to reduce the complexity and improve the performance of MC-CDMA TWR systems with ESA signaling.

1) Optimization of the ESA-Based FDE: Since we consider a practical scenario where each user only has the CSI between itself and the relay and thereby does not know the interference power, we employ SNR as the optimization goal. Specifically, we optimize the users' transmit vectors to maximize the SNR under the MUI-free constraint. In other words, we try to find an optimal ESA signaling procedure that satisfies (13) in a sense of maximizing the SNR, which is an effective way to deal with the power-loss problem arising in SA signaling.

Substituting (13) into (1), it is not difficult to derive the received SNR of the kth superimposed symbol in the MAC phase as $\alpha_k P_{\mathcal{U}}/\sigma_{\mathcal{R}}^2$. Given the signal and noise power, maximizing the SNR is equivalent to maximizing α_k. Under the ESA constraint in (13) and the transmit power constraint at the users, the optimization problem is formulated as

$$\max_{\boldsymbol{w}_{\mathcal{A}_k t}, \boldsymbol{w}_{\mathcal{B}_k t}} \quad \alpha_k \tag{14a}$$

$$\text{s.t.} \quad \boldsymbol{C}^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k} = \sqrt{\alpha_k}\boldsymbol{e}_k \tag{14b}$$

$$\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{w}_{\mathcal{B}_k} = \sqrt{\alpha_k}\boldsymbol{e}_k \tag{14c}$$

$$\boldsymbol{w}_{\mathcal{A}_k t}^H\boldsymbol{w}_{\mathcal{A}_k t} \leq 1 \quad \boldsymbol{w}_{\mathcal{B}_k t}^H\boldsymbol{w}_{\mathcal{B}_k t} \leq 1. \tag{14d}$$

It is not hard to solve the optimal $\boldsymbol{w}_{\mathcal{A}_k t}$ and $\boldsymbol{w}_{\mathcal{B}_k t}$, which are, respectively, the least-squares solutions of (14b) and (14c), i.e.,

$$\begin{aligned} \boldsymbol{w}_{\mathcal{A}_k t} &= \sqrt{\alpha_k}\boldsymbol{H}_{\mathcal{A}_k}^H\boldsymbol{C}\left(\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{H}_{\mathcal{A}_k}^H\boldsymbol{C}\right)^{-1}\boldsymbol{e}_k \\ \boldsymbol{w}_{\mathcal{B}_k t} &= \sqrt{\alpha_k}\boldsymbol{H}_{\mathcal{B}_k}^H\boldsymbol{C}\left(\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{H}_{\mathcal{B}_k}^H\boldsymbol{C}\right)^{-1}\boldsymbol{e}_k \end{aligned} \tag{15}$$

where $\alpha_k = \min\{\alpha_{\mathcal{A}_k}^{\text{ESA}}, \alpha_{\mathcal{B}_k}^{\text{ESA}}\}$, and

$$\begin{aligned} \alpha_{u_k}^{\text{ESA}} &= \frac{1}{\left\|\boldsymbol{H}_{u_k}^H\boldsymbol{C}\left(\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}\right)^{-1}\boldsymbol{e}_k\right\|^2} \\ &= \frac{1}{\boldsymbol{e}_k^H\left(\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}\right)^{-1}\boldsymbol{e}_k}, \qquad u_k = \mathcal{A}_k, \mathcal{B}_k. \end{aligned} \tag{16}$$

We refer to (15) as *Max-SNR ESA signaling* or *Max-SNR ESA-based FDE*. Substituting (15) and (11) into (13), we can see that both SA and ESA are feasible solutions of optimization problem (14).

Denote $N = M/K$ as the *spreading factor* and K/M as the *system load factor*. Considering the processing capability of the linear transceiver, we do not investigate the case of $N < 1$, i.e., the system is overloaded. Therefore, in this paper, spreading factor N is always greater than or equal to 1. When $N = 1$, i.e., the system is fully loaded, Max-SNR ESA signaling is equivalent to SA signaling. If $N > 1$, i.e., the system is underloaded, ESA signaling is the optimal solution, which implies that Max-SNR ESA signaling outperforms SA signaling.

2) ESA-Based One-Tap FDE: In (15), $\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}$ is no longer a diagonal matrix. This means that the Max-SNR ESA-based FDE is of high complexity. To develop an ESA-based one-tap FDE, we design the spreading sequence in the following.

Note that an arbitrary $M \times K$ matrix $\boldsymbol{C}$ can be decomposed into an $M \times K$ matrix $\boldsymbol{P}$ and a $K \times K$ unitary matrix $\boldsymbol{U}$, i.e.,

$$\boldsymbol{C} = \boldsymbol{P}\boldsymbol{U}. \tag{17}$$

If we regard $\boldsymbol{C}$ as the spreading sequence matrix and each column of $\boldsymbol{U}$ as the orthogonal sequence after despreading, then $\boldsymbol{P}$ reflects the relationship between the sequences before and after despreading. Since $\boldsymbol{U}$ denotes how to separate the users to provide *multiplexing gain* and $\boldsymbol{P}$ denotes how to spread signals to provide *frequency diversity gain*, we call $\boldsymbol{U}$ the *orthogonal code matrix* and $\boldsymbol{P}$ the *spreading matrix*. Therefore, (17) implies that the spreading sequence plays two different roles, and these roles can be decoupled.

Proposition 1: We obtain an ESA-based one-tap FDE when the elements of spreading matrix $\boldsymbol{P}$ satisfy

$$P_{m,j}P_{m,i} = 0 \qquad \forall j \neq i \tag{18}$$

i.e., there is no more than one nonzero element in each row of $\boldsymbol{P}$.

Proof: Substituting (17) into (15), the ESA-based FDE can be rewritten as

$$\boldsymbol{w}_{u_k t} = \sqrt{\alpha_k}\boldsymbol{H}_{u_k}^H\boldsymbol{P}\left(\boldsymbol{P}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{P}\right)^{-1}\boldsymbol{U}\boldsymbol{e}_k, \quad u_k = \mathcal{A}_k, \mathcal{B}_k. \tag{19}$$

From (18), the (i,j)th element of $\boldsymbol{P}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{P}$ becomes

$$\sum_{m=1}^{M} P_{m,i}P_{m,j}|H_{u_{k,m}}|^2 = 0 \qquad \forall i \neq j \tag{20}$$

for arbitrary $|H_{u_{k,1}}|^2, \ldots, |H_{u_{k,M}}|^2$, which means that $\boldsymbol{P}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{P}$ is always diagonal. Therefore, (19) becomes the ESA-based one-tap FDE. ■

Comparing (15), (19), and (11), we can see that the complexity of the ESA-based FDE mainly depends on the inversion of the $K \times K$ matrix, which is of order $\mathcal{O}(K^3)$. In contrast, the complexity of the SA-based FDE depends on M scalar divisions and that of the ESA-based one-tap FDE depends on K divisions, which are of orders $\mathcal{O}(M)$ and $\mathcal{O}(K)$, respectively. Obviously, the ESA-based one-tap FDE can reduce the complexity of the ESA-based FDE significantly. Compared with the SA-based FDE, the ESA-based one-tap FDE has similar complexity but can improve performance effectively.

Proposition 1 suggests that a low-complexity ESA-based FDE can be obtained by constructing $\boldsymbol{C}$ from a spreading matrix $\boldsymbol{P}$ satisfying (18) and an arbitrary unitary matrix $\boldsymbol{U}$. In fact, we can further optimize $\boldsymbol{P}$ to improve the spectral efficiency, as will be shown in the next section.

C. Duality Between the MAC and the BC

Thus far, we have investigated the transmitters of the users and the receiver of the relay for the MAC phase. Based on the duality between the MAC and BC phases and the reciprocity of their channel responses, when the relay employs its receiver in the MAC phase as the transmitter in the BC phase, i.e.,

$$\boldsymbol{W}_{\mathcal{R}t} = \boldsymbol{W}_{\mathcal{R}r}^* = \boldsymbol{C}^* \tag{21}$$

and the users apply the receivers

$$\boldsymbol{w}_{\mathcal{A}_k r} = \boldsymbol{w}_{\mathcal{A}_k t}^* \quad \boldsymbol{w}_{\mathcal{B}_k r} = \boldsymbol{w}_{\mathcal{B}_k t}^* \tag{22}$$

from (13), we have

$$\boldsymbol{w}_{\mathcal{A}_k r}^H \boldsymbol{H}_{\mathcal{A}_k}^T \boldsymbol{C} = \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{C} = \sqrt{\alpha_k} \boldsymbol{e}_k^T. \tag{23}$$

It indicates that we can achieve MUI-free transmission in the BC phase.

IV. Performance Analysis and Further Optimization

Here, we will analyze the instantaneous and average spectral efficiency of MC-CDMA TWR systems using Max-SNR ESA signaling. With the analysis, we can compare the performance of the Max-SNR ESA-based FDE with that of the SA-based FDE and NSA signaling and can further optimize spreading matrix $\boldsymbol{P}$ and the power allocation among the relay and users, given the overall power in the TWR network.

A. Instantaneous Spectral Efficiency and Optimal Spreading Matrix

1) Max-SNR ESA Versus SA: In MC-CDMA TWR systems, there are K pairs i.i.d. data streams forwarded by the relay over M subcarriers in two phases; hence, the instantaneous spectral efficiency is

$$\eta = \frac{1}{2M} \sum_{k=1}^{K} \log(1 + \gamma_{\mathcal{A}_k}) + \log(1 + \gamma_{\mathcal{B}_k}) \tag{24}$$

where $\gamma_{\mathcal{A}_k}$ and $\gamma_{\mathcal{B}_k}$ are the end-to-end SNRs of symbols $d_{\mathcal{A}_k}$ and $d_{\mathcal{B}_k}$, respectively.

In the EF relay, analogous to the analysis for the decode-and-forward (DF) relay in [1], the end-to-end SNR can be obtained as

$$\gamma_{\mathcal{A}_k}^{\text{EF}} = \min\{\gamma_{\mathcal{A}_k,\mathcal{B}_k \to \mathcal{R}}, \gamma_{\mathcal{R} \to \mathcal{B}_k}\} \tag{25a}$$

$$\gamma_{\mathcal{B}_k}^{\text{EF}} = \min\{\gamma_{\mathcal{A}_k,\mathcal{B}_k \to \mathcal{R}}, \gamma_{\mathcal{R} \to \mathcal{A}_k}\} \tag{25b}$$

where $\gamma_{\mathcal{A}_k,\mathcal{B}_k \to \mathcal{R}}$ denotes the SNR of the user–relay link in the MAC phase, and $\gamma_{\mathcal{R} \to \mathcal{A}_k}$ and $\gamma_{\mathcal{R} \to \mathcal{B}_k}$ represent the SNRs of the relay–user $\mathcal{A}_k$ and relay–user $\mathcal{B}_k$ links, respectively.

Substituting (15) and (22) into (1) and (6), the SNRs of the Max-SNR ESA-based FDE can be derived as

$$\gamma_{\mathcal{A}_k,\mathcal{B}_k \to \mathcal{R}} = \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \frac{\left|\boldsymbol{c}_k^H \boldsymbol{H}_{\mathcal{A}_k} \boldsymbol{w}_{\mathcal{A}_k t}\right|^2}{\boldsymbol{c}_k^H \boldsymbol{c}_k} = \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k \tag{26a}$$

$$\gamma_{\mathcal{R} \to \mathcal{A}_k} = \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \frac{\left|\boldsymbol{w}_{\mathcal{A}_k r}^H \boldsymbol{H}_{\mathcal{A}_k}^T \boldsymbol{c}_k^*\right|^2}{\boldsymbol{w}_{\mathcal{A}_k r}^H \boldsymbol{w}_{\mathcal{A}_k r}} = \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{A}_k} \tag{26b}$$

$$\gamma_{\mathcal{R} \to \mathcal{B}_k} = \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \frac{\left|\boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{c}_k^*\right|^2}{\boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{w}_{\mathcal{B}_k r}} = \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{B}_k} \tag{26c}$$

where $\alpha_k = \min\{\alpha_{\mathcal{A}_k}, \alpha_{\mathcal{B}_k}\}$, and $\alpha_{\mathcal{A}_k}$ and $\alpha_{\mathcal{B}_k}$ are obtained from (16).

Similarly, upon substituting (11) and (22), the SNRs of the SA-based FDE can be derived, which has the same form as (26), except that $\alpha_{\mathcal{A}_k}$ and $\alpha_{\mathcal{B}_k}$ are obtained from (12).

In the AF relay, the end-to-end SNR of the kth pair of users can be derived as (see the Appendix)

$$\gamma_{\mathcal{A}_k}^{\text{AF}} = \frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_k \alpha_{\mathcal{B}_k}}{\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{B}_k} + \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k + \frac{1}{2}} \tag{27a}$$

$$\gamma_{\mathcal{B}_k}^{\text{AF}} = \frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_k \alpha_{\mathcal{A}_k}}{\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{A}_k} + \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k + \frac{1}{2}}. \tag{27b}$$

Substituting (25a) and (25b) or (27a) and (27b) into (24), we can obtain the instantaneous spectral efficiency of the MC-CDMA TWR system using the EF or AF relay, which is

$$\eta^{\text{EF}} = \frac{1}{2M} \sum_{k=1}^{K} \log\left(1 + \min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k, \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{B}_k}\right\}\right) + \frac{1}{2M} \sum_{k=1}^{K} \log\left(1 + \min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k, \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{A}_k}\right\}\right) \tag{28a}$$

$$\eta^{\rm AF} = \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\alpha_k\alpha_{\mathcal{B}_k}}{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\alpha_k+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\alpha_{\mathcal{B}_k}+\frac{1}{2}}\right) + \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\alpha_k\alpha_{\mathcal{A}_k}}{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\alpha_k+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\alpha_{\mathcal{A}_k}+\frac{1}{2}}\right). \tag{28b}$$

When substituting $\alpha_{u_k}^{\rm SA}$ in (12) or $\alpha_{u_k}^{\rm ESA}$ in (16) into (28a) and (28b), we can obtain the spectral efficiency of the SA-based FDE or that of the Max-SNR ESA-based FDE.

Proposition 2: When the employed spreading sequences are orthogonal, i.e., $\boldsymbol{C}^H\boldsymbol{C} = \boldsymbol{I}_K$, the spectral efficiency of the Max-SNR ESA-based one-tap FDE is equal to or greater than that of the SA-based one-tap FDE.

Proof: Upon substituting (17), (12), and (16) can be rewritten as

$$\alpha_{u_k}^{\rm SA} = \left(\boldsymbol{e}_k^H\boldsymbol{U}^H\boldsymbol{P}^H\left(\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\right)^{-1}\boldsymbol{P}\boldsymbol{U}\boldsymbol{e}_k\right)^{-1} = \left(\sum_{j=1}^{K}|U_{k,j}|^2\sum_{m=1}^{M}\frac{|P_{m,j}|^2}{|H_{u_{k,m}}|^2}\right)^{-1} \tag{29a}$$

$$\alpha_{u_k}^{\rm ESA} = \left(\boldsymbol{e}_k^H\boldsymbol{U}^H\left(\boldsymbol{P}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{P}\right)^{-1}\boldsymbol{U}\boldsymbol{e}_k\right)^{-1} = \left(\sum_{j=1}^{K}\frac{|U_{k,j}|^2}{\sum_{m=1}^{M}|P_{m,j}|^2|H_{u_{k,m}}|^2}\right)^{-1} \tag{29b}$$

where $U_{k,j}$ is the (j,k)th element of $\boldsymbol{U}$.

Obviously, (29a) and (29b) can be expressed as the following unified form:

$$\alpha_{u_k} = \left(\sum_{j=1}^{K}|U_{k,j}|^2\Pi_{u_{k,j}}^{-1}\right)^{-1} \tag{30}$$

where

$$\Pi_{u_{k,j}}^{\rm SA} = \left(\sum_{m=1}^{M}|P_{m,j}|^2|H_{u_{k,m}}|^{-2}\right)^{-1} \tag{31a}$$

$$\Pi_{u_{k,j}}^{\rm ESA} = \sum_{m=1}^{M}|P_{m,j}|^2|H_{u_{k,m}}|^2. \tag{31b}$$

When $\boldsymbol{C}^H\boldsymbol{C} = \boldsymbol{I}_K$, i.e., $\boldsymbol{U}^H\boldsymbol{P}^H\boldsymbol{P}\boldsymbol{U} = \boldsymbol{I}_K$, we have $\boldsymbol{P}^H\boldsymbol{P} = \boldsymbol{U}\boldsymbol{U}^H = \boldsymbol{I}_K$, i.e., $\sum_{m=1}^{M}|P_{m,j}|^2 = 1$. Consequently, $\Pi_{u_{k,j}}^{\rm SA}$ in (31a) is a weighted *harmonic mean* of $\{|H_{u_{k,m}}|^2\}$, whereas $\Pi_{u_{k,j}}^{\rm ESA}$ in (31b) is their weighted *arithmetic mean*. The weighted arithmetic mean of nonnegative numbers is always greater than or equal to their weighted harmonic mean [22], i.e.,

$$\Pi_{u_{k,j}}^{\rm ESA} \geq \Pi_{u_{k,j}}^{\rm SA}, \qquad u_k = \mathcal{A}_k, \mathcal{B}_k. \tag{32}$$

Substituting (32) into (30), we have $\alpha_{u_k}^{\rm ESA} \geq \alpha_{u_k}^{\rm SA}$. Further substituting this inequality into (28a) and (28b) and after some regular manipulations, we can obtain $\eta^{\rm ESA} \geq \eta^{\rm SA}$ for both the EF and AF relays. ■

2) Optimization of the Spreading Matrix: As shown in (29b), spreading matrix $\boldsymbol{P}$ affects the performance of the Max-SNR ESA-based one-tap FDE. Therefore, we can improve the performance of the one-tap FDE by designing $\boldsymbol{P}$.

The spreading matrix essentially represents how to change the equivalent channel after despreading by exploiting the redundant subcarriers. From (31b), we can see that the number of the nonzero elements in each column of $\boldsymbol{P}$, their indexes, and weighting coefficients, respectively, denote how many and which subcarriers should be combined and how to combine them. Considering that the relay has no CSI, to provide the maximal diversity gain, the spreading matrix should be designed to combine the maximal number of subcarriers with the maximal spacing and equal gain weighting, i.e., $\boldsymbol{P}^o$, whose elements satisfy

$$\left|P_{m,j}^o\right| = \begin{cases} 1/\sqrt{N}, & m = nK+j \\ & \quad n\in[0,N),\ j\in[1,K] \\ 0, & \text{otherwise.}\end{cases} \tag{33}$$

Proposition 3: When $N \geq L$, where L is the number of resolvable paths, the Max-SNR ESA-based one-tap FDE with $\boldsymbol{P}^o$ can achieve the matched filter bound (MFB), i.e.,

$$\alpha_{u_k} = \sum_{l=0}^{L-1}|h_{u_{k,l}}|^2, \qquad u_k = \mathcal{A}_k, \mathcal{B}_k \tag{34}$$

where $h_{u_{k,l}}$ denotes the lth resolvable path in the time domain for $l = 0,\ldots,L-1$.

Proof: The channel response at the mth subcarrier can be expressed as $H_{u_{k,m}} = \sum_{l=0}^{L-1}h_{u_{k,l}}\exp(-i(2\pi(m-1)l/M))$. For optimal spreading matrix $\boldsymbol{P}^o$, when $N \geq L$, we have

$$\begin{aligned}\Pi_{u_{k,j}}^{\rm ESA} &= \frac{1}{N}\sum_{n=0}^{N-1}|H_{u_{k,nK+j}}|^2 \\ &= \sum_{n=0}^{N-1}\left|\frac{1}{\sqrt{N}}\sum_{l=0}^{L-1}h_{u_{k,l}}\exp\left(-i\frac{2\pi(nK+j-1)l}{NK}\right)\right|^2 \\ &= \sum_{n=0}^{N-1}\left|\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1}\tilde{h}_{u_{k,l}}\exp\left(-i\frac{2\pi(j-1)l}{NK}-i\frac{2\pi nl}{N}\right)\right|^2\end{aligned} \tag{35}$$

where $\tilde{h}_{u_{k,l}} = h_{u_{k,l}}$ if $l < L$; otherwise, $\tilde{h}_{u_{k,l}} = 0$.

By using Parseval's theorem, (35) becomes

$$\Pi_{u_{k,j}}^{\rm ESA} = \sum_{n=0}^{N-1}\left|\tilde{h}_{u_{k,l}}\exp\left(-i\frac{2\pi(j-1)l}{NK}\right)\right|^2 = \sum_{l=0}^{L-1}|h_{u_{k,l}}|^2. \tag{36}$$

Substituting (36) into (30), we obtain (34). ■

Note that spreading factor $N \geq L$ is a special case for MC-CDMA systems, and the conclusion seems trivial. Nonetheless, it has been shown that if there is one data stream (i.e., $K = 1$, $N = M$), the MC-CDMA system with Walsh–Hadamard codes can achieve the MFB [23]. In MC-CDMA systems, usually more than one data stream is simultaneously transmitted to support multiple pairs of users. Proposition 3 implies that, for any orthogonal codes $\boldsymbol{U}$ and for any number of $K \leq M/L$, the ESA-based one-tap FDE with $\boldsymbol{P}^o$ can always achieve the MFB.

B. Average Spectral Efficiency and Optimal Power Allocation

Now, we analyze the average spectral efficiency of Max-SNR ESA signaling in fading channels, with which we can show the impact of the spreading sequences and further optimize the optimal power allocation.

1) Asymptotic Analysis: The study in [24] indicates that, when $K > 2$, it is intractable to derive an explicit expression of the average spectral efficiency. Fortunately, as $K \to \infty$, we can obtain the closed-form asymptotic spectral efficiency, which converges to the average spectral efficiency. Moreover, as shown in the later simulation results, as $M/K \to N$, the asymptotic spectral efficiency is close to the average spectral efficiency with finite K and M. In the following, we will analyze the asymptotic spectral efficiency.

In (28a) and (28b), only term α_{u_k} depends on the channel response. To analyze the asymptotic spectral efficiency, it is important to derive the asymptotic value of α_{u_k}, i.e., $\bar{\alpha}_k = \lim_{K\to\infty} \alpha_{u_k}$. Since based on the forthcoming analysis $\bar{\alpha}_k$ only depends on the statistics of fading channels that are assumed to be identical for all users, i.e., $\bar{\alpha}_k$ is independent of k, $\bar{\alpha}_k$ is denoted by $\bar{\alpha}$ in the sequel.

For Max-SNR ESA signaling, the asymptotic value can be obtained from (16) by using the similar analysis in [25], i.e.,

$$\begin{aligned}\bar{\alpha} &= \lim_{K\to\infty} \left(\boldsymbol{e}_k^H \left(\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C} \right)^{-1} \boldsymbol{e}_k \right)^{-1} \\ &= \left(1/K \operatorname{tr} \left(\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C} \right)^{-1} \right)^{-1} \\ &= \left(\int_0^\infty x^{-1} f_{\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C}}(x) dx \right)^{-1} \\ &= \frac{1}{\varphi \left(\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C} \right)} \end{aligned} \tag{37}$$

where $\varphi(\boldsymbol{X}) \triangleq \int f_{\boldsymbol{X}}(t)/t dt$, and $f_{\boldsymbol{X}}(x)$ is the probability density function (pdf) of the eigenvalues of $\boldsymbol{X}$.

Since (37) is related to the characteristics of spreading sequences and fading channels, we consider several typical spreading sequences for MC-CDMA TWR systems in Rayleigh fading channels, including the nonorthogonal random sequences whose entries are i.i.d. random variables, the orthogonal random sequences that are uniformly distributed over the orthogonal space, and the spreading sequences constructed from (17) for the one-tap FDE with the optimal spreading matrix in (33). They are respectively referred to as i.i.d., orthogonal, and one-tap codes for short in the sequel.

i.i.d. Codes: When $\boldsymbol{C}$ is an $M \times K$ matrix whose entries are i.i.d. complex random variables with zero mean and variance $1/M$, and $\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H$ is a Hermitian random matrix and independent of $\boldsymbol{C}$, [25, Th. 2.42 and Lemma 2.28] showed that as $K, M \to \infty$ with $M/K \to N$, the empirical distribution of the eigenvalues of $\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C}$ converges with a probability of 1 to a distribution that satisfies

$$\mathcal{G}\left(\frac{1}{N} \varphi \left(\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C} \right) \right) = 1 - \frac{1}{N} \tag{38}$$

where

$$\mathcal{G}(x) \triangleq \int \frac{f_{\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H}(t)}{1 + xt} dt \tag{39}$$

and $f_{\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H}(x)$ is the pdf of the eigenvalues of $\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H$.

In Rayleigh fading channels, $\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H$ is an $M \times M$ diagonal matrix whose diagonal elements are exponentially distributed, i.e., $f_{\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H}(x) = e^{-x}$. Substituting into (39), we have

$$\mathcal{G}(x) = (1/x) \exp(1/x) \mathrm{Ei}(-1/x) \tag{40}$$

where $\mathrm{Ei}(x) \triangleq \int_{-\infty}^{x} e^t/t dt$ denotes the exponential integral function.

Substituting (40) into (38) and computing the inverse function of $\mathcal{G}(x)$, the asymptotic value of α_{u_k} with i.i.d. codes is

$$\bar{\alpha}^{\text{i.i.d.}} = \frac{1}{N \mathcal{H}(1 - 1/N)} \tag{41}$$

where $\mathcal{H}(x) = \mathcal{G}^{-1}(x)$ is the inverse function of $\mathcal{G}(x)$.

Orthogonal codes: When $\boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H$ is the same as the preceding case but $\boldsymbol{C}$ becomes an $M \times K$ matrix uniformly distributed over the manifold of $M \times K$ complex matrices such that $\boldsymbol{C}^H \boldsymbol{C} = \boldsymbol{I}$, [25, Ex. 2.51 and Lemma 2.28] indicated that as $K, M \to \infty$ with $M/K \to N$, the empirical distribution of the eigenvalues of $\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C}$ converges with a probability of 1 to a distribution that satisfies

$$\mathcal{G}\left(\frac{1}{N-1} \varphi \left(\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C} \right) \right) = 1 - \frac{1}{N}. \tag{42}$$

Substituting (40) into (42) and computing the inverse function of $\mathcal{G}(x)$ again, the asymptotic value of α_{u_k} with orthogonal codes is

$$\bar{\alpha}^{\text{Orth}} = \frac{1}{(N-1) \mathcal{H}(1 - 1/N)}. \tag{43}$$

One-tap codes: When $\boldsymbol{C}$ is constructed from (17) for the one-tap FDE, $\boldsymbol{U}$ is a unitary matrix and $\boldsymbol{\Pi}_{u_k} = \boldsymbol{P}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{P}$ is a diagonal matrix. Therefore, $\boldsymbol{U}^H \boldsymbol{\Pi}_{u_k} \boldsymbol{U}$ is an eigendecomposition of $\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C}$, i.e., $\boldsymbol{\Pi}_{u_k}$ is the eigenvalue matrix of $\boldsymbol{C}^H \boldsymbol{H}_{u_k} \boldsymbol{H}_{u_k}^H \boldsymbol{C}$. When the optimal spreading matrix in (33) is considered, from (31b), we have

$$\Pi_{u_k,j}^{\text{ESA}} = \frac{1}{N} \sum_{n=0}^{N-1} |H_{u_k, nK+j}|^2 \tag{44}$$

where $|H_{u_{k,nK+j}}|^2$ is exponentially distributed. Then, the pdf of $\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}$ is

$$f_{\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}}(x) = \frac{N}{(N-1)!}(Nx)^{N-1}e^{-Nx}. \tag{45}$$

Substituting (45) into (37) and applying [26, eq. (3.351.3)], we have $\varphi(\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}) = N/(N-1)$. Substituting into (37), the asymptotic value of α_{u_k} with one-tap codes is

$$\bar{\alpha}^{\text{One-tap}} = 1 - 1/N. \tag{46}$$

Note that $\bar{\alpha}$ is only related to the eigenvalue of $\boldsymbol{C}^H\boldsymbol{H}_{u_k}\boldsymbol{H}_{u_k}^H\boldsymbol{C}$ but independent of the eigenvector matrix, i.e., $\boldsymbol{U}$. Therefore, it is not necessary to design $\boldsymbol{U}$; in other words, $\boldsymbol{U}$ can be chosen as an arbitrary unitary matrix.

From (28a) and (28b), the asymptotic spectral efficiency of MC-CDMA TWR systems using EF and AF relays is obtained as

$$\bar{\eta}_{\text{ESA}}^{\text{EF}} = \frac{1}{N}\log\left(1 + \min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}, \frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right\}\bar{\alpha}\right) \tag{47a}$$

$$\bar{\eta}_{\text{ESA}}^{\text{AF}} = \frac{1}{N}\log\left(1 + \frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\bar{\alpha}^2}{\left(\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} + \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right)\bar{\alpha} + \frac{1}{2}}\right) \tag{47b}$$

where $\bar{\alpha}$ is obtained from (41), (43), and (46) for i.i.d., orthogonal, and one-tap codes, respectively.

We will show the impact of various spreading sequences on the spectral efficiency of MC-CDMA TWR systems with the Max-SNR ESA-based FDE via numerical analysis in the next section.

Now, we have obtained the closed-form expression of the asymptotic spectral efficiency, as shown in (47a) and (47b). The spectral efficiency depends on spreading factor $N = M/K$ and the maximal transmit powers at the relay and the users, i.e., $P_{\mathcal{U}}$ and $P_{\mathcal{R}}$. Due to the complicated relationship between spectral efficiency and load factor $1/N = K/M$, it is rather involved to directly derive optimal load factor K/M. Nonetheless, in the practical MC-CDMA TWR systems where the number of subcarriers M is given, the number of user pair K is an integer in the range of $[1, M]$. Therefore, it is not hard to find the optimal value of K by $M-1$ comparisons, and then, optimal load factor $1/N = K/M$ is obtained. In contrast, there exists an explicit relationship between the spectral efficiency and the maximal transmit powers. We can optimize the maximal transmit powers of the relay and the users to maximize the spectral efficiency.

2) Optimal Power Allocation: To avoid frequent information exchange between the users and the relay, the maximal powers are allocated by optimizing the average spectral efficiency instead of the instantaneous spectral efficiency. Assuming that the network overall power per symbol $P = P_{\mathcal{U}} + P_{\mathcal{R}}$ is limited.

When $\sigma_{\mathcal{R}}^2 = \sigma_{\mathcal{U}}^2 = \sigma^2$, from (47a), we know that the average spectral efficiency of the EF relay is monotonically increasing with $\min\{P_{\mathcal{U}}, 2P_{\mathcal{R}}\}$. Then, the optimal power allocation can be derived by maximizing $\min\{P_{\mathcal{U}}, 2P_{\mathcal{R}}\}$, which is

$$P_{\mathcal{U}} = 2P/3 \quad P_{\mathcal{R}} = P/3 \tag{48}$$

and the corresponding maximal spectral efficiency is

$$\bar{\eta}_{\text{ESA}}^{\text{EF}} = \frac{1}{N}\log\left(1 + \frac{2P}{3\sigma^2}\bar{\alpha}\right). \tag{49}$$

Substituting $P_{\mathcal{U}} = P - P_{\mathcal{R}}$ into (47b) and letting $\partial\bar{\eta}_{\text{ESA}}^{\text{AF}}/\partial P_{\mathcal{R}} = 0$, the optimal power allocation that maximizes the average spectral efficiency of the AF relay is

$$P_{\mathcal{U}} = P/2 \quad P_{\mathcal{R}} = P/2 \tag{50}$$

and the corresponding maximal spectral efficiency is

$$\begin{aligned}\bar{\eta}_{\text{ESA}}^{\text{AF}} &= \frac{1}{N}\log\left(1 + \frac{P}{4\sigma^2}\bar{\alpha}\left(1 + \frac{1}{2\bar{\alpha}}\frac{\sigma^2}{P}\right)^{-1}\right)\\ &\leq \frac{1}{N}\log\left(1 + \frac{P}{4\sigma^2}\bar{\alpha}\right).\end{aligned} \tag{51}$$

Comparing (49) with (51), we can see that the EF relay is more spectral efficient than the AF relay.

C. Performance Comparison Between ESA and NSA

To investigate the performance gain of ESA signaling over NSA signaling, we first analyze the asymptotic spectral efficiency of MC-CDMA TWR systems using NSA signaling.

When each user only has the CSI between the relay and itself while the relay has no CSI as we have assumed, we can design NSA signaling by solving the same optimization problem as in (14), except that the interference-free constraints now become $\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{w}_{\mathcal{A}_k} = \sqrt{\alpha_{\mathcal{A}_k}}\boldsymbol{e}_{\mathcal{A}_k}$ and $\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{w}_{\mathcal{B}_k} = \sqrt{\alpha_{\mathcal{B}_k}}\boldsymbol{e}_{\mathcal{B}_k}$. Using the same procedure as we derive the spectral efficiency of ESA signaling in Section IV-B, we can derive the spectral efficiency of NSA signaling. It shows that the spectral efficiency of NSA signaling is only 1/2 of that of ESA signaling when 1/2 of the users are supported by NSA signaling, as compared with those served by ESA signaling. However, under the considered CSI assumption, such NSA signaling is not a good solution since we can always employ the proposed ESA that performs much better.

In the literature of NSA signaling [8], there is a popular CSI assumption where the relay has the CSI of all users but the users have no CSI. To ensure MUI-free transmission, the relay employs the ZF transceiver [8] for NSA signaling. Although such a CSI assumption is inconsistent with that for SA or ESA signaling, the comparison is relatively fair since the CSI is available either at the users or at the relay. When the EF relay is employed by SA or ESA signaling, the DF relay is considered for NSA signaling, which is comparable. In this scenario, the TWR system with K pairs of users is equivalent to a two-hop relay system with $2K$ pairs of users. Therefore, we can obtain the performance of MC-CDMA TWR systems with NSA signaling from the well-established results of two-hop relay systems [1], [20].

For NSA signaling, from the conclusions in [1] and [20] and after using the similar derivations with SA signaling, the instantaneous spectral efficiency with the DF or AF relay can be obtained as

$$\eta_{\rm NSA}^{\rm DF} = \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\beta_{\mathcal{A}_k}, \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{B}_k}\right\}\right) + \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\beta_{\mathcal{B}_k}, \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{A}_k}\right\}\right) \quad (52a)$$

$$\eta_{\rm NSA}^{\rm AF} = \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{A}_k}\beta_{\mathcal{B}_k}}{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\beta_{\mathcal{A}_k}+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{B}_k}+1}\right) + \frac{1}{2M}\sum_{k=1}^{K}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{B}_k}\beta_{\mathcal{A}_k}}{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\beta_{\mathcal{B}_k}+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\beta_{\mathcal{A}_k}+1}\right) \quad (52b)$$

where

$$\beta_{\mathcal{A}_k} = \left(\boldsymbol{e}_k^H\left(\bar{\boldsymbol{H}}_{\mathcal{AB}}^H\bar{\boldsymbol{H}}_{\mathcal{AB}}\right)^{-1}\boldsymbol{e}_k\right)^{-1} \quad (53a)$$

$$\beta_{\mathcal{B}_k} = \left(\boldsymbol{e}_{k+K}^H\left(\bar{\boldsymbol{H}}_{\mathcal{AB}}^H\bar{\boldsymbol{H}}_{\mathcal{AB}}\right)^{-1}\boldsymbol{e}_{k+K}\right)^{-1} \quad (53b)$$

and $\bar{\boldsymbol{H}}_{\mathcal{AB}} = [\boldsymbol{H}_{\mathcal{A}_1}\boldsymbol{c}_1,\ldots,\boldsymbol{H}_{\mathcal{A}_K}\boldsymbol{c}_K,\boldsymbol{H}_{\mathcal{B}_1}\boldsymbol{c}_1,\ldots,\boldsymbol{H}_{\mathcal{B}_K}\boldsymbol{c}_K]$ is the $M\times 2K$ equivalent channel matrix.

To simplify the analysis, here we assume that the diagonal elements of $\boldsymbol{H}_{\mathcal{A}_1},\ldots,\boldsymbol{H}_{\mathcal{A}_K},\boldsymbol{H}_{\mathcal{B}_1},\ldots,\boldsymbol{H}_{\mathcal{B}_K}$ are zero-mean i.i.d. random variables with normalized variance. This implies that we assume that the channels of all links are independent frequency-selective fading. We will show in later simulations that the obtained conclusion is still valid without this assumption. As a result, no matter what the spreading sequences $\boldsymbol{c}_k$, $k=1,\ldots,K$ are, the elements of equivalent channel matrix $\bar{\boldsymbol{H}}_{\mathcal{AB}}$ are i.i.d. random variables. Since the diagonal entries of $\boldsymbol{H}_{u_k}$, $u_k=\mathcal{A}_k,\mathcal{B}_k$, are zero mean and with normalized variance, and the spreading sequences satisfy $\boldsymbol{c}_k^H\boldsymbol{c}_k=1$, the elements of $\bar{\boldsymbol{H}}_{\mathcal{AB}}$ are zero mean and with variance $1/M$.

Then, according to Marcenko–Pastur law [25] in the random matrix theory, as $K,M\to\infty$ with $M/K\to N$, the empirical distribution of $\bar{\boldsymbol{H}}_{\mathcal{AB}}^H\bar{\boldsymbol{H}}_{\mathcal{AB}}$ converges with a probability of 1 to a distribution that satisfies

$$\left(\varphi\left(\bar{\boldsymbol{H}}_{\mathcal{AB}}^H\bar{\boldsymbol{H}}_{\mathcal{AB}}\right)\right)^{-1} = (1-2K/M)^+ = (1-2/N)^+. \quad (54)$$

Consequently, the asymptotic value of β_{u_k} can be obtained as $\bar{\beta}=\lim_{K\to\infty}\beta_{u_k}=(1-2/N)^+$. Substituting into (52a) and (52b), the asymptotic spectral efficiency of MC-CDMA TWR systems using NSA signaling becomes

$$\bar{\eta}_{\rm NSA}^{\rm DF} = \frac{1}{N}\log\left(1+\min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2},\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right\}\left(1-\frac{2}{N}\right)^+\right) \quad (55a)$$

$$\bar{\eta}_{\rm NSA}^{\rm AF} = \frac{1}{N}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\left(\left(1-\frac{2}{N}\right)^+\right)^2}{\left(\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right)\left(1-\frac{2}{N}\right)^++1}\right). \quad (55b)$$

Considering that the asymptotic spectral efficiency of MC-CDMA TWR systems with Max-SNR ESA signaling depends on the spreading sequence, we compare the performance when the one-tap code is employed, because it is of low complexity and of practical interest.

Substituting (46) into (47a) and (47b), the asymptotic spectral efficiency of MC-CDMA TWR systems using ESA signaling is

$$\bar{\eta}_{\rm ESA}^{\rm EF} = \frac{1}{N}\log\left(1+\min\left\{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2},\frac{2P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right\}\left(1-\frac{1}{N}\right)\right) \quad (56a)$$

$$\bar{\eta}_{\rm ESA}^{\rm AF} = \frac{1}{N}\log\left(1+\frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\left(1-\frac{1}{N}\right)^2}{\left(\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2}+\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2}\right)\left(1-\frac{1}{N}\right)+\frac{1}{2}}\right). \quad (56b)$$

Comparing (55a) with (56a), and (55b) with (56b), we have

$$\bar{\eta}_{\rm ESA}^{\rm AF}(N) > \bar{\eta}_{\rm NSA}^{\rm AF}(N) \quad (57a)$$
$$\bar{\eta}_{\rm ESA}^{\rm EF}(N) \geq \bar{\eta}_{\rm NSA}^{\rm DF}(N). \quad (57b)$$

It follows that for an arbitrary spreading factor N, ESA signaling is more spectral efficient than NSA signaling in MC-CDMA TWR systems.

Setting the spreading factor in (56a) and (56b) as N, and that in (55a) and (55b) as $2N$, then we have

$$\eta_{\rm ESA}^{\rm AF}(N) > 2\eta_{\rm NSA}^{\rm AF}(2N) \quad (58a)$$
$$\eta_{\rm ESA}^{\rm EF}(N) \geq 2\eta_{\rm NSA}^{\rm DF}(2N). \quad (58b)$$

It means that when the number of users is doubled, ESA signaling achieves more than twice of the spectral efficiency of NSA signaling. In addition, denote $N_{\rm ESA}^o$ and $N_{\rm NSA}^o$ as the optimal spreading factors to achieve the maximal spectral efficiency of the MC-CDMA TWR system using the ESA and NSA signaling procedures, respectively, from (58a) and (58b), it is not difficult to derive that

$$\eta_{\rm ESA}^{\rm AF}(N_{\rm ESA}^o) \geq \eta_{\rm ESA}^{\rm AF}\left(N_{\rm NSA}^o/2\right) > 2\eta_{\rm NSA}^{\rm AF}\left(N_{\rm NSA}^o\right) \quad (59a)$$
$$\eta_{\rm ESA}^{\rm EF}(N_{\rm ESA}^o) \geq \eta_{\rm ESA}^{\rm EF}\left(N_{\rm NSA}^o/2\right) \geq 2\eta_{\rm NSA}^{\rm DF}\left(N_{\rm NSA}^o\right). \quad (59b)$$

As a result, when the one-tap code is employed, the maximal spectral efficiency of ESA signaling is twice more than that of NSA signaling. Moreover, from our forthcoming simulation and numerical analysis, we can see that the performance of orthogonal codes is better than that of one-tap codes. Therefore, the results in (59a) and (59b) are also valid for orthogonal codes.

V. Numerical and Simulation Results

Here, we demonstrate the performance of the MC-CDMA TWR systems with the Max-SNR ESA-based FDE. In the simulations, the MC-CDMA signals over $M=32$ subcarriers are transmitted over frequency-selective Rayleigh fading channels with $L=8$ resolvable multiple paths. Assuming that the noises at both the relay and all the users have the same variance, i.e., $\sigma_{\mathcal{R}}^2=\sigma_{\mathcal{U}}^2=\sigma^2$, then the SNR per symbol is P/σ^2.

To show the spectral efficiency gain provided by SA or ESA signaling, the performance of the MC-CDMA TWR system with NSA signaling is shown as a reference. The NSA signaling procedure we simulated is the same as that we have analyzed in Section IV. We first illustrate the data rate per user pair of MC-CDMA TWR systems using the Max-SNR ESA, SA, and NSA signaling procedures, where the orthogonal and nonorthogonal random spreading sequences are considered in

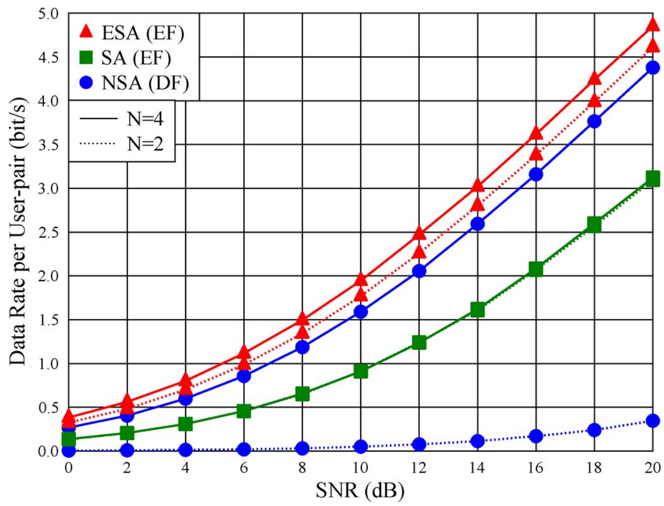

Fig. 2. Data rate per user pair versus SNR of MC-CDMA TWR systems using orthogonal codes $P_{\mathcal{R}} = P_{\mathcal{U}} = P/2$. The results of SA signaling with $N = 4$ and $N = 2$ overlap.

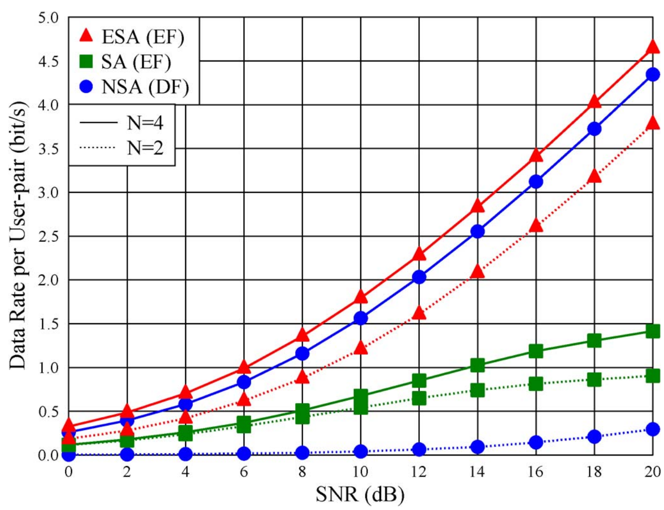

Fig. 3. Data rate per user pair versus SNR of MC-CDMA TWR systems using i.i.d. codes $P_{\mathcal{R}} = P_{\mathcal{U}} = P/2$.

Figs. 2 and 3, respectively. From these figures, we see that SA signaling is superior to NSA signaling when $N = 2$ but is inferior to NSA signaling when $N = 4$. This can be explained as follows: When $N = 2$, the TWR system with NSA signaling is fully loaded but that with SA signaling is underloaded. Therefore, SA signaling outperforms NSA signaling. When $N = 4$, the TWR system with both NSA and SA signaling procedures is underloaded. Due to the signal power loss, SA signaling suffers from severe spectral efficiency degradation. By contrast, Max-SNR ESA signaling achieves the highest spectral efficiency among them.

Comparing Fig. 2 with Fig. 3, we can see that when the spreading sequences become nonorthogonal, there exists severe performance degradation for SA signaling but only a slight performance loss for ESA signaling. When orthogonal spreading sequences are used, SA signaling can achieve MUI-free transmission. From (11), we know that the SA-base FDE does not depend on the spreading factor; hence, the results of $N = 2$ and $N = 4$ overlap. On the other hand, when nonorthogonal spreading sequences are employed, SA signaling introduces

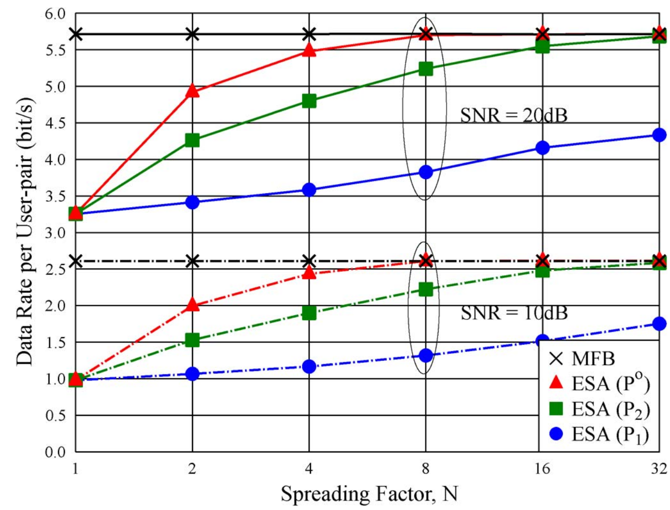

Fig. 4. Data rate per user pair versus spreading factor N of MC-CDMA TWR systems using the Max-SNR ESA-based one-tap FDE, EF relay, $P_{\mathcal{R}} = P_{\mathcal{U}} = P/2$.

interference among multiple user pairs; thereby, the case of $N = 2$ performs worse due to more severe interference among user pairs. It is shown that ESA signaling is more robust to the orthogonality of spreading sequences than SA signaling.

We then investigate the data rate per user pair of the MC-CDMA TWR systems using the Max-SNR ESA-based one-tap FDE in Fig. 4, where different spreading matrices are compared and the MFB is provided as a reference. Spreading matrix $\boldsymbol{P}_1$ chooses only K subcarriers to transmit K symbols, whereas $\boldsymbol{P}_2$ and $\boldsymbol{P}^o$ exploit the whole bandwidth, i.e., all M subcarriers to transmit the symbols. Different from $\boldsymbol{P}^o$, which combines the subcarriers with the maximal spacing, $\boldsymbol{P}_2$ combines the adjacent subcarriers.

It is shown that the data rates with all spreading matrices increase with N. When $N = 1$, all spreading matrices achieve the same performance. When $N > 1$, $\boldsymbol{P}^o$ and $\boldsymbol{P}_2$ outperform $\boldsymbol{P}_1$. This is because $\boldsymbol{P}^o$ and $\boldsymbol{P}_2$ combine more subcarriers to provide more diversity gain than $\boldsymbol{P}_1$. Since the channel correlation between adjacent subcarriers will reduce the diversity gain provided by $\boldsymbol{P}_2$, it is inferior to $\boldsymbol{P}^o$. Furthermore, $\boldsymbol{P}^o$ achieves the MFB when $N \geq L$. These results agree with our analysis very well.

Fig. 5 shows the spectral efficiency of Max-SNR ESA signaling obtained from the asymptotic analysis and simulation, respectively. It shows that the asymptotic analysis results are close to the simulation results in a finite number of users and subcarriers for an arbitrary SNR. We also show the impact of different spreading sequences on Max-SNR ESA signaling. It is clear that the orthogonal codes achieve the maximal spectral efficiency, whereas the i.i.d. codes are the opposite. The performance of one-tap codes is quite close to that of orthogonal codes. Therefore, the proposed one-tap Max-SNR ESA-based FDE achieves a good tradeoff between performance and complexity for MC-CDMA TWR systems.

From the foregoing results, we can see that Max-SNR ESA signaling performs well when $N > 1$, i.e., the system is underloaded. Intuitively, the large value of N (or equivalently the small value of K) means a waste of spectrum resource.

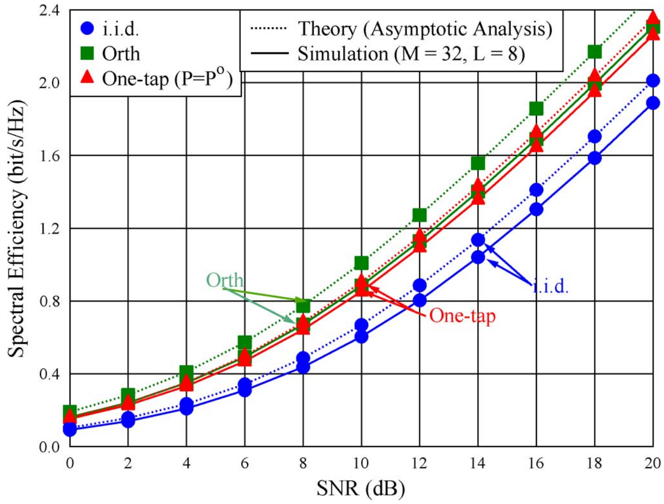

Fig. 5. Spectral efficiency versus SNR of MC-CDMA TWR systems using the Max-SNR ESA-based FDE with different codes, EF relay and $P_{\mathcal{R}} = P_{\mathcal{U}} = P/2$.

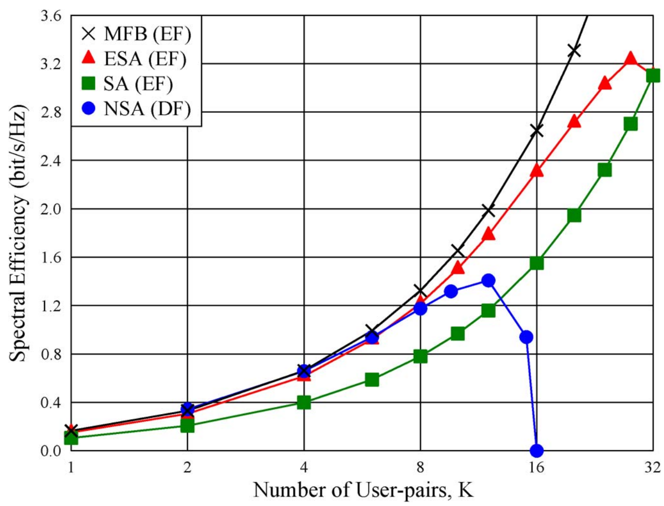

Fig. 6. Impact of K on the spectral efficiency of MC-CDMA TWR systems using the EF or DF relay and $P_{\mathcal{R}} = P_{\mathcal{U}} = P/2$.

To see if this is true, we investigate the spectral efficiency of MC-CDMA TWR systems with different K in Fig. 6, where the orthogonal codes are employed. We observe that when $K \leq 12$, NSA signaling can achieve higher spectral efficiency than SA signaling. However, when $K > 12$, the observation is the opposite. For NSA signaling with a ZF relay, the system is fully loaded or overloaded when $K \geq 16$ given $M = 32$; thereby, the spectral efficiency will reduce to zero. ESA signaling always outperforms other signaling procedures no matter what the value of K is. Moreover, the maximal spectral efficiency of NSA signaling is 1.4 bit/s/Hz $(K = 12)$, whereas that of ESA signaling is 3.2 bit/s/Hz $(K = 30)$. Therefore, the maximal spectral efficiency of ESA signaling is about 2.3 times more than that of NSA signaling, which agrees with our conclusion shown in (59b) very well.

Fig. 7 shows the impact of the power allocation among the relay and the users on the spectral efficiency of MC-CDMA TWR systems with Max-SNR ESA signaling, where $K = 30$ is chosen from Fig. 6 to achieve the maximal spectral efficiency. We can see that equal-power allocation, i.e., $P_{\mathcal{U}} = P_{\mathcal{R}}$, is

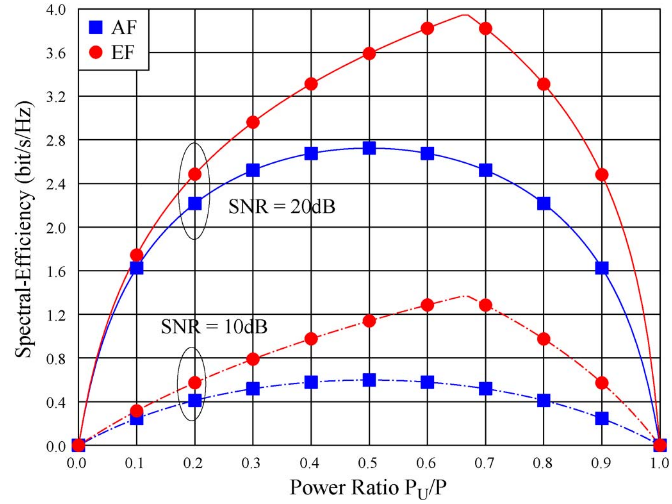

Fig. 7. Impact of the power allocation between the relay and the users on the spectral efficiency of MC-CDMA TWR systems using the Max-SNR ESA-based FDE and $K = 30$.

optimal for the AF relay, whereas $P_{\mathcal{U}} = 2P_{\mathcal{R}}$ is optimal for the EF relay, which is consistent with our theoretical analysis. Comparing the spectral efficiency of MC-CDMA TWR systems using different forwarding schemes, we can see that the EF relay is more spectral efficient than the AF relay.

VI. Conclusion

In this paper, we have developed an ESA signaling procedure to improve the spectral efficiency of MC-CDMA TWR systems through exploiting the feature of frequency-spreading systems. By aligning the received symbols after despreading rather than aligning the received signals before despreading, ESA signaling relaxes the alignment constraint and allows more feasible solutions to align the signals from each pair of users. Based on the observation that the signal power loss is a bottleneck to improve the performance of SA signaling, we developed an optimal ESA signaling procedure to maximize the SNR. Considering that the complexity and performance of ESA signaling depend on the spreading sequence, we designed the spreading sequences to obtain a low-complexity one-tap FDE to implement ESA signaling and to provide the maximal diversity gain for the FDE. Through analytical analysis, we showed that the spectral efficiency of the proposed ESA signaling with the optimal load factor is twice more than that of NSA signaling. Simulation results demonstrate that ESA signaling is superior to SA and NSA signaling procedures in various SNR levels, spreading factors, and different numbers of user pairs and is robust to the nonorthogonal spreading sequences.

Appendix
Derivation of the SNR With the AF Relay

Substituting (15) into (4) and then (6), after signal-level SIC, the symbol from user $\mathcal{A}_k$ to user $\mathcal{B}_k$ can be estimated as

$$\hat{d}_{\mathcal{A}_k} = \sqrt{\alpha_k^{\mathrm{AF}}}\boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{c}_k^* \boldsymbol{c}_k^H \boldsymbol{H}_{\mathcal{A}_k} \boldsymbol{w}_{\mathcal{B}_k t} d_{\mathcal{A}_k} + \sqrt{\alpha_k^{\mathrm{AF}}}\boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{c}_k^* \boldsymbol{c}_k^H \boldsymbol{n}_{\mathcal{R}} + \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{n}_{\mathcal{B}_k}. \quad (60)$$

Then, it is not difficult to derive that the signal power and noise power, respectively, in terms of $\hat{d}_{\mathcal{A}_k}$ are

$$P_{s\mathcal{A}_k} = P_{\mathcal{U}} \left| \sqrt{\alpha_k^{\mathrm{AF}}} \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{c}_k^* \boldsymbol{c}_k^H \boldsymbol{H}_{\mathcal{A}_k} \boldsymbol{w}_{\mathcal{B}_k t} \right|^2 = P_{\mathcal{U}} \alpha_k^{\mathrm{AF}} \alpha_k^2 \quad (61)$$

$$P_{n\mathcal{A}_k} = \alpha_k^{\mathrm{AF}} \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{H}_{\mathcal{B}_k}^T \boldsymbol{c}_k^* \boldsymbol{c}_k^T \boldsymbol{H}_{\mathcal{B}_k}^* \boldsymbol{w}_{\mathcal{B}_k r} \sigma_{\mathcal{R}}^2 + \boldsymbol{w}_{\mathcal{B}_k r}^H \boldsymbol{w}_{\mathcal{B}_k r} \sigma_{\mathcal{U}}^2 = \alpha_k^{\mathrm{AF}} \alpha_k \sigma_{\mathcal{R}}^2 + \alpha_k / \alpha_{\mathcal{B}_k} \sigma_{\mathcal{U}}^2 \quad (62)$$

where $\alpha_k^{\mathrm{AF}} = 2P_{\mathcal{R}} / (2P_{\mathcal{U}} \alpha_k + \sigma_{\mathcal{R}}^2)$.

Then, the corresponding received SNR becomes

$$\gamma_{\mathcal{A}_k}^{\mathrm{AF}} = \frac{P_{s\mathcal{A}_k}}{P_{n\mathcal{A}_k}} = \frac{P_{\mathcal{U}} \alpha_k^{\mathrm{AF}} \alpha_k}{\alpha_k^{\mathrm{AF}} \sigma_{\mathcal{R}}^2 + 1/\alpha_{\mathcal{B}_k} \sigma_{\mathcal{U}}^2} = \frac{2P_{\mathcal{U}} P_{\mathcal{R}} \alpha_k \alpha_{\mathcal{B}_k}}{2P_{\mathcal{R}} \alpha_{\mathcal{B}_k} \sigma_{\mathcal{R}}^2 + 2P_{\mathcal{U}} \alpha_k \sigma_{\mathcal{U}}^2 + \sigma_{\mathcal{R}}^2 \sigma_{\mathcal{U}}^2} = \frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_k \alpha_{\mathcal{B}_k}}{\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{B}_k} + \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k + \frac{1}{2}}. \quad (63)$$

Similarly, we can derive the SNR of $\hat{d}_{\mathcal{B}_k}$ as

$$\gamma_{\mathcal{B}_k}^{\mathrm{AF}} = \frac{\frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_k \alpha_{\mathcal{A}_k}}{\frac{P_{\mathcal{R}}}{\sigma_{\mathcal{U}}^2} \alpha_{\mathcal{A}_k} + \frac{P_{\mathcal{U}}}{\sigma_{\mathcal{R}}^2} \alpha_k + \frac{1}{2}}. \quad (64)$$

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Tingting Liu (M'11) received the B.S. and Ph.D. degrees in signal and information processing from Beihang University, Beijing, China, in 2004 and 2011, respectively.

From December 2008 to January 2010, she was a Visiting Student with the School of Electronics and Computer Science, University of Southampton, Southampton, U.K. She is currently a Postdoctoral Fellow with the School of Electronics and Information Engineering, Beihang University. Her research interests are in the areas of wireless communications and signal processing, including multicarrier code-division multiple-access, multiple-input–multiple-output, and distributed cooperative communications.

Chenyang Yang (SM'08) received the M.S.E. and Ph.D. degrees in electrical engineering from Beihang University (formerly Beijing University of Aeronautics and Astronautics), Beijing, China, in 1989 and 1997, respectively.

She is currently a Full Professor with the School of Electronics and Information Engineering, Beihang University. She has published various papers and filed many patents in the fields of signal processing and wireless communications. Her recent research interests include signal processing in network multiple-input multiple-output, cooperative communication, energy-efficient transmission, and interference management.

Prof. Yang is the Chair of the IEEE Communications Society Beijing Chapter. She has served as the Technical Program Committee Member for many IEEE conferences, such as the IEEE International Conference on Communications and the IEEE Global Telecommunications Conference. She currently serves as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, an Associate Editor-in-Chief of the *Chinese Journal of Communications*, and an Associate Editor-in-Chief of the *Chinese Journal of Signal Processing*. She was nominated as an Outstanding Young Professor of Beijing in 1995 and was supported by the First Teaching and Research Award Program for Outstanding Young Teachers of Higher Education Institutions by the Ministry of Education (P.R.C. "TRAPOYT") during 1999–2004.