Signal Alignment for Multicarrier Code Division Multiple User Two-Way Relay Systems

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***Abstract*—Two-way relay (TWR) transmission provides high spectral efficiency when one-pair of two users exchange information via a single relay. However, in multiuser relay systems where multi-pair of users exchange messages, if the relay does not have sufficient degrees of freedom, the gain of TWR communication may vanish. Fortunately, signal alignment (SA) signaling can recover the spectral efficiency of TWR transmission in multiuser scenarios, which is originally proposed for multiple-input-multiple-output (MIMO) systems. In this paper we investigate signal alignment for multicarrier code division multiple access (MC-CDMA) TWR systems. Due to the difference in channel characteristics and degrees of freedom, the existing SA signalings designed for MIMO TWR systems do not always perform well in MC-CDMA TWR systems. By exploiting the unique features of both TWR systems and MC-CDMA channels, we propose a spectral-efficient SA signaling for MC-CDMA TWR systems, where each pair of users employ a maximal ratio transmitter of its counterpart to align their signals at the relay. We then analyze the spectral efficiency of the designed SA signaling, compare it with non-SA (NSA) signaling, and optimize the power allocation among the relay and users. It is shown from asymptotic analysis and simulation results that the proposed SA signaling can support more users and achieve higher spectral efficiency than NSA signaling.**

***Index Terms*—MC-CDMA, two-way relay (TWR), signal alignment (SA), amplify-and-forward (AF), estimation-and-forward (EF), spectral efficiency.**

I. Introduction

TWO-HOP communication systems using half-duplex relays suffer from a pre-log factor $1/2$ in the capacity expression [1, 2]. To mitigate such a loss in spectral efficiency, significant efforts have been devoted to develop spectral-efficient relaying techniques [3–5]. Two-way relay (TWR) communication is one of the attractive techniques, which completes the data exchange between one or more pairs of users through two phases: multiple access channel (MAC) phase and broadcast channel (BC) phase [3, 6–8].

In multiuser TWR systems where multi-pair of two users exchange their messages, the received signals at the users consist of both self-interference (SI) and multiuser interference (MUI). SI is the previously transmitted information of a user that returns to itself, which can be removed by self-interference cancelation (SIC) [6]. Since the users usually do not know the channel state information (CSI) of other pairs of users, they can not avoid the MUI, and the relay needs to combat the MUI in both MAC and BC phases. Various techniques have been developed for multiuser TWR systems, e.g., using code division multiple access (CDMA) [8], frequency or time division multiple access [9, 10], and space division multiple access (SDMA) [7, 11–14]. The two-way relay methods developed in [7, 11, 12] treat the signals of K pairs of users as $2K$ independent signals. To remove the MUI thoroughly, the relay needs at least $2K$ degrees of freedom (DoFs), such as $2K$ codes, subcarriers or antennas. Yet in conventional relay systems, the relay requires K DoFs to assist K pairs of users [7]. Given $2K$ DoFs at the relay, the two-hop relay transmission needs four phases to support $2K$ pairs of users, which achieves the same spectral efficiency as the TWR methods proposed in [7, 11, 12]. This indicates that the spectral efficiency advantage of the TWR vanishes.

If one-pair of users compress their signals at the relay in MAC phase employing *signal alignment (SA)* signaling proposed in [4], the relay can use *network coding* to forward the superimposed signal in BC phase. Then each user is capable of extracting its desired information from the superimposed signal using SIC without any performance loss. As a result, in order to support information exchange between K pairs of users with K DoFs at the relay, the TWR system with the SA signaling needs two phases, but the TWR system without the SA signaling and the two-hop relay system require four phases [7]. Based on the observation that the signals from one-pair of users are not really the MUI, the SA signaling recovers the spectral efficiency of TWR systems in multiuser scenarios, which is in fact a special case of interference alignment (IA) [15].

In CDMA [8], orthogonal frequency division multiple access (OFDMA) [9], and frequency or time division multiple access (F/TDMA) [10] TWR systems under flat fading channels, the signals between one-pair of users are naturally aligned by sharing the same resource, such as the same code, subcarrier, or time slot. However, in SDMA or multicarrier CDMA (MC-CDMA) systems under frequency-selective fading channels, we need to design the transmit vectors of each pair of users to align their signals at the relay based on the CSI. In [4], the SA principle is first proposed and a SA signaling is designed for multiple-input-multiple-output (MIMO) systems. In [13, 14], the precoder at the base station is developed and analyzed for signal alignment in MIMO cellular systems.

Although multi-carrier systems have similar receive signal

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models to multi-antenna systems, existing SA algorithms for MIMO are not efficient for MC-CDMA owing to their different DoFs and channel characteristics.

On one hand, it has been shown that if the sum DoFs of one-pair of users are greater than the DoFs of the relay, there will be multiple SA solutions [4]. In the MIMO system considered in [4], the users are equipped with less antennas than the relay. Consequently, there is only one SA solution and the study focuses on how to find the feasible solution. By contrast, in MC-CDMA systems, each user may have the same number of subcarriers as the relay. There is more than one SA solution and different solutions perform distinctively. Therefore, the goal of SA design in MC-CDMA systems is to find the optimal solution or at least an effective one among multiple solutions.

On the other hand, the channel matrices of MC-CDMA systems are always diagonal, but those of MIMO systems are not. Recently, the impact of channel characteristics on the design and performance of IA signaling have been recognized [15]. Similarly, a SA signaling designed for MIMO does not necessarily perform well for MC-CDMA.

In this paper, we study spectral efficient SA signaling for MC-CDMA TWR systems. We start from analyzing two solutions of SA signaling, one of which was proposed for MIMO TWR systems. To mitigate their performance loss introduced by the signal power reduction, we proceed to find an optimal SA solution that maximizes each user's own signal-to noise-ratio (SNR). By exploiting the fact that the signals from each pair of users are useful for each other in TWR and by exploiting the diagonal channel structure in MC-CDMA systems, we propose a channel exchange SA (CE-SA) signaling, where each user maximizes SNR of its counterpart. Though very simple, the CE-SA signaling can achieve a good trade-off between enhancing signal power and mitigating interference comparing with the existing SA signalings. By analyzing and comparing the spectral efficiency of CE-SA signaling with that of non-SA (NSA) signaling, we show that the designed SA signaling can improve the spectral efficiency significantly.

The remainder of the paper is organized as follows. In Section II, we describe the signal model and transceiver schemes. The CE-SA signaling is introduced in Section III. In Section IV, we analyze the spectral efficiency of the proposed CE-SA signaling and compare with that of NSA signaling. Simulation results are provided in Section V and finally our conclusions are drawn in Section VI.

Notations Bold uppercase and lowercase variables are used to denote matrices and vectors. Conjugation, transpose, Hermitian transpose and expectation are represented by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $\mathbb{E}\{\cdot\}$, respectively. The trace of a square matrix is denoted as $\mathrm{tr}\{\cdot\}$. $\mathrm{diag}\{\cdots\}$ denotes the diagonal matrix, and $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^H\boldsymbol{x}}$ denotes the norm of the vector $\boldsymbol{x}$.

II. System Description

In this section, we will introduce the signal models and describe the transceiver schemes.

A. Signal Models

We consider an MC-CDMA TWR system with M subcarriers as shown in Fig. 1, where $K (K \leq M)$ pairs of users

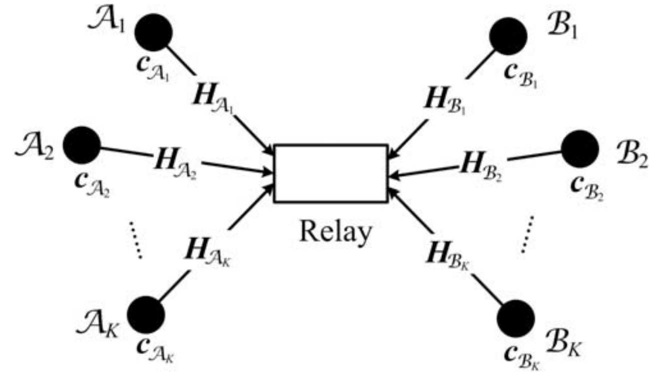

Fig. 1. Multiuser TWR systems with K pairs of users assisted by a single relay.

exchange their messages through a single half-duplex relay.

In MAC phase, the kth-pair of users, $\mathcal{A}_k$ and $\mathcal{B}_k$, use two M-length frequency domain transmit vectors $\boldsymbol{c}_{\mathcal{A}_k}$ and $\boldsymbol{c}_{\mathcal{B}_k}$ to convey their symbols $d_{\mathcal{A}_k}$ and $d_{\mathcal{B}_k}$, respectively. The transmit symbols $d_{\mathcal{A}_1}, \cdots, d_{\mathcal{A}_K}, d_{\mathcal{B}_1}, \cdots, d_{\mathcal{B}_K}$ are independent and identically distributed (i.i.d.) random variables satisfying $\mathbb{E}\{d_{\mathcal{A}_k}\} = \mathbb{E}\{d_{\mathcal{B}_k}\} = 0$ and $\mathbb{E}\{|d_{\mathcal{A}_k}|^2\} = \mathbb{E}\{|d_{\mathcal{B}_k}|^2\} = P_{\mathcal{U}}$, where $P_{\mathcal{U}}$ is the maximal transmit power per symbol at each user. To meet the transmit power constraint, the transmit vectors should satisfy $\boldsymbol{c}_{\mathcal{A}_k}^H\boldsymbol{c}_{\mathcal{A}_k} \leq 1$ and $\boldsymbol{c}_{\mathcal{B}_k}^H\boldsymbol{c}_{\mathcal{B}_k} \leq 1$ [1]. Then, the frequency domain received signal at the relay is expressed as

$$\begin{aligned}\boldsymbol{y}_{\mathcal{R}} &= \sum\nolimits_{k=1}^{K} (\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_{\mathcal{A}_k}d_{\mathcal{A}_k} + \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_{\mathcal{B}_k}d_{\mathcal{B}_k}) + \boldsymbol{n}_{\mathcal{R}} \\ &= \boldsymbol{H}_{\mathcal{A}}\boldsymbol{d}_{\mathcal{A}} + \boldsymbol{H}_{\mathcal{B}}\boldsymbol{d}_{\mathcal{B}} + \boldsymbol{n}_{\mathcal{R}} \end{aligned} \tag{1}$$

where $\boldsymbol{d}_{\mathcal{A}} = [d_{\mathcal{A}_1}, \cdots, d_{\mathcal{A}_K}]^T$ and $\boldsymbol{d}_{\mathcal{B}} = [d_{\mathcal{B}_1}, \cdots, d_{\mathcal{B}_K}]^T$, $\boldsymbol{H}_{\mathcal{A}} = [\boldsymbol{H}_{\mathcal{A}_1}\boldsymbol{c}_{\mathcal{A}_1}, \cdots, \boldsymbol{H}_{\mathcal{A}_K}\boldsymbol{c}_{\mathcal{A}_K}] \in \mathbb{C}^{M\times K}$, $\boldsymbol{H}_{\mathcal{B}} = [\boldsymbol{H}_{\mathcal{B}_1}\boldsymbol{c}_{\mathcal{B}_1}, \cdots, \boldsymbol{H}_{\mathcal{B}_K}\boldsymbol{c}_{\mathcal{B}_K}] \in \mathbb{C}^{M\times K}$, $\boldsymbol{H}_{\mathcal{A}_k} = \mathrm{diag}\{h_{\mathcal{A}_{1,k}}, \cdots, h_{\mathcal{A}_{M,k}}\}$, $\boldsymbol{H}_{\mathcal{B}_k} = \mathrm{diag}\{h_{\mathcal{B}_{1,k}}, \cdots, h_{\mathcal{B}_{M,k}}\}$, whose diagonal elements denote the frequency domain channel responses over the M subcarriers from the kth-pair of users to the relay, $\boldsymbol{n}_{\mathcal{R}}$ is an M-length zero-mean Gaussian noise vector at the relay with a covariance matrix $\mathbb{E}\{\boldsymbol{n}_{\mathcal{R}}\boldsymbol{n}_{\mathcal{R}}^H\} = \sigma_{\mathcal{R}}^2\boldsymbol{I}_M$, and $\sigma_{\mathcal{R}}^2$ is the noise variance.

We assume that the TWR channels are reciprocal, i.e., the channel matrices from the relay to the kth-pair of users are $\boldsymbol{H}_{\mathcal{A}_k}^T$ and $\boldsymbol{H}_{\mathcal{B}_k}^T$, respectively. Then in BC phase the received signals at the kth-pair of users are respectively expressed as

$$\boldsymbol{y}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{A}_k}^T\boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{A}_k} \tag{2a}$$

$$\boldsymbol{y}_{\mathcal{B}_k} = \boldsymbol{H}_{\mathcal{B}_k}^T\boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{B}_k} \tag{2b}$$

where $\boldsymbol{x}_{\mathcal{R}}$ is the forwarded signal vector satisfying $\mathrm{tr}\{\boldsymbol{x}_{\mathcal{R}}\boldsymbol{x}_{\mathcal{R}}^H\}/(2K) \leq P_{\mathcal{R}}$, $P_{\mathcal{R}}$ is the maximal transmit power per symbol at the relay, $\boldsymbol{n}_{\mathcal{A}_k}$ and $\boldsymbol{n}_{\mathcal{B}_k}$ are the M-length zero-mean Gaussian noise vectors with $\mathbb{E}\{\boldsymbol{n}_{\mathcal{A}_k}\boldsymbol{n}_{\mathcal{A}_k}^H\} = \mathbb{E}\{\boldsymbol{n}_{\mathcal{B}_k}\boldsymbol{n}_{\mathcal{B}_k}^H\} = \sigma_{\mathcal{U}}^2\boldsymbol{I}_M$, and $\sigma_{\mathcal{U}}^2$ is the noise variance at each user.

B. Transceiver Scheme

In this paper, we assume that each pair of users only have the CSI between this pair of users and the relay, i.e., $\boldsymbol{H}_{\mathcal{A}_k}$

[1]Note that if there is no SA constraint, $\boldsymbol{c}_{\mathcal{A}_k}^H\boldsymbol{c}_{\mathcal{A}_k} = 1$ and $\boldsymbol{c}_{\mathcal{B}_k}^H\boldsymbol{c}_{\mathcal{B}_k} = 1$.

and $\boldsymbol{H}_{\mathcal{B}_k}$, and the relay has the CSIs between all users and itself. This requirement for CSIs is reasonable and is common in TWR systems [5].

In MAC phase, each pair of users align their signals at the relay with judiciously designed transmit vectors, and the relay mitigates the interference between multiple user-pairs with a linear detector. In BC phase, the relay forwards the superimposed signals by network coding and the users extract the desired information from the superimposed signals by SIC.

1) SA at the Users: In order to align the signals coming from one-pair of users at the relay, the transmit vectors need to satisfy

$$\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_{\mathcal{B}_k} = \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_k}} \tag{3}$$

where $\boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_k}}$ is the common equivalent channel vector of the kth-pair of users after employing SA signaling.

When SA signaling is considered, the received signal at the relay becomes

$$\boldsymbol{y}_{\mathcal{R}} = \sum\nolimits_{k=1}^{K} \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_k}} d_{\underline{\mathcal{A}+\mathcal{B}_k}} + \boldsymbol{n}_{\mathcal{R}} = \boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{d}_{\underline{\mathcal{A}+\mathcal{B}}} + \boldsymbol{n}_{\mathcal{R}} \tag{4}$$

where $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}} = [\boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_1}}, \cdots, \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_K}}]$, $\boldsymbol{d}_{\underline{\mathcal{A}+\mathcal{B}}} = [d_{\underline{\mathcal{A}+\mathcal{B}_1}}, \cdots, d_{\underline{\mathcal{A}+\mathcal{B}_K}}]^T$, and $d_{\underline{\mathcal{A}+\mathcal{B}_k}} = d_{\mathcal{A}_k} + d_{\mathcal{B}_k}$ is the superimposed symbol of the kth-pair of users.

By comparing (1) and (4), we can find that now $2K$ symbols are compressed into K symbols. Thereby the relay with M subcarriers can suppress the MUI in MAC phase. To allow the relay to mitigate the MUI in BC phase, each pair of users also need to align their received signals. Based on the duality between the MAC and BC phases and the reciprocity of their channels, the users can apply their transmit vectors in MAC phase as the receive vectors in BC phase [11], i.e.,

$$z_{\mathcal{A}_k} = \boldsymbol{c}_{\mathcal{A}_k}^T\boldsymbol{y}_{\mathcal{A}_k} = \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_k}}^T\boldsymbol{x}_{\mathcal{R}} + \boldsymbol{c}_{\mathcal{A}_k}^T\boldsymbol{n}_{\mathcal{A}_k} \tag{5a}$$

$$z_{\mathcal{B}_k} = \boldsymbol{c}_{\mathcal{B}_k}^T\boldsymbol{y}_{\mathcal{B}_k} = \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}_k}}^T\boldsymbol{x}_{\mathcal{R}} + \boldsymbol{c}_{\mathcal{B}_k}^T\boldsymbol{n}_{\mathcal{B}_k} \tag{5b}$$

2) Network Coding at the Relay: After aligning the signals, the relay needs to mitigate the MUI and forward the superimposed symbols by network coding. Various network coding strategies can be applied for TWR systems. Two of the most popular strategies are analogue network coding (ANC) [16, 17] and physical-layer network coding (PNC) [14, 18]. ANC can be applied for amplify-and-forward (AF), while PNC can be used for estimate-and-forward (EF) [16].

When using ANC, the AF relay employs a linear processor to forward the signal. The forwarded signal can be expressed as

$$\boldsymbol{x}_{\mathcal{R}}^{\mathrm{AF}} = \sqrt{\alpha_{\mathrm{AF}}}\boldsymbol{W}_{\mathcal{R}}\boldsymbol{y}_{\mathcal{R}} \tag{6}$$

where $\alpha_{\mathrm{AF}} = 2KP_{\mathcal{R}}/\mathrm{tr}\{\boldsymbol{W}_{\mathcal{R}}\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}}\boldsymbol{y}_{\mathcal{R}}^H\}\boldsymbol{W}_{\mathcal{R}}^H\}$ is an amplification factor to meet the relay transmit power constraint.

The analysis in [19] indicates that the transceiver at the AF relay can be decoupled into a linear multiuser detector (MUD) for MAC phase and a linear multiuser transmitter (MUT) for BC phase without performance loss, i.e.,

$$\boldsymbol{W}_{\mathcal{R}} = \boldsymbol{W}_{\mathcal{R}t}\boldsymbol{W}_{\mathcal{R}r}^H \tag{7}$$

where $\boldsymbol{W}_{\mathcal{R}r}$ and $\boldsymbol{W}_{\mathcal{R}t}$ are the MUD and the MUT, respectively.

When using PNC, the EF relay first employs an MUD to receive the superimposed symbols, i.e.,

$$\hat{\boldsymbol{d}}_{\underline{\mathcal{A}+\mathcal{B}}} = \boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{y}_{\mathcal{R}} = \boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{d}_{\underline{\mathcal{A}+\mathcal{B}}} + \boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{n}_{\mathcal{R}} \tag{8}$$

Applying the PNC mapping principle in [18], the bitwise XORed message $\hat{\boldsymbol{b}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}$ can be decoded from (8), where $\boldsymbol{b}_{\underline{\mathcal{A}\oplus\mathcal{B}}} = [b_{\underline{\mathcal{A}\oplus\mathcal{B}_1}}, \cdots, b_{\underline{\mathcal{A}\oplus\mathcal{B}_K}}]^T$, $b_{\underline{\mathcal{A}\oplus\mathcal{B}_k}} = b_{\mathcal{A}_k} \oplus b_{\mathcal{B}_k}$, $b_{\mathcal{A}_k}$ and $b_{\mathcal{B}_k}$ are the messages coming from the user $\mathcal{A}_k$ and $\mathcal{B}_k$, respectively. Then the EF relay modulates $\hat{\boldsymbol{b}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}$ into $\hat{\boldsymbol{d}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}$ and employs an MUT to broadcast $\hat{\boldsymbol{d}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}$ to all users. The forwarded signal is

$$\boldsymbol{x}_{\mathcal{R}}^{\mathrm{EF}} = \sqrt{\alpha_{\mathrm{EF}}}\boldsymbol{W}_{\mathcal{R}t}\hat{\boldsymbol{d}}_{\underline{\mathcal{A}\oplus\mathcal{B}}} \tag{9}$$

where $\alpha_{\mathrm{EF}} = 2KP_{\mathcal{R}}/\mathrm{tr}\{\boldsymbol{W}_{\mathcal{R}t}\mathbb{E}\{\hat{\boldsymbol{d}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}\hat{\boldsymbol{d}}_{\underline{\mathcal{A}\oplus\mathcal{B}}}^H\}\boldsymbol{W}_{\mathcal{R}t}^H\}$ is an amplification factor to meet the relay transmit power constraint.

Since egocentric-altruistic (E-A) relay optimization is proved to yield a linear optimal transceiver under minimum mean-square error (MMSE) criterion and meanwhile the performance of the transceiver is analytical tractable [19], we apply the E-A relay optimization to design $\boldsymbol{W}_{\mathcal{R}r}$ and $\boldsymbol{W}_{\mathcal{R}t}$. From (4), (5a) and (5b), the equivalent channel matrices in the MAC and BC phases are $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}$ and $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^T$, respectively. With similar derivations as in [19], we have

$$\boldsymbol{W}_{\mathcal{R}r} = \left(\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^H + \rho\sigma_{\mathcal{R}}^2/(2P_{\mathcal{U}})\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}} \tag{10}$$

$$\boldsymbol{W}_{\mathcal{R}t} = \left(\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^*\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^T + \rho\sigma_{\mathcal{U}}^2/(2P_{\mathcal{R}})\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^* \tag{11}$$

where ρ is the noise-suppression factor [20], $\boldsymbol{W}_{\mathcal{R}r}$ and $\boldsymbol{W}_{\mathcal{R}t}$ constitute an MMSE transceiver when $\rho = 1$ or a zero-forcing (ZF) transceiver when $\rho \to 0$.

3) SIC at the Users: Using different network coding, the signals of one-pair of users are superimposed in different ways, then the users need to employ different SIC methods to extract the desired signals or messages.

When ANC is considered, the user $\mathcal{A}_k$ and $\mathcal{B}_k$ demodulate the superimposed signal $d_{\underline{\mathcal{A}+\mathcal{B}_k}} = d_{\mathcal{A}_k} + d_{\mathcal{B}_k}$ from (5a) and (5b), respectively. Let $\hat{d}_{\underline{\mathcal{A}+\mathcal{B}_k}}^{\mathcal{A}_k}$ and $\hat{d}_{\underline{\mathcal{A}+\mathcal{B}_k}}^{\mathcal{B}_k}$ denote their demodulated signals, the users can estimate their desired signals by the signal-level SIC [5], i.e.,

$$\hat{d}_{\mathcal{B}_k} = \hat{d}_{\underline{\mathcal{A}+\mathcal{B}_k}}^{\mathcal{A}_k} - d_{\mathcal{A}_k} \tag{12a}$$

$$\hat{d}_{\mathcal{A}_k} = \hat{d}_{\underline{\mathcal{A}+\mathcal{B}_k}}^{\mathcal{B}_k} - d_{\mathcal{B}_k} \tag{12b}$$

When PNC is applied, the user $\mathcal{A}_k$ and $\mathcal{B}_k$ decode the bitwise XORed message $b_{\underline{\mathcal{A}\oplus\mathcal{B}_k}} = b_{\mathcal{A}_k} \oplus b_{\mathcal{B}_k}$ from (5a) and (5b), respectively. Let $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}_k}}^{\mathcal{A}_k}$ and $\hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}_k}}^{\mathcal{B}_k}$ represent the decoded messages, the users can recover their desired message by the bit-level SIC [5], i.e.,

$$\hat{b}_{\mathcal{B}_k} = \hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}_k}}^{\mathcal{A}_k} \oplus b_{\mathcal{A}_k} \tag{13a}$$

$$\hat{b}_{\mathcal{A}_k} = \hat{b}_{\underline{\mathcal{A}\oplus\mathcal{B}_k}}^{\mathcal{B}_k} \oplus b_{\mathcal{B}_k} \tag{13b}$$

III. Signal Alignment Signaling Design

In this section, we will design SA signaling for MC-CDMA TWR systems. Before introducing our method, we will first address several possible solutions of the SA signaling, then we present an optimal solution and discuss its deficiencies.

A. SA Solutions in MC-CDMA TWR Systems

From (3), the SA transmit vectors can be obtained by solving the following linear equation,

$$\begin{bmatrix} \boldsymbol{H}_{\mathcal{A}_k} & -\boldsymbol{H}_{\mathcal{B}_k} \end{bmatrix} \begin{bmatrix} \boldsymbol{c}_{\mathcal{A}_k} \\ \boldsymbol{c}_{\mathcal{B}_k} \end{bmatrix} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{c}} = \boldsymbol{0} \tag{14}$$

where $\tilde{\boldsymbol{H}} = [\boldsymbol{H}_{\mathcal{A}_k}, -\boldsymbol{H}_{\mathcal{B}_k}] \in \mathbb{C}^{M_{\mathcal{R}} \times (M_{\mathcal{A}_k}+M_{\mathcal{B}_k})}$, $\tilde{\boldsymbol{c}} = [\boldsymbol{c}_{\mathcal{A}_k}^T, \boldsymbol{c}_{\mathcal{B}_k}^T]^T \in \mathbb{C}^{(M_{\mathcal{A}_k}+M_{\mathcal{B}_k})\times 1}$, $M_{\mathcal{A}_k}$, $M_{\mathcal{B}_k}$ and $M_{\mathcal{R}}$ denote the DoFs of the user $\mathcal{A}_k$, $\mathcal{B}_k$, and the relay $\mathcal{R}$, respectively.

Since the matrix $\tilde{\boldsymbol{H}}$ has rank of $\min\{M_{\mathcal{A}_k} + M_{\mathcal{B}_k}, M_{\mathcal{R}}\}$ with probability one [4], the dimension of its null space is

$$\begin{aligned} &\{M_{\mathcal{A}_k} + M_{\mathcal{B}_k}\} - \min\{M_{\mathcal{A}_k} + M_{\mathcal{B}_k}, M_{\mathcal{R}}\} \\ =&(M_{\mathcal{A}_k} + M_{\mathcal{B}_k} - M_{\mathcal{R}})^+ \end{aligned} \tag{15}$$

Therefore, there are $(M_{\mathcal{A}_k} + M_{\mathcal{B}_k} - M_{\mathcal{R}})^+$ nonzero solutions for the equation in (14), where $(x)^+ = \max\{x, 0\}$. This indicates that we have $(M_{\mathcal{A}_k} + M_{\mathcal{B}_k} - M_{\mathcal{R}})^+$ ways to align the signals.

In MIMO systems, $M_{\mathcal{A}_k}$, $M_{\mathcal{B}_k}$, and $M_{\mathcal{R}}$ respectively denote the antenna number of the user $\mathcal{A}_k$, $\mathcal{B}_k$, and the relay $\mathcal{R}$. In [4] $M_{\mathcal{A}_k} = M_{\mathcal{B}_k} = 2$ and $M_{\mathcal{R}} = 3$, then $(M_{\mathcal{A}_k}+M_{\mathcal{B}_k}-M_{\mathcal{R}})^+ = 1$. Consequently, there is only one way to align the signal, and the goal of SA design is to find the feasible solution.

By contrast, in MC-CDMA systems with M subcarriers, $M_{\mathcal{A}_k} = M_{\mathcal{B}_k} = M_{\mathcal{R}} = M$, then $(M_{\mathcal{A}_k}+M_{\mathcal{B}_k}-M_{\mathcal{R}})^+ = M$. Therefore, there are M independent SA transmit vectors and these vectors span the entire signal space[2]. As a result, the signals of each pair of users can be aligned to an arbitrary direction at the relay. For example, the signals of the kth-pair of users can be aligned to a direction $\boldsymbol{x}$ by simply choosing the transmit vectors as

$$\boldsymbol{c}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{x},\ \boldsymbol{c}_{\mathcal{B}_k} = \boldsymbol{H}_{\mathcal{B}_k}^{-1}\boldsymbol{x} \tag{16}$$

However, different aligned directions generated by different SA signalings perform distinctively. Some SA signalings may even be inferior to the NSA signaling. We can see this from the following examples.

Let $\boldsymbol{x} = \alpha_k \boldsymbol{c}_k$, where $\boldsymbol{c}_k$ is a frequency spreading sequence and α_k is a scaling factor to satisfy the transmit power constraint of the kth users. Then from (16) we have

$$\boldsymbol{c}_{\mathcal{A}_k} = \alpha_k \boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{c}_k,\ \boldsymbol{c}_{\mathcal{B}_k} = \alpha_k \boldsymbol{H}_{\mathcal{B}_k}^{-1}\boldsymbol{c}_k \tag{17}$$

where $\alpha_k = \min\{1/\|\boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{c}_k\|, 1/\|\boldsymbol{H}_{\mathcal{B}_k}^{-1}\boldsymbol{c}_k\|\}$, both users convert their equivalent channels into additive white Gaussian noise (AWGN) channel with same kind of pre-processing. We call the transmitters in (17) as *two-side SA* (TS-SA) signaling.

Let $\boldsymbol{x} = \alpha_k \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k$, we have

$$\boldsymbol{c}_{\mathcal{A}_k} = \alpha_k \boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k,\ \boldsymbol{c}_{\mathcal{B}_k} = \alpha_k \boldsymbol{c}_k \tag{18}$$

where $\alpha_k = \min\{1/\|\boldsymbol{H}_{\mathcal{A}_k}^{-1}\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k\|, 1/\|\boldsymbol{c}_k\|\}$. Since only one user adjusts its equivalent channel to align to its counterpart, the transmitters in (18) is referred to as *one-side SA* (OS-SA) signaling.

The OS-SA signaling was developed and analyzed for MIMO TWR systems in [13, 14]. However, the analysis in [21] shows that it performs even worse than the NSA signaling when $K \le M/2$. This is because in this case, the TWR systems are under-loaded and able to remove the MUI thoroughly even without SA signaling, but the OS-SA signaling suffers from severe signal power loss due to the channel inversion operation in (18). Similarly, from (17) we know that the performance of the TS-SA signaling will also be degraded by the signal power loss. These examples indicate that SA solutions can remove the interference between each pair of users but may not improve the signal power. This motivates us to optimize the SA signaling to maximize SNR.

B. Max Signal-to-Noise Ratio SA Signaling

Due to the lack of CSIs of other pairs of users, the users do not know the interference power. Consequently, we can only optimize their transmit vectors to maximize SNR instead of signal-to-interference-plus-noise ratio (SINR).

Given the noise power of the relay, to maximize SNR is equivalent to maximize the received signal power. According the SA principle shown in (3), we need to align the direction to the common equivalent channel of the kth-pair of users, i.e., $\boldsymbol{x} = \boldsymbol{h}_{\mathcal{A}\cdot\mathcal{B}_k}$. The received signal power at the relay is $\boldsymbol{x}^H\boldsymbol{x}$. From (16), the transmit power constraints can be expressed as $\boldsymbol{c}_{\mathcal{A}_k}^H\boldsymbol{c}_{\mathcal{A}_k} = \boldsymbol{x}^H\left(\boldsymbol{H}_{\mathcal{A}_k}^H\boldsymbol{H}_{\mathcal{A}_k}\right)^{-1}\boldsymbol{x} \le 1$ and $\boldsymbol{c}_{\mathcal{B}_k}^H\boldsymbol{c}_{\mathcal{B}_k} = \boldsymbol{x}^H\left(\boldsymbol{H}_{\mathcal{B}_k}^H\boldsymbol{H}_{\mathcal{B}_k}\right)^{-1}\boldsymbol{x} \le 1$. Then the optimization problem can be formulated as

$$\begin{aligned} &\max_{\boldsymbol{x}}\ \boldsymbol{x}^H\boldsymbol{x} \\ &s.t.\ \boldsymbol{x}_k^H\boldsymbol{A}_k\boldsymbol{x} \le 1,\ \boldsymbol{x}^H\boldsymbol{B}_k\boldsymbol{x} \le 1 \end{aligned} \tag{19}$$

where $\boldsymbol{A}_k = \left(\boldsymbol{H}_{\mathcal{A}_k}^H\boldsymbol{H}_{\mathcal{A}_k}\right)^{-1}$ and $\boldsymbol{B}_k = \left(\boldsymbol{H}_{\mathcal{B}_k}^H\boldsymbol{H}_{\mathcal{B}_k}\right)^{-1}$.

Both the objective function and constraints are quadratical, hence this is a quadratically constrained quadratic program (QCQP) problem [22]. Since $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are positive semidefinite matrices, this problem can be converted into a semidefinite programming (SDP) problem by using the semidefinite relaxation (SDR) technique [22]. Then a computationally efficient approximated solution can be obtained by using the convex optimization toolbox CVX [23]. By substituting the solution of $\boldsymbol{x}$ into (16), we can obtain the optimal transmit vectors for the maximal SNR (Max-SNR) SA signaling.

To investigate the performance of the Max-SNR SA signaling, we first consider a special case of $\boldsymbol{H}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{B}_k} = \boldsymbol{H}_k$, where the closed-form solution is available. In this case, the optimal $\boldsymbol{x}$ turns out to be the eigenvector of the minimal eigenvalue of $\boldsymbol{A}_k = \left(\boldsymbol{H}_k^H\boldsymbol{H}_k\right)^{-1}$, i.e., the eigenvector of the maximal eigenvalue of $\boldsymbol{H}_k^H\boldsymbol{H}_k$. This signaling has been employed as the transmit or receive vector at the user side for a multiuser MIMO system in [24]. In MIMO systems, since the entries of $\boldsymbol{H}_k$ are i.i.d. random variables, the dominant eigenvectors of different users have low correlation and hence cause less MUI. Consequently, from the result in [24], we see that this signaling performs well. In MC-CDMA systems, however, since $\boldsymbol{H}_k$ is diagonal, its eigenvectors are the basis vectors with only one entry being one while the other entries being zeros. As a result, each user will select the strongest subcarrier to achieve its own maximal SNR. It is highly probable that the selected subcarriers of different user-pairs

[2]The dimension of signal space is M, hence any M independent vectors can span the entire signal space.

collide to each other, which will lead to a severe performance degradation.

For the general cases where $\boldsymbol{H}_{\mathcal{A}_k} \neq \boldsymbol{H}_{\mathcal{B}_k}$, several subcarriers are chosen to transmit signals, which can be shown in the following proposition.

Proposition 1: For a subcarrier i, if one can find another subcarrier j satisfying $|h_{\mathcal{A}_{j,k}}|^2 > |h_{\mathcal{A}_{i,k}}|^2$ and $|h_{\mathcal{B}_{j,k}}|^2 > |h_{\mathcal{B}_{i,k}}|^2$, the ith subcarrier will not be selected for transmission by the optimal SA signaling.

Proof: See appendix A. ■

It indicates that there are always some subcarriers not being used by the Max-SNR signaling. In fact, our forthcoming simulations show that only one or two subcarriers will transmit signals in most cases. Again, the MUI caused by subcarrier collision will reduce the performance of the Max-SNR signaling.

C. CE-SA Signaling

The TS-SA and OS-SA signalings can remove the MUI between each pair of users but suffer from signal power loss due to the channel inversion operation, whereas the Max-SNR SA signaling can obtain the maximal signal power but is not immune to the MUI due to the usage of a few subcarriers. This suggests that a SA signaling without the channel inversion operation and using all subcarriers can achieve a good trade-off between enhancing the signal power and mitigating the interference.

Comparing (17) with (18), we have an interesting observation on the SA signaling. Specifically, if one user employs a maximal ratio transmitter (MRT) of its counterpart to construct its own transmit vector, its counterpart no longer needs the channel inversion operation for transmitting. For example, in (18) the user $\mathcal{A}_k$ employs the MRT of the user $\mathcal{B}_k$ as a component of its transmit vector, then the user $\mathcal{B}_k$ does not employ a ZF precoder to align the signals.

Inspired by this observation, we propose an altruistic transmission strategy: each user employs the MRT of its counterpart as its own transmit vector. That is to say, each user helps its counterpart to achieve the maximal SNR, instead of maximizing its own SNR as in the Max-SNR SA signaling. This is reasonable in TWR systems, because the signals from one-pair of users are all useful information for the two users. When every user assists its counterpart to optimize the performance, its own performance can be improved and finally the overall performance will be enhanced. The transmit vectors of the kth-pair of users can be expressed as

$$\boldsymbol{c}_{\mathcal{A}_k} = \alpha_k \boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k,\ \boldsymbol{c}_{\mathcal{B}_k} = \alpha_k \boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_k \tag{20}$$

where $\alpha_k = 1/\max\{\|\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_k\|, \|\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k\|\}$.

Considering the diagonal channel structure in MC-CDMA systems, we have $\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{H}_{\mathcal{A}_k} = \boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{H}_{\mathcal{B}_k}$. Then these transmit vectors are able to align the signals to the direction of $\alpha_k\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_k$ at the relay. Since each user needs to exchange its CSI to its counterpart, the proposed transmission strategy is called *channel exchange SA* (CE-SA) signaling.

There is no channel inversion operation in (20), therefore such a SA signaling can provide higher signal power than the TS-SA and OS-SA signalings. All subcarriers are used for transmission, therefore the CE-SA signaling is expected to be more robust to the inter-user-pair interference than the Max-SA signaling. In later simulations, we will show that the proposed CE-SA signaling provides substantial performance gain over the TS-SA, OS-SA and Max-SA signalings in MC-CDMA TWR systems.

IV. Performance Analysis

In order to investigate how much performance gain can be achieved by the CE-SA signaling, in this section we will analyze its spectral efficiency and compare with that of NSA signaling.

A. Spectral Efficiency

1) CE-SA: When SA signaling is employed, there are K i.i.d. data streams seen by the relay. Each stream conveys two symbols from one-pair of users over two phases, hence the spectral efficiency can be obtained as

$$\eta_{\mathrm{SA}} = \frac{1}{M}\sum\nolimits_{k=1}^{K}\log\left(1+\gamma_{k,\mathrm{SA}}\right) \tag{21}$$

where $\gamma_{k,\mathrm{SA}}$ is the end-to-end SINR of the two phase transmission, which depends on the forwarding strategy and the relay transceiver.

Although we can obtain the exact SINR expression following similar derivations as in [19], it is too complicated to analyze. Here we introduce approximation to simplify the SINR expression and validate the results via simulations later.

The study in [25] shows that in AWGN channels the end-to-end SNR γ is a function of the SNR in MAC phase γ^{MAC} and the SNR in BC phase γ^{BC}, i.e., $\gamma = \gamma^{\mathrm{MAC}}\cdot\gamma^{\mathrm{BC}}/(2\gamma^{\mathrm{MAC}}+\gamma^{\mathrm{BC}}+1)$. In our study, the received interference signals of the kth-pair of users at both the relay and the users are not Gaussian distributed. However, after using the linear receivers, they can be approximated as Gaussian noise. Applying the Gaussian approximation, the end-to-end SINR of AF relay can be approximated as a function of the SINRs in two phases, i.e.,

$$\gamma_{k,\mathrm{SA}}^{\mathrm{AF}} \approx \frac{\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{MAC}}\cdot\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{BC}}}{2\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{MAC}}+\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{BC}}+1},\ k=1,2,\ldots,K \tag{22}$$

where $\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{MAC}}$ and $\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{BC}}$ are the SINRs of the kth-pair of users in the MAC and BC phases, respectively.

In MAC phase, from (4) we obtain

$$\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{MAC}} = \frac{\left|\boldsymbol{e}_k^H\boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{e}_k\right|^2}{\sum_{j\neq k}2\left|\boldsymbol{e}_k^H\boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{e}_j\right|^2+\frac{\sigma_{\mathcal{R}}^2}{P_{\mathcal{U}}}\boldsymbol{e}_k^H\boldsymbol{W}_{\mathcal{R}r}^H\boldsymbol{W}_{\mathcal{R}r}\boldsymbol{e}_k} \tag{23}$$

where $\boldsymbol{e}_k$ is a basis vector with the kth entry being one while all the other entries being zeros.

Upon substituting (10), (23) becomes,

$$\gamma_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}^{\mathrm{MAC}} = \frac{P_{\mathcal{U}}/\sigma_{\mathcal{R}}^2}{\boldsymbol{e}_k^H\left(\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}^H\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}+\rho\sigma_{\mathcal{R}}^2/(2P_{\mathcal{U}})\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \frac{\rho}{2} \tag{24}$$

where $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}} = [\boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}}_1}, \cdots, \boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}}_K}]$ and $\boldsymbol{h}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k} = \alpha_k \boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k$, and (24) is the SINR of the MMSE-MUD when $\rho = 1$ or the SINR of the ZF-MUD when $\rho \to 0$.

In BC phase, from (5a) or (5b) we obtain

$$\gamma^{\mathrm{BC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k} = \frac{\left|\boldsymbol{e}_k^H \boldsymbol{H}^T_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{W}_{\mathcal{R}t}\boldsymbol{e}_k\right|^2}{\sum_{j\neq k}\left|\boldsymbol{e}_k^H \boldsymbol{H}^T_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{W}_{\mathcal{R}t}\boldsymbol{e}_j\right|^2 + \frac{\sigma^2_{\mathcal{U}}}{2KP_{\mathcal{R}}}\mathrm{tr}\{\boldsymbol{W}^H_{\mathcal{R}t}\boldsymbol{W}_{\mathcal{R}t}\}} \tag{25}$$

When substituting the preprocessing matrix $\boldsymbol{W}_{\mathcal{R}t}$ in (11) into (25), the expression of $\gamma^{\mathrm{BC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}$ will be too complicated to analysis. Based on the equivalency between the MUT and the MUD optimization as shown in [20], the SINR achieved by the MUT with $\boldsymbol{W}_{\mathcal{R}t}$ can be approximated by the SINR achieved by the MUD with $\boldsymbol{W}_{\mathcal{R}r} = \boldsymbol{W}^*_{\mathcal{R}t}$, when the noise variances at the relay and at each user are identical. Then (25) can be approximated as,

$$\gamma^{\mathrm{BC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k} \approx \frac{2P_{\mathcal{R}}/\sigma^2_{\mathcal{U}}}{\boldsymbol{e}_k^H\left(\boldsymbol{H}^T_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{H}^*_{\underline{\mathcal{A}\cdot\mathcal{B}}} + \rho\sigma^2_{\mathcal{U}}/(2P_{\mathcal{R}})\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \rho \tag{26}$$

which denote the SINR of the MMSE-MUT and ZF-MUT when $\rho = 1$ and $\rho \to 0$, respectively.

Similar to the decode-and-forward (DF) schemes [1], the performance of the EF relay is limited by the worse link of two phases. Hence the SINR of EF relay can be obtained as

$$\gamma^{\mathrm{EF}}_{k,\mathrm{SA}} = \min\left\{\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}, \gamma^{\mathrm{BC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k}\right\},\ k = 1, 2, \ldots, K \tag{27}$$

Substituting (22) or (27) into (21), we can obtain the instantaneous spectral efficiency of the MC-CDMA TWR system using the SA signaling for AF or EF relay.

2) NSA: When using NSA signaling, the TWR system with K pairs of users is equivalent to a two-hop relay system with $2K$ pairs of users. Therefore, we can obtain the performance of TWR systems with NSA signaling from the well-established results of two-hop relay systems [1, 19].

In this case, $2K$ i.i.d. data streams are received by the relay. Since each stream only conveys one symbol over two phases, the spectral efficiency can be obtained as

$$\eta_{\mathrm{NSA}} = \frac{1}{2M}\sum_{k=1}^{2K}\log\left(1 + \gamma_{k,\mathrm{NSA}}\right) \tag{28}$$

where $\gamma_{k,\mathrm{NSA}}$ is the SINR of the kth symbol in the combined symbol vector $\boldsymbol{d}_{\underline{\mathcal{A}\cup\mathcal{B}}} = [\boldsymbol{d}^T_{\mathcal{A}}, \boldsymbol{d}^T_{\mathcal{B}}]^T$, which depends on the forwarding operation and the transceiver at the relay.

Similarly, the SINR of AF relay can be approximated as

$$\gamma^{\mathrm{AF}}_{k,\mathrm{NSA}} \approx \frac{\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k}\cdot\gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k}}{\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k} + \gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k} + 1},\ k = 1, 2, \ldots, 2K \tag{29}$$

where $\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k}$ and $\gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k}$ are the SINRs of the kth symbol in the MAC and BC phases, respectively.

From the results in [19], we have

$$\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k} = \frac{P_{\mathcal{U}}/\sigma^2_{\mathcal{R}}}{\boldsymbol{e}_k^H\left(\boldsymbol{H}^H_{\underline{\mathcal{A}\cup\mathcal{B}}}\boldsymbol{H}_{\underline{\mathcal{A}\cup\mathcal{B}}} + \rho\sigma^2_{\mathcal{R}}/P_{\mathcal{U}}\boldsymbol{I}_{2K}\right)^{-1}\boldsymbol{e}_k} - \rho \tag{30a}$$

$$\gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k} \approx \frac{P_{\mathcal{R}}/\sigma^2_{\mathcal{U}}}{\boldsymbol{e}_k^H\left(\boldsymbol{H}^T_{\underline{\mathcal{B}\cup\mathcal{A}}}\boldsymbol{H}^*_{\underline{\mathcal{B}\cup\mathcal{A}}} + \rho\sigma^2_{\mathcal{U}}/P_{\mathcal{R}}\boldsymbol{I}_{2K}\right)^{-1}\boldsymbol{e}_k} - \rho \tag{30b}$$

where $\boldsymbol{H}_{\underline{\mathcal{A}\cup\mathcal{B}}} = [\boldsymbol{H}_{\mathcal{A}}, \boldsymbol{H}_{\mathcal{B}}]$ and $\boldsymbol{H}_{\underline{\mathcal{B}\cup\mathcal{A}}} = [\boldsymbol{H}_{\mathcal{B}}, \boldsymbol{H}_{\mathcal{A}}]$.

The SINR of DF relay is obtained as

$$\gamma^{\mathrm{DF}}_{k,\mathrm{NSA}} = \min\left\{\gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k}, \gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k}\right\},\ k = 1, 2, \ldots, 2K \tag{31}$$

Substituting (29) or (31) into (28), we can obtain the spectral efficiency of the MC-CDMA TWR system with NSA signaling when AF or DF is applied at the relay.

Since the equivalent channel matrices of the CE-SA and NSA signalings $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}$ and $\boldsymbol{H}_{\underline{\mathcal{A}\cup\mathcal{B}}}$ are different, it is hard to compare their instantaneous spectral efficiency. In the following, we further derive average spectral efficiency by asymptotical analysis.

B. Asymptotic Spectral Efficiency

The study in [26] indicates that when $K > 2$, it is intractable to derive an explicit expression of average spectral efficiency. Fortunately, as $K \to \infty$, we can obtain closed-form asymptotic spectral efficiency, which converges to the average spectral efficiency. Moreover, as shown in the later simulation results, as $K/M \to \beta$, the asymptotic spectral efficiency is very close to the average spectral efficiency with finite K, M, where β is defined as the system load factor. In the following, we will analyze the asymptotic spectral efficiency instead of the average spectral efficiency.

1) CE-SA: From (20), we know that the (m,k)th entry of $M \times K$ channel matrix $\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}$ is

$$h_{\underline{\mathcal{A}\cdot\mathcal{B}}_{m,k}} = \frac{h_{\mathcal{A}_{m,k}}h_{\mathcal{B}_{m,k}}c_{m,k}}{\max\{\|\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_k\|, \|\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k\|\}}, \quad k = 1, 2, \ldots, K,\ m = 1, 2, \ldots, M \tag{32}$$

where $c_{m,k}$ is the mth entry of $\boldsymbol{c}_k$.

Assuming that the channel response of each subcarrier is an i.i.d. random variable with normalized average energy, i.e., $\mathbb{E}\{|h_{\mathcal{A}_{m,k}}|^2\} = \mathbb{E}\{|h_{\mathcal{B}_{m,k}}|^2\} = 1$, then we have $\lim_{M\to\infty}\|\boldsymbol{H}_{\mathcal{A}_k}\boldsymbol{c}_k\| = \mathbb{E}\{|h_{\mathcal{A}_{m,k}}|^2\}\|\boldsymbol{c}_k\| = \|\boldsymbol{c}_k\|$ and $\lim_{M\to\infty}\|\boldsymbol{H}_{\mathcal{B}_k}\boldsymbol{c}_k\| = \|\boldsymbol{c}_k\|$. Therefore, as $M \to \infty$, we have $h_{\underline{\mathcal{A}\cdot\mathcal{B}}_{m,k}} \to h_{\mathcal{A}_{m,k}}h_{\mathcal{B}_{m,k}}c_{m,k}/\|\boldsymbol{c}_k\|$. Considering that $h_{\mathcal{A}_{m,k}}$ and $h_{\mathcal{B}_{m,k}}$ are zero-mean i.i.d. random variables, $\mathbb{E}\{h_{\mathcal{A}_{m,k}}h_{\mathcal{B}_{m,k}}c_{m,k}/\|\boldsymbol{c}_k\|\} = \mathbb{E}\{h_{\mathcal{A}_{m,k}}\}\mathbb{E}\{h_{\mathcal{B}_{m,k}}\}\mathbb{E}\{c_{m,k}/\|\boldsymbol{c}_k\|\} = 0$. Consequently, $h_{\underline{\mathcal{A}\cdot\mathcal{B}}_{m,k}}$ is a zero-mean random variable with variance $1/M$. According to the Marcenko-Pastur law [27], as $K,\ M \to \infty$ with $K/M \to \beta$, the empirical distribution of the eigenvalues of $\boldsymbol{H}^H_{\underline{\mathcal{A}\cdot\mathcal{B}}}\boldsymbol{H}_{\underline{\mathcal{A}\cdot\mathcal{B}}}$ converges almost surely. Therefore, the SINR in each phase also converges to a deterministic variable.

Analogous to the derivations in [27], we can derive the asymptotic SINRs from (24) and (26) as

$$\lim_{M,K\to\infty} \gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k} = \frac{1}{2}\mathcal{F}_\rho\left(\frac{2P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \beta\right) \tag{33a}$$

$$\lim_{M,K\to\infty} \gamma^{\mathrm{BC}}_{\underline{\mathcal{A}\cdot\mathcal{B}}_k} = \mathcal{F}_\rho\left(\frac{2P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, \beta\right) \tag{33b}$$

where $\mathcal{F}_\rho(x,z) \triangleq x - \frac{1}{4}\left(\sqrt{\rho + x\left(1+\sqrt{z}\right)^2} - \sqrt{\rho + x\left(1-\sqrt{z}\right)^2}\right)^2$.

Substituting (33a) and (33b) into (22) or (27), from (21) we obtain the asymptotic spectral efficiency of AF or EF relay as

$$\eta^{\mathrm{AF}}_{\mathrm{CE-SA}} = \beta \log\left(1 + \frac{1}{2}\frac{\mathcal{F}_\rho\left(\frac{2P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \beta\right)\mathcal{F}_\rho\left(\frac{2P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, \beta\right)}{\mathcal{F}_\rho\left(\frac{2P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \beta\right) + \mathcal{F}_\rho\left(\frac{2P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, \beta\right) + 1}\right) \tag{34a}$$

$$\eta^{\mathrm{EF}}_{\mathrm{CE-SA}} = \beta \log\left(1 + \min\left\{\frac{1}{2}\mathcal{F}_\rho\left(\frac{2P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \beta\right), \mathcal{F}_\rho\left(\frac{2P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, \beta\right)\right\}\right) \tag{34b}$$

2) NSA: For NSA signaling, the entries of the $M \times 2K$ channel matrix $\boldsymbol{H}_{\underline{\mathcal{A}\cup\mathcal{B}}}$ are also zero-mean with variance $1/M$. Again, as $K, M \to \infty$ with $2K/M \to 2\beta$, the empirical distribution of the eigenvalues of $\boldsymbol{H}^H_{\underline{\mathcal{A}\cup\mathcal{B}}}\boldsymbol{H}_{\underline{\mathcal{A}\cup\mathcal{B}}}$ converges almost surely and the SINR converges to a deterministic variable. Similarly, the asymptotic SINRs in (30a) and (30b) can be derived as

$$\lim_{M,K\to\infty} \gamma^{\mathrm{MAC}}_{\underline{\mathcal{A}\cup\mathcal{B}}_k} = \mathcal{F}_\rho\left(\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, 2\beta\right) \tag{35a}$$

$$\lim_{M,K\to\infty} \gamma^{\mathrm{BC}}_{\underline{\mathcal{B}\cup\mathcal{A}}_k} = \mathcal{F}_\rho\left(\frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, 2\beta\right) \tag{35b}$$

Substituting (35a) and (35b) into (29) or (31), then from (28) we can derive the asymptotic spectral efficiency of AF or DF relay as

$$\eta^{\mathrm{AF}}_{\mathrm{NSA}} = \beta \log\left(1 + \frac{\mathcal{F}_\rho\left(\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, 2\beta\right)\mathcal{F}_\rho\left(\frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, 2\beta\right)}{\mathcal{F}_\rho\left(\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, 2\beta\right) + \mathcal{F}_\rho\left(\frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}, 2\beta\right) + 1}\right) \tag{36a}$$

$$\eta^{\mathrm{DF}}_{\mathrm{NSA}} = \beta \log\left(1 + \mathcal{F}_\rho\left(\min\left\{\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\right\}, 2\beta\right)\right) \tag{36b}$$

3) Performance Comparison: Since $\mathcal{F}_\rho(x,z)$ is a complicated function of x when $\rho = 1$, it is rather involved to analyze the spectral efficiency of the MC-CDMA TWR system with the MMSE transceiver at the relay. By contrast, $\mathcal{F}_\rho(x,z) = x(1-z)^+$ as $\rho \to 0$, thereby the spectral efficiency of the TWR system with the ZF transceiver has simple expression and is easy for comparison. Since the performance of ZF transceiver asymptotically approaches to that of MMSE transceiver as SNR increases, we will focus on the performance comparison when the ZF transceiver is applied.

Let $\rho \to 0$, (34a) and (34b) become

$$\eta^{\mathrm{AF}}_{\mathrm{CE-SA}} = \beta \log\left(1 + \frac{\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}\frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\left((1-\beta)^+\right)^2}{\frac{1}{2} + \left(\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}} + \frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\right)(1-\beta)^+}\right) \tag{37a}$$

$$\eta^{\mathrm{EF}}_{\mathrm{CE-SA}} = \beta \log\left(1 + \min\left\{\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \frac{2P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\right\}(1-\beta)^+\right) \tag{37b}$$

and (36a) and (36b) are

$$\eta^{\mathrm{AF}}_{\mathrm{NSA}} = \beta \log\left(1 + \frac{\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}\frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\left((1-2\beta)^+\right)^2}{1 + \left(\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}} + \frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\right)(1-2\beta)^+}\right) \tag{38a}$$

$$\eta^{\mathrm{DF}}_{\mathrm{NSA}} = \beta \log\left(1 + \min\left\{\frac{P_{\mathcal{U}}}{\sigma^2_{\mathcal{R}}}, \frac{P_{\mathcal{R}}}{\sigma^2_{\mathcal{U}}}\right\}(1-2\beta)^+\right) \tag{38b}$$

Comparing (37a) with (38a), and (37b) with (38b), we have

$$\eta^{\mathrm{AF}}_{\mathrm{CE-SA}}(\beta) > \eta^{\mathrm{AF}}_{\mathrm{NSA}}(\beta) \tag{39a}$$

$$\eta^{\mathrm{EF}}_{\mathrm{CE-SA}}(\beta) \geq \eta^{\mathrm{DF}}_{\mathrm{NSA}}(\beta) \tag{39b}$$

It follows that for any load factor β, the CS-SA signaling is more spectral-efficient than the NSA signaling in MC-CDMA TWR systems.

Moreover, when $1 > \beta \geq 1/2$, the MC-CDMA system with the NSA signaling becomes fully- or over-loaded, whereas the system with the CE-SA signaling is still under-loaded. Therefore, the CS-SA signaling can support more users than the NSA signaling. This can be more clearly seen as following. Setting the load factor in (37a) and (37b) as 2β, and that in (38a) and (38b) as β, then we have

$$\eta^{\mathrm{AF}}_{\mathrm{CE-SA}}(2\beta) > 2\eta^{\mathrm{AF}}_{\mathrm{NSA}}(\beta) \tag{40a}$$

$$\eta^{\mathrm{EF}}_{\mathrm{CE-SA}}(2\beta) \geq 2\eta^{\mathrm{DF}}_{\mathrm{NSA}}(\beta) \tag{40b}$$

It means that when the number of users is doubled, the CE-SA signaling achieves more than twice of the spectral efficiency of NSA signaling. Furthermore, from (40a) and (40b), it is not difficult to prove that $\max_\beta\{\eta^{\mathrm{AF}}_{\mathrm{CE-SA}}\} > 2\max_\beta\{\eta^{\mathrm{AF}}_{\mathrm{NSA}}\}$ and $\max_\beta\{\eta^{\mathrm{EF}}_{\mathrm{CE-SA}}\} > 2\max_\beta\{\eta^{\mathrm{DF}}_{\mathrm{NSA}}\}$. Therefore, if we are allowed to choose the optimal β, the maximal spectral efficiency of CE-SA signaling is twice more than that of NSA signaling.

C. Global Power Allocation

From the previous analysis, we can see that the spectral efficiency of MC-CDMA TWR systems depends on the transmit power constraints at the relay and at the users, i.e., $P_{\mathcal{U}}$ and $P_{\mathcal{R}}$. Therefore, given the network overall power per symbol $P = P_{\mathcal{U}} + P_{\mathcal{R}}$, we can further improve the spectral efficiency by optimizing the power allocation among the relay and the users.

From (37a) it is not hard to derive the optimal power allocation that maximizes the spectral efficiency of AF relay, which is,

$$P_{\mathcal{U}} = \frac{\sigma^2_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_{\mathcal{U}}}P,\ P_{\mathcal{R}} = \frac{\sigma^2_{\mathcal{U}}}{\sigma^2_{\mathcal{R}} + \sigma^2_{\mathcal{U}}}P \tag{41}$$

Then the maximal spectral efficiency is

$$\begin{aligned}\eta_{\mathrm{CE-SA}}^{\mathrm{AF}} =&\beta \log \left(1+\frac{\left(\frac{P}{\sigma_{\mathcal{U}}^2+\sigma_{\mathcal{R}}^2}(1-\beta)^+\right)^2}{\frac{1}{2}+\frac{2P}{\sigma_{\mathcal{U}}^2+\sigma_{\mathcal{R}}^2}(1-\beta)^+}\right)\\ \leq&\beta \log\left(1+\frac{P}{2\sigma_{\mathcal{U}}^2+2\sigma_{\mathcal{R}}^2}(1-\beta)^+\right) \end{aligned}\tag{42}$$

From (37b) we know that the spectral efficiency of EF relay is monotonically increasing with $\min\left\{P_{\mathcal{U}}/\sigma_{\mathcal{R}}^2, 2P_{\mathcal{R}}/\sigma_{\mathcal{U}}^2\right\}$. Then the optimal power allocation can be derived by maximizing $\min\left\{P_{\mathcal{U}}/\sigma_{\mathcal{R}}^2, 2P_{\mathcal{R}}/\sigma_{\mathcal{U}}^2\right\}$, which is,

$$P_{\mathcal{U}}=\frac{2\sigma_{\mathcal{R}}^2}{2\sigma_{\mathcal{R}}^2+\sigma_{\mathcal{U}}^2}P,\ P_{\mathcal{R}}=\frac{\sigma_{\mathcal{U}}^2}{2\sigma_{\mathcal{R}}^2+\sigma_{\mathcal{U}}^2}P \tag{43}$$

and then the maximal spectral efficiency is

$$\eta_{\mathrm{CE-SA}}^{\mathrm{EF}}=\beta\log\left(1+\frac{2P}{\sigma_{\mathcal{U}}^2+2\sigma_{\mathcal{R}}^2}(1-\beta)^+\right) \tag{44}$$

From (42) and (44), we can further compare the spectral efficiency with different forwarding schemes. Since

$$\frac{P}{2\sigma_{\mathcal{U}}^2+2\sigma_{\mathcal{R}}^2}<\frac{2P}{\sigma_{\mathcal{U}}^2+2\sigma_{\mathcal{R}}^2} \tag{45}$$

we know that EF relay is more spectral-efficient than AF relay.

V. Simulation and Numerical Results

In this section, we will compare the performance of MC-CDMA TWR systems using SA signaling for AF relay with that using NSA signaling for AF relay, and compare that using SA signaling for EF relay with that using NSA signaling for DF relay. As in [11], equal gain random spreading sequences are employed. To validate our analysis for MIMO and MC-CDMA TWR systems with SA or NSA signaling, we compare their achievable data rates via simulation. As a reference, we also present the performance of an MC-CDMA TWR system with global CSI at the relay and at all the users.

In the simulations, the signals are transmitted over frequency selective Rayleigh fading channels with L i.i.d. resolvable multiple paths. We assume $\sigma_{\mathcal{R}}^2=\sigma_{\mathcal{U}}^2=\sigma^2$, then the SNR per symbol was P/σ^2.

A. MIMO Versus MC-CDMA

In Figs. 2 and 3, we show the cumulative distribution function (CDF) of the achievable data rate per user-pair in MIMO and MC-CDMA TWR systems, respectively. We compare the NSA signaling with various SA signalings of AF relay. We also compare these signalings of AF relay in MC-CDMA system with an MC-CDMA system with global CSI.

From Fig. 2, we see that the performance of the TS-SA and OS-SA signalings is even worse than that of the NSA signaling in the MIMO system when $K=M/2$. This can be explained as follows. On one hand, in this scenario the TWR systems with both NSA and SA signalings are not overloaded, hence the MUI can be suppressed thoroughly. On the other hand, due to the signal power loss caused by the channel inversion operation, the TS-SA and OS-SA signalings suffer from severe performance degradation, which finally results in their lower spectral efficiency than the NSA signaling.

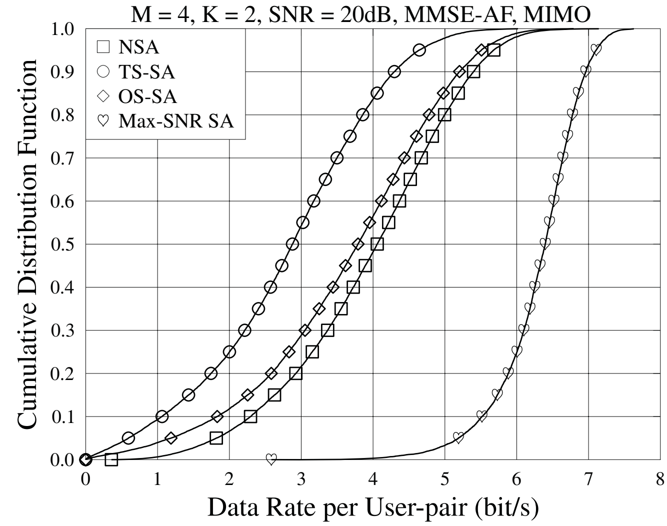

Fig. 2. CDF of per user-pair data rate in MIMO TWR systems using different signalings, AF relay, $P_{\mathcal{U}}=P_{\mathcal{R}}=P/2$.

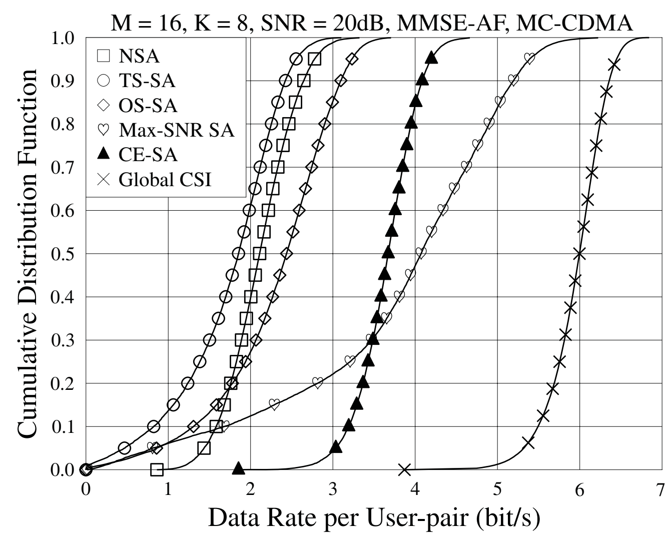

Fig. 3. CDF of per user-pair data rate in MC-CDMA TWR systems using different signalings, AF relay, $P_{\mathcal{U}}=P_{\mathcal{R}}=P/2$, and the CDF of per user-pair data rate in an MC-CDMA system with global CSI at both the relay and all the users.

By contrast, the Max-SNR SA signaling achieves the highest spectral efficiency among them because it improves the signal power significantly.

From Fig. 3, we find that when the Max-SNR SA signaling is applied for MC-CDMA TWR systems, the data rates of some user-pairs are improved, but those of others are reduced and become even worse than that of using the NSA signaling. As previously discussed, the Max-SNR SA signaling suggests that each user chooses several subcarriers to transmit signals. Consequently, the unexpected subcarrier collision leads to the lower spectral efficiency and the worse fairness among users. This demonstrates that a SA signaling which is efficient for MIMO systems is not necessarily efficient for MC-CDMA systems. By contrast, the proposed CE-SA signaling is able to improve spectral efficiency for all user-pairs. By converting the problem of maximizing each user's own SNR into the problem of maximizing its counterpart's SNR, there are no

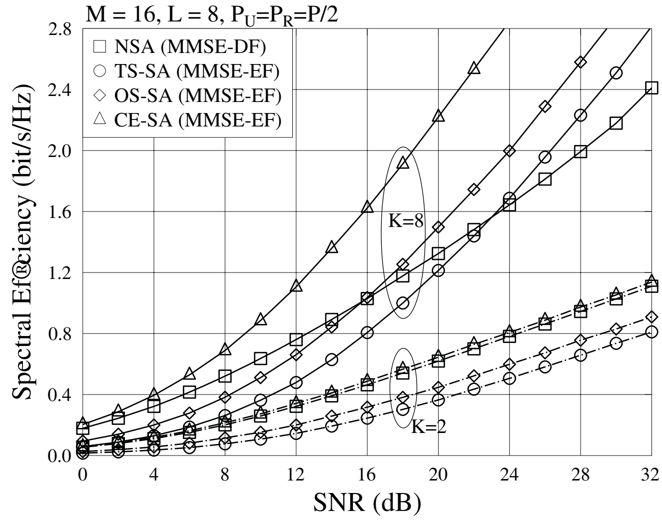

Fig. 4. Spectral efficiency of MC-CDMA TWR systems versus SNR, EF and DF relay, $P_{\mathcal{U}} = P_{\mathcal{R}} = P/2$.

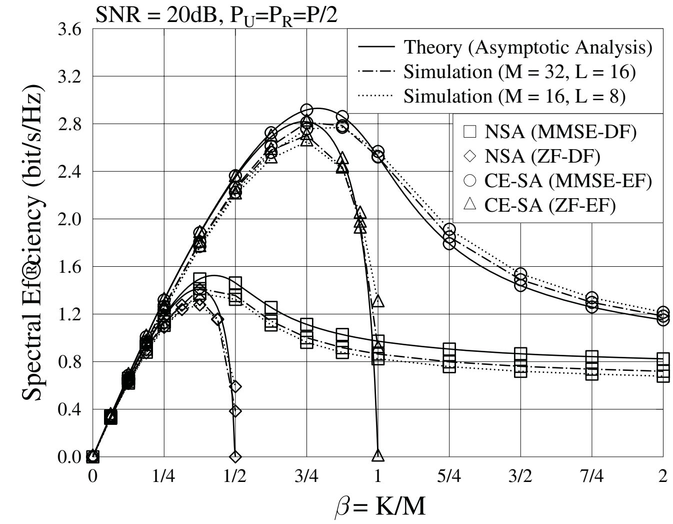

Fig. 6. Spectral efficiency of MC-CDMA TWR systems versus the system load factor β, EF and DF relay, $P_{\mathcal{U}} = P_{\mathcal{R}} = P/2$.

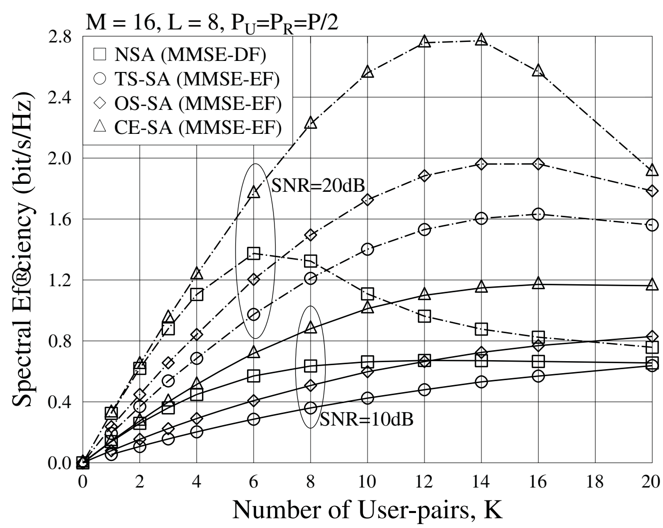

Fig. 5. Spectral efficiency of MC-CDMA TWR systems versus the number of user-pairs K, EF and DF relay, $P_{\mathcal{U}} = P_{\mathcal{R}} = P/2$.

longer signal power loss and inter-user-pair interference. Such a simple manner of cooperation between the two users in each pair yields a good trade-off between the average spectral efficiency and fairness among users. To provide a performance upper-bound of various SA signaling, we also present the performance of an MC-CDMA system with global CSI at all nodes. Under such a CSI assumption, a jointly optimized transceiver for the relay and users can be applied, which aims at maximizing the bi-directional sum rate but is suboptimal due to the non-convexity of the sum rate [21]. As expected, the transceiver with the global CSI at every node is superior to all the SA signaling, however, gathering global CSI will occupy considerable overhead in practice.

B. NSA Versus SA in MC-CDMA TWR Systems

Figs. 4 and 5 respectively show the impact of SNR and the number of user-pairs K on the spectral efficiency. When $K \geq M/2$, i.e., the TWR system with NSA signaling is over-loaded but that with SA signaling is still under-loaded, the TS-SA and OS-SA signalings outperform the NSA signaling. Otherwise, the TS-SA and OS-SA signalings are even inferior to the NSA signaling. Since the CE-SA signaling can avoid signal power loss and can support more users than the NSA signaling, it always outperforms the NSA, TS-SA and OS-SA signalings for any SNR and K.

In Fig. 6, we compare the simulation results with the numerical results of the asymptotic analysis. It shows that the results of the asymptotic analysis are quite close to the simulation results in finite number of users and subcarriers. Therefore, the performance comparison and the power allocation based on the asymptotic analysis are also valid for practical systems. For an arbitrary β, no matter whether the MMSE or ZF transceiver is considered, the CE-SA signaling achieves higher spectral efficiency than the NSA one. Moreover, when the MMSE transceiver is employed, the maximal spectral efficiency of the CE-SA signaling of EF relay is 2.8bit/s/Hz (at $\beta = 3/4$), while that of the NSA signaling of DF relay is only 1.4bit/s/Hz (at $\beta = 3/8$). These results agree with our earlier analysis very well.

C. Impact of Power Allocation

Fig. 7 shows the impact of the power allocation among the relay and the users on the spectral efficiency of the MC-CDMA TWR systems, where the load factor is chosen to maximize the spectral efficiency and is obtained by searching from previous simulation results. When $\sigma^2_{\mathcal{R}} = \sigma^2_{\mathcal{U}}$, we can see that equal power allocation, i.e., $P_{\mathcal{U}} = P_{\mathcal{R}}$, is optimal for the AF relay, while $P_{\mathcal{U}} = 2P_{\mathcal{R}}$ is optimal for the EF relay in high SNR level, which is consistence with our theoretical analysis. Comparing the spectral efficiency of the MC-CDMA TWR systems using different forwarding schemes, we can see that the EF relay is more spectral-efficient than the AF relay.

VI. Conclusions

In this paper we have studied SA signaling for MC-CDMA two-way relay systems. Since some feasible solutions for aligning the signals from each pair of users may suffer from

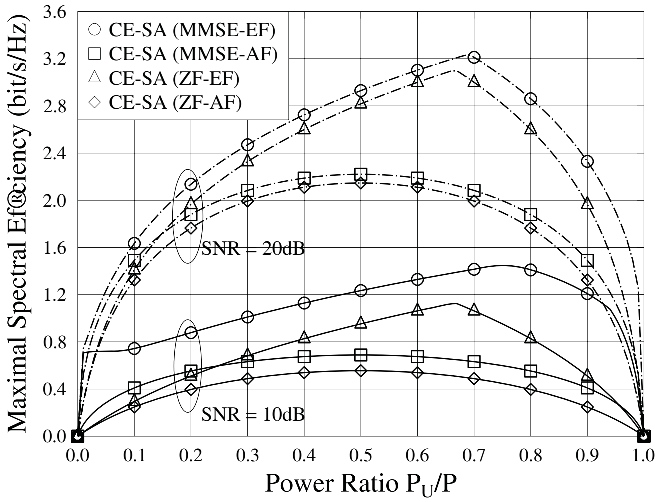

Fig. 7. Impact of the power allocation between the relay and the users on the spectral efficiency of MC-CDMA TWR systems with the optimal load factor.

signal power loss, we presented an optimal SA signaling in a sense of maximizing each user's own SNR. To avoid the inter-user-pair interference caused by such a SA signaling, we proposed a channel exchange SA signaling where each user assists its counterpart to achieve the maximal SNR, considering the fact that each pair of users in two-way relay systems could cooperative and by exploiting the diagonal channel structure of MC-CDMA systems. We analyzed asymptotic spectral efficiency of the MC-CDMA TWR system with the channel exchange SA and the NSA signalings. It showed from both analytical and simulation results that when the load factor of the proposed SA signaling is twice of that of the NSA one, the channel exchange signaling can achieve at least twice the spectral efficiency of the NSA signaling. The spectral efficiency can be further improved by optimizing the power allocation among the relay and the users.

APPENDIX

A. Proof of Proposition 1

In MC-CDMA systems, $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are diagonal matrices. Denote $y_m = |x_m|^2$ where x_m is the mth entry of $\boldsymbol{x}$ for $m = 1, 2, \ldots, M$, we can rewritten (19) as

$$\begin{aligned} \max_{\boldsymbol{y}} \quad & \sum\nolimits_{m=1}^{M} y_m \\ s.t. \quad & \sum\nolimits_{m=1}^{M} a_m y_m \le 1, \ \sum\nolimits_{m=1}^{M} b_m y_m \le 1 \\ & y_m \ge 0, \ m = 1, 2, \ldots, M \end{aligned} \tag{46}$$

where $a_m = |h_{\mathcal{A}_{m,k}}|^{-2}$ and $b_m = |h_{\mathcal{B}_{m,k}}|^{-2}$ for $m = 1, 2, \ldots, M$.

Denote $\boldsymbol{y}^o = [y_1^o, \cdots, y_M^o]^T$ as the optimal solution of (46), then we have $\sum_{m=1}^{M} a_m y_m^o \le 1$, $\sum_{m=1}^{M} b_m y_m^o \le 1$. Assume that the ith subcarrier is employed by the optimal signaling, $y_i^o \neq 0$, and there exists another subcarrier j satisfying $|h_{\mathcal{A}_{j,k}}|^2 > |h_{\mathcal{A}_{i,k}}|^2$ and $|h_{\mathcal{B}_{j,k}}|^2 > |h_{\mathcal{B}_{i,k}}|^2$. We construct a new vector $\boldsymbol{z} = [z_1, \cdots, z_M]^T$ with $z_i = 0$, where

$$z_m = \begin{cases} 0 & m = i \\ y_m^o + \min\left\{\dfrac{a_i}{a_j}, \dfrac{b_i}{b_j}\right\} y_i^o & m = j \\ y_m^o & m \neq i, j \end{cases} \tag{47}$$

It is not hard to show that $\boldsymbol{z}$ is a feasible solution of (46) since

$$\begin{aligned} \sum_{m=1}^{M} a_m z_m &= \sum_{m \neq i,j} a_m y_m^o + a_j \left(y_j^o + \min\left\{\frac{a_i}{a_j}, \frac{b_i}{b_j}\right\} y_i^o \right) \\ &= \sum_{m=1}^{M} a_m y_m^o + a_i y_i^o \left(\min\left\{1, \frac{a_j}{a_i}\frac{b_i}{b_j}\right\} - 1 \right) \\ &\le \sum_{m=1}^{M} a_m y_m^o \le 1 \end{aligned} \tag{48}$$

and similarly we have $\sum_{m=1}^{M} b_m z_m \le 1$.

The objective function of $\boldsymbol{z}$ is

$$\sum_{m=1}^{M} z_m = \sum_{m=1}^{M} y_m^o + \left(\min\left\{\frac{a_i}{a_j}, \frac{b_i}{b_j}\right\} - 1 \right) y_i^o \tag{49}$$

When $|h_{\mathcal{A}_{j,k}}|^2 > |h_{\mathcal{A}_{i,k}}|^2$ and $|h_{\mathcal{B}_{j,k}}|^2 > |h_{\mathcal{B}_{i,k}}|^2$, we have $a_j < a_i$ and $b_j < b_i$. Therefore, $\min\{a_i/a_j, b_i/b_j\} - 1 > 0$. From (49), we have

$$\sum_{m=1}^{M} z_m > \sum_{m=1}^{M} y_m^o \tag{50}$$

It means that $\boldsymbol{z}$ is better than $\boldsymbol{y}^o$, which is in conflict to the fact that $\boldsymbol{y}^o$ is the optimal solution of (46). Therefore, the optimal SA signaling will not use the ith subcarrier.

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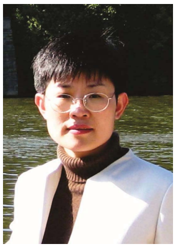

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