Spatial User Capacity of UWB Networks with Space-Time Focusing Transmission

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***Abstract*— Space-time focusing transmission in impulse-radio ultra-wideband (IR-UWB) systems resorts to the large number of resolvable paths to reduce the inter-pulse interference as well as the multiuser interference, and to simplify the receiver design. In this paper, we study the spatial user capacity of IR-UWB systems with space-time focusing transmission where the users are randomly distributed. We will derive the power distribution of the aggregate interference and investigate the collision probability between the desired focusing peak signal and interference signals. The closed-form expressions of the upper and lower bound of the outage probability and the spatial user capacity are obtained. Analysis results reveal the connections between the spatial user capacity and various system and channel parameters such as antenna gain, frame length, path loss factor, and multipath delay spread, which provide design guidelines for IR-UWB networks.**

I. Introduction

Impulse-radio ultra-wideband (IR-UWB) signals have large bandwidth, which can resolve a large number of multipath components in densely scattered channels. For communication links connecting different pairs of users, the correlation between multipath channel coefficient vectors are weak even when the user positions are very close [1]. Exploiting these characteristics, time-reversal (TR) prefiltering technique was proposed in IR-UWB communications [2], [3], which can focus the signal energy to a specific time instant and geometrical position.

The space-time focusing transmission has been widely studied in underwater acoustic communications [4], [5], and UWB radar and imaging areas [6], [7]. In UWB communications, TR technique is usually used to provide low complexity receiver [8]. By prefiltering the signal at the transmitter side with a temporally reversed channel impulse response, the received signal will have a peak at the desired time and location. The physical channel behaves as a spatial-temporal matched filter. In time domain, the focused peak is a low duty-cycle signal, thus inter-pulse interference reduces and a simple one-tap receiver can be used. In space domain, the strong signal only appears at one spot, thus mutual interference among coexisting users can be mitigated. TR techniques are also evolved for multiuser and multi-antenna transmission in recent years [9]–[12].

IR-UWB communications are favorable for *ad hoc* networks with randomly distributed nodes, where transmission links are built in a peer-to-peer mode. Although experiment results demonstrate that space-time focusing transmission leads to much lower sidelobes of the transmitted signal, the impact of such kind of interference on the user capacity has not been studied, as far as the authors known. Analyzing the spatial user capacity can provide some insight into the problem of how and why each system or channel parameter affect the network performance, and will provide design guidelines for IR-UWB networks.

In a landmark paper on the *ad hoc* network capacity [13], the authors showed that the throughput for each node vanishes with $\sqrt{n}$, where the channel was shared by n identical randomly located nodes with the random access scheme. Some results on user capacity for direct sequence code-division multi-access (DS-CDMA) and frequency hopping (FH)-CDMA systems were presented in [14], [15]. Essentially, space-time focusing transmission in IR-UWB systems accesses the channel with a combined random time-division and random code-division scheme. The random propagation delay of the low duty-cycle signal leads to a random accessing time, and the random multipath response of the communication link induces a random "spreading code". Larger number of multipath components will bring higher "spreading gain", but may also lead to higher collision probability. The combined impact on the user capacity is still not well-understood.

In this paper, we model the aggregate interference powers as two heavy-tailed distributions, *i.e.*, Cauchy and Lévy distributions, according to the path loss factor being 2 or 4. This yields explicit upper and lower bounds of the user capacity, which shows clearly the connections between the user capacity and the frame length, multipath delay spread, pulse width, transmit antenna number, link distance and the outage probability constraint, *etc*.

The rest of this paper is organized as follows. Section II introduces the network setting and the UWB space-time focusing transmission system. Then in Section III the upper and lower bounds of the outage probability in AWGN channels and in multipath and multi-antenna channels are respectively derived, and the closed-form expressions of the spatial user capacity are obtained. Simulation results are provided in Section IV to verify the theoretical analysis in various conditions. The paper is concluded in Section V.

II. System Description

We consider *ad hoc* networks without coordinators, where half-duplex nodes distributed uniformly within a circle, as

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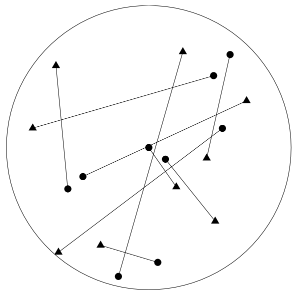

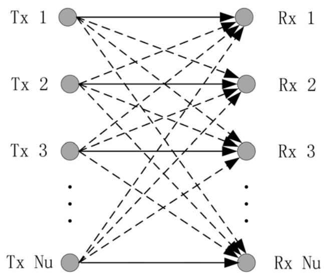

Fig. 1. (a) Nodes distribution in *ad hoc* networks, where the solid triangles denote transmitters and the solid circles denote receivers. (b) Interference channel model.

shown in Fig. 1(a). Each node is either a transmitter or a receiver. Without loss of generality, we regard the receiver at the center as the desired user, and all transmitters except the desired one as the interference users. This is an interference channel problem, whose equivalent model is shown in Fig. 1(b). The link distance of the desired transmitter and receiver is r_D, while the link distances between the interference transmitters and the desired receiver are random variables whose values are less than a threshold distance r_T, where $r_T \gg r_D$. The weak interference outside r_T are neglected.

In AWGN channels, the channel response $h(t) = \delta(t)$, then the TR prefilter is also $\delta(t)$. The transmitted signal of the kth user is

$$s^{(k)}(t) = \sum_i \sqrt{P_t} x_i^{(k)} p(t - iT_s), \quad (1)$$

where $\sqrt{P_t}$ is the transmit power, $x_i^{(k)}$ is the ith data symbol, $p(t)$ is the UWB short pulse with width T_p and normalized energy, T_s is the pulse repetition period or the frame length in UWB terminology. In each frame, there are $N_s = T_s/T_p$ time slots.

In multipath channels, define the channel response between the transmitter j and receiver k as $h_{j,k}(t)$. Then the TR prefilter at the kth transmitter for the kth receiver is $h^*_{k,k}(-t)$, and the transmitted signal is

$$\tilde{s}^{(k)}(t) = s^{(k)}(t) * h^*_{k,k}(-t), \quad (2)$$

where "$*$" denotes convolution operation.

At the intended receiver k the received signal is a summation of the signals from all N_u coexisting users that are further filtered by the multipath channels, *i.e.*,

$$\begin{aligned} r^{(k)}(t) &= \sum_{j=0}^{N_u-1} A_{j,k} \tilde{s}^{(j)}(t - \tau_{j,k}) * h_{j,k}(t) + z(t) \\ &= A_{k,k} s^{(k)}(t - \tau_{k,k}) * h^*_{k,k}(-t) * h_{k,k}(t) \\ &\quad + \underbrace{\sum_{j=0, j\neq k}^{N_u-1} A_{j,k} s^{(j)}(t - \tau_{j,k}) * h^*_{j,j}(-t) * h_{j,k}(t)}_{\text{Cochannel Interference}} \\ &\quad + z(t), \end{aligned} \quad (3)$$

where $A_{j,k}$ and $\tau_{j,k}$ are the signal amplitude attenuation and random propagation delay from the transmitter j to the receiver k, respectively, and $z(t)$ is additive white Gaussian noises.

Since the prefilter $h^*_{k,k}(-t)$ matches with the channel response $h_{k,k}(t)$, there will be a focusing peak at $t = iT_s + \tau_{k,k}$ that is the desired signal from transmitter k. The unintended cochannel interference from other transmitters behave as random dispersions since $h_{j,j}(t)$ and $h_{j,k}(t)$ are weakly correlated.

Assume that there is no intersymbol interference (ISI). Then the receiver k can simply sample the focusing peak for detection. The sampled signal is

$$r^{(k)}[i] = r^{(k)}(iT_s + \tau_{k,k}). \quad (4)$$

In these samples, the signal energy from the desired transmitter k is fully collected, while only parts of the energy from interferers are present due to the dispersion of interference signals. This leads to a power gain which is referred to as *spreading gain* because of its similarity with the gain obtained in conventional spreading systems. The value of this gain depends on the delay spread and cross-correlation properties of channel responses $h_{j,j}(t)$ and $h_{j,k}(t)$.

When the duration of $h^*_{j,j}(-t) * h_{j,k}(t)$ is less than the frame length T_s, the signal from transmitter j may not collide with the focusing peak, thereby does not degrade the detection performance of the desired user k. Longer T_s will cause lower collision probability. This leads to another gain to mitigate the interference which is referred to as *time-focusing gain*.

When each transmitter equips with M antennas, the channel responses from each transmit antenna to the receive antenna are different. As a result, the prefilters at different transmit antennas are different. From each antenna of transmitter k, there is a focused signal. The M focusing peaks will all arrive at the same time instant and accumulate coherently, thus an array gain M can be obtained, which is in fact a *space-focusing gain*.

III. Spatial User Capacity

We will first derive the outage probabilities of IR-UWB systems in AWGN channels, the random interference powers with different path loss factors are addressed by two kinds of heavy-tailed distributions. We will then derive the outage probabilities of IR-UWB systems in multipath channels and with multiple transmit antennas, considering the impact of channel delay spread on the spreading gain and collision probability. The closed-form expressions of the spatial user capacity are finally developed.

A. Outage Probability in AWGN Channels

In AWGN channels, the received signals are the combined pulse trains from all users with different delays. When the pulses from different users fall in the same time slot, mutual interference will appear. Consider one interference user whose distance to the desired user is r. Since the interference users are uniformly distributed inside a circle with the radius r_T, the probability density function (PDF) of r is

$$f_r(x) = \frac{2x}{r_T^2}, \quad x \le r_T. \tag{5}$$

The interference power depends on the propagation distance r and the path loss factor α, *i.e.*,

$$P_r = P_t \left(\frac{4\pi f_c r_0}{c}\right)^{-2} \left(\frac{r}{r_0}\right)^{-\alpha} = P_0 r^{-\alpha}, \tag{6}$$

where $P_0 = P_t c^2 r_0^\alpha / (4\pi f_c r_0)^2$ is the received power at a reference distance r_0, f_c is the carrier frequency, c is the light speed. In free space propagation, the path loss factor $\alpha = 2$, while in urban propagation environments, the path loss factor can be as large as 4. Other values of α between 2 and 4 can reflect various propagation environments in suburban and rural areas.

For the convenience of derivation, we normalize P_r with $P_0 r_T^{-\alpha}$ and define a normalized interference power as

$$\lambda = \frac{P_r}{P_0 r_T^{-\alpha}}. \tag{7}$$

The PDF of λ can then be obtained as

$$f_\lambda(x) = \frac{2}{\alpha} x^{-(2/\alpha)-1} = \begin{cases} \frac{1}{x^2}, & \alpha = 2 \\ \frac{2}{3x^{5/3}}, & \alpha = 3 \\ \frac{1}{2x^{3/2}}, & \alpha = 4 \end{cases}, \tag{8}$$

where $x \ge 1$. It shows that λ has a heavy-tailed distribution, which means that its tail decays with the power law instead of the exponential law [16].

When there are more than one interference users, the PDF of the aggregate interference power is the multi-fold convolutions of (8). It is hard to obtain its closed-form expression. Observing (8), we find that when $\alpha = 2$, the PDF of λ can be approximated by a Cauchy distribution

$$f_\lambda(x; x_0, b) = \frac{2}{\pi}\left[\frac{b}{(x-x_0)^2 + b^2}\right] = \frac{1}{x^2 + \pi^2/4}, \tag{9}$$

where the location parameter $x_0 = 0$, the scale parameter $b = \pi/2$. The coefficient outside the bracket is $1/\pi$ in standard Cauchy distribution, here it is replaced by $2/\pi$ because of the single-sided constraint $\lambda \ge 1$.

Using this approximation, the sum of n independent copies of λ, defined as Λ_n, still follows the Cauchy distribution without considering the constraint $\lambda \ge 1$, where its scale parameter increases to nb. The cumulative distribution function (CDF) of Λ_n can be developed as [16]

$$F_{\Lambda_n}(x; 0, \frac{\pi}{2}) = \frac{2}{\pi} \arctan\left(\frac{2x}{n\pi}\right). \tag{10}$$

When the constraint is considered, the practical PDF of Λ_n has heavier tail than that obtained by Cauchy distribution, and thus the practical CDF of Λ_n is smaller than $F_{\Lambda_n}(x; 0, \frac{\pi}{2})$. However, we will see in the later simulations that (10) is a quite tight bound when few interference users exist.

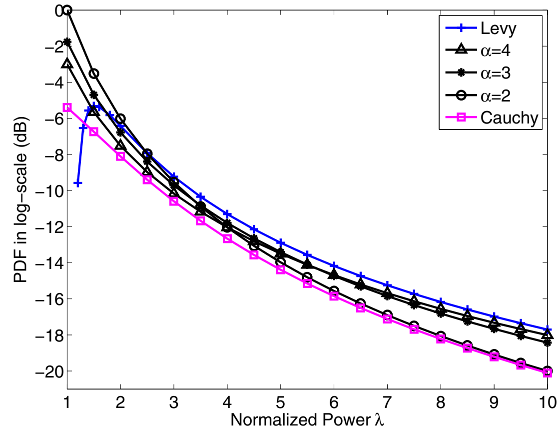

Fig. 2. The PDFs of Cauchy and Lévy distribution, as well as the practical PDFs of the normalized interference power λ when α=2, 3, and 4.

When $\alpha = 4$, the PDF of λ can be approximated by a Lévy distribution

$$f_\lambda(x; x_0, c) = \sqrt{\frac{c}{2\pi}} \frac{e^{-c/2(x-x_0)}}{(x-x_0)^{3/2}} = \frac{1}{2}\left[\frac{e^{-\pi/4(x-1)}}{(x-1)^{3/2}}\right], \tag{11}$$

where the location parameter $x_0 = 1$, the scale parameter $c = \pi/2$, and $\lambda \ge 1$.

Using this approximation, Λ_n still subjects to Lévy distribution, where its location parameter becomes nx_0 and its scale parameter increases to $n^2 c$. The CDF of Λ_n can be derived as [16]

$$F_{\Lambda_n}(x; 1, \frac{\pi}{2}) = \text{erfc}\left(\sqrt{\frac{n^2\pi}{4(x-n)}}\right), \tag{12}$$

where $\text{erfc}(\cdot)$ is the complementary error function.

Fig. 2 shows the practical PDFs of the normalized interference power λ when $\alpha = 2, 3, 4$, as well as their two approximations. We can see that, Cauchy distribution serves as a lower bound and Lévy distribution serves as an upper bound.

Similar to (7), we define the normalized signal power as

$$\lambda_D = \frac{P_0 r_D^{-\alpha}}{P_0 r_T^{-\alpha}} = \frac{r_T^\alpha}{r_D^\alpha}. \tag{13}$$

Assume that the required signal-to-interference-plus-noise-ratio (SINR) for reliable transmission is $\beta = \lambda_D/(\lambda_N + \lambda_I)$, where $\lambda_N = P_N/(P_0 r_T^{-\alpha})$ is the normalized noise power. If the SNR of the desired user is given as γ, *i.e.*, $\lambda_D/\lambda_N = \gamma$, then the interference power tolerance will be

$$\lambda_I = \left(\frac{1}{\beta} - \frac{1}{\gamma}\right)\lambda_D = \mu \frac{r_T^\alpha}{r_D^\alpha}, \tag{14}$$

where $\mu = \frac{1}{\beta} - \frac{1}{\gamma}$. The communication will break when the interference power exceed λ_I.

We first consider that the pulses from n interference users arrive at the same time slot with that of the desired user, then the outage probability of the desired user is

$$P(\Lambda_n > \lambda_I) = 1 - F_{\Lambda_n}(\lambda_I) = \begin{cases} \operatorname{erf}\left(\sqrt{\frac{n^2\pi}{4(\lambda_I - n)}}\right), & \text{UB} \\ \frac{2}{\pi}\arctan\left(\frac{n\pi}{2\lambda_I}\right), & \text{LB} \end{cases} \quad (15)$$

where $\operatorname{erf}(x) = 1 - \operatorname{erfc}(x)$ is the error function, 'UB' stands for upper bound and 'LB' stands for lower bound. The upper bound is derived from Lévy distribution and the lower bound is derived from Cauchy distribution.

Since there are N_s time slots in a frame, if there are N_u interference users in total, the probability that n users occupy the same time slot with the desired user is

$$p_{N_u}(n) = \frac{C_{N_u}^n (N_s - 1)^{N_u - n}}{N_s^{N_u}}. \quad (16)$$

It is apparent that increasing N_s will reduce the collision probability and thus reduce the average outage probability. This is the benefit brought by the low duty-cycle characteristic of the IR-UWB signals.

The average outage probability is the summation of all the possibilities that n users generate interference and their aggregate power exceeds the designed tolerance, *i.e.*,

$$\begin{aligned} P_{\text{out}}(N_u) &= \sum_{n=1}^{N_u} p_{N_u}(n) P(\Lambda_n > \lambda_I) \\ &= \begin{cases} \sum_{n=1}^{N_u} \frac{C_{N_u}^n (N_s-1)^{N_u-n}}{N_s^{N_u}} \left[\operatorname{erf}\left(\sqrt{\frac{n^2\pi}{4(\lambda_I-n)}}\right)\right], & \text{UB} \\ \sum_{n=1}^{N_u} \frac{C_{N_u}^n (N_s-1)^{N_u-n}}{N_s^{N_u}} \left[\frac{2}{\pi}\arctan\left(\frac{n\pi}{2\lambda_I}\right)\right]. & \text{LB} \end{cases} \end{aligned} \quad (17)$$

B. Outage Probability in Multipath Channels

It is known that the small-scale fading of UWB channels are very small [1]. Therefore, it is reasonable to assume that the received signal power only depends on the path loss and the shadowing. Assume that $\int_0^\infty |h_{i,j}(t)|^2 = 1$, *i.e.*, the energy of multipath channel is normalized, and $\tau_{\max} < T_s$, *i.e.*, there is no ISI.

Assume that the channel's power delay profile subjects to exponential decaying, *i.e.*,

$$D(\tau) = \frac{1}{\tau_{\text{RMS}}} e^{-\frac{\tau}{\tau_{\text{RMS}}}}, \quad \tau > 0, \quad \int_0^\infty D(\tau) d\tau = 1, \quad (18)$$

where τ_{RMS} is the root-mean-square (RMS) delay spread of the channel.

From (3), we know that the composite response of the desired channel is $\tilde{h}_{k,k}(t) = h_{k,k}^*(-t) * h_{k,k}(t)$, which has a focusing peak at $t = 0$ and the energy of the peak is $\int_0^\infty |h_{k,k}(t)|^2 dt = 1$. The duration of the peak signal is $2T_p$, thus its power is $1/2T_p$.

Similarly, the composite response of the interference channel is $\tilde{h}_{j,k}(t) = h_{j,j}^*(-t) * h_{j,k}(t)$, which is a random process and its average power is obtained as

$$E\left[|\tilde{h}_{j,k}(t)|^2\right] = \int_0^\infty D(\tau - t) D(\tau) d\tau = \frac{1}{2\tau_{\text{RMS}}} e^{-\frac{|t|}{\tau_{\text{RMS}}}}, \quad (19)$$

where the first equality comes from the uncorrelated property of the two channels.

We can see that the average interference channel power subjects to double-sided exponential decaying. To obtain explicit expressions of the spreading gain and the collision probability, we approximate the profile of the average interference power by a rectangle with the same area. The effect of this approximation will be shown through simulations.

Since the sum power of the interference channel is

$$\int_{-\infty}^{\infty} \frac{1}{2\tau_{\text{RMS}}} e^{-\frac{|t|}{\tau_{\text{RMS}}}} dt = 1, \quad (20)$$

and the maximal value of (19) is $1/2\tau_{\text{RMS}}$, the rectangle has a length $2\tau_{\text{RMS}}$ given the height $1/2\tau_{\text{RMS}}$. Then the approximated interference channel power will always be $1/2\tau_{\text{RMS}}$ in a duration of $2\tau_{\text{RMS}}$.

Since the desired channel has a power $1/2T_p$ and the interference channel has a power $1/2\tau_{\text{RMS}}$, the *spreading gain* can be obtained as

$$G_S = \frac{1/2T_p}{1/2\tau_{\text{RMS}}} = \frac{\tau_{\text{RMS}}}{T_p}, \quad (21)$$

which reflects the interference suppression capability of the TR prefiltering scheme through random code-division.

Since the frame length is T_s and the approximated interference duration is $2\tau_{\text{RMS}}$, the probability that the signal of one interference user collides with the focusing peak of the desired user is approximatly

$$\delta = \frac{2\tau_{\text{RMS}}}{T_s} = \frac{2G_S}{N_s}. \quad (22)$$

The reciprocal of δ is actually the *time-focusing gain*, *i.e.*,

$$G_T = \frac{T_s}{2\tau_{\text{RMS}}} = \frac{N_s}{2G_S}, \quad (23)$$

which reflects the interference mitigation capability of TR prefiltering scheme through random time-division.

When totally N_u users exist, the probability that n users simultaneously interfere with the desired user is

$$p_{N_u}(n) = C_{N_u}^n \delta^n (1 - \delta)^{N_u - n}. \quad (24)$$

Due to the spreading gain, the influence of interference on the decision statistics in multipath channels reduces to $1/G_S$ of that in AWGN channels when the same interference power is received. Consequently, when there are n interference signals, an outage happens when the sum power of the interference signals Λ_n exceeds $G_S\lambda_I$. Then the average outage probability in multipath channels is

$$P_{\text{out}}(N_u) = \sum_{n=1}^{N_u - 1} p_{N_u}(n) P(\Lambda_n > G_S \lambda_I). \quad (25)$$

When each transmitter equips with M antennas, the output power at each antenna reduces to $1/M$ of that in single-antenna case. The desired signal will be increased by the array gain M while both the interference power and the collision probability between the interference and the desired signals will not change. Therefore, the *space-focusing gain* $G_A = M$.

Considering the antenna gain, spreading gain, and the collision probability, the average outage probability in multipath channel with multiple transmit antennas is obtained as

$$P_{\text{out}}(N_u) = \sum_{n=1}^{N_u-1} p_{N_u}(n) P(\Lambda_n > G_A G_S \lambda_I)$$
$$= \begin{cases} \sum_{n=1}^{N_u} C_{N_u}^n \delta^n (1-\delta)^{N_u-n} \left[\text{erf}\left(\sqrt{\frac{n^2\pi}{4(G_A G_S \lambda_I - n)}} \right) \right], \text{UB} \\ \sum_{n=1}^{N_u} C_{N_u}^n \delta^n (1-\delta)^{N_u-n} \left[\frac{2}{\pi} \arctan\left(\frac{n\pi}{2G_A G_S \lambda_I} \right) \right]. \text{LB} \end{cases} \quad (26)$$

C. Spatial User Capacity

Given a required outage probability ϵ, the spatial user capacity U is obtained as the maximal accommodable user number in the network, which is

$$U = \arg\max_{N_u} \{ P_{\text{out}}(N_u) \le \epsilon \}. \quad (27)$$

When the outage probability is small, both the error function and the arctangent function can be approximated as linear functions, *i.e.*,

$$\text{erf}(x) \approx \frac{2}{\sqrt{\pi}} x, \quad \arctan(x) \approx x. \quad (28)$$

Using these approximations, (26) can be simplified as

$$P_{\text{out}}(N_u) = \begin{cases} \frac{N_u}{G_T \sqrt{G_A G_S \lambda_I}} = \frac{2N_u \sqrt{\tau_{\text{RMS}} T_p}}{\sqrt{M\lambda_I} T_s}, & \text{UB} \\ \frac{N_u}{G_T G_A G_S \lambda_I} = \frac{2N_u T_p}{M\lambda_I T_s}, & \text{LB} \end{cases}. \quad (29)$$

Therefore, given the outage probability constraint $P_{\text{out}}(N_u) = \epsilon$, the spatial user capacity can be expressed as

$$U = \begin{cases} \epsilon G_T G_A G_S \lambda_I = \frac{\epsilon M \lambda_I T_s}{2T_p}, & \text{UB} \\ \epsilon G_T \sqrt{G_A G_S \lambda_I} = \frac{\epsilon \sqrt{M\lambda_I} T_s}{2\sqrt{\tau_{\text{RMS}} T_p}}, & \text{LB} \end{cases}. \quad (30)$$

By contrast to that in the outage probability, the upper bound of spatial user capacity is obtained from Cauchy distribution which can be achieved when $\alpha = 2$ and the lower bound is obtained from Lévy distribution which can be achieved when $\alpha = 4$.

It is shown from (30) that increasing the time-focusing gain, the space-focusing gain and the spreading gain all lead to higher spatial user capacity. However, the increasing rates are different in the upper and lower bound.

We can see that the capacity obtained by Lévy bound is less than that obtained by Cauchy bound. This indicates that large path loss factor will reduce user capacity. With larger path loss factor, despite that both the desired signal power and the interference power attenuate faster, the aggregate inference power is more likely to exceed the interference tolerance given the total user number.

It is shown that the spatial user capacity U always grows linearly with T_s whenever $\alpha = 2$ or 4. However, U grows linearly with G_A when $\alpha = 2$ and grows sublinearly with $\sqrt{G_A}$ when $\alpha = 4$.

It can be observed that the delay spread does not affect the user capacity when $\alpha = 2$, whereas the user capacity decreases with $\sqrt{\tau_{\text{RMS}}}$ when $\alpha = 4$. As we have analyzed earlier, large delay spread will introduce high spreading gain, while it will also increase the collision probability among users. When $\alpha = 2$, there exists a balance between these two competing factors. However, when $\alpha = 4$, the gain obtained from spreading can not compromise the performance degradation led by collisions.

IV. Simulation Results

In this section, the outage probability expressions derived in AWGN and multipath channels are verified through simulations. We set the link distance of the desired user $r_D = 100$ m, and the threshold distance of the interference users $r_T = 1000$ m. Consider that the SNR of the desired user is 10 dB, and the required SINR is 4 dB, then the normalized interference power tolerance $\lambda_I = 0.3\lambda_D$.

1) Outage probability in AWGN channels: We first verify the outage probability obtained in AWGN channel with $\alpha = 2$. The number of time slots in each frame is set to be $N_s = 10$. The outage probabilities obtained through theoretical analysis and simulations are shown in Fig. 3. The results of Cauchy bound and Lévy bound are obtained from (17). The curves labeled '0 dB', '3 dB' and '6 dB' are simulation results with corresponding standard derivations of shadowing. We can see that Cauchy bound is quite tight as a lower bound when the user number is less than 10 and the shadowing is low. When more users coexist in the network, the lower bound becomes loose. Lévy bound is an upper bound. With the increase of the shadowing standard derivation, the outage probability will gradually approach the upper bound.

The theoretical and simulated outage probabilities when $\alpha = 4$ are presented in Fig. 4. We can see that Cauchy bound is loose now, but Lévy bound is quite tight. Although with the increase of the shadowing standard derivation the simulated outage probabilities will exceed the upper bound, the differences between them are very small. The results shown in Fig. 3 and Fig. 4 are consistent with our analysis in Section III. Since the CDF of the standard Cauchy distribution is used for that of the single-sided Cauchy distribution with constraint $\lambda \ge 1$, the lower bound has bias when user number is large.

2) Outage probability in multipath channels: IEEE 802.15.4a channel model is used to generate the multipath channel response [17], where "CM3" environment is considered and the multipath delay spread $\tau_{\text{RMS}} = 10$ ns. In multipath channels, both the power and the duration of the interference signals are random variables in different channel realizations. The theoretical results are obtained from (25), where the rectangle approximation of the average interference power profile is used. Fig. 5 shows the theoretical and simulation results, where the pulse width $T_p = 1$ ns, the frame length $T_s = 100$ ns, and other conditions are the same with those in AWGN channels. Here, $\alpha = 2$ and $\alpha = 4$ are used respectively for Cauchy bound and Lévy bound, and shadowing is not considered. The theoretical results are shown to agree well with the simulation results, hence can be used for further analysis.

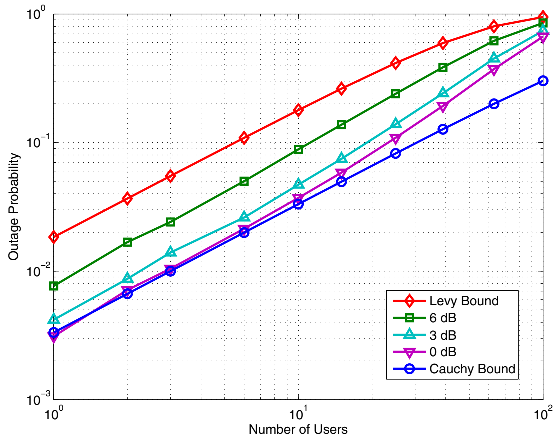

Fig. 3. The outage probability in AWGN channels when $\alpha = 2$, $N_s = 10$, the shadowing standard derivation are respectively 0, 3, and 6 dB.

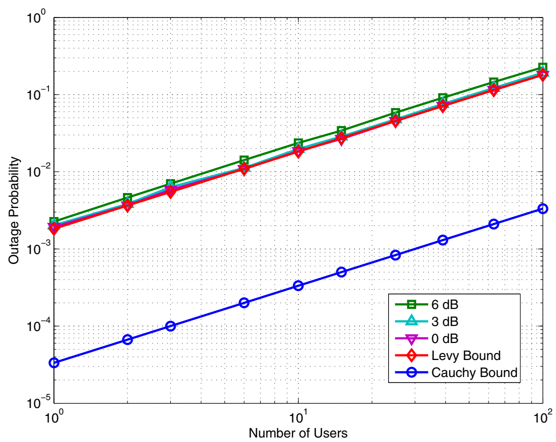

Fig. 4. The outage probability in AWGN channels when $\alpha = 4$, $N_s = 10$, the shadowing standard derivation are respectively 0, 3, and 6 dB.

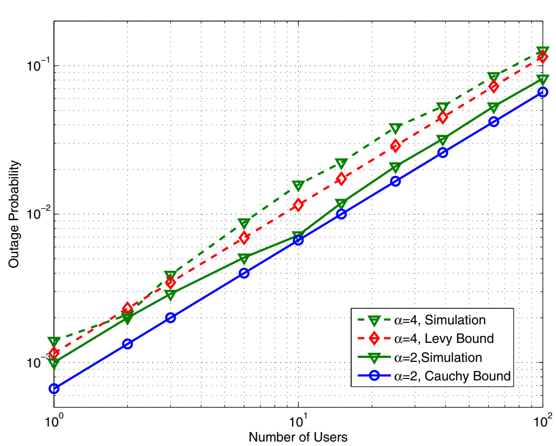

Fig. 5. The outage probability in multipath channels, where T_s=100 ns, τ_{RMS}=10 ns, α=2 and 4 are respectively simulated.

V. Conclusion

In this paper, the spatial user capacity of the IR-UWB networks with space-time focusing transmission is analyzed. We derived the upper and lower bounds of the outage probability according to different path loss factors, and then developed the closed-form expressions of the spatial user capacity. Analysis results showed that, the spatial user capacity grows linearly with the frame length, and reduces with large path loss factors. Depending on the path loss factor being 2 or 4, the spatial user capacity grows either linearly or sublinearly with the array gain and the spreading gain.

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References

[1] J. Keignart, C. Abou-Rjeily, C. Delaveaud, and N. Daniele, "UWB SIMO channel measurements and simulations," *IEEE Trans. Microwave Theory Tech.*, vol. 54, pp. 1812–1819, June 2006.

[2] M. Emami, M. Vu, J. Hansen, A. J. Paulraj, and G. Papanicolaou, "Matched filtering with rate back-off for low complexity communications in very large delay spread channels," in *ACSSC 2004*, pp. 218–222.

[3] T. Strohmer, M. Emami, J. Hansen, G. Papanicolaou, and A. J. Paulraj, "Application of time-reversal with MMSE equalizer to UWB communications," in *IEEE GLOBECOM 2004*, pp. 3123–3127.

[4] A. Derode, P. Roux, and M. Fink, "Robust acoustic time reversal with high-order multiple scattering," *Phys. Rev. Letters*, vol. 75, pp. 4206–4209, 1995.

[5] M. Tanter, J. F. Aubry, J. Gerber, J. L. Thomas, and M. Fink, "Optimal focusing by spatio-temporal inverse filter," *J. Acoust. Soc. Am.*, vol. 110, pp. 37–47, July 2001.

[6] J. L. Schwartz and B. D. Steinberg, "Ultrasparse, ultrawideband arrays," *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, pp. 376–393, Mar. 1998.

[7] M. G. M. Hussain, "Principles of space-time array processing for ultrawide-band impulse radar and radio communications," *IEEE Trans. Veh. Technol.*, vol. 51, pp. 393–403, May 2002.

[8] N. Guo, B. M. Sadler, and R. C. Qiu, "Reduced-complexity UWB time-reversal techniques and experimental results," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 4221–4226, Dec. 2007.

[9] H. T. Nguyen, J. B. Andersen, and G. F. Pedersen, "The potential use of time reversal techniques in multiple element antenna systems," *IEEE Commun. Lett.*, vol. 9, pp. 40–42, Jan. 2005.

[10] H. T. Nguyen, I. Z. Kovcs, and P. C. F. Eggers, "A time reversal transmission approach for multiuser UWB communications," *IEEE Trans. Antennas Propagat.*, vol. 54, pp. 3216–3224, Nov. 2006.

[11] R. C. Qiu, C. Zhou, N. Guo, and J. Q. Zhang, "Time reversal with MISO for ultrawideband communications: experimental results," *IEEE Antennas Wireless Propagat. Lett.*, vol. 5, pp. 269–273, Dec. 2006.

[12] C. Zhou, N. Guo, and R. C. Qiu, "Time-reversed ultra-wideband (UWB) multiple input multiple output (MIMO) based on measured spatial channels," *IEEE Trans. Veh. Technol.*, vol. 58, pp. 2884–2898, July 2009.

[13] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.

[14] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Trans. Inform. Theory*, vol. 51, pp. 4091–4102, Dec. 2005.

[15] C. Comaniciu and H. V. Poor, "On the capacity of mobile ad hoc networks with delay constraints," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 2061–2071, Aug. 2006.

[16] J. P. Nolan, *Stable Distributions - Models for Heavy Tailed Data*. Boston: Birkhäuser, 2010, in progress, Chapter 1 online at academic2.american.edu/ ∼jpnolan.

[17] A. F. Molisch, "Ultrawideband propagation channels - theory, measurement, and modeling," *IEEE Trans. Veh. Technol.*, vol. 54, pp. 1528–1545, Sept. 2005.