Interference Alignment Transceiver Design for MIMO Interference Broadcast Channels

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***Abstract*— In this paper, we develop linear interference alignment (IA) approach for multi-input-multi-output interference broadcast channel (MIMO-IBC). Since multiple data streams from each base station (BS) to multiple mobile stations (MSs) experience identical channel, it is hard to ensure the rank constraint to the intended MSs in the desired cell meanwhile ensure the interference aligned at the unintended MSs in other cells. Considering the difficulty in aligning interference at the receiver in MIMO-IBC, we design a transceiver to align and eliminate the interference at the transmitter. Specifically, we first design receive vectors of all MSs to align the inter-cell interference at the BS side, and then design the precoder of each BS to eliminate all the intra- and inter-cell interference. The proposed approach can be applied for general MIMO-IBC and has closed-from solutions for some antenna configurations. Simulation results validate our analysis and show that the proposed IA transceiver can achieve the maximal degrees of freedom of MIMO-IBC and attain a good trade-off between the maximal number of data streams and signal-to-noise ration gain.**

I. Introduction

Inter-cell interference (ICI) is a major bottleneck to achieve high spectral efficiency of universal frequency reuse cellular networks, especially for multi-input-multi-output (MIMO) systems. When multiple base stations (BSs) share both the data and the channel state information (CSI) among each other, coherent BS cooperation transmission, i.e., network MIMO, can improve system throughput remarkably [1]. If only CSI is shared, such a system setting becomes an interference broadcast channel (IBC) when each BS transmits to multiple MSs with same time-frequency resource and an interference channel (IC) when each BS transmits to a single MS, in the information theoretic terminology.

Interference alignment (IA) provides a novel solution to manage the ICI in the interference channels. It has been shown that in a MIMO-IC each cell can achieve $1/2$ degrees of freedom (DoF) using the principle of IA [2]. Encouraged by its great potential, significant research efforts have been devoted to both the DoF analysis [3–5] and the IA transceiver design [6–10] recently.

Priori studies mainly focus on the IA for IC [6–8]. Inspired by the exciting results for the DoF of the two-cell MIMO-IBC [3], we are interested in IA for MIMO-IBC systems. The IA algorithm for IBC is more complicated than that for IC, since the alignment of ICI at the receiver of a mobile station (MS) does not ensure the alignment at other MSs [9]. Until now, only a few studies have designed the IA transceivers for MIMO-IBC [9–11]. In [9, 10], two closed-from linear IA algorithms were designed for two-cell MIMO-IBC with a special number of transmit and receive antennas. In [11], a weighted minimum mean squared error (MMSE) method was developed for a general MIMO-IBC, which requires iteration between the transceivers at the BSs and MSs.

In this paper, we provide a unified linear IA approach for general MIMO-IBC with either symmetric or asymmetric antenna configuration in each cell. Different from priori closed-form algorithms that develop IA transceiver for a given dimension of interference subspace, we develop IA transceiver to achieve maximal DoF, or equivalently to employ minimal antenna resources. Instead of directly extend the IA transceiver in MIMO-IC to MIMO-IBC, we exploit the inherent feature of MIMO-IBC. Considering the difficulty in aligning interference at the MS sides, we propose to design the IA transceiver where the MSs first align the ICI at the BS side, then the BS eliminates both intra- and inter-cell interference. Existing closed-from IA algorithms [9, 10] are special forms of the proposed method. Simulation results show that the proposed IA transceiver can achieve the maximal DoF for MIMO-IBC and achieve a trade-off between the maximal number of data steams and signal-to-noise ratio (SNR) gain when either the BSs or the MSs have redundant antennas resources.

II. System model

Consider a G-cell MIMO-IBC, where BS_i equipped with M_i antennas simultaneously transmits $d_{i_1}, \cdots, d_{i_{K_i}}$ data streams to its K_i MSs each with $N_{i_1}, \cdots, N_{i_{K_i}}$ antennas, $i = 1, \cdots, G$. Assume that there are no data sharing among the BSs. The total number of data streams in cell i is $d_i = \sum_{k=1}^{K_i} d_{i_k}$ and the overall number of data streams in G cells is $d^{\text{tot}} = \sum_{i=1}^{G} d_i = \sum_{i=1}^{G}\sum_{k=1}^{K_i} d_{i_k}$.

Assume that every BS and MS have the CSIs of all links as in the priori literature [10, 11]. In downlink transmission, BS_i employs an $M_i \times d_i$ precoding matrix $\boldsymbol{V}_i = [\boldsymbol{V}_{i_1}, \cdots, \boldsymbol{V}_{i_{K_i}}]$ to convey the symbol vector $\boldsymbol{x}_i = [\boldsymbol{x}_{i_1}^T, \cdots, \boldsymbol{x}_{i_{K_i}}^T]^T$, where $\boldsymbol{V}_{i_k} \in \mathbb{C}^{M_i \times d_{i_k}}$ and $\boldsymbol{x}_{i_k} \in \mathbb{C}^{d_{i_k} \times 1}$ respectively denote the precoder and symbol vector for MS_{i_k} satisfying $\text{Tr}\{\boldsymbol{V}_{i_k}^H \boldsymbol{V}_{i_k}\} = d_{i_k}$ and $E\{\boldsymbol{x}_{i_k}^H \boldsymbol{x}_{i_k}\} = P d_{i_k}$, and P is the transmit power of each data stream.

The desired symbols of MS_{i_k} can be estimated as

$$\hat{\boldsymbol{x}}_{i_k} = \boldsymbol{U}_{i_k}^H \boldsymbol{H}_{i_k,i} \boldsymbol{V}_i \boldsymbol{x}_i + \sum_{j=1, j\neq i}^{G} \boldsymbol{U}_{i_k}^H \boldsymbol{H}_{i_k,j} \boldsymbol{V}_j \boldsymbol{x}_j + \boldsymbol{U}_{i_k}^H \boldsymbol{n}_{i_k} \quad (1)$$

where $\boldsymbol{U}_{i_k} \in \mathbb{C}^{N_{i_k} \times d_{i_k}}$ is the receive matrix of MS_{i_k}, $\boldsymbol{H}_{i_k,j} \in \mathbb{C}^{N_{i_k} \times M_j}$ is the channel matrix from BS_j to MS_{i_k} whose elements are independent and identically distributed (i.i.d.) complex Gaussian variables with zero-mean and unit variance, $\boldsymbol{n}_{i_k} \in \mathbb{C}^{N_{i_k} \times 1}$ is a zero-mean additive white Gaussian noise (AWGN) vector with variance σ_n^2, the first term consists of both desired signal and multiuser interference (MUI) that comes from MSs in the same cell (i.e., intra-cell interference), and the second term is ICI.

III. IA Transceiver Design for MIMO-IBC

To develop an IA transceiver to achieve the maximal DoF of MIMO-IBC, we first investigate the IA feasibility.

A. IA Feasibility for MIMO-IBC

The linear IA conditions for MIMO-IC were first introduced in [6] and later investigated by many recent works such as [4, 5], which include an interference-free constraint and a signal space rank constraint. Similarly, from (1) the linear IA conditions for MIMO-IBC are

$$\boldsymbol{U}_{i_k}^H \boldsymbol{H}_{i_k,j} \boldsymbol{V}_j = 0,\ \forall i \neq j \tag{2a}$$

$$\mathrm{rank}\left(\begin{bmatrix} \boldsymbol{U}_{i_1}^H \boldsymbol{H}_{i_1,i} \\ \cdots \\ \boldsymbol{U}_{i_{K_i}}^H \boldsymbol{H}_{i_{K_i},i} \end{bmatrix} \boldsymbol{V}_i\right) = d_i,\ \forall i \tag{2b}$$

The first equation guarantees ICI-free at each MS. The second equation suggests that we need to reserve enough spatial resources for transmitting the desired signals in cell i, which is automatically satisfied with probability one in i.i.d. MIMO channels [4, 6] when $\mathrm{rank}(\boldsymbol{V}_i) = d_i$ and $\mathrm{rank}(\boldsymbol{U}_{i_k}) = d_{i_k}$.

From the IA conditions, we can study the IA feasibility. The necessary conditions of feasible IA for MIMO-IBC can be directly extended from that for MIMO-IC by applying the knowledge of algebraic geometry and field theory as in [5]. However, the sufficient conditions can not be extended from that for MIMO-IC due to the unique channel characteristic of MIMO-IBC where multiple data streams to different MSs from one BS experience the same channel response. After non-trivial derivation in [12], we obtain the necessary and sufficient conditions for some special G-cell MIMO-IBC systems in the following.

Suppose that all MSs have the same number of data streams $d_{i_k} = d$, M_i and N_{i_k} are divisible by d and the channel matrices $\boldsymbol{H}_{i_k,j}$ are generic (e.g., drawn from a continuous probability distribution), we can achieve the required number of data streams $\{K_1 d, \cdots, K_G d\}$ for G cells if and only if the following conditions are satisfying

$$M_i \geq K_i d,\ \forall i \tag{3a}$$

$$N_{i_k} \geq d,\ \forall i, k \tag{3b}$$

$$\sum_{i:(i,j)\in\mathcal{I}} K_i(M_i/d - K_i) + \sum_{j:(i,j)\in\mathcal{I}} \sum_{k\in\mathcal{K}_j} (N_{j_k}/d - 1) \geq \sum_{(i,j)\in\mathcal{I}} K_i |\mathcal{K}_j|,\ \forall \mathcal{I} \subseteq \mathcal{J} \tag{3c}$$

where $\mathcal{J} = \{(i,j) | 1 \leq i \neq j \leq G\}$, $\mathcal{K}_i \subseteq \{1, \ldots, K_i\}$, and $|\mathcal{K}_j|$ denotes the cardinality of $\mathcal{K}_j$.

When each cell has the same number of transmit and receive antennas, i.e., $M_i = M$ and $N_{i_k} = N$, based on the above necessary and sufficient conditions and some tedious derivations in [12], we obtain the maximal achievable DoF in MIMO-IBC[1] for given d, i.e.,

$$d^{\mathrm{tot}} = \begin{cases} M, & \forall N = d \\ \begin{matrix} 2\min\{M, \lfloor (M+N-d)/2 \rfloor\}, \forall G = 2 \\ \min\{GM, M+N-d\}, \quad \forall G > 2 \end{matrix}, & \forall N > d \end{cases} \tag{4}$$

In order to investigate how to achieve the maximal DoF, we design the transceiver for the given number of data streams with the minimum number of antennas satisfying (3a)$\sim$(3c).

B. Transceiver Design

Here, we design the linear IA transceiver for the G-cell MIMO-IBC system where all MSs have the same number of data streams d, M_i and N_{i_k} are divisible by d. For brevity, we introduce the transceiver design through a case of $d = 1$, which is not difficult to extend to general cases of $d > 1$. When $d = 1$, the receiver of MS_{i_k} reduces to vector $\boldsymbol{u}_{i_k}$.

1) Transceiver Design Order: Note that, there are two kinds of interference alignment. When the interference is aligned at the receivers by designing proper precoders, the receivers can eliminate the aligned interference. This is referred to as *IA at the receivers (IAR)* in this work. If the interference is pre-aligned at the transmitters by designing receive vectors, the transmitters can avoid the pre-aligned interference. This is referred to as *IA at the transmitters (IAT)*.

For example, in a two-cell MIMO-IBC where MS_{i_k} has N_{i_k} antennas, to reserve at least one-dimensional space to receive the desired signal, it needs to reserve at most $(N_{i_k}-1)$-dimensional space to eliminate ICI. Hence, the IAR precoder of BS_j $(j \neq i)$ should satisfy

$$\begin{cases} \mathrm{rank}\left(\boldsymbol{H}_{i_1,j}\boldsymbol{V}_j\right) \leq N_{i_1} - 1 \\ \cdots \\ \mathrm{rank}\left(\boldsymbol{H}_{i_{K_i},j}\boldsymbol{V}_j\right) \leq N_{i_{K_i}} - 1 \end{cases} \forall j \neq i \tag{5a}$$

$$\mathrm{rank}\left(\boldsymbol{V}_j\right) = K_j \tag{5b}$$

Similarly, for BS_j with M_j antennas to reserve at least K_j-dimensional space to transmit the desired signals, it needs to reserve at most $(M_j - K_j)$-dimensional space. Therefore, the IAT receive vectors of $\mathrm{MS}_{i_1}, \cdots \mathrm{MS}_{i_{K_i}}$ $(\forall i \neq j)$ should satisfy

$$\mathrm{rank}\left(\boldsymbol{H}_{i_1,j}^H \boldsymbol{u}_{i_1}, \cdots, \boldsymbol{H}_{i_{K_i},j}^H \boldsymbol{u}_{i_{K_i}}\right) \leq M_i - K_i\ \forall i \neq j \tag{6a}$$

$$\mathrm{rank}\left(\boldsymbol{u}_{i_k}\right) = 1 \tag{6b}$$

In MIMO-IC where each BS only supports one MS, the designed transceivers of IAR and IAT are equivalent due to the uplink-downlink duality [6]. However, in MIMO-IBC where each BS supports multiple MSs, the transceivers designed

[1]We assume that there are enough MSs. In fact, the required total number of MSs in all cells is no more than $(M+N)/d$ [12].

for IAR and IAT are quite different. From (5a), we see that the IAR requires BS_j to compress multiple data streams experiencing identical channel response. This leads to coherent transmit vectors for different MSs, which is contradictory to the constraint in (5b). In contrast, from (6a), we know that with IAU the MSs compress their data streams experiencing different channels, which will not causes contradictory to (6b). We will focus on the IAT in the sequel.

2) Transmitter Design with Given Receiver: Once $\{\boldsymbol{u}_{1_1}, \cdots, \boldsymbol{u}_{G_{K_G}}\}$ are obtained, we can design the precoders to eliminate the ICI and MUI. We can use zero-forcing (ZF) criterion to keep interference-free. To improve the performance in low SNR region, we can also use the maximal signal-to-leakage-and-noise-ratio (Max-SLNR) criterion as in [6]. It is not hard to derive the precoder of d_{i_k} at BS_i as

$$\boldsymbol{v}_{i_k} = \left(\sum_{j=1}^{G} \sum_{l=1}^{K_j} \boldsymbol{H}_{j_l,i}^H \boldsymbol{u}_{j_l} \boldsymbol{u}_{j_l}^H \boldsymbol{H}_{j_l,i} + \rho\sigma_n^2 \boldsymbol{I} \right)^{-1} \boldsymbol{H}_{i_k,i}^H \boldsymbol{u}_{i_k} \quad (7)$$

which is the Max-SLNR precoder when $\rho = 1$ or the ZF precoder when $\rho \to 0$.

3) Receiver Design: In G-cell MIMO-IBC, for IAT, the receive vectors need to satisfy

$$\mathrm{rank}\,(\boldsymbol{Q}_i) \le M_i - K_i, \; i = 1\ldots, G \quad (8)$$

where $\boldsymbol{Q}_i = [\boldsymbol{P}_{1,i}, \cdots, \boldsymbol{P}_{i-1,i}, \boldsymbol{P}_{i+1,i}, \cdots, \boldsymbol{P}_{G,i}] \in \mathbb{C}^{M_i \times (K^{\mathrm{tot}} - K_i)}$, $\boldsymbol{P}_{j,i} = [\boldsymbol{H}_{j_1,i}^H \boldsymbol{u}_{j_1}, \cdots, \boldsymbol{H}_{j_{K_j},i}^H \boldsymbol{u}_{j_{K_j}}] \in \mathbb{C}^{M_i \times K_j}$ denotes the equivalent channel at the MSs in cell j from BS_i, and $K^{\mathrm{tot}} = \sum_{i=1}^{G} K_i$ is the total number of MSs in all cells.

In (8), $\boldsymbol{Q}_i$ has $K^{\mathrm{tot}} - K_i$ column vectors, then $\mathrm{rank}\,(\boldsymbol{Q}_i) \le K^{\mathrm{tot}} - K_i$. When $M_i \ge K^{\mathrm{tot}}$, it is undoubted $\mathrm{rank}\,(\boldsymbol{Q}_i) \le M_i - K_i$, i.e., (8) always holds. In this case, each BS is capable of eliminating all ICI while the MSs do not need to align the interference. In fact, the precoding now degenerates to the coordinated beamforming (CB) [1].

When $M_i < K^{\mathrm{tot}}$, we need to design the receive vectors to satisfy (8). There should be at most $D_i = M_i - K_i$ linear independent column vectors in $\boldsymbol{Q}_i$, and other $M_i - K_i$ column vectors fall in the space spanned by these D_i vectors. Without loss of generality, we assume that the first D_i column vectors are independent. Then we have

$$\begin{cases} [\boldsymbol{A}_i]_{1,1}[\boldsymbol{Q}_i]_1 + \cdots + [\boldsymbol{A}_i]_{1,D_i}[\boldsymbol{Q}_i]_{D_i} - [\boldsymbol{Q}_i]_{D_i+1} = 0 \\ [\boldsymbol{A}_i]_{2,1}[\boldsymbol{Q}_i]_1 + \cdots + [\boldsymbol{A}_i]_{2,D_i}[\boldsymbol{Q}_i]_{D_i} - [\boldsymbol{Q}_i]_{D_i+2} = 0 \\ \qquad \cdots \\ [\boldsymbol{A}_i]_{K^{\mathrm{tot}}-M_i,1}[\boldsymbol{Q}_i]_1 + \cdots + [\boldsymbol{A}_i]_{K^{\mathrm{tot}}-M_i,D_i}[\boldsymbol{Q}_i]_{D_i} \\ \qquad\qquad - [\boldsymbol{Q}_i]_{K^{\mathrm{tot}}-K_i} = 0 \end{cases} \quad (9)$$

for $i = 1, \ldots, G$, where $[\boldsymbol{Q}_i]_l$ is defined as the lth colum vector of $\boldsymbol{Q}_i$, $\boldsymbol{A}_i$ is a $(K^{\mathrm{tot}} - M_i) \times (M_i - K_i)$ nonzero matrix and $[\boldsymbol{A}_i]_{j,l}$ is the (j,l)-th element of $\boldsymbol{A}_i$.

Substituting $\boldsymbol{Q}_1 = [\boldsymbol{H}_{2_1,1}^H \boldsymbol{u}_{2_1}, \cdots, \boldsymbol{H}_{G_{K_G},1}^H \boldsymbol{u}_{G_{K_G}}]$, $\cdots$, $\boldsymbol{Q}_G = [\boldsymbol{H}_{1_1,G}^H \boldsymbol{u}_{1_1}, \cdots, \boldsymbol{H}_{G-1_{K_{G-1}},G}^H \boldsymbol{u}_{G-1_{K_{G-1}}}]$ into (9), we can obtain a linear equation in term of the receive vectors,

$$\overline{\boldsymbol{H}} \boldsymbol{u} = 0 \quad (10)$$

where $\boldsymbol{u} = [\boldsymbol{u}_{1_1}^T, \cdots, \boldsymbol{u}_{1_{K_1}}^T, \cdots, \boldsymbol{u}_{G_1}^T, \cdots, \boldsymbol{u}_{G_{K_G}}^T]^T \in \mathbb{C}^{N^{\mathrm{tot}} \times 1}$, $N^{\mathrm{tot}} = \sum_{i=1}^{G} \sum_{k=1}^{K_i} N_{i_k}$ is the total number of receive antennas of all MSs, and $\overline{\boldsymbol{H}}$ is of size $\sum_{i=1}^{G} (K^{\mathrm{tot}} - M_i) M_i \times N^{\mathrm{tot}}$, whose entries are functions of $\{\boldsymbol{H}_{i_k,j}\}$ and $\{\boldsymbol{A}_i\}$.

We can design the receive vectors by letting $\boldsymbol{u} \in \mathrm{null}\{\overline{\boldsymbol{H}}\}$, $\|\boldsymbol{u}_{i_k}\|^2 \ne 0$, where $\mathrm{null}\{\overline{\boldsymbol{H}}\}$ means the null space of the matrix $\overline{\boldsymbol{H}}$. Considering that $\boldsymbol{u}_{i_k}$ and $\alpha \boldsymbol{u}_{i_k}$ (α is a non-zero scale factor) align the interference into the same space, for simplicity, we restrict the receive vectors to follow $\|\boldsymbol{u}_{i_k}\|^2 = 1$.

To illustrate the idea of receive vector design, we consider a case where $M_i = K_i$. From (3a)$\sim$(3c), we know that the required minimal value of N_{i_k} is $N_{i_k} = K^{tot} - K_i + 1$. Then $\overline{\boldsymbol{H}}$ in (10) is of size $\sum_{i=1}^{G} (K^{\mathrm{tot}} - M_i) K_i \times \sum_{i=1}^{G} (K^{\mathrm{tot}} - M_i + 1) K_i$. The number of rows of $\overline{\boldsymbol{H}}$ is less than its number of columns, hence there exists at least one non-zero solution of (10), from which we can obtain the receive vectors of the MSs.

For example, in a two-cell MIMO-IBC where $K_1 = 3$, $K_2 = 2$, when $M_1 = 3$ and $M_2 = 2$, from (3a)$\sim$(3c), the required minimal value of N_{i_k} satisfies $N_{1_1} = N_{1_2} = N_{1_3} = 3$ and $N_{2_1} = N_{2_2} = 4$. Then the corresponding $\overline{\boldsymbol{H}}$ is

$$\overline{\boldsymbol{H}} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{H}_{2_1,1}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{H}_{2_2,1}^H \\ -\boldsymbol{H}_{1_1,2}^H & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{H}_{1_2,2}^H & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{H}_{1_3,2}^H & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

which is of size 13×17. Let $\boldsymbol{u} \in \mathrm{null}\{\overline{\boldsymbol{H}}\}$, we can obtain the receive vectors. After some elementary transformations, $\overline{\boldsymbol{H}}$ can be converted into a block diagonal matrix. Then the receive vector of each MS can be further obtained as

$$\boldsymbol{u}_{1_1} = \mathrm{null}\{\boldsymbol{H}_{1_1,2}^H\}, \; \boldsymbol{u}_{1_2} = \mathrm{null}\{\boldsymbol{H}_{1_2,2}^H\}, \; \boldsymbol{u}_{1_3} = \mathrm{null}\{\boldsymbol{H}_{1_3,2}^H\}, \; \boldsymbol{u}_{2_1} = \mathrm{null}\{\boldsymbol{H}_{2_1,1}^H\}, \; \boldsymbol{u}_{2_2} = \mathrm{null}\{\boldsymbol{H}_{2_2,1}^H\}$$

In this example, the MSs are able to eliminate all the ICI, i.e., they align the ICI into $D_i = 0$-dimensional subspace. Then the BSs only need to eliminate the MUI. Similar idea has been proposed in [9] for downlink IA in a two-cell MIMO-IBC. However, [9] requires $M_i = K_i + 1$ transmit antennas whereas our method only requires $M_i = K_i$ antennas. Actually, the extra one antenna is unnecessary since $M_i = K_i$ is enough for BS_i to transmit its desired signals. Since our IAT method can also implement IA for other antenna configurations, the downlink IA in [9] is a special case of our method.

In the example addressed above, $D_i = 0$ (i.e., $M_i = K_i$), where $\overline{\boldsymbol{H}}$ does not depend on the matrix of combination coefficients $\{\boldsymbol{A}_i\}$. When $D_i > 0$ (i.e., $M_i > K_i$), $\overline{\boldsymbol{H}}$ will be associate with $\{\boldsymbol{A}_i\}$. In the following, we discuss the design of $\{\boldsymbol{A}_i\}$.

In (10), $\overline{\boldsymbol{H}}$ is of size $\sum_{i=1}^{G} (K^{\mathrm{tot}} - M_i) M_i \times N^{\mathrm{tot}}$. Since different relationship between the number of rows of $\overline{\boldsymbol{H}}$ and the number of columns leads to different design of $\{\boldsymbol{A}_i\}$, we investigate $\{\boldsymbol{A}_i\}$ in the following two cases.

Case I: $\sum_{i=1}^{G}(K^{\text{tot}}-M_i)M_i < N^{\text{tot}}$

In this case, $\text{rank}(\overline{\boldsymbol{H}}) < N^{\text{tot}}$ always holds. For arbitrary non-zero matrix $\{\boldsymbol{A}_i\}$, the matrix $\overline{\boldsymbol{H}}^H\overline{\boldsymbol{H}}$ always has at least one zero eigenvalue. Therefore, we can choose $\boldsymbol{u}$ directly as the corresponding eigenvector.

Take a two-cell MIMO-IBC as an example, where BS_1 and BS_2 support $K_1 = K_2 = K$ MSs. When $M_i = K+1$, from (3a)∼(3c) the required minimal value of N_{i_k} is $N_{i_k} = K$. Then $\overline{\boldsymbol{H}}$ in (10) now becomes

$$\overline{\boldsymbol{H}} = \begin{pmatrix} \boldsymbol{0} & \overline{\boldsymbol{H}}_1 \\ \overline{\boldsymbol{H}}_2 & \boldsymbol{0} \end{pmatrix}$$

where

$$\overline{\boldsymbol{H}}_i = \begin{pmatrix} [\boldsymbol{A}_i]_{1,1}\boldsymbol{H}_{j_1,i}^H & -\boldsymbol{H}_{j_2,i}^H & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ [\boldsymbol{A}_i]_{2,1}\boldsymbol{H}_{j_1,i}^H & \boldsymbol{0} & -\boldsymbol{H}_{j_3,i}^H & & \\ \vdots & \vdots & & \ddots & \vdots \\ [\boldsymbol{A}_i]_{K-1,1}\boldsymbol{H}_{j_1,i}^H & \boldsymbol{0} & \boldsymbol{0} & \cdots & -\boldsymbol{H}_{j_K,i}^H \end{pmatrix}$$

is of size $(K^2-1)\times K^2$.

Since the number of rows in $\overline{\boldsymbol{H}}$ is less than the number of columns, we can design $\boldsymbol{u}$ as the eigenvector corresponds to the zero eigenvalue of $\overline{\boldsymbol{H}}^H\overline{\boldsymbol{H}}$.

In this example, the constraint shown in (10) can be expressed as another form,

$$\boldsymbol{H}_{j_1,i}^H\boldsymbol{u}_{j_1} = [\boldsymbol{A}_i]_{1,1}^{-1}\boldsymbol{H}_{j_2,i}^H\boldsymbol{u}_{j_2} = \cdots = [\boldsymbol{A}_i]_{K-1,1}^{-1}\boldsymbol{H}_{j_K,i}^H\boldsymbol{u}_{j_K} \tag{11}$$

It indicates that all the ICIs are aligned into a one-dimensional subspace. In fact, both the downlink IA in [10] and the uplink IA in [3] considered the IA constraint in (11), i.e., these IA transceivers are designed given one-dimensional interference subspace.

We can see from (11) that the value of $[\boldsymbol{A}_i]_{j,l}$ never affects the directions of the aligned space only if $[\boldsymbol{A}_i]_{j,l} \neq 0$. Therefore, under the constraint of $\|\boldsymbol{u}_{i_k}\|^2 = 1$, the obtained receive vectors $\{\boldsymbol{u}_{i_k}\}$ do not depend on $\{\boldsymbol{A}_i\}$. Consequently, we obtain a surprising but reasonable conclusion that $[\boldsymbol{A}_i]_{j,l}$ can be set arbitrary except zero.

Take another two-cell MIMO-IBC as an example, where $K_1 = K_2 = K$. When $M_i = 2K-1$, from (3a)∼(3c) the required minimal value of N_{i_k} is $N_{i_k} = 2$. Then, $\overline{\boldsymbol{H}}$ in (10) is

$$\overline{\boldsymbol{H}}_i = \begin{pmatrix} [\boldsymbol{A}_i]_{1,1}\boldsymbol{H}_{j_1,i}^H & \cdots & [\boldsymbol{A}_i]_{1,K-1}\boldsymbol{H}_{j_{K-1},i}^H & -\boldsymbol{H}_{j_K,i}^H \end{pmatrix}$$

which is a $(2K-1)\times 2K$ matrix. Similarly, $[\boldsymbol{A}_i]_{j,l}$ can be set arbitrarily. Then we can design $\boldsymbol{u}$ as the eigenvector corresponds to the zero eigenvalue of $\overline{\boldsymbol{H}}^H\overline{\boldsymbol{H}}$.

Case II: $\sum_{i=1}^{G}(K^{\text{tot}}-M_i)M_i \geq N^{\text{tot}}$

In this case, $\text{rank}(\overline{\boldsymbol{H}}) < N^{\text{tot}}$ does not always hold for arbitrary $\{\boldsymbol{A}_i\}$. In order to obtain the IA transceiver, we need to design the coefficient matrix $\{\boldsymbol{A}_i\}$.

So far, only the authors of [8] have investigated the design of $\{\boldsymbol{A}_i\}$ for a special case of $\sum_{i=1}^{G}(K^{\text{tot}}-M_i)M_i = N^{\text{tot}}$, where $\overline{\boldsymbol{H}}$ is a square matrix. To ensure $\text{rank}(\overline{\boldsymbol{H}}) < N^{\text{tot}}$, $\boldsymbol{A}_1,\cdots,\boldsymbol{A}_G$ need to satisfy

$$\det\{\overline{\boldsymbol{H}}\} = 0 \tag{12}$$

Take the typical case of a three-cell MIMO-IC with $M = N = 2$ as an example, which has been addressed widely in MIMO-IC [2, 8], the corresponding matrix is

$$\overline{\boldsymbol{H}} = \begin{pmatrix} \boldsymbol{0} & [\boldsymbol{A}_1]_{1,1}\boldsymbol{H}_{2_1,1}^H & -\boldsymbol{H}_{3_1,1}^H \\ [\boldsymbol{A}_2]_{1,1}\boldsymbol{H}_{1_1,2}^H & \boldsymbol{0} & -\boldsymbol{H}_{3_1,2}^H \\ [\boldsymbol{A}_3]_{1,1}\boldsymbol{H}_{1_1,3}^H & -\boldsymbol{H}_{2_1,3}^H & \boldsymbol{0} \end{pmatrix}$$

which is a 6×6 matrix. After some regular derivations, the determinant of $\overline{\boldsymbol{H}}$ can be obtained as a function of $[\boldsymbol{A}_1]_{1,1}$, $[\boldsymbol{A}_2]_{1,1}$ and $[\boldsymbol{A}_3]_{1,1}$, i.e.,

$$\det\left(\overline{\boldsymbol{H}}\right) = a\det\left([\boldsymbol{A}_1]_{1,1}[\boldsymbol{A}_3]_{1,1}[\boldsymbol{A}_2]_{1,1}^{-1}\boldsymbol{I} - \boldsymbol{\Omega}\right) \tag{13}$$

where $\boldsymbol{\Omega} = -\left(\boldsymbol{H}_{2_1,1}^H\boldsymbol{H}_{3_1,2}^H\boldsymbol{H}_{1_1,3}^H\right)^{-1}\boldsymbol{H}_{3_1,1}^H\boldsymbol{H}_{1_1,2}^H\boldsymbol{H}_{2_1,3}^H$ and $a = [\boldsymbol{A}_2]_{1,1}\det\left(\boldsymbol{H}_{2_1,1}^H\boldsymbol{H}_{3_1,2}^H\boldsymbol{H}_{1_1,3}^H\right)$.

When $[\boldsymbol{A}_1]_{1,1}[\boldsymbol{A}_3]_{1,1}[\boldsymbol{A}_2]_{1,1}^{-1}$ is equal to an eigenvalue of $\boldsymbol{\Omega}$, we have $\det\{\overline{\boldsymbol{H}}\} = 0$, i.e., $\text{rank}(\overline{\boldsymbol{H}}) < N^{\text{tot}}$.

This method proposed in [8] can be easily extended to MIMO-IBC when $\overline{\boldsymbol{H}}$ is a square matrix, however it can not be applied to more general cases.

To design the linear IA for the case where $\sum_{i=1}^{G}(K^{\text{tot}} - M_i)M_i > N^{\text{tot}}$, in the following we find $\{\boldsymbol{A}_i\}$ when $\overline{\boldsymbol{H}}$ is not a square matrix. Since $\text{rank}(\overline{\boldsymbol{H}}) < N^{\text{tot}}$ is equivalent to the case where there exists a N^{tot}-dimensional vector $\boldsymbol{x}$ satisfying $\|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2 = 0$, we can convert this rank constraint problem into the following optimization problem,

$$\min_{\boldsymbol{x},\boldsymbol{A}_1,\cdots,\boldsymbol{A}_G} \|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2 \tag{14}$$

where $\overline{\boldsymbol{H}}$ is a function of $\boldsymbol{A}_1,\cdots,\boldsymbol{A}_G$.

For the given values of $\boldsymbol{A}_1,\cdots,\boldsymbol{A}_G$, the optimal solution $\boldsymbol{x}$ of the problem (14) is

$$\boldsymbol{x} = \arg\min_{\boldsymbol{x}}\{\|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2\} \tag{15}$$

It is easy to show that the optimal $\boldsymbol{x}$ is the minimum dominant eigenvector of $\overline{\boldsymbol{H}}^H\overline{\boldsymbol{H}}$.

Moreover, note that (9) can be expressed as a linear equation of $\{\boldsymbol{A}_i\}$, i.e.,

$$\boldsymbol{B}_i\boldsymbol{A}_i^T - \boldsymbol{C}_i = 0 \tag{16}$$

where $\boldsymbol{B}_i = [[\boldsymbol{Q}_i]_1, [\boldsymbol{Q}_i]_2, \cdots, [\boldsymbol{Q}_i]_{D_i}] \in \mathbb{C}^{M_i\times D_i}$ and $\boldsymbol{C}_i = [[\boldsymbol{Q}_i]_{(D_i+1)}, [\boldsymbol{Q}_i]_{(D_i+2)}, \cdots, [\boldsymbol{Q}_i]_{K^{\text{tot}}-K_i}] \in \mathbb{C}^{M_i\times(K^{\text{tot}}-M_i)}$.

Then for a given $\boldsymbol{x}$, the objective function of (14) becomes $\|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2 = \sum_{i=1}^{G}\|\boldsymbol{B}_i\boldsymbol{A}_i^T - \boldsymbol{C}_i\|^2$. Therefore, it is not hard to show that the optimal $\{\boldsymbol{A}_i\}$ for (14) becomes

$$\boldsymbol{A}_i = \arg\min_{\boldsymbol{A}_i}\{\|\boldsymbol{B}_i\boldsymbol{A}_i^T - \boldsymbol{C}_i\|^2\} = \boldsymbol{C}_i^T\boldsymbol{B}_i^*\left(\boldsymbol{B}_i^T\boldsymbol{B}_i^*\right)^{-1} \tag{17}$$

Using alternating minimization algorithm as in [7], we can employ an iterative algorithm to design $\{\boldsymbol{A}_i\}$, which includes the following steps:

1) Initialize $\{\boldsymbol{A}_i\}$ as arbitrary matrices for $i = 1,\ldots,G$.

2) Let $\boldsymbol{x}$ be the minumum dominant eigenvector of $\overline{\boldsymbol{H}}^H \overline{\boldsymbol{H}}$.
3) Let $\boldsymbol{A}_i = \boldsymbol{C}_i^T \boldsymbol{B}_i^* \left(\boldsymbol{B}_i^T \boldsymbol{B}_i^*\right)^{-1}$.
4) Repeat steps 2), 3) until $\{\|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2\} \le \varepsilon$, where ε is a given non-negative number.

Note that the objective function in (14) is non-negative, and the objective function will reduce by optimizing $\boldsymbol{x}$ with given values of $\{\boldsymbol{A}_i\}$, and vice versa. Therefore, this iterative algorithm is convergent. When (3a)$\sim$(3c) are satisfied, the IA feasibility conditions guarantee that there exists at least one IA solution. Although our iterative algorithm may not converge to an IA solution, setting different initial values in parallel can increase the probability of convergence. In fact, from our forthcoming simulation, we find that it is enough to set one initial value to achieve $\varepsilon = 10^{-5}$. Compared with the existing iterative IA algorithms [6, 11], our method only iteratively calculates the coefficient matrix $\{\boldsymbol{A}_i\}$ rather than iteratively finds the precoders and receive vectors, which avoids the trouble of IAU in the precoder design and reduces the complexity.

The proposed IAT algorithm is summarized as follows,

1) Construct $\overline{\boldsymbol{H}}$ in find $\{\boldsymbol{A}_i\}$ to meet $\|\overline{\boldsymbol{H}}\boldsymbol{x}\|^2 \le \varepsilon$.
2) Design $\boldsymbol{u} = [\boldsymbol{u}_{1_1}^T, \cdots, \boldsymbol{u}_{G_{K_G}}^T]^T$ as the minimum dominant eigenvector of $\overline{\boldsymbol{H}}^H \overline{\boldsymbol{H}}$.
3) Design precoder $\boldsymbol{v}_{i_k}$ from (7).

C. Trade-off between number of data streams and SNR gain

The study in [14] indicates that IA is in demand if a system is operated at or close to the maximum achievable rate, while for low-rate applications the desirable strategy is to exploit all antennas to achieve high diversity gain rather than to align the interference. If an IA transceiver developed for high-rate applications is directly applied for low-rate applications, the resulting signal power loss may reduce the performance of the system.

Fortunately, since our IAT algorithm is designed to employ the minimal antenna resource rather than designed for a given dimensional of interference subspace, it can adaptively adjust the dimension of interference subspace according to the number of MSs. When the number of MSs decreases, the constraint for the receive vectors will be relaxed. Then the MSs can employ redundant spatial resources to receive the signals under the interference-free condition, resulting in a good performance in low-rate cases.

To see this, consider a two-cell MIMO-IBC where $M = 5$ and $N = 4$. From the feasibility conditions of IA in MIMO-IBC, we know that each BS can support at most $K = 4$ MSs in each cell, and the MSs need to align the ICI into $D = M - K = 1$-dimensional subspace. When $K = 3$, if we employ existing IA algorithms that align the interference into a pre-determined subspace, i.e., still align the ICI into a one-dimensional subspace, $\overline{\boldsymbol{H}}$ in (10) is

$$\overline{\boldsymbol{H}}_i = \begin{pmatrix} [\boldsymbol{A}_i]_{1,1}\boldsymbol{H}_{j_1,i}^H & -\boldsymbol{H}_{j_2,i}^H & \boldsymbol{0} \\ [\boldsymbol{A}_i]_{2,1}\boldsymbol{H}_{j_1,i}^H & \boldsymbol{0} & -\boldsymbol{H}_{j_3,i}^H \end{pmatrix} \in \mathbb{C}^{10\times 12}$$

and there exist two non-zero solutions for $\boldsymbol{u}$.

When $K = 3$, however, with our algorithm the receive vectors at the MSs align the ICI into $D = M - K = 2$-dimensional subspace. Then $\overline{\boldsymbol{H}}$ in (10) becomes

$$\overline{\boldsymbol{H}}_i = \left([\boldsymbol{A}_i]_{1,1}\boldsymbol{H}_{j_1,i}^H \quad [\boldsymbol{A}_i]_{1,2}\boldsymbol{H}_{j_2,i}^H \quad -\boldsymbol{H}_{j_3,i}^H\right) \in \mathbb{C}^{5\times 12}$$

and there exist seven non-zero solutions for $\boldsymbol{u}$. This provides the MSs a subspace of higher dimension to receive the desired signals without any interference. Thereby the MSs have the opportunity to exploit these redundant spatial resources to enhance the signal power, which finally yields a trade-off between the number of data streams and SNR gain.

IV. Numerical and Simulation Results

In this section, we verify our analysis and evaluate the performance of the proposed IAT method via simulation and numerical results.

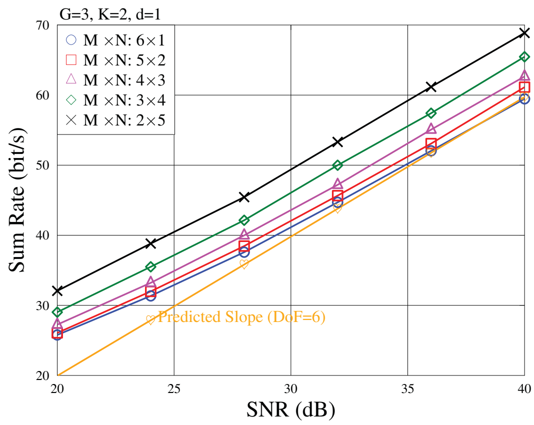

Fig. 1. Sum rate versus SNR in a three-cell MIMO-IBC, $G = 3$, $K = 2$.

Figure 1 shows the simulated sum rate versus SNR for a symmetric three-cell MIMO-IBC where each BS supports $K = 2$ MSs. From (3a)$\sim$(3c), we know that there is more than one antenna configuration to achieve the required number of data streams. To achieve $d^{\text{tot}} = 6$, the required minimum number of antennas needs to satisfy the following configurations of $(M \times N)$: (6×1), (5×2), (4×3), (3×4) and (2×5). Based on these antenna configurations, our IAT method respectively aligns the interference into $D = 4, 3, 2, 1, 0$-dimensional subspace. We can see that the slope of all the simulated curves is equal to the required number of data streams, which is with legend “Predicted slope”. It suggests that our IAT method can achieve the required number of data streams with the minimum antenna configurations. Note that although the number of data streams of all cases are identical, their sum rates are different. The sum rate is higher for a system with larger number of receive antennas, which comes from the array gain (i.e., the SNR gain) in a MIMO receiver.

In Fig. 2, we show the data rate per user in a two-cell MIMO-IBC where $M = 5$ and $N = 4$. In this antenna configuration, each cell can support at most $K = 4$ MSs. When $K = 4$, the receive vectors of the IAT align the ICI

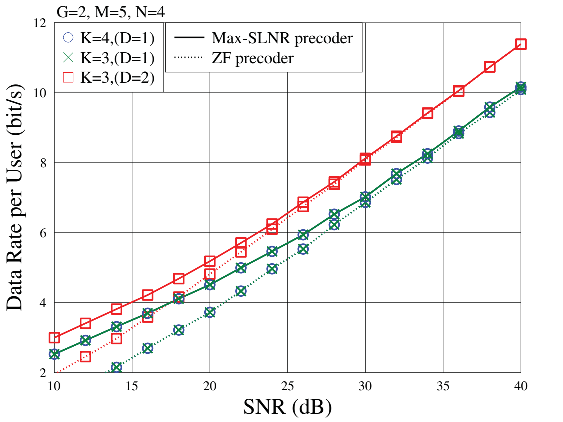

Fig. 2. Data rate per user versus SNR, $G = 2$, $M = 5$ and $N = 4$.

into $D = 1$-dimensional subspace. When $K = 3$, the receive vectors of the IAT can align the ICI into $D = 1$ or $D = 2$-dimensional subspace. From these results, we can see that when $K = 3$, if the system still aligns the ICI into $D = 1$-dimensional space, the data rate per user of $K = 3(D = 1)$ is the same as that of $K = 4(D = 1)$. However, it is clear that if we adjust the dimension of the aligned interference subspace according to K, e.g., let $D = M - K = 2$, the data rate per user of $K = 3(D = 2)$ is higher than that of $K = 3(D = 1)$. It is because compared with IA for $D = 1$, the IA for $D = 2$ relaxes the constraint for receive vectors, as we have explained in last section. This shows that the proposed method achieves a flexible trade-off between the number of data streams and SNR gain and makes a good use of antennas.

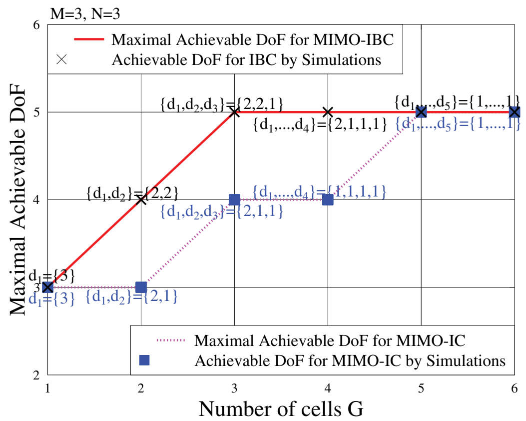

Fig. 3. DoF versus the number of cells G, $M = 3, N = 3$.

In Fig. 3, we show the maximal achievable DoF when the number of antennas is given as $M = N = 3$, which are obtained by simulation using our IAT method and by the maximal achievable DoF in (4). Note that our IAT algorithm can be applied in both MIMO-IC and MIMO-IBC. In MIMO-IC, each cell supports only one MS who transmits multiple data streams, while in MIMO-IBC, each cell serves multiple MSs and each MS transmits only one data stream. To show how the maximal DoF is achieved, we also provide the number of data streams in all cells $\{d_1, \cdots, d_G\}$ in the figure. For example, in the case of $G = 3$, $\{d_1, d_2, d_3\} = \{2, 2, 1\}$ denotes that BS_1 and BS_2 respectively transmit two data streams and BS_3 transmits one data stream. From these results, we can see that no matter in MIMO-IC or MIMO-IBC, our IAT algorithm always achieves the maximal achievable DoF.

V. Conclusion

In this paper, we developed a linear interference alignment transceiver for general MIMO interference broadcast channels. By designing the transceiver that achieves the maximal degrees of freedom, the proposed method is able to adaptively adjust the dimension of the aligned interference subspace according to the number of active MSs. Therefore it can achieve a good trade-off between the data stream number and the SNR gain that improves the performance of interference alignment in low-rate applications. To accommodate the difficulty in aligning interference at MS sides for MIMO interference broadcast channels, with our transceiver the MSs first align the inter-cell interference at the base station and then the base stations pre-eliminate the aligned inter-cell interference as well as the intra-cell interference before downlink transmission.

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