Low Complexity Scheduling for Downlink Multiuser MIMO Systems in Correlated Channels

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***Abstract*—Many successive scheduling schemes associated with linear transmit beamforming have been proposed which can asymptotically achieve the sum rate of dirty paper coding (DPC) in *i.i.d.* channels. In this paper, the performance of the successive scheduling in spatially correlated channels is studied. Then we propose a low complexity alternating user scheduling (AUS), which can be applied to both zero-forcing beamforming (ZFBF) and regularized ZFBF (R-ZFBF). The new scheduling algorithm can achieve a comparable sum rate with that of the exhaustive searching in both *i.i.d.* and spatially correlated channels with the same order of complexity as the successive scheduling.**

I. Introduction

Dirty paper coding is the optimal transmit strategy for downlink multiuser MIMO systems [1] if perfect channel state information (CSI) is available at transmitter, but in practice it is infeasible due to its prohibitive computational complexity. Some suboptimal transmit strategies apply transmit beamforming [2] and user scheduling [3] [4] separately to achieve the multiplexing gain and multiuser diversity gain, both of which require much lower complexity. These suboptimal transmit strategies can achieve the same scaling of the sum rate of DPC when the number of users approaches infinity [5].

For a given transmit beamforming, the optimal user scheduling needs exhaustive searching over all possible user sets, which may result in a huge complexity, especially when the number of users is very large. Successive searching is an effective method to reduce the searching space by converting a multi-dimensional searching problem into multiple one-dimensional searching problems. Many low complexity user scheduling algorithms bear this spirit, such as the tree-based sorting algorithm [3] and semiorthogonal user scheduling (SUS) [4]. Furthermore, it has been shown that the successive scheduling associated with zero-forcing beamforming (ZFBF) can achieve the same sum rate as that of DPC asymptotically when the number of users approaches to infinity in *i.i.d.* channels [4].

The performance of the successive scheduling has been compared in [6] under *i.i.d.* channels through analyzing the statistics of the channel gain and the angle between user's channels. To the best of our knowledge, however, its performance in spatially correlated channels was not investigated in earlier works. Moreover, since existing successive scheduling algorithms are essentially designed for ZFBF [4], [7], the performance will degrade when some non-ZFBF beamforming algorithms such as regularized-ZFBF (R-ZFBF) [2] are used due to the mismatch between the scheduling and beamforming.

In this paper we'll first show that the performance gap between the successive scheduling and the exhaustive searching will increase in spatially correlated channels with respect to that in *i.i.d.* channels. This will be demonstrated by characterizing the statistics of the upper bound of the angle between the channels of users in a possible user set for these two scheduling algorithms. Then we'll propose a low complexity alternating user scheduling, which can be applied to both ZFBF and R-ZFBF. The alternating scheduling can approximately achieve the same sum rate as that of the exhaustive searching in both *i.i.d.* and spatially correlated channels.

This work was supported by the International S&T Cooperation Program of China (ISCP) (No. 2008DFA12100).

II. System Model

A. Signal Model

Consider a downlink multiuser MISO system supporting K users. The transmitter is equipped with M antennas and each user with only one antenna. The received signal for user i can be written as

$$y_i = \mathbf{h}_i \sum_{j=1, j\in\mathcal{S}}^{K} \mathbf{w}_j^H x_j + n_i, \qquad (1)$$

where for user i, $\mathbf{h}_i \in \mathbb{C}^{1\times M}$ is the channel vector, and $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, x_i is the transmitted signal, $\mathbf{w}_i \in \mathbb{C}^{1\times M}$ is the transmit beamforming vector, n_i is the additive white Gaussian noise with zero mean and variance σ^2. $\mathcal{S} \subset \{1, 2, \ldots, K\}$ is the index set of users scheduled concurrently.

With the assumption that perfect CSI is available at the transmitter and equal power is allocated among users, the transmit beamforming vector of ZFBF and R-ZFBF can be written as [2]

$$\mathbf{w}_i = \frac{\mathbf{h}_i \left(\mathbf{H}_\mathcal{S}^H \mathbf{H}_\mathcal{S} + \beta \mathbf{I}\right)^{-1}}{\left\| \mathbf{h}_i \left(\mathbf{H}_\mathcal{S}^H \mathbf{H}_\mathcal{S} + \beta \mathbf{I}\right)^{-1} \right\|} \sqrt{\frac{P}{|\mathcal{S}|}}, \qquad (2)$$

where $\mathbf{H}_\mathcal{S}$ is the submatrix of $\mathbf{H} = [\mathbf{h}_1^T \cdots \mathbf{h}_K^T]^T$ whose row vectors are selected from $\mathbf{H}$ according to $\mathcal{S}$, P is the total

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transmit power, $|\mathcal{S}|$ is the size of $\mathcal{S}$, $\beta \to 0$ is for ZFBF, and $\beta = |\mathcal{S}|\sigma^2$ is for R-ZFBF. Therefore, the sum rate for the system with user set $\mathcal{S}$ is

$$R(\mathcal{S}) = \sum_{i,l \in \mathcal{S}} log\left(1 + \frac{P\left|\mathbf{h}_i \mathbf{w}_i^H\right|^2}{\sum_{l \neq i} P\left|\mathbf{h}_i \mathbf{w}_l^H\right|^2 + |\mathcal{S}|\sigma^2}\right). \quad (3)$$

B. Scheduling

When the total users' number K is larger than M, the multiuser scheduling problem can be formulated as follows,

$$\begin{aligned} &\max_{\mathcal{S}} \;\; R(\mathcal{S}) \\ &s.t. \quad |\mathcal{S}| = M. \end{aligned} \quad (4)$$

Assume that the number of scheduled users equals to M, *i.e.*, let the transmitter serve as many users as possible by fully using all of the available degrees of freedom. Although this may reduce the sum rate, it is more suitable for the scenarios where a large number of users compete for the traffic of delay-sensitive applications. Without this assumption, the number of scheduled users can be determined adaptively. Then the sum rate will depend on the multiplexing gain, transmit diversity gain and multiuser diversity gain, which will hinder the analysis of the influence of spatial correlation on multiuser scheduling.

For the user scheduling problem of (4), exhaustive searching can achieve the maximal sum rate by examining all $\binom{K}{M}$ possible user sets. As a suboptimal solution, successive scheduling [7] selects only one user at each step to maximize the sum rate of the user set consisted of the new user and the users already selected. After M steps, M users will be selected. It has a low complexity by reducing the searching space.

Let $\mathcal{T} = \{1, 2, \ldots, K\}$ denote the set of indexes of all users, and $\mathcal{S}_m = \{s_1, \cdots, s_m\}$ denote the scheduling result at m-th step, $m = 1, 2, \ldots, M$. The successive scheduling can be summarized as follows,

1) Initialize by selecting the user with the maximum channel gain as the first user,

$$s_1 = \arg\max_{k \in \mathcal{T}} \|\mathbf{h}_k\|, \quad (5)$$

and $\mathcal{S}_1 = \{s_1\}$.

2) For $m = 2, \ldots, M$, compute the sum rate with (3) for each user left and the users already scheduled. Then select the user with the maximal sum rate,

$$s_m = \arg\max_{k \in \mathcal{T} \setminus \mathcal{S}_{m-1}} R(\mathcal{S}_{m-1} \cup \{k\}). \quad (6)$$

Update $\mathcal{S}_m \Leftarrow \mathcal{S}_{m-1} \cup \{s_m\}$, where the notation of $Y \setminus X$ denotes the set of elements of Y that are not elements of X, and k is the index of a possible new user.

One can see from (2) and (3) that in order to obtain the sum rate, the successive scheduling not only involves many matrix inversion operations but also needs to compute the data rate of M users. The resulting complexity can be significantly reduced if ZFBF is employed. Since the inter-user interference can be completely cancelled for ZFBF, the signal-to-noise ratio (SNR) of user i at m-th step is

$$SNR_{i,m} = \frac{\|\mathbf{h}_i\|^2 - \|\mathbf{v}_i\|^2}{m\sigma^2} = \frac{\|\mathbf{h}_i\|^2 \sin^2\theta_i}{m\sigma^2}, \quad (7)$$

where $\mathbf{h}_i$ is the channel vector of user i, $\mathbf{v}_i$ is the projection of $\mathbf{h}_i$ onto the subspace spanned by the channel vectors of selected users, θ_i denotes the angle between $\mathbf{h}_i$ and $\mathbf{v}_i$, $\cos\theta_i = \frac{|\mathbf{v}_i \mathbf{h}_i^H|}{\|\mathbf{v}_i\|\|\mathbf{h}_i\|}$. $\mathbf{v}_i$ can be effectively computed by iterative projection algorithm. Such a successive scheduling (SS) tailed for ZFBF is referred to SS-ZFBF in the sequel.

SS-ZFBF can be further simplified by using SNR of the new user rather than sum rate as the selection criterion. Moreover, a threshold can be introduced to evaluate the orthogonality between the new user and the selected users. The user with worse orthogonality will be excluded in order to reduce the searching space. This is the so-called SUS [4].

III. Performance Analysis

In this section the performance of SS-ZFBF will be analyzed in spatially correlated channels. We can see from (7) that the performance depends on the channel gain and the angle between the channels of users. To avoid complicated analysis with sum rate, we will analyze the performance by investigating the impact of using SS-ZFBF on the statistics of the angle θ_i. Furthermore, considering the fact that θ_i is upperly bounded by the angle between the channel of the new user and the channel of any one user already selected, we will analyze the case of $M = 2$ in the sequel of this section.

To compare the performance of SS-ZFBF with that of the exhaustive scheduling, we first present one theorem revealing the statistic behavior of the angle between the channel vectors as follows.

Theorem 1: For K random vectors $\mathbf{u}_k \in \mathbb{C}^{1\times 2}$ and $\mathbf{u}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, let θ_1 denote the angle between any two of the vectors and θ_2 denote the angle between the vector with maximum norm and any other vector. Then the probability density function (*pdf*) of $\sin^2\theta_1$ and $\sin^2\theta_2$ can be derived as

$$\begin{aligned} f(x) \;=\; &\int \frac{\Upsilon_{\mathbf{v}}\left(1-\rho^2\right) d_1 d_2}{\pi\lambda_1\lambda_2\left[\left(d_2-d_1\right)x+d_1\right]^2} \\ &\cdot\left[1-\frac{4\rho^2 d_1 d_2 x\left(1-x\right)}{\left[\left(d_2-d_1\right)x+d_1\right]^2}\right]^{-3/2} d\mathbf{v}, \end{aligned} \quad (8)$$

where $\mathbf{v} \in \mathbb{C}^{1\times 2}$, $\mathbf{v}\mathbf{v}^H = 1$, $\mathbf{v}_\perp$ is the orthogonal complement of $\mathbf{v}$, λ_1 and λ_2 are the different eigenvalues of $\mathbf{R}$, $d_1 = \mathbf{v}_\perp \mathbf{R} \mathbf{v}_\perp^H$, $d_2 = \mathbf{v}\mathbf{R}\mathbf{v}^H$, $\rho = \mathbf{v}_\perp \mathbf{R} \mathbf{v}^H \mathbf{v} \mathbf{R} \mathbf{v}_\perp^H / (d_1 d_2)$. $\Upsilon_{\mathbf{v}}$ is a function of $\mathbf{v}$, which is different for θ_1 and θ_2. For θ_1,

$$\Upsilon_{\mathbf{v}} = 1. \quad (9)$$

For θ_2, $\Upsilon_{\mathbf{v}}$ is defined as

$$\Upsilon_{\mathbf{v}} = \int_0^\infty K\alpha e^{-\alpha \mathbf{v}\mathbf{R}^{-1}\mathbf{v}^H}\left[1 + \frac{\lambda_2 e^{\frac{-\alpha}{\lambda_2}} - \lambda_1 e^{\frac{-\alpha}{\lambda_1}}}{\lambda_1 - \lambda_2}\right]^{K-1} d\alpha. \quad (10)$$

The proof is shown in [8].

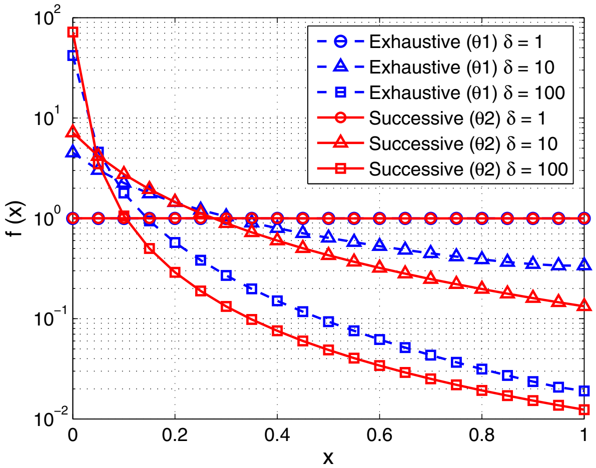

Fig. 1. The *pdf* of $\sin^2 \theta_1$ and $\sin^2 \theta_2$ with different δ, which illustrates the performance of the exhaustive searching and successive scheduling respectively, $K = 10$.

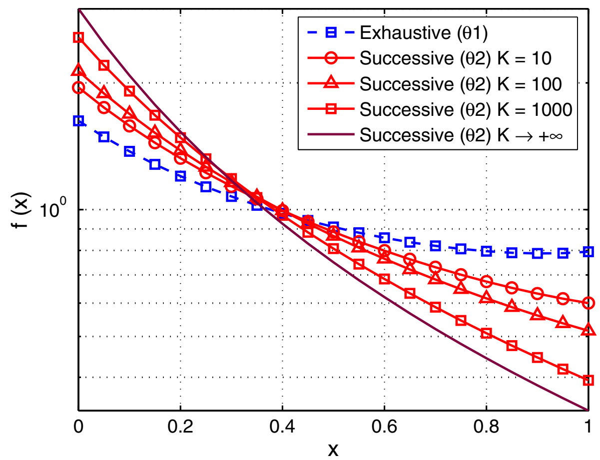

Fig. 2. The *pdf* of $\sin^2 \theta_1$ and $\sin^2 \theta_2$ with different K, which illustrates the performance of the exhaustive searching and successive scheduling respectively, $\delta = 3$.

Remark 1: When $\mathbf{R} = \mathbf{I}$, $f(x) = 1$ for θ_1 and θ_2, which means that both $\sin^2 \theta_1$ and $\sin^2 \theta_2$ are uniformly distributed in $[0, 1]$. The proof is shown in [8].

Remark 2: When $K \rightarrow \infty$, the *pdf* of $\sin^2 \theta_2$ can be simplified as

$$f(x) = \frac{\delta}{\left[(\delta - 1)\, x + 1\right]^2}. \tag{11}$$

where δ is the conditional number of $\mathbf{R}$. The proof is shown in [8]. Note that θ_1 is independent of K.

For the exhaustive searching, a possible user set consists of any two users, while for SS-ZFBF a possible user set includes the user with the maximum channel gain and any other user. When the random vectors in Theorem 1 are replaced by the channel vectors, θ_1 and θ_2 will correspond to the angle between the two channel vectors in a possible user set for the exhaustive searching and SS-ZFBF, respectively. Therefore, Theorem 1 and its remarks can illustrate the relationship between the performance of these two scheduling algorithms.

Fig.1 and Fig.2 show the *pdf* of $\sin^2 \theta_1$ and $\sin^2 \theta_2$ for different channel correlation defined by δ and the total users' number K. In Fig.1 we can see that for *i.i.d* channels, *i.e.*, when $\delta = 1$, θ_1 and θ_2 have the identical distribution as shown in Remark 1, which is coincident with the result for *i.i.d.* channels given by [9]. This means that using SS-ZFBF will not change the orthogonality between the channels in a possible user set. While for spatially correlated channels, *i.e.*, when $\delta > 1$, both θ_1 and θ_2 statistically decrease with the increase of the spatial correlation. In Fig.2, θ_2 is statistically less than θ_1 in spatially correlated channels, and the difference between them increases when K grows. When K is reasonably large, the lower bound given by Remark 2 can be achieved asymptotically. This indicates that SS-ZFBF may destroy the orthogonality between the channels in a possible user set when the channels are spatially correlated. This will reduce the SNR according to (7). Therefore, there will exist a larger performance gap between SS-ZFBF and the exhaustive searching in spatially correlated channels than that in *i.i.d.* channels. In next section, we will propose a low complexity user scheduling, which can achieve a comparable sum rate with that of the exhaustive searching in spatially correlated channels.

IV. Alternating Scheduling

By introducing an alternating searching procedure on the basis of the results of the successive scheduling or SUS, we present an alternating user scheduling which can improve the performance. The alternating searching is a sort of relaxation algorithm for multi-variable optimization problems [10]. Although in some cases AUS may converge to a local maximum, the probability of the occurrence of these cases can be significantly reduced by judiciously choosing the initial scheduling algorithms. For example, we can use the successive scheduling or SUS as the initial algorithm.

AUS is an iterative algorithm. At each step of the iteration, we rechoose a new user based on the other $M - 1$ users already selected. Let $\mathcal{S}_M = \{s_1^0, \cdots, s_M^0\}$ denote the initial scheduling result and R_0 denote the corresponding sum rate. The selection of m-th user at $(i + 1)$-th iteration can be described as

$$s_m^{i+1} = \underset{k \in \mathcal{T} \setminus \mathcal{S}_M \cup \{s_m^i\}}{\arg\max} R\left(\mathcal{S}_M \cup \{k\} \setminus \{s_m^i\}\right), \tag{12}$$

$$\mathcal{S}_M \Leftarrow \mathcal{S}_M \cup \{s_m^{i+1}\} \setminus \{s_m^i\}, m = 1, \ldots, M. \tag{13}$$

When all the M users have been rechosen, we compute the corresponding sum rate. The iteration will stop when

$$R_{i+1} - R_i < \epsilon, \tag{14}$$

where ϵ is a specific threshold.

The scheduling algorithm shown in (12)-(14) is as complicated as the successive scheduling, both involve many matrix

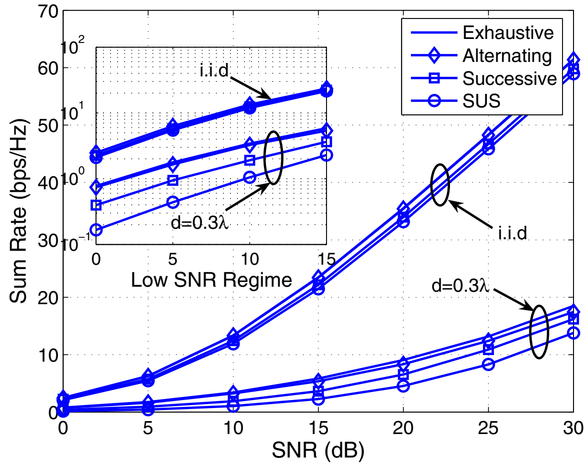

Fig. 3. Sum rate vs SNR for different channel correlation defined by the antenna space d, $K = 10$, $M = 8$, ZFBF. The subfigure depicts the logarithm of sum rate at the low SNR regime.

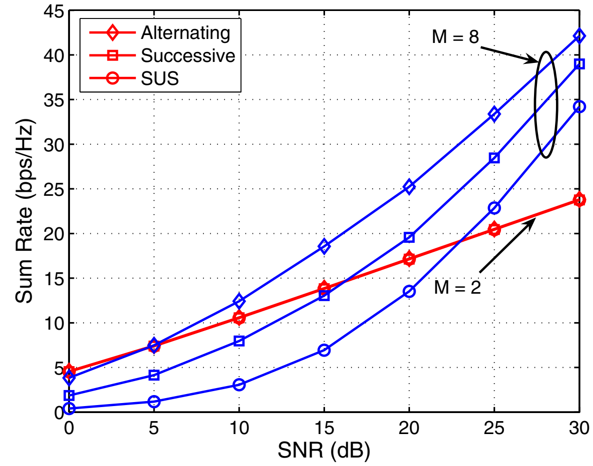

Fig. 4. Sum rate vs SNR for different M, $K = 100$, $d = 0.3\lambda$, ZFBF.

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY

Scheduling Algorithm	**Order of Complex multiplications**
Exha. searching	$O\left(\binom{K}{M}M^3\right)$
Succ. scheduling	$O\left(KM^4\right)$
SUS	$O\left(KM^2\right)$
Succ.-initialized AUS	$O\left((i+1)KM^4\right)$
SUS-initialized AUS	$O\left(iKM^4\right)$

* i is the iteration number in AUS, $i \geq 1$

inversion operations. By using the matrix inversion lemma [10], we will develop a unified low complexity AUS for both ZFBF and R-ZFBF.

In the initialization step using the successive scheduling, the matrix inversion $\mathbf{G}_{m+1,k} = (\mathbf{H}_{\mathcal{S}_m}^H\mathbf{H}_{\mathcal{S}_m} + \mathbf{h}_k^H\mathbf{h}_k + \beta\mathbf{I})^{-1}$ is required to select $(m+1)$-th user in (6). It can be iteratively obtained as

$$\mathbf{G}_{m+1,k} = \mathbf{G}_m - \mathbf{G}_m\mathbf{h}_k^H\mathbf{h}_k\mathbf{G}_m / \left(1 + \mathbf{h}_k^H\mathbf{G}_m\mathbf{h}_k\right), \quad (15)$$

where $\mathbf{G}_m = (\mathbf{H}_{\mathcal{S}_m}^H\mathbf{H}_{\mathcal{S}_m} + \beta\mathbf{I})^{-1}$.

Then, in the alternating searching step, the matrix inversion $\mathbf{G}_{M,k} = (\mathbf{H}_{\mathcal{S}_M}^H\mathbf{H}_{\mathcal{S}_M} - \mathbf{h}_m^H\mathbf{h}_m + \mathbf{h}_k^H\mathbf{h}_k + \beta\mathbf{I})^{-1}$ which is necessary to update m-th user in (12) can be iteratively computed as

$$\mathbf{G}_{M-1} = \mathbf{G}_M + \mathbf{G}_M\mathbf{h}_m^H\mathbf{h}_m\mathbf{G}_M / \left(1 - \mathbf{h}_m^H\mathbf{G}_M\mathbf{h}_m\right), \quad (16)$$

$$\mathbf{G}_{M,k} = \mathbf{G}_{M-1} - \mathbf{G}_{M-1}\mathbf{h}_k^H\mathbf{h}_k\mathbf{G}_{M-1} / \left(1 + \mathbf{h}_k^H\mathbf{G}_{M-1}\mathbf{h}_k\right), \quad (17)$$

where $\mathbf{G}_M = (\mathbf{H}_{\mathcal{S}_M}^H\mathbf{H}_{\mathcal{S}_M} + \beta\mathbf{I})^{-1}$.

The AUS can be summarized as follows,

1) Initialize using the successive scheduling or SUS. For the successive scheduling, use (15) to obtain $\mathcal{S}_M = \{s_1^1, \cdots, s_M^1\}$ in (12).
2) For $m = 1, \ldots, M$, use (12), (13), (16) and (17) to update the results of scheduling.
3) Repeat the iterations until (14) is satisfied.

The computational complexity of five scheduling algorithms, including exhaustive searching, successive scheduling, SUS, AUS initialized by successive scheduling and AUS initialized by SUS, in terms of complex multiplications is compared in Table I. The complexity of AUS can be controlled by the iteration number. More specifically, it depends on the initial scheduling results and the threshold ϵ. Initialization using the successive scheduling or SUS can effectively improve the convergence speed. The threshold reflects the tradeoff between the performance and complexity. In next section we will show that the sum rate is not sensitive to the threshold through simulations. Generally speaking, only one iteration is enough to achieve the most benefit of AUS.

V. SIMULATION RESULTS

A. Sum Rate

In this subsection we'll evaluate the performance of the successive scheduling, SUS and AUS in spatially correlated channels. In the simulation, the virtual channel representation is used, where 50 departure paths and 5° angle spread are considered. We assume that the transmitter is equipped with a uniform linear array. The angles of departure (AOD) are independently and uniformly distributed in $[-\pi, \pi]$, and the directions of departure for each user are independently and uniformly distributed in the angle spread centered at its AOD.

The performance degradation of successive scheduling and SUS caused by channel correlation is shown in Fig.3. While they can approach the optimal performance in *i.i.d.* channels, there exists a large performance gap between the exhaustive searching and the successive scheduling in spatially correlated channels. SUS is inferior to the successive scheduling due to the change of scheduling criteria. The proposed AUS can achieve the optimal performance obtained by the exhaustive searching in both *i.i.d.* and spatially correlated channels. Fig.4 illustrates the sum rate for different transmit antenna numbers.

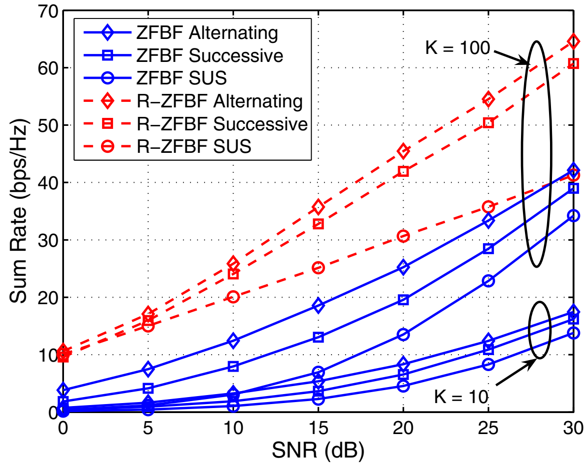

Fig. 5. Sum rate vs SNR for different K and transmit beamforming algorithms, $M = 8$, $d = 0.3\lambda$.

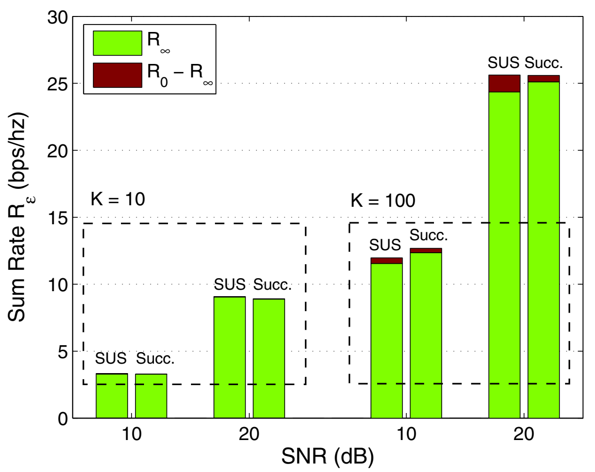

Fig. 6. Sum rate vs K and SNR for different threshold ϵ, $M = 8$, $d = 0.3\lambda$, ZFBF.

It's shown that the influence of channel correlation becomes more severe with the increase of transmit antenna number. We can also find that the sum rate for 8 transmit antennas is lower than that for 2 transmit antennas at low SNR. This is due to the fact that the multiplexing gain is reduced at the low SNR domain and the power allocated to each user decreases with the increase of scheduled users. Fig.5 depicts the sum rate for different user numbers and beamforming algorithms. It's shown that the more the total users are, the larger the sum rate will be, and the more obvious the advantage of AUS will become. The gain of AUS over the successive scheduling for R-ZFBF is less than that for ZFBF. This is because R-ZFBF can mitigate the impact of channel correlation by using a well designed diagonal loading.

B. Tradeoff between performance and complexity of AUS

Iteration number is a critical factor that affects both sum rate and complexity of AUS. By analyzing the sum rate for two extreme cases of iteration number, we can reveal the potential performance gain that can be obtained by increasing the complexity. We consider the AUS for ZFBF initialized by the successive scheduling and SUS, respectively. We assume that the transmitter is equipped with 8 equally spaced antennas, and d is 0.3 wavelength. Fig.6 illustrates the tradeoff between the performance and complexity. R_0 denotes the sum rate when the threshold $\epsilon = 0$, which shows the highest sum rate that AUS can achieve at the expense of the maximal iteration number. R_∞ denotes the sum rate when $\epsilon = \infty$, which is achieved by only one iteration but at a sum rate loss. We can see that the gap of sum rate between $\epsilon = 0$ and $\epsilon = \infty$ is negligible for AUS initialized by the successive scheduling or SUS. SUS-initialized AUS has approximately the same sum rate as that of the AUS initialized by the successive scheduling. Therefore, the SUS-initialized AUS with only one iteration is recommended, which requires the same order of complexity as the successive scheduling as shown in Table I.

VI. Conclusion

In this paper, we analyzed the performance of the successive scheduling in spatially correlated channels. The results show a large performance loss since the successive scheduling may destroy the channel orthogonality. We proposed a low complexity alternating user scheduling algorithm, which can be applied to both ZFBF and R-ZFBF. It can achieve a comparable sum rate with that of the exhaustive searching in both *i.i.d.* and spatially correlated channels with the same order of complexity as the successive scheduling.

Acknowledgment

The authors would like to thank Prof. Eduard Jorswieck and Dr. Xi Zhang for their helpful comments and discussions.

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