Performance Analysis of MRT and Transmit Antenna Selection with Feedback Delay and Channel Estimation Error

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***Abstract*—In this paper, we investigate the performance of four closed-loop multiple-input multiple-output (MIMO) schemes with channel state information (CSI) feedback delay and channel estimation error. These schemes are conventional maximal ratio transmission scheme (C-MRT), improved MRT (I-MRT), conventional transmit antenna selection with space-time block code scheme (C-TAS/STBC) and improved TAS/STBC (I-TAS/STBC). Exact bit error rate (BER) expressions of C-MRT and C-TAS/STBC and approximate BER expressions of I-MRT and I-TAS/STBC for two specific scenarios are derived for binary phase shift keying (BPSK) modulation. The performance of the schemes with CSI feedback delay and channel estimation error are validated and compared with each other by numerical and simulation results.**

I. INTRODUCTION

Spatial transmit diversity is an effective technology to combat fading in wireless channels.

Compared with space-time codes, the closed-loop MIMO schemes employing MRT [1] or TAS/STBC [2] can achieve not only full diversity but also more array gain, when perfect knowledge of CSI in both transmitter and receiver (CSIT and CSIR) is available. In practical FDD and TDD system, however, accurate CSI is hard to get especially in time-varying channels. In this study, we model the factors resulting in imperfect CSIT and CSIR as CSI feedback delay and channel estimation error. CSI feedback delay leads to an outdated CSIT, and channel estimation error introduces equivalent additive noise to perfect CSIT and CSIR. A receiver can exploit either real-time or outdated CSIR with channel estimation error depending on whether channel tracking or prediction [3, 4] is used or not. In this paper, the schemes using outdated CSIR are referred to conventional schemes, including C-MRT and C-TAS/STBC. The schemes using real-time CSIR are named as improved schemes, including I-MRT and I-TAS/STBC.

From both theoretical and practical viewpoint, it is necessary to quantify the impact of CSI feedback delay and channel estimation error on the performance of the four schemes. The BER of MISO C-MRT in the presence of CSI feedback delay and channel estimation error is investigated in [5, 6]. Since the decision variable of MIMO MRT is different from that of MISO MRT, the performance analysis of MISO MRT can not be directly extended to MIMO MRT. The performance of MIMO MRT only with channel estimation error is investigated in [7].

By contrast to MRT, transmit antenna selection (TAS) [8] can reduce the overhead of CSI feedback. On the premise of achieving full diversity order, combining TAS and STBC (TAS/STBC) can further reduce the feedback overhead [2, 9]. There have been, however, few studies on the performance analysis of TAS/STBC. The performance analysis in [2] does not provide an explicit expression. In [10], the performance of TAS/STBC under the assumption of perfect CSIT and CSIR is evaluated. In [11], merely channel estimation error is considered for the performance of TAS/STBC, and explicit BER expression is derived only for MISO system.

In this study, a decision variable different from [7] is employed to analyze the performance of MRT and TAS/STBC in MIMO system with both CSI feedback delay and channel estimation error. The contribution lies in both theoretical analysis and performance comparison, where exact BER expressions of C-MRT and C-TAS/STBC and approximate BER expressions of I-MRT and I-TAS/STBC for two specific scenarios are derived for BPSK modulation.

This paper is organized as follows. Section Ⅱ introduces the system model, and the channel model with CSI feedback delay and channel estimation error. Section Ⅲ presents the performance analysis of C-MRT, I-MRT, C-TAS/STBC and I-TAS/STBC, followed by the numerical analysis, simulation and discussion in Section Ⅳ. Finally, the conclusion remarks are made in Section Ⅴ.

The following notations are used in this paper. $\mathbf{X}^{\mathrm{T}}$ and $\mathbf{X}^{\mathrm{H}}$ express the transpose and conjugate transpose of matrix $\mathbf{X}$, respectively. $\|\mathbf{x}\|_2$ stands for the 2-norm of vector $\mathbf{x}$, and $\|\mathbf{X}\|_{\mathrm{F}}$ represents the Frobenius norm of matrix $\mathbf{X}$. $J_0(\bullet)$ denotes the zeroth-order Bessel function of the first kind. $\mathbf{X}_{i,j}$ expresses the *(i,j)*-th entry of matrix $\mathbf{X}$. $\mathbf{X} \sim CN(\mathbf{m}, \mathbf{R})$ and $\mathbf{X} \sim N(\mathbf{m}, \mathbf{R})$ stand for complex and real-valued Gaussian random matrix $\mathbf{X}$ with mean $\mathbf{m}$ and covariance matrix $\mathbf{R}$. $\mathrm{E}_x[\bullet]$ denotes the expectation of random variable x.

II. SYSTEM MODEL AND CHANNEL MODEL

Consider a closed-loop MIMO system with M transmit and N receive antennas, which transmits a single data stream. Then the estimated signal can be expressed as

$$\hat{x} = \mathbf{z}^{\mathrm{H}}(\mathbf{H}\mathbf{w}x + \mathbf{n}) = \mathbf{z}^{\mathrm{H}}\mathbf{H}\mathbf{w}x + \tilde{n}, \qquad (1)$$

where x is the transmitted signal with energy E_s, $\mathbf{w}$ and $\mathbf{z}$ are the transmit and receive weighting vector, respectively, $\tilde{n}$ is the additive white Gaussian noise (AWGN) with zero-mean and variance $\sigma_{\tilde{n}}^2$ and $\sigma_{\tilde{n}}^2 = 2\|\mathbf{z}\|_2^2 \cdot \sigma_n^2$, $\mathbf{n} \sim CN(\mathbf{0}, 2\sigma_n^2\mathbf{I})$ is the AWGN vector, $\mathbf{H} \sim CN(\mathbf{0}, \sigma_h^2\mathbf{I})$ is the channel matrix of size $\mathrm{N}\times\mathrm{M}$.

Consider a time-varying independently and identically distributed (i.i.d.) Rayleigh fading channel, which can be modeled as vector AR model. The relationship of the channel at time t and $t-\tau$ is given by [5]

$$\mathbf{H}_t = \rho_d \cdot \mathbf{H}_{t-\tau} + \mathbf{F}\,, \tag{2}$$

where $\rho_d = J_0(2\pi f_D \tau)$ is the time correlation coefficient, f_D is the Doppler frequency shift, τ is the time delay owing to CSI feedback, $\mathbf{F} \sim CN(\mathbf{0}, \sigma_f^2\mathbf{I})$ is a driving noise matrix, and $\sigma_f^2 = (1-\rho_d^2)\sigma_h^2$.

In order to detect the received signal coherently, receiver needs to estimate the channel $\mathbf{H}_t$. The channel estimation $\hat{\mathbf{H}}_t$ can be expressed as

$$\hat{\mathbf{H}}_t = \mathbf{H}_t + \mathbf{E}\,, \tag{3}$$

where $\mathbf{E} \sim CN(\mathbf{0}, \sigma_e^2\mathbf{I})$ is the channel estimation error matrix, which is independent of $\mathbf{H}_t$, and $\hat{\mathbf{H}}_t \sim CN(\mathbf{0}, \sigma_{\hat{h}}^2\mathbf{I})$ with $\sigma_{\hat{h}}^2 = \sigma_h^2 + \sigma_e^2$. Channel response $\mathbf{H}_t$ can be represented by [7]

$$\mathbf{H}_t = \rho_e \cdot \hat{\mathbf{H}}_t + \mathbf{D}\,, \tag{4}$$

where $\rho_e = \sigma_h^2 / \sigma_{\hat{h}}^2$, $\mathbf{D} \sim CN(\mathbf{0}, \sigma_d^2\mathbf{I})$ is a random matrix , which is independent of $\hat{\mathbf{H}}_t$, and $\sigma_d^2 = \sigma_h^2\sigma_e^2/\sigma_{\hat{h}}^2$.

The CSI at time $t-\tau$ is required to demodulate the received signal at time t in the C-MRT and C-TAS/STBC receivers. Thus it is necessary to find the relationship between the channel estimation at time $t-\tau$ $\hat{\mathbf{H}}_{t-\tau}$ and the actual channel at time t $\mathbf{H}_t$. It follows from (2) and (4) that $\mathbf{H}_t$ can be rewritten as

$$\mathbf{H}_t = \rho_d \cdot \left(\rho_e \hat{\mathbf{H}}_{t-\tau} + \mathbf{D}\right) + \mathbf{F} \triangleq \rho \hat{\mathbf{H}}_{t-\tau} + \mathbf{B}\,, \tag{5}$$

where $\mathbf{B} \sim CN(\mathbf{0}, \sigma_b^2\mathbf{I})$ is the random matrix, which is independent of $\hat{\mathbf{H}}_{t-\tau}$, and $\sigma_b^2 = \rho_d^2\sigma_h^2\sigma_e^2/\sigma_{\hat{h}}^2 + (1-\rho_d^2)\sigma_h^2$.

III. BER ANALYSIS OF MRT AND TAS/STBC SCHEMES

Based on the system model and channel model described in Section Ⅱ, this section analyzes the performance of the four schemes with CSI feedback delay and channel estimation error for BPSK modulation. The decision variable ζ in (1) is the real part of $\hat{x}$, i.e., $\zeta = \mathrm{Re}(\mathbf{z}^{\mathrm{H}}\mathbf{H}\mathbf{w}x) + \mathrm{Re}(\tilde{\mathbf{n}})$. Hence, the instantaneous BER is

$$P_b = Q\left(\frac{\mathrm{Re}(\mathbf{z}^{\mathrm{H}}\mathbf{H}\mathbf{w})}{\|\mathbf{z}\|_2}\sqrt{\frac{E_s}{\sigma_n^2}}\right). \tag{6}$$

A. Average BER of C-MRT

In C-MRT transceiver, $\hat{\mathbf{H}}_{t-\tau}$ is employed to perform both transmit beamforming and maximal ratio combining. The singular value decomposition (SVD) of $\hat{\mathbf{H}}_{t-\tau}$ can be written as

$$\hat{\mathbf{H}}_{t-\tau} = \mathbf{U}_{\mathrm{N}\times\mathrm{N}}\mathbf{\Lambda}_{\mathrm{N}\times\mathrm{M}}\mathbf{V}_{\mathrm{M}\times\mathrm{M}}^{\mathrm{H}}\,, \tag{7}$$

where $\mathbf{U}_{\mathrm{N}\times\mathrm{N}} = [\mathbf{u}_1\ \mathbf{u}_2 \cdots \mathbf{u}_{\mathrm{N}}]$, $\mathbf{\Lambda}_{\mathrm{N}\times\mathrm{M}} = diag\{\delta_1, \delta_2, \cdots, \delta_r, 0, \cdots\}$, $\mathbf{V}_{\mathrm{M}\times\mathrm{M}} = [\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_{\mathrm{M}}]$, and r is the rank of $\hat{\mathbf{H}}_{t-\tau}$. If the maximal singular value $\delta_{\max} = \delta_1$, the transmit weighting vector $\mathbf{w} = \mathbf{v}_1$, and the receive weighting vector $\mathbf{z} = \mathbf{u}_1$. Based on (6) the instantaneous BER of C-MRT is

$$P_{C-MRT,b} = Q\left(\mathrm{Re}\left(\mathbf{u}_1^{\mathrm{H}}\mathbf{H}_t\mathbf{v}_1\right)\cdot\gamma_0\right). \tag{8}$$

Substituting (5) into (8), $P_{C-MRT,b}$ can be rewritten as

$$P_{C-MRT,b} = Q\left((\rho\delta_{\max} + \alpha)\cdot\gamma_0\right) \triangleq Q(\gamma_1)\,, \tag{9}$$

where $\gamma_0 = \sqrt{E_s/\sigma_n^2}$, $\alpha = \mathrm{Re}\left(\mathbf{u}_1^{\mathrm{H}}\mathbf{B}\mathbf{v}_1\right)$, and $\alpha \sim N(0, \sigma_b^2/2)$.

The average BER of C-MRT can be obtained by

$$\begin{aligned}\bar{P}_{C-MRT,b} &= \mathrm{E}\left[P_{C-MRT,b}\right] \\ &= \int_0^{+\infty}\int_0^{+\infty} Q(\gamma)\cdot f_{\gamma_1}(\delta,\alpha)\,d\alpha\, d\delta \\ &= \int_0^{+\infty} f_{\delta_{\max}}(\delta)\,d\delta \int_0^{+\infty} Q(\gamma)\cdot f_{\gamma_1|\delta_{\max}}(\gamma|\delta)\,d\gamma \\ &= \mathrm{E}_{\delta_{\max}}\left[\mathrm{E}_{\gamma_1|\delta_{\max}}[Q(\gamma)]\right].\end{aligned} \tag{10}$$

When the probability density function (p.d.f.) of $\delta_{\max}$ and the conditional probability density function (c.p.d.f.) of γ_1 given $\delta_{\max}$ are available, the average BER can be obtained from (10).

With the p.d.f. of the maximal eigenvalue of random matrix $\mathbf{H}_{t-\tau}$ provided in [12], the p.d.f. of the maximal eigenvalue $\lambda_{\max}$ of $\hat{\mathbf{H}}_{t-\tau}$ can be given by

$$f_{\lambda_{\hat{\mathbf{H}},\max}}(u) = \sum_{i=1}^{\mathrm{L}}\sum_{m=\mathrm{S-L}}^{(\mathrm{S+L})i-2i^2} d_{i,m}\frac{i^{m+1}u^m e^{-iu^2/\sigma_{\hat{h}}^2}}{\sigma_{\hat{h}}^{2(m+1)}\cdot m!}\,, \tag{11}$$

where $d_{i,m} = m!c_{i,m}\left(i^{m+1}\prod_{i=1}^{\mathrm{L}}(\mathrm{L}-i)!(\mathrm{N}-i)!\right)^{-1}$, $c_{i,m}$ is the coefficient of $u^m e^{-iu}$ in the expansion of $\frac{d}{du}\det(\mathcal{S}(u))$, $\mathcal{S}(u)$ is the $\mathrm{L}\times\mathrm{L}$ *Hankel* matrix, $\mathrm{L} = \min(\mathrm{M},\mathrm{N})$, $\mathrm{S} = \max(\mathrm{M},\mathrm{N})$.

It is shown from (11) that the p.d.f. of $\lambda_{\max}$ is the linear combination of a finite χ^2 distributed p.d.f.. The p.d.f. of $\delta_{\max} = \sqrt{\lambda_{\max}}$ is

$$f_{\delta_{\max}}(u) = \sum_{i=1}^{\mathrm{L}}\sum_{m=\mathrm{S-L}}^{(\mathrm{S+L})i-2i^2} d_{i,m}\frac{2\cdot i^{m+1}u^{2m+1}e^{-iu^2/\sigma_{\hat{h}}^2}}{\sigma_{\hat{h}}^{2(m+1)}\cdot m!}. \tag{12}$$

Given $\delta_{\max}$, γ_1 is a real-valued Gaussian random variable, which c.p.d.f. is

$$f_{\gamma_1|\delta_{\max}}(u) = \frac{1}{\sqrt{2\pi}\gamma_0\sigma_\alpha}e^{-\frac{(u-\rho\delta_{\max}\gamma_0)^2}{2\gamma_0^2\sigma_\alpha^2}}. \tag{13}$$

Substituting (12) and (13) into (10), the average BER of C-MRT $\bar{P}_{C-MRT,b}$ can be derived. Due to the lack of space, $\bar{P}_{C-MRT,b}$ is directly given by

$$\bar{P}_{C-MRT,b} = \sum_{i=1}^{\mathrm{L}}\sum_{m=\mathrm{S-L}}^{(\mathrm{S+L})i-2i^2}\frac{1}{2}d_{i,m}\mathcal{G}(m,b)\,, \tag{14}$$

where $\mathcal{G}(m,b) = 1 - \frac{1}{2}\sum_{j=0}^{m}\sum_{k=0}^{j}\left(-\frac{1}{2}\right)^k\cdot\binom{j}{k}\cdot\frac{\mathcal{L}_k\left(\frac{a}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{(k+1)/2}}$, $\mathcal{L}_k(x)$ is the k-th order *Legendre* polynomial, $b = \frac{(\gamma_0^2\sigma_b^2+2)\cdot i}{4\gamma_0^2\rho^2\sigma_{\hat{h}}^2}$, $a = b + 0.5$. [5, (26)] is applied during the derivation of (14).

B. Average BER of I-MRT

In I-MRT, the receive weighting vector $\mathbf{z}=\hat{\mathbf{H}}_t\mathbf{v}_1$, where $\hat{\mathbf{H}}_t$ is the channel estimation at time t. Applying (3), (4), (5) and (6), the instantaneous BER of I-MRT can be expressed as

$$\begin{aligned}P_{I-MRT,b}&=Q\left(\left(\rho_e\cdot\|\rho\mathbf{u}_1\delta_{\max}+\mathbf{A}\|_2+\alpha\right)\cdot\gamma_0\right)\\&\triangleq Q\left(\left(\rho_e\cdot\Omega+\alpha\right)\cdot\gamma_0\right)\triangleq Q(\gamma_2)\end{aligned}. \quad (15)$$

In (15), $\mathbf{A}\sim CN(\mathbf{0},\sigma_{\mathbf{A}}^2\mathbf{I})$, $\alpha\sim N(0,\sigma_b^2/2)$, where $\mathbf{A}=(\mathbf{B}+\mathbf{E})\,\mathbf{v}_1$, $\sigma_{\mathbf{A}}^2=\rho_d^2\sigma_h^2\sigma_e^2/\sigma_{\hat{h}}^2+(1-\rho_d^2)\sigma_h^2+\sigma_e^2$, $\alpha=\mathrm{Re}\left(\mathbf{v}_1^{\mathrm{H}}\hat{\mathbf{H}}_t^{\mathrm{H}}\mathbf{D}\mathbf{v}_1\right)/\|\hat{\mathbf{H}}_t\mathbf{v}_1\|_2$.

In the same way as deriving BER for C-MRT, the average BER of I-MRT can be obtained with knowledge of the p.d.f. of Ω and the c.p.d.f. of γ_2 given Ω. Unfortunately, it is very difficult to derive the p.d.f. of Ω. Here two specific scenarios are considered as follows.

When $\rho_d\to 1$ and for high SNR, which means little CSI feedback delay and less channel estimation error, Ω can be approximated as

$$\Omega\approx\rho\delta_{\max}. \quad (16)$$

Based on (10), (12) and (16), the average BER of I-MRT $\overline{P}_{I-MRT,b}$ can be still expressed by (14) with $b=\dfrac{(\gamma_0^2\sigma_d^2+2)\cdot i}{4\gamma_0^2\rho_e^2\rho^2\sigma_{\hat{h}}^2}$.

If $\rho_d\to 0$, which means large CSI feedback delay, Ω can be approximated as

$$\Omega\approx\|\mathbf{A}\|_2. \quad (17)$$

Therefore, the average BER of I-MRT in this case is

$$\overline{P}_{I-MRT,b}=\frac{1}{2}\mathcal{G}\left(\mathrm{N}-1,\frac{\gamma_0^2\sigma_d^2+2}{4\gamma_0^2\rho_e^2\sigma_{\mathbf{A}}^2}\right). \quad (18)$$

C. Average BER of C-TAS/STBC

In order to analyze the performance of C-TAS/STBC and I-TAS/STBC, we'll derive the decision variable of TAS/STBC with perfect CSIT and CSIR at first.

If transmitter selects i-th and j-th transmit antennas, the channels between the two selected transmit antennas and N receive antennas can be expressed as

$$\breve{\mathbf{H}}=\begin{bmatrix}\mathbf{h}_i & \mathbf{h}_j\end{bmatrix}, \quad (19)$$

where $\mathbf{h}_i$ and $\mathbf{h}_j$ are i-th and j-th column vectors of $\mathbf{H}$, respectively.

After selecting two transmit antennas, the considered MIMO system becomes STBC system with two transmit and N receive antennas. The received signal is

$$\mathbf{y}=\breve{\mathbf{G}}\cdot\mathbf{x}+\mathbf{n}, \quad (20)$$

where $\mathbf{y}=\begin{bmatrix}y_{1,1} & y_{1,2}^* & \cdots & y_{\mathrm{N},1} & y_{\mathrm{N},2}^*\end{bmatrix}^{\mathrm{T}}$, $\mathbf{x}=\begin{bmatrix}x_1 & x_2^*\end{bmatrix}^{\mathrm{T}}$, $y_{m,k}$ and x_k denote the received signal on the m-th receive antenna and the transmitted signal in the k-th signal duration, respectively. $\breve{\mathbf{G}}=\begin{bmatrix}h_{1,i} & h_{1,j}^* & \cdots & h_{\mathrm{N},i} & h_{\mathrm{N},j}^*\\ -h_{1,j} & h_{1,i}^* & \cdots & -h_{\mathrm{N},j} & h_{\mathrm{N},i}^*\end{bmatrix}^{\mathrm{T}}$, $\mathbf{n}\sim CN(\mathbf{0},2\sigma_n^2\mathbf{I})$ is AWGN vector.

In receiver $\breve{\mathbf{G}}^{\mathrm{H}}$ is used as the weighting matrix, and the estimation of transmitted signal is

$$\hat{\mathbf{x}}=\breve{\mathbf{G}}^{\mathrm{H}}\mathbf{y}=\begin{bmatrix}\|\breve{\mathbf{H}}\|_{\mathrm{F}}^2 & 0\\ 0 & \|\breve{\mathbf{H}}\|_{\mathrm{F}}^2\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2^*\end{bmatrix}+\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}, \quad (21)$$

where η_1 and η_2 are independent complex Gaussian random variables with zero-mean and covariance $2\|\breve{\mathbf{H}}\|_{\mathrm{F}}^2\sigma_n^2$.

According to (21), the instantaneous BER of TAS/STBC can be expressed as

$$P_{TAS/STBC,Ideal,b}=Q\left(\|\breve{\mathbf{H}}\|_{\mathrm{F}}\cdot\gamma_0\right). \quad (22)$$

Under the assumption that the fixed total power is equally allocated to the two transmit antennas, we have $\gamma_0=\sqrt{E_s/2\sigma_n^2}$.

It can be confirmed from (22) that in order to minimize the average BER, the system should select the two transmit antennas such that the Frobenius norm of $\breve{\mathbf{H}}$ is maximal [2].

In C-TAS/STBC, the receiver selects two transmit antennas based on the channel estimation $\hat{\mathbf{H}}_{t-\tau}$ at time $t-\tau$, and feedbacks the result to transmitter. Utilizing the channel model defined in Section Ⅱ, the estimated signal of C-TAS/STBC is

$$\hat{\mathbf{x}}=\left(\rho\cdot\begin{bmatrix}\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2 & 0\\ 0 & \left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2\end{bmatrix}+\hat{\breve{\mathbf{G}}}_{t-\tau}^{\mathrm{H}}\cdot\mathbf{B}\right)\cdot\begin{bmatrix}x_1\\ x_2^*\end{bmatrix}+\hat{\breve{\mathbf{G}}}_{t-\tau}^{\mathrm{H}}\cdot\mathbf{n}. \quad (23)$$

Because x_1 and x_2 have the same BER, without loss of generality, only x_1 is considered here. From (23) the estimation of x_1 is

$$\hat{x}_1=\rho\cdot\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2x_1+\alpha_1x_1+\alpha_2x_2^*+\eta_1, \quad (24)$$

where $\eta_1=\left[\hat{\breve{\mathbf{G}}}_{t-\tau}^{\mathrm{H}}\cdot\mathbf{n}\right]_{1,1}$, $\alpha_1=\left[\hat{\breve{\mathbf{G}}}_{t-\tau}^{\mathrm{H}}\cdot\mathbf{B}\right]_{1,1}$, $\alpha_2=\left[\hat{\breve{\mathbf{G}}}_{t-\tau}^{\mathrm{H}}\cdot\mathbf{B}\right]_{1,2}$, and $\eta_1\sim CN\left(0,2\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2\sigma_n^2\right)$, $\alpha_1,\alpha_2\sim CN\left(0,\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2\sigma_b^2\right)$.

For BPSK modulation, since x_2 can be identical to x_1 or $-x_1$ with equal probability, the instantaneous BER of C-TAS/STBC $P_{C-T/S,b}$ can be expressed as

$$P_{C-T/S,b}=\frac{1}{2}\left[P_{C-T/S,b}(x_2=x_1)+P_{C-T/S,b}(x_2=-x_1)\right]. \quad (25)$$

In the case of $x_2=x_1$, the instantaneous BER is

$$P_{C-T/S,b}(x_2=x_1)=Q\left(\left(\rho\cdot\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}+\alpha\right)\gamma_0\right)\triangleq Q(\gamma_3). \quad (26)$$

where $\alpha=\mathrm{Re}(\alpha_1+\alpha_2)/\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}$, and $\alpha\sim N(0,\sigma_b^2)$.

Similarly, in the case of $x_2=-x_1$, the instantaneous BER is the same as (26). Hence, $P_{C-T/S,b}$ can still be expressed by (26).

Like C-MRT, in order to obtain the average BER of C-TAS/STBC the p.d.f. of $\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}$ and the c.p.d.f. of γ_3 given $\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}$ are required. Due to space limitations, the p.d.f. of

$\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^{2}$ is directly given by (27), where $c_{i,j}$ is the coefficient of u^j in the expansion of $\left(\sum_{k=0}^{\mathrm{N}-1} u^k / \left(\sigma_{\hat{h}}^{2k}\cdot k!\right)\right)^i$. Substituting (27) into (10), the average BER of C-TAS/STBC $\bar{P}_{C-T/S,b}$ can be expressed by (28), where $b_1 = \dfrac{\gamma_0^2\sigma_b^2+1}{2\gamma_0^2\rho^2\sigma_{\hat{h}}^2}$, $b_2 = \dfrac{\left(\gamma_0^2\sigma_b^2+1\right)\cdot(i+2)}{4\gamma_0^2\rho^2\sigma_{\hat{h}}^2}$.

It's worth to note that although the expression of (28) is complicated, it only consists of finite and simple arithmetic operations which can be used to evaluate BER efficiently.

D. Average BER of I-TAS/STBC

In I-TAS/STBC, the receiver utilizes the channel estimation $\hat{\breve{\mathbf{H}}}_t$ to demodulate the transmitted signal. Based on the channel model in Section Ⅱ, the estimated signal can be expressed as

$$\hat{\mathbf{x}} = \left(\rho_e\cdot\begin{bmatrix}\left\|\hat{\breve{\mathbf{H}}}_t\right\|_{\mathrm{F}}^2 & 0\\ 0 & \left\|\hat{\breve{\mathbf{H}}}_t\right\|_{\mathrm{F}}^2\end{bmatrix} + \hat{\breve{\mathbf{G}}}_t^{\mathrm{H}}\cdot\mathbf{D}\right)\cdot\begin{bmatrix}x_1\\ x_2^*\end{bmatrix} + \hat{\breve{\mathbf{G}}}_t^{\mathrm{H}}\cdot\mathbf{n}. \quad (29)$$

The instantaneous BER of I-TAS/STBC can be written as

$$\begin{aligned} P_{I-T/S,b} &= Q\left(\left(\rho_e\cdot\left\|\rho\cdot\breve{\mathbf{H}}_{t-\tau}+\breve{\mathbf{A}}\right\|_{\mathrm{F}}+\alpha\right)\gamma_0\right)\\ &\triangleq Q\left(\left(\rho_e\cdot\breve{\Omega}+\alpha\right)\gamma_0\right)\triangleq Q(\gamma_4)\end{aligned}, \quad (30)$$

where $\breve{\mathbf{A}} \sim CN\left(\mathbf{0},\sigma_{\breve{\mathbf{A}}}^2\mathbf{I}\right)$, $\sigma_{\breve{\mathbf{A}}}^2 = \rho_d^2\sigma_{\hat{h}}^2\sigma_e^2/\sigma_{\hat{h}}^2 + \left(1-\rho_d^2\right)\sigma_h^2 + \sigma_e^2$, $\alpha \sim N\left(0,\sigma_d^2\right)$, $\alpha = \mathrm{Re}(\alpha_1+\alpha_2)\Big/\left\|\hat{\breve{\mathbf{H}}}_t\right\|_{\mathrm{F}}$, and α_1 and α_2 are independent complex Gaussian random variables with zero-mean and covariance $\left\|\hat{\breve{\mathbf{H}}}_t\right\|_{\mathrm{F}}^2\sigma_d^2$.

The same specific scenarios in Subsection B are considered here. When $\rho_d\to 1$ and for high SNR, $\breve{\Omega}$ can be approximated as

$$\breve{\Omega} \approx \rho\cdot\left\|\breve{\mathbf{H}}_{t-\tau}\right\|_{\mathrm{F}}. \quad (31)$$

As a result, the average BER of I-TAS/STBC $\bar{P}_{I-T/S,b}$ can still be written as (28) with $b_1 = \dfrac{\gamma_0^2\sigma_d^2+1}{2\gamma_0^2\rho_e^2\rho^2\sigma_{\hat{h}}^2}$, $b_2 = \dfrac{\left(\gamma_0^2\sigma_d^2+1\right)\cdot(i+2)}{4\gamma_0^2\rho_e^2\rho^2\sigma_{\hat{h}}^2}$.

By contrast, if $\rho_d \to 0$, $\breve{\Omega}$ can be approximated as

$$\breve{\Omega} \approx \left\|\breve{\mathbf{A}}\right\|_{\mathrm{F}}. \quad (32)$$

Therefore, the average BER of I-TAS/STBC in this case is

$$\bar{P}_{I-MRT,b} = \frac{1}{2}\mathcal{G}\left(2\mathrm{N}-1, \frac{\gamma_0^2\sigma_d^2+1}{2\gamma_0^2\rho_e^2\sigma_{\mathbf{A}}^2}\right). \quad (33)$$

IV. Numerical Analysis

Numerical results and Monte-Carlo simulations are provided to analyze the performance of C-MRT, I-MRT, C-TAS/STBC and I-TAS/STBC in Figs. 1-2. The considered MIMO system equips with 4 transmit and 2 receive antennas. Least-Square channel estimation based on training sequence is applied in simulations. The normalized Doppler frequency $f_d\tau$ is used to scale the CSI feedback delay, which is chosen as 0.03 and 1. Thus, the corresponding time correlation coefficients ρ_d are 0.9911 and 0.2203, respectively.

Fig. 1 illustrates the numerical and simulation results of the average BER of the four schemes with little CSI feedback delay and channel estimation error, in which the time correlation coefficient $\rho_d = 0.9911$. It's shown that for C-MRT and C-TAS/STBC the numerical results match exactly with the simulation results, and for I-MRT and I-TAS/STBC the numerical results can match the simulation results very well under the condition of little feedback and high SNR. It's also shown that channel estimation error will not lead to error floor, and the performance degradation caused by CSI feedback delay is trivial since $\rho_d \to 1$.

Fig. 2 depicts the performance degradation due to the large CSI feedback delay and channel estimation error, where the time correlation coefficient $\rho_d = 0.2203$. There are obvious error floors for C-MRT and C-TAS/STBC, whereas I-MRT and I-TAS/STBC can eliminate error floors effectively by exploiting real-time CSIR. When $\rho_d \to 1$, however, the improved schemes only have slight advantage over conventional schemes for TAS/STBC, and even have worse performance for MRT (see Fig. 1). It can be inferred that if the perfect CSIT and CSIR are known, conventional schemes and

$$\begin{aligned} f_{\left\|\hat{\breve{\mathbf{H}}}_{t-\tau}\right\|_{\mathrm{F}}^2}(z) = {} & \frac{\mathrm{M}(\mathrm{M}-1)}{\sigma_{\hat{h}}^{4\mathrm{N}}\cdot[(\mathrm{N}-1)!]^2}\cdot\sum_{i=1}^{\mathrm{M}-2}\sum_{j=0}^{(\mathrm{N}-1)\cdot i}\sum_{k=0}^{\mathrm{N}+j-1}\sum_{m=0}^{\mathrm{N}+k-1}\left[(-1)^{i+k+m}\cdot\binom{\mathrm{M}-2}{i}\cdot\binom{\mathrm{N}+j-1}{k}\cdot\frac{(\mathrm{N}+k-1)!}{(\mathrm{N}+k-1-m)!}\cdot c_{i,j}\cdot\left(\frac{\sigma_{\hat{h}}^2}{i}\right)^{m+1}\right.\\ & \left.\cdot z^{2\mathrm{N}+j-m-2}\cdot\left(e^{-\frac{z}{\sigma_{\hat{h}}^2}} - \frac{e^{-\frac{(i+2)z}{2\sigma_{\hat{h}}^2}}}{2^{\mathrm{N}+k-1-m}}\right)\right] + \frac{\mathrm{M}(\mathrm{M}-1)}{\sigma_{\hat{h}}^{4\mathrm{N}}\cdot 2\cdot(2\mathrm{N}-1)!}\cdot z^{2\mathrm{N}-1}\cdot e^{-\frac{z}{\sigma_{\hat{h}}^2}} \end{aligned} \quad (27)$$

$$\begin{aligned} \bar{P}_{C-T/S,b} = {} & \sum_{i=1}^{\mathrm{M}-2}\sum_{j=0}^{(\mathrm{N}-1)\cdot i}\sum_{k=0}^{\mathrm{N}+j-1}\sum_{m=0}^{\mathrm{N}+k-1}\left\{(-1)^{i+k+m}\cdot\binom{\mathrm{M}-2}{i}\cdot\binom{\mathrm{N}+j-1}{k}\cdot\frac{\mathrm{M}(\mathrm{M}-1)\cdot c_{i,j}\cdot\sigma_{\hat{h}}^{2j}}{i^{m+1}}\cdot\frac{(\mathrm{N}+k-1)!\cdot(2\mathrm{N}+j-m-2)!}{[(\mathrm{N}-1)!]^2\cdot(\mathrm{N}+k-1-m)!}\cdot\right.\\ & \left.[\mathcal{G}(2\mathrm{N}+j-m-2,b_1) - \frac{2^{\mathrm{N}+j-k}}{(i+2)^{2\mathrm{N}+j-m-1}}\cdot\mathcal{G}(2\mathrm{N}+j-m-2,b_2)]\right\} + \frac{\mathrm{M}(\mathrm{M}-1)}{4}\cdot\mathcal{G}(2\mathrm{N}-1,b_1) \end{aligned} \quad (28)$$

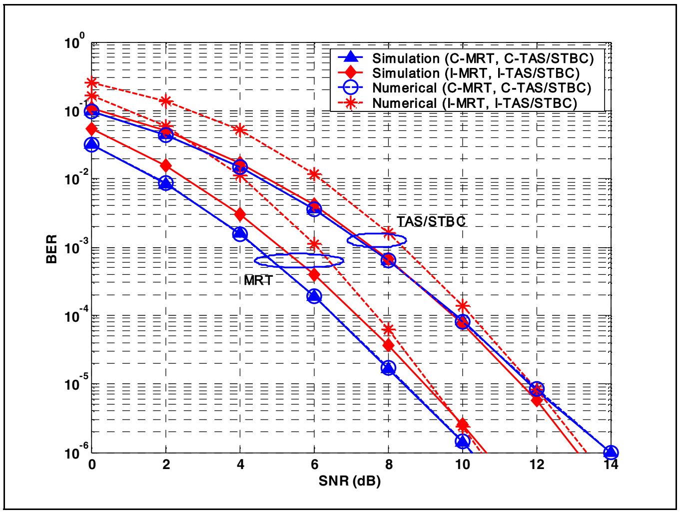

Fig. 1 BER with CSI feedback delay and channel estimation error ($\rho_d = 0.9911$)

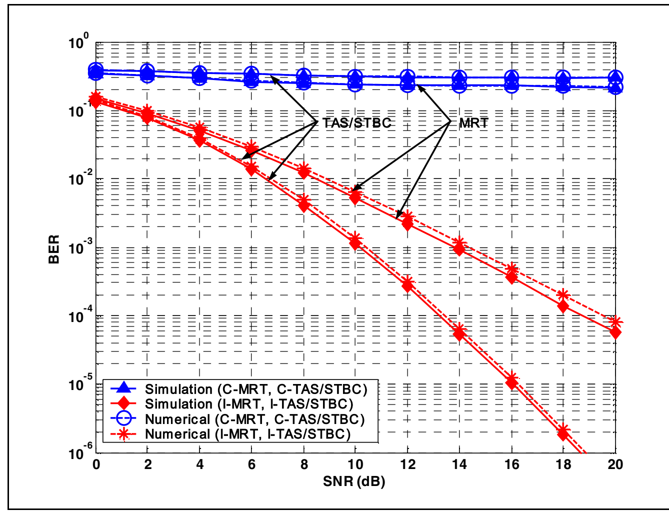

Fig. 2 BER with CSI feedback delay and channel estimation error ($\rho_d = 0.2203$)

improved schemes will achieve same performance. Otherwise, conventional schemes will outperform the improved schemes when only channel estimation error is in presence, yet for large CSI feedback delay the improved schemes performs better.

It's shown from Fig.1 that I-MRT outperforms I-TAS/STBC. Nevertheless, it's shown in Fig. 2 that I-TAS/STBC has superior performance to I-MRT. It can be forecasted that for little CSI feedback delay I-MRT will have better performance, but when CSI feedback delay increases to some extent, I-TAS/STBC will outperform I-MRT. This is due to the fact that when CSI feedback delay approaches to zero, I-MRT can obtain more array gain than I-TAS/STBC, which leads to better performance. In another extreme case, when CSI feedback delay increases to infinity, I-MRT in (M, N) system (i.e. a MIMO system with M transmit and N receive antennas) is equivalent to MRC in (1, N) system, whereas I-TAS/STBC in (M, N) system is equivalent to STBC in (2, N) system. Obviously, the latter can achieve larger diversity order, thus has better performance.

V. CONCLUSIONS

In this paper, we investigate the performance of four schemes in MIMO systems with CSI feedback delay and channel estimation error. For C-MRT and C-TAS/STBC exact BER expressions are derived, and for I-MRT and I-TAS/STBC approximate BER expressions are provided for two specific scenarios. The performance of the considered schemes are validated and compared with each other by numerical analysis and simulations in various scenarios. The results show that for little CSI feedback delay the performance of conventional schemes are superior to the improved schemes, yet for large CSI feedback delay the improved schemes perform better, and when CSI feedback delay increases to a certain extent, I-TAS/STBC outperforms I-MRT.

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