Interference-Alignment-Embedded Iterative Beamforming Design for Multi-cell MIMO Broadcasting

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***Abstract*—In this paper, we consider downlink multi-cell multi-user multi-antenna systems, where the users experience interference from adjacent cells as well as other users in the same cell. We design an iterative beamforming algorithm towards sum-rate maximization. Specifically, we propose an interference-alignment-embedded structure in the iteration, where a two-layer precoder approximately aligns the intra-cell interference with the inter-cell interference, and a max-SINR detector maximizes the achievable data rate. Simulation results show that the algorithm converges rapidly and achieves superior performance in noise-limited environments.**

Keywords – beamforming; interference alignment; multiuser MIMO; sum-rate maximization; transceiver design

I. INTRODUCTION

Multi-user multiple-input multiple-output (MIMO) is a spectral-efficient downlink transmission technique in Long Term Evolution (LTE)-Advanced systems, where one base station (BS) serves multiple users simultaneously through spatial multiplexing. Unfortunately, in full frequency reuse cellular systems without coordination among the BSs, the users under multi-user MIMO transmission will confront with inter-cell interference from other BSs as well as intra-cell multi-user interference from its serving BS. In the information theoretic terminology, this is a typical MIMO interference broadcasting channel (MIMO-IBC) problem.

Although the problems of interference channel have been studied for several decades, there is no answer yet for optimal transmit/receive strategies to achieve the capacity region of a multi-cell multi-user MIMO system. Recently, the research focus lies in deriving various approximations of the capacity region and developing corresponding achievable transmission schemes. Among them, a widely used first-order approximation, degrees of freedom (DoF), can reflect the system performance when the signal to noise ratio (SNR) goes to infinity [1].

With a breakthrough concept named interference alignment (IA) [2], $K/2$ DoF can be achieved in a K-user interference channel with time-varying or frequency-selective fading. Through elaborated design of precoding matrices, at each receiver, the interference from all other users are aligned in half of its subspace, thus the desired signal can be conveyed in the other half subspace. The surprising DoF result reveals that interference channels are not essentially interference-limited [3].

The IA concept was subsequently extended to multiple-antenna and multi-cell scenarios. For MIMO-IBC, a structural downlink interference alignment (DIA) scheme for a two-cell scenario was presented in [4], where K users exist in each cell and K+1 antennas are equipped both at each BS and at each user. By designing two layers of precoding, the intra-cell interference are aligned with the inter-cell interference and K/ K+1 DoF is achieved. Since zero-forcing (ZF) criterion was used both for the precoder and the detector, the achievable data rate is degraded at low-SNR level. Moreover, the DIA scheme does not applicable for more than two cells scenarios.

In general multi-cell multi-user cases, several iterative algorithms were developed [5-8]. In [5], the reciprocity of wireless network was exploited, and the detector based on the max-signal to noise and interference ratio (SINR) criterion was used for precoding in each iteration when each BS or each user operates from receiving mode to transmitting mode. In [6] and [7], the sum-rate maximization problem was transformed into a weighted minimum mean square error (MMSE) minimization problem, then the detectors, weighting matrices, and precoders were alternatively optimized. In [8], the gradient descent method was applied to search for a locally maximized sum-rate. In fact, the optimization problem in MIMO interference channel is usually non-convex. Without considering the possible IA structure, the optimization procedure can only reach a local optimum.

To improve the data rate in noise-limited environments, we propose an iterative algorithm with an embedded IA structure. We first design a max-SINR detector for each user. Then, by feeding back the detector vectors of all users to their serving BS, we design a ZF precoder for each BS. Different with existing DIA algorithm, with our algorithm the intra-cell interference is not exactly aligned with the inter-cell interference after convergence. Instead, the two spaces keeps an angle so that the SINR can be maximized. This allows the proposed algorithm applicable in general multi-cell scenarios.

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we introduce the structure of DIA and present the algorithm. Simulations are shown in Section IV, and Section V concludes the paper.

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II. System Model

Consider a G-cell network, where each BS serves K_g users. The BS and the k-th user in cell g are equipped with M_g and N_{k_g} antennas, respectively. Define S as the set of all receivers, which is,

$$S = \{k_g | k_g \in 1, 2, \ldots, K_g, g \in 1, 2, \ldots, G\} \tag{1}$$

Let $\mathbf{V}_{k_g} \in \mathbb{C}^{M_g \times d_{k_g}}$ denote the precoding matrix for BS g to transmit the symbol $\mathbf{s}_{k_g} \in \mathbb{C}^{d_{k_g} \times 1}$ to the k-th user in its serving cell, i.e., user k_g. Then, the transmit signal of BS is

$$\mathbf{x}_g = \sum_{k_g=1}^{K_g} \mathbf{V}_{k_g} \mathbf{s}_{k_g} \tag{2}$$

where we assume $E[\mathbf{s}_{k_g} \mathbf{s}_{k_g}^H] = \mathbf{I}$, and $\mathbf{I}$ is the identity matrix.

The received signal at the k_gth user can be written as

$$\mathbf{y}_{k_g} = \underbrace{\mathbf{H}_{k_g g} \mathbf{V}_{k_g} \mathbf{s}_{k_g}}_{\text{desired signal}} + \underbrace{\sum_{m_g=1, m_g \neq k_g}^{K_g} \mathbf{H}_{k_g g} \mathbf{V}_{m_g} \mathbf{s}_{m_g}}_{\text{intracell interference}}$$
$$+ \underbrace{\sum_{j \neq g, j=1}^{G} \sum_{l_j=1}^{K_j} \mathbf{H}_{k_g j} \mathbf{V}_{l_j} \mathbf{s}_{l_j} + \mathbf{n}_{k_g}}_{\text{intercell interference plus noise}}, \forall k_g \in S \tag{3}$$

where $\mathbf{H}_{k_g j}$ is the channel matrix from BS j to user k_g whose elements are independent and identically distributed random varables, and $\mathbf{n}_{k_g} \in \mathbb{C}^{N_{k_g} \times 1}$ is the additive white Gaussian noise with distribution $CN(0, \sigma_{k_g}^2 \mathbf{I})$.

Consider linear detector $\mathbf{U}_{k_g}$, then the estimated symbols are given by

$$\hat{\mathbf{s}}_{k_g} = \mathbf{U}_{k_g}^H \mathbf{y}_{k_g}, \forall k_g \in S \tag{4}$$

The problem of interest is to find the transmit and receive matrices $\{\mathbf{U}_{k_g}, \mathbf{V}_{k_g}\}$ that maximizes a certain utility of the system. The transmit power is constrained as follows

$$\sum_{k_g=1}^{K_g} \mathrm{Tr}(\mathbf{V}_{k_g} \mathbf{V}_{k_g}^H) \leq P_g \tag{5}$$

Given the set of $\{\mathbf{U}_{k_g}, \mathbf{V}_{k_g}\}$ for multiple users, from (2) and (4) we can obtain the SINR for user k_g as

$$SINR_{k_g} = \frac{\| \mathbf{U}_{k_g}^H \mathbf{H}_{k_g g} \mathbf{V}_{k_g} \|^2}{\| \mathbf{U}_{k_g}^H (\sum\limits_{(l_j, j) \neq (k_g, g)} \mathbf{H}_{k_g j} \mathbf{V}_{l_j} \mathbf{V}_{l_j}^H \mathbf{H}_{k_g j}^H + \sigma_{k_g}^2 \mathbf{I}) \mathbf{U}_{k_g} \|^2} \tag{6}$$

The achievable data rate of user k_g can be expressed as

$$R_{k_g} = \log_2 \left| \mathbf{I} + \frac{\mathbf{H}_{k_g g} \mathbf{V}_{k_g} \mathbf{V}_{k_g}^H \mathbf{H}_{k_g g}^H}{\sum\limits_{(l_j, j) \neq (k_g, g)} \mathbf{H}_{k_g j} \mathbf{V}_{l_j} \mathbf{V}_{l_j}^H \mathbf{H}_{k_g j}^H + \sigma_{k_g}^2 \mathbf{I}} \right| \tag{7}$$

III. IA-Embedded Iterative Algorithm

In this section, we first introduce the precoding and detection structure of the DIA algorithms. Then, we present our iterative algorithm with the DIA structure embedded. We assume in this section that each cell serves K users and each user receives one data stream. Symmetric antenna configuration is considered, where each BS and each user have the same numbers of antennas, i.e., M=N=K+1.

A. Downlink Interference Alignment

In [4], a two-layer precoding structure was proposed to align the intra-cell interference with the inter-cell interference, and a ZF detector was used to completely remove these interference. As shown in Fig. 1, the inner layer precoder is a ZF precoder $\mathbf{B}_g \in \mathbb{C}^{K \times K}$ which we will show the detail later, and the outer layer precoder is a fixed matrix $\mathbf{P}_g \in \mathbb{C}^{M \times K}$. Thus the transmit signal of BS g is

$$\mathbf{x}_g = \mathbf{P}_g \mathbf{B}_g \mathbf{s}_g \tag{8}$$

where $\mathbf{s}_g \in \mathbb{C}^{K \times 1}$ are the data streams for K users.

The received signal at the k_gth user is

$$\mathbf{y}_{k_g} = \mathbf{H}_{k_g 1} \mathbf{P}_1 \mathbf{B}_1 \mathbf{s}_1 + \mathbf{H}_{k_g 2} \mathbf{P}_2 \mathbf{B}_2 \mathbf{s}_2 + \mathbf{n}_{k_g}, \forall k_g \in S \tag{9}$$

From the view point of user k_g, the inter-cell interference is constrained in a subspace spanned by $\mathbf{H}_{k_g j} \mathbf{P}_j$ where $j \neq g$. The ZF detector is the vector $\mathbf{u}_{k_g j}$ that is in the null space of $\mathbf{H}_{k_g j} \mathbf{P}_j$. Since the dimension of $\mathbf{H}_{k_g j} \mathbf{P}_j$ is M-by-K, the null space always exists. The inter-cell interference is thus eliminated.

To design the inner layer precoder, we need to feed back the detector $\mathbf{u}_{k_g}$ to its serving BS g. Then a composite matrix is constructed and the inner layer precoding matrix is obtained using ZF criterion as

$$\mathbf{B}_g = \begin{bmatrix} \mathbf{u}_{1_g}^H \mathbf{H}_{1_g g} \mathbf{P}_g \\ \mathbf{u}_{2_g}^H \mathbf{H}_{2_g g} \mathbf{P}_g \\ \cdots\cdots \\ \mathbf{u}_{K_g}^H \mathbf{H}_{K_g g} \mathbf{P}_g \end{bmatrix}^{-1} \tag{10}$$

The combined precoding matrix at BS g is $\mathbf{P}_g \mathbf{B}_g$. We can check that with this precoder the intra-cell interference is zero at the receiver. Since the inter-cell interference occupies a subspace with dimension K and the intra-cell interference occupies a subspace with dimension K-1, and both space are orthogonal with the receive vector $\mathbf{u}_{k_g}$, the intra-cell interference is aligned with the inter-cell interference.

It follows that each BS can transmit K data streams using K+1 dimensional subspace, therefore the achievable DoF per cell is $K/K+1$. As K approaches infinity, we asymptotically have DoF = 1.

The DIA algorithm uses ZF criterion both in the precoder and in the detector. Although the interference is perfectly aligned and removed, the signal power may also be severely reduced if the angle between the signal vector and the interference subspace is too small. When SNR is high, such a power loss can be neglected. When SNR is low, however, the power loss will lead to a dramatic data rate reduction.

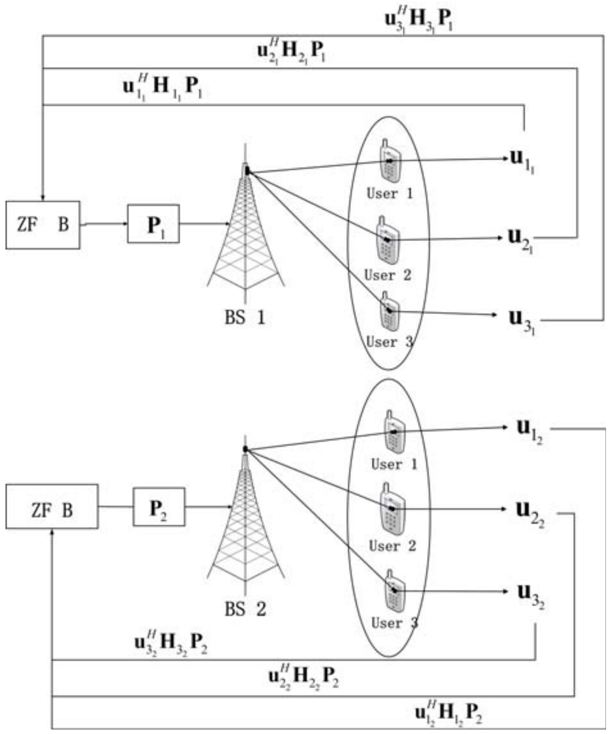

Fig. 1. A cellular network example with two cells each serving three users.

B. IA-Embedded Iterative Algorithm

To retrieve the power loss caused by ZF processing, we can use max-SINR criterion to design the detector at each user, but this will destroy the perfect interference alignment presented in last subsection. In the sequel, we derive an iteration algorithm to gradually maximize the data rate of each user, where the intra-cell interference and inter-cell interference are approximately aligned.

From (6), we can derive the detector that maximizes the SINR at user k_g as follows

$$\mathbf{u}_{k_g} = \mathbf{D}_{k_g}^{-1}\mathbf{H}_{k_g g}\mathbf{v}_{k_g} \tag{11}$$

where $\mathbf{D}_{k_g} = \sum_{(l_j,j)\neq(k_g,g)}^{K} \mathbf{H}_{k_g j}\mathbf{V}_{l_j}\mathbf{V}_{l_j}^H\mathbf{H}_{k_g j}^H + \sigma_{k_g}^2\mathbf{I}$ is the interference plus noise covariance matrix at user k_g.

Note that the inter-cell interference is not ensured orthogonal with the detection vector $\mathbf{u}_{k_g}$ any more. According to (7), $\mathbf{u}_{k_g}$ will maximize the data rate of user k_g.

To simplify the precoder design, we still use ZF criterion to design the inner layer multiuser precoder, and the expression of $\mathbf{B}_g$ is the same as (10). The outer layer precoding matrix $\mathbf{P_g}$ is obtained from the singular value decomposition of channel matrix $\mathbf{H}_{k_g g}$. To be specific, define $\mathbf{p}_{k_g}$ as the singular vector of $\mathbf{H}_{k_g g}$ corresponding to its largest singular value. Then $\mathbf{P}_g$ is obtained as

$$\mathbf{P}_g = [\mathbf{p}_{1_g} \ \ \mathbf{p}_{2_g} \ \ \dots \ \ \mathbf{p}_{K_g}] \tag{12}$$

The combined precoding matrix should satisfy the transmit power constraint, hence there is a weighting factor α_g such that

$$\mathbf{V}_g = \alpha_g\mathbf{P}_g\mathbf{B}_g, \ \text{ and } \ \mathrm{Tr}(\mathbf{V}_g\mathbf{V}_g^H) \leq P_g \tag{13}$$

The detailed procedure of the proposed iterative algorithm is listed in Table I.

TABLE I
THE DETAILED STEPS OF IA-EMBEDDED ITERATIVE ALGORITHM

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Algorithm 1
1. Set outer layer precoding matrix $\mathbf{P}_g, g = 1, 2, \dots, G$ as in (13)
2. Initialize $\mathbf{V}_g = \mathbf{P}_g, g = 1, 2, \dots, G$
3. Compute the detection vector at each user,
       $\mathbf{u}_{k_g} = \mathbf{D}_{k_g}^H\mathbf{H}_{k_g g}\mathbf{v}_{k_g}$
4. Calculate the inner layer precoding matrix at BS $g$,
       $\mathbf{B}_g = \begin{bmatrix} \mathbf{u}_{1_g}^H\mathbf{H}_{1_g g}\mathbf{P}_g \\ \mathbf{u}_{2_g}^H\mathbf{H}_{2_g g}\mathbf{P}_g \\ \cdots\cdots \\ \mathbf{u}_{K_g}^H\mathbf{H}_{K_g g}\mathbf{P}_g \end{bmatrix}^{-1}$
5. Calculate the combined precoding matrix at BS $g$
       $\mathbf{V}_g = \alpha_g\mathbf{P}_g\mathbf{B}_g, \ \text{and} \ \mathrm{Tr}(\mathbf{V}_g\mathbf{V}_g^H) \leq P_g$
6. Repeat to Step 3 until convergence.
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When the iteration converges, the intra-cell interference should be approximately aligned with the inter-cell interference, and the detector is adapted to maximize the SINR of each user. Because interference alignment is embedded in the iteration, we call this algorithm as IA-embedded iterative beamforming algorithm.

IV. SIMULATION RESULTS

In this section, we will first verify the convergence of the proposed iterative algorithm, and then evaluate the sum rate performance in a cellular network by comparing with other three algorithms.

To facilitate the comparison, we first consider two cells, i.e., $G = 2$. Each BS serves seven users, and each BS and each user are equipped with eight antennas, i.e., $K = 7, M = N = K + 1 = 8$.

The convergence property is shown in Fig. 2, from which we can see that the power of residual inter-cell interference reduces with the iteration times. Since the inner layer ZF precoding is calculated in the final step of the iterations, there is no residual intra-cell interference. We can find that the converging rate and the converged performance depend on the SNR conditions. For higher SNR level, the converging rate is slow, but the residual interference is small.

The performance comparison among different transmission schemes are shown in Fig. 3, where IA-E is our proposed IA-embedded iterative algorithm, DIA is the ZF-based algorithm introduced in [4], WMMSE is the weighted MMSE iteration algorithm developed in [7], and max-SINR is the reciprocally iterated algorithm presented in [5]. It is shown that the iterative IA-E algorithm provides substantial gains over the non-iterative DIA algorithm in low-SNR level. Moreover, with the IA-embedded structural design, we achieve a better performance than the sum-rate optimized WMMSE algorithm and avoid the heavy computation burden of its complicated

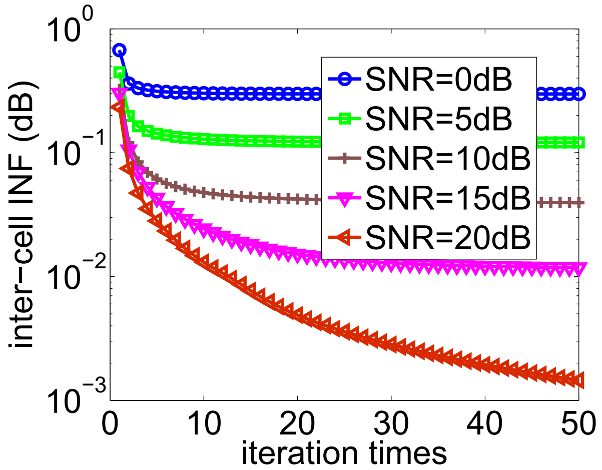

Fig. 2. The residual power of inter-cell interference versus the iteration times in different SNR conditions, where $G = 2, K = 7, M = N = K + 1 = 8$.

power allocation process. Although max-SINR criterion is also used in the reciprocally iterated algorithm, it does not perform well in the considered multi-cell multi-user configurations.

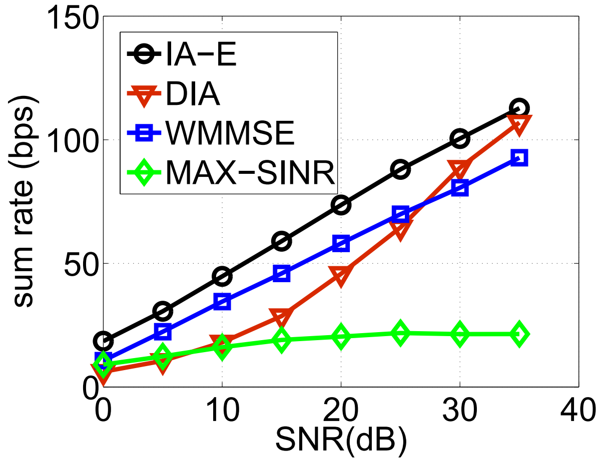

Fig. 3. The comparison of sum rate performance among different transmission schemes. Where $G = 2, K = 7, M = N = K + 1 = 8$.

With more than two cells, DIA algorithm in [4] can not apply anymore. The comparison of other three algorithms is shown in Fig. 4. The proposed IA-E algorithm always outperforms the max-SINR iterated algorithm in [5], and is superior to the WMMSE algorithm at low and moderate SNR levels, but becomes inferior to the WMMSE algorithm at high SNR. Nonetheless, the IA-E algorithm only requires feedback within a cell, but the WMMSE algorithm needs to exchange the transceiver and channel information across all cells.

V. CONCLUSION

In this paper, an interference-alignment-embedded iterative algorithm was proposed for multi-cell multi-user MIMO transmission. By relaxing the strict alignment requirement between the intra-cell interference and the inter-cell interference, a trade-off between the signal power loss and the interference mitigation is achieved. Simulation results showed that the proposed algorithm provides substantial gain in data rate in noise-limited scenarios over the DoF oriented DIA algorithm. Compared with other iteration-based algorithms, the proposed algorithm has lower implementational complexity and better performance.

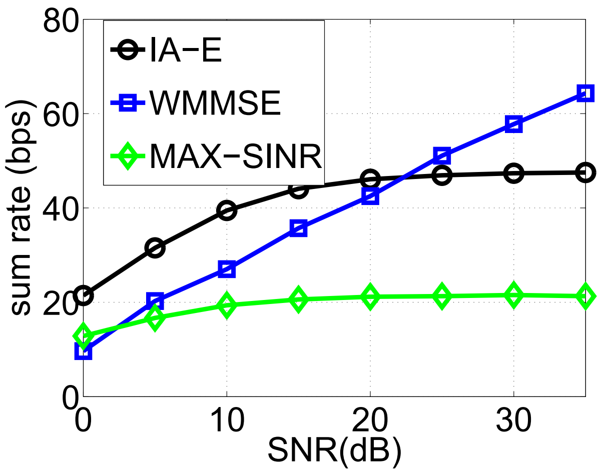

Fig. 4. The comparison of sum rate performance among different transmission schemes with more than two cells, where $G = 3, K = 7, M = N = K + 1 = 8$.

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