Impact of Nonorthogonal Training on Performance of Downlink Base Station Cooperative Transmission

Xueying Hou, Student Member, IEEE, Chenyang Yang, Senior Member, IEEE, and Buon Kiong Lau, Senior Member, IEEE

Abstract—Base station (BS) cooperative transmission is a promising technique to improve the spectral efficiency of cellular systems, and by using it, the channels become asymmetric in average gain. In this paper, we study the impact of the asymmetric channel gains on the performance of coherent cooperative transmission systems, when minimum mean square error (MMSE) and least square (LS) channel estimators are applied to jointly estimate the channel state information (CSI) under nonorthogonal training. We first derive an upper bound of rate loss caused by both channel estimation errors and CSI delay. We then analyze the mean square errors of the MMSE and LS estimators under both orthogonal and nonorthogonal training, which finally reveals the impact of different kinds of training on the precoding performance. It is shown that nonorthogonal training for the users in different cells leads to minor performance degradation for the MMSE channel-estimator-assisted downlink precoding. The performance degradation induced by channel estimation errors is almost independent of the user’s location. By contrast, the performance loss caused by the CSI delay is more severe for users located at the cell center than for users located at the cell edge. Our analysis is verified via simulation results.

Index Terms—Base station (BS) cooperative transmission, channel asymmetry, channel estimation, nonorthogonal training.

I. INTRODUCTION

Coherent base station (BS) cooperative transmission, which is a popular form of coordinated multipoint transmission (CoMP), can provide high spectral efficiency for cellular systems when both data and channel state information (CSI) are available at a central unit (CU) [1]–[4].

To facilitate downlink (DL) precoding in such CoMP systems, both local and cross channels, i.e., the channels between the cooperative BSs and mobile stations (MSs), which are in the same cell and in different cells, need to be estimated. In time-division duplexing (TDD) systems, the required CSI can be estimated through uplink (UL) training by exploiting channel reciprocity. When training signals for all MSs are orthogonal, conventional estimators such as that proposed in [5] can be directly applied with good performance. However, in current cellular systems, such as those complying with the Long-Term Evolution standard [6], the training sequences of MSs in the same cell are orthogonal, but those for the MSs in different cells are not. Moreover, the independent UL frequency scheduling among cells may lead to partially overlapped training sequences for the MSs in different cells, which results in high cross correlation of training sequences. To improve channel estimation performance, we can simply apply orthogonal training sequences for the MSs in different cells. However, this not only introduces large overhead, which occupies expensive UL resources [7], but also demands intercell signaling and coordination protocol to coordinate the training sequences and UL scheduling among multiple cells [8]. Such a burden will become more noticeable when the cooperative clusters are dynamically formed [4]. From the viewpoint of system compatibility, scalability, and complexity, the training sequences of MSs in different cells are highly preferred to be nonorthogonal. Nonetheless, it is not known whether nonorthogonal training can provide acceptable performance or not. In fact, even if we employ orthogonal training, the cross channels that experience severe attenuation seem to be hard to estimate due to the limited transmit power of MSs. On the other hand, when the channel is time varying, the CSI employed by CoMP transmission will become outdated due to the delay between UL channel estimation and DL data transmission. Such a channel distortion will inevitably lead to the deterioration of DL transmission performance. Therefore, the extent to which the channel distortion degrades the performance of DL CoMP precoding is of great interest.

In this paper, we study the impact of nonorthogonal training and CSI delay on the performance of DL CoMP transmission. A CoMP channel is an aggregation of multiple single-cell channels and is inherently asymmetric, i.e., the average channel gains from different BSs to one MS and those from MSs in different cells to one BS are different. Such an asymmetric channel feature is fundamental in CoMP systems since the difference of the average channel gains cannot be compensated by a power control mechanism. Specifically, if the MSs in different cells compensate their average channel gain differences toward one BS by power control, their receive signal energy differences toward other BSs will increase. This is analogous to the interference asynchrony feature, which cannot be dealt with by time-advanced techniques [3].

We consider coherent CoMP multiuser multiple antenna orthogonal frequency-division multiplexing (OFDM) systems. To show the connection of the performance between DL transmission and channel distortion caused by channel estimation and CSI delay, we first derive a lower bound of the achievable per-user rate of CoMP system using zero-forcing beamforming (ZF/BF). The impact of channel estimation errors on DL CoMP transmission has been studied in [7] and [8]. In [7], the authors considered DL channel estimation for feedback, but they did not address the impact of the channel asymmetry and nonorthogonal training. In [8], it is assumed that the cooperative BSs only serve one user on each subcarrier, which is different from our analysis for the multiuser case. The impact of CSI delay on DL CoMP systems has been investigated in [9] in the context of noncoherent CoMP with no data sharing among the BSs. We then analyze the performance of minimum mean square error (MMSE) and least square (LS) channel estimators, which are widely applied and have been extensively studied in single-cell systems (see [10] and references therein). In multicell systems, Jose et al. [11] showed that, when the MSs in different cells use identical sequences for UL training, the estimation performance of the desired channels will be severely degraded when a traditional channel estimator is used.

REFERENCES


[11]
where only the local channel is estimated and where the received signals of the cross channels are treated as interference. Considering the channel asymmetry and by jointly estimating the local and cross channels, we will show that the MMSE estimator is robust to the nonorthogonality of training sequences in different cells, whereas the LS estimator is quite sensitive to them. Moreover, our analysis shows that, although the cross channels are weak, their estimation errors induce even less rate loss than that of the local channel. In fact, we find that the average per-user rate loss caused by channel estimation errors is nearly independent of the MS’s location when the MMSE estimator is applied under intercell nonorthogonal training. By contrast, the performance loss caused by CSI delay is more severe for MSs located at the cell center than that for MSs located at the cell edge.

The rest of the paper is organized as follows: Section II introduces the system models. Section III analyzes the impact of nonorthogonal training and CSI delay on the performance of DL precoding. Simulation and numerical results are provided in Section IV, and conclusions are given in Section V.

Notations: \((X)^T\), \((X)^*\), and \((X)^H\) denote the transpose, conjugate, and conjugate transpose of \(X\), respectively. \(\text{vec}(X)\) is the vector column obtained by stacking the columns of \(X\), and \(X(i, i)\) and \(X(:, i)\) represent the \((i, i)\)th element and the \(i\)th column of matrix \(X\), respectively. \(\parallel \cdot \parallel\) represents the two-norm, \(\otimes\) is the Kronecker product, and \(\text{diag}\{\cdot\}\) is a diagonal matrix. \(\mathbb{E}\{\cdot\}\) is the expectation operator. \(I_N\) denotes the identity matrix of size \(N\), and \(0\) is the matrix of zeros.

II. System Models

Consider a coherent CoMP-OFDM system, where \(B\) BSs, each equipped with \(N_t\) antennas, cooperatively serve \(M\) single-antenna MSs using multicell ZFBF.

We consider TDD systems, where the CSI is estimated through UL training by exploiting channel reciprocity. A time-varying block-fading channel is considered, where the channel remains constant for an OFDM symbol duration and changes from symbol to symbol. During the UL training period, all MSs send training sequences, and each BS estimates the CSI between it and all MSs. Then, the CU collects the estimated CSI from each BS and computes the precoders based on the estimated CSI. Finally, the CU sends the DL data and precoders to each BS, and all BSs cooperatively serve MSs using multicell precoding.

We denote the delay between UL channel estimation and DL data transmission as \(D\) symbols. Denote the number of subcarriers as \(K\) and the number of resolvable paths of channel impulse response (CIR) as \(L\). To simplify the notations and for the sake of clarity, we assume the number of resolvable paths to be 1, with no loss of generality. Under this assumption, the channel frequency responses over all subcarriers are identical. We will verify in Section IV that the following analysis based on the flat-fading assumption also holds for frequency-selective-fading channels. In the following, we will omit the index of subcarrier for brevity.

Denote the composite channel vector between BS\(_b\) and MS\(_m\) at the \(n\)th discrete-time instant as \(h_{m,b}[n] = \alpha_{m,b} \vec{g}_{m,b}[n]\), where \(\alpha_{m,b}\) is the large-scale fading coefficient including both path loss and shadowing and \(\vec{g}_{m,b}[n] \in \mathbb{C}^{N_t \times 1}\) is the small-scale fading channel vector, whose entries are assumed to be independently identically distributed (i.i.d.) unit variance complex Gaussian variables. The first-order Gauss–Markov model is considered to characterize the time-varying property of small-scale fading channels by which the current and delayed small-scale fading channel vectors are related as

\[
\vec{g}_{m,b}[n] = \rho \vec{g}_{m,b}[n-D] + \sqrt{1-\rho^2} \vec{e}_{m,b}[n]
\]  

where \(\vec{e}_{m,b}[n] \in \mathbb{C}^{N_t \times 1}\) is the channel error vector whose entries are i.i.d. unit variance complex Gaussian variables, \(\vec{e}_{m,b}[n]\) is uncorrelated with \(\vec{g}_{m,b}[n-D]\) and \(\vec{g}_{m,b}[n]\), and \(\rho \in [0,1]\) is the fading correlation coefficient that characterizes the extent of time variation.

The DL global channel of MS\(_m\) at time \(n\) is the aggregation of \(B\) single-cell channel vectors, which is

\[
\vec{h}_{m}[n] = [\alpha_{m,1} \vec{g}_{m,1}[n], \ldots, \alpha_{m,B} \vec{g}_{m,B}[n]]^T
\]

\[
= \rho [\alpha_{m,1} \vec{g}_{m,1}[n-D], \ldots, \alpha_{m,B} \vec{g}_{m,B}[n-D]]^T
\]

\[
+ \sqrt{1-\rho^2} [\alpha_{m,1} \vec{e}_{m,1}[n], \ldots, \alpha_{m,B} \vec{e}_{m,B}[n]]^T
\]

\[
= \rho \vec{h}_{m}[n-D] + \sqrt{1-\rho^2} \vec{e}_{m}[n]
\]

where \(\vec{h}_{m}[n-D] \triangleq [\alpha_{m,1} \vec{g}_{m,1}[n-D], \ldots, \alpha_{m,B} \vec{g}_{m,B}[n-D]]^T\) is the DL global channel of MS\(_m\) at time \((n-D)\), and \(\vec{e}_{m}[n] \triangleq [\alpha_{m,1} \vec{e}_{m,1}[n], \ldots, \alpha_{m,B} \vec{e}_{m,B}[n]]^T\) is the channel error vector of the DL global channel caused by the delay of CSI.

A. UL Training

During the UL training period, each BS needs to estimate the CSI between itself and all MSs. We consider that all the MSs send training sequences at the \((n-D)\)th discrete-time instant. Assume that the transmit power is the same for all the MSs, and it is denoted as \(p_u\). Denote the frequency domain training sequence of MS\(_m\) as \(s_m \in \mathbb{C}^{K \times 1}\). Then, the received signal matrix at BS\(_b\) during the UL training phase can be expressed as

\[
\vec{Y}_b[n-D] = \sqrt{p_u} \vec{h}_{m}[n-D] \vec{S}^T + \vec{N}[n-D]
\]  

where \(\vec{Y}_b[n-D] = [\vec{y}_b^1[n-D], \ldots, \vec{y}_b^K[n-D]] \in \mathbb{C}^{N_t \times K}\)

\(\vec{y}_b[n-D] \in \mathbb{C}^{N_t \times 1}\) is the received signal vector of BS\(_b\) at the \(k\)th subcarrier, \(\vec{h}_{m,b}[n-D] = [\vec{h}_{1,b,1}[n-D], \ldots, \vec{h}_{b,M_b}[n-D]] \in \mathbb{C}^{N_t \times M}\) is the channel matrix between BS\(_b\) and all MSs at the \((n-D)\)th discrete-time instant when channels are estimated, \(\vec{S} = [s_1, \ldots, s_M] \in \mathbb{C}^{K \times M}\) is the training matrix formed by the training sequences of all MSs, and \(\vec{N}[n-D] \in \mathbb{C}^{N_t \times K}\) is the additive white Gaussian noise (AWGN) matrix, whose elements are random variables with zero mean and covariance \(\sigma_n^2\).

By vectorizing the received signal in (3) and applying \(\text{vec}(ABC) = (C^T \otimes A) \text{vec}(B)\), we can obtain the vectorization of \(\vec{Y}_b[n-D]\) as

\[
\text{vec}(\vec{Y}_b[n-D]) = \sqrt{p_u} \text{vec}(\vec{H}_b[n-D]) + \text{vec}(\vec{N}[n-D]),
\]

where \(\bar{S} = \vec{S} \otimes I_{N_t}\). Then, \(\vec{H}_b[n-D]\) can be estimated as

\[
\hat{\text{vec}}(\vec{H}_b[n-D]) = \left( \bar{S}^H \bar{S} + \mu \cdot \frac{\sigma_n^2}{p_u} \mathbf{R}_b^{-1} \right)^{-1} \bar{S}^H \text{vec}(\vec{Y}_b[n-D])
\]

where \(\mathbf{R}_b = \mathbb{E}[(\text{vec}(\vec{H}_b[n-D]) \text{vec}(\vec{H}_b[n-D])^H)]\) is the correlation matrix of the vectorized channel matrix \(\vec{H}_b[n-D]\). We assume that the channels from multiple MSs to BS\(_b\) are uncorrelated, and the separation distance of antennas at each BS are large enough that the channels from multiple antennas of one BS to one MS are spatially uncorrelated. Then, we have \(\mathbf{R}_b = \Lambda_b \otimes I_{N_t}\), where \(\Lambda_b = \text{diag}\{\sigma_1^2, \ldots, \sigma_K^2\}\). The estimator in (4) is the MMSE estimator when \(\mu = 1\) and LS estimator when \(\mu = 0\).
B. Downlink Transmission

The CU collects the estimated CSI from both BS to calculate the multicell precoders. Denote the estimated DL channel matrix from all cooperative BSs to MSs as $\hat{\mathbf{H}}[n-D] = [\hat{\mathbf{H}}_1^T[n-D], \ldots, \hat{\mathbf{H}}_M^T[n-D]]$; then, a multicell ZFBF is $\mathbf{V} = \mathbf{H}^H[n-D](\mathbf{H}[n-D]\mathbf{H}^H[n-D])^{-1}$. Denote $\hat{\mathbf{h}}_{m,n}$ and $p_d(m)$ as the data and power allocated for MS$_m$. For simplicity, we assume that $E\{\hat{\mathbf{h}}_{m,n}[n]\hat{\mathbf{h}}_{m,n}[n]\} = 1$ and the transmit power is equally allocated to each MS, i.e., $p_d(m) = p_d$. Then, the received signal of MS$_m$ is

$$y_{m}[n] = \sum_{j=1}^{M} \sqrt{p_d} \hat{\mathbf{h}}_{m,n}^H[n] \mathbf{v}_j[n] d_j[n] + z_{m,n} \tag{5}$$

where $\mathbf{v}_j[n] = \mathbf{V}[n(:, j)]/\| \mathbf{V}[n(:, j)] \|$ is the precoding vector for MS$_j$, and $z_{m,n}$ is the AWGN with zero mean and covariance $\sigma_d^2$.

For CoMP transmission, the ZFBF precoder should be designed under per-BS power constraint [1], but its performance is hard to analyze. In this paper, we consider a suboptimal but more tractable under per-BS power constraint, which is the per-user power constraint as in [3]. It has been shown that the ZFBF with different power constraints closely perform when the number of users in each cell is large [12].

III. IMPACT OF DELAYED NONORTHOGONAL TRAINING ON COOPERATIVE TRANSMISSION

In this section, we first show the connection between the performance of DL transmission and the distortion of the global composite channel, which is caused by both channel estimation and CSI delay. Then, we derive the mean square error (MSE) of the MMSE estimator and LS estimator under both orthogonal and nonorthogonal training and analyze the impact of the resulting channel estimation errors on the performance of multicell ZFBF.

A. Relationship Between Precoding Performance and Channel Distortion

We analyze the average per-user rate loss of CoMP transmission using ZFBF caused by both channel estimation errors and CSI delay.

The average rate of MS$_m$ achieved by ZFBF with delayed estimation of CSI is obtained as $E\{R_{m}[n]\}$, where $R_{m}[n]$ is the data rate of MS$_m$ at the nth time interval.

Using similar techniques as in [13], we can derive the upper bound of the rate loss of MS$_m$ caused by both channel estimation errors and CSI delay, which yields

$$E\{|\Delta R_{m}[n]\} = E\{R_{\text{ideal},m}[n]\} - E\{R_{m}[n]\} \leq \log_2 \left( 1 + \frac{p_d}{\sigma_d^2} E\{I_{m}[n]\} \right) \tag{6}$$

where $E\{R_{\text{ideal},m}[n]\}$ is the average rate achieved under perfect CSI, and $I_{m}[n] = \sum_{j=1, j \neq m}^{M} \| \hat{\mathbf{h}}_{m,n}^H[n] \mathbf{v}_j[n] \|^2$ is the interference power experienced by MS$_m$ caused by channel estimation errors and CSI delay.

Denote the estimation of DL channel vector for MS$_m$ as $\hat{\mathbf{h}}_{m}[n-D]$ and its estimation error as

$$\hat{\mathbf{h}}_{m}[n-D] = \hat{\mathbf{h}}_{m,m}[n-D] - \hat{\mathbf{h}}_{m}[n-D] = [\hat{\mathbf{h}}_{m,1}[n-D]^T, \ldots, \hat{\mathbf{h}}_{m,B}[n-D]^T]^T \tag{7}$$

where $\hat{\mathbf{h}}_{m,b}[n-D] = [\hat{\mathbf{h}}_{m,b,1}[n-D], \ldots, \hat{\mathbf{h}}_{m,b,N_t}[n-D]]^T$ is the estimation error vector of the composite channel between MS$_m$ and BS$_b$. Denote the MSE of the channel estimate between MS$_m$ and the $b$th antenna of BS$_b$ as $\varepsilon_{m,b,a}^2 = E\{\|\hat{\mathbf{h}}_{m,b,a}[n-D]\|^2\}$. Since the MSEs of the CSI estimate between different antennas of BS$_b$ and MS$_m$ are equal, we define $\varepsilon_{m,b,1}^2 = \cdots = \varepsilon_{m,b,N_t}^2 \triangleq \varepsilon_{m,b}^2$. When the MMSE estimator is used, the channel estimation errors are independent of the channel estimates. Since the precoders are functions of the channel estimates, the channel estimation errors $\hat{\mathbf{h}}_{m}[n-D]$ and the precoders $\mathbf{v}_j[n], j = 1, \ldots, K$, are mutually independent. Then, the average interference power can be derived as follows:

$$E\{I_{m}[n]\} = \mathbb{E}\left\{ \sum_{j=1, j \neq m}^{M} \| \hat{\mathbf{h}}_{m,n}^H[n] \mathbf{v}_j[n] \| \right\} \leq \sum_{j=1, j \neq m}^{M} \mathbb{E}\{\| \hat{\mathbf{h}}_{m,n}^H[n] \mathbf{v}_j[n] \|^2\} \tag{8}$$

where (a) follows because $\mathbf{h}_{m}[n] = \rho \mathbf{h}_{m}[n-D] + \sqrt{1-\rho^2} \mathbf{e}_{m}[n] = \rho \mathbf{h}_{m}[n-D] + \rho \mathbf{e}_{m}[n-D]$ and $\mathbf{e}_{m}[n-D]$ is orthogonal to $\mathbf{v}_j[n]$ due to ZFBF; (b) comes by the definitions of $\mathbf{e}_{m}[n]$ and $\mathbf{h}_{m}[n-D]$ in (2) and (7); (c) is obtained by averaging over $\mathbf{h}_{m}[n-D]$ and $\mathbf{e}_{m,b}[n]$, in which the statistical property of mutual independence among channel estimation errors $\mathbf{h}_{m}[n-D]$, the channel error caused by CSI delay $\mathbf{e}_{m}[n]$, and the precoder vectors $\mathbf{v}_j[n], j = 1, \ldots, K$, is used; and (d) comes from the assumption of per-user power constraint, which leads to $\| \mathbf{v}_j[n] \|^2 \leq \sum_{j=1}^{B} \| \mathbf{v}_j[n] \|^2 \leq 1$.

Then, the upper bound of the rate loss is

$$\Delta R_{m}^{\text{UB}} = \log_2 \left[ 1 + \frac{p_d}{\sigma_d^2} \sum_{b=1}^{B} \varepsilon_{m,b} \right] \mathbb{I}_{m}^{\text{CEE}} \triangleq \Delta R_{m}^{\text{UB}}$$

$$\Delta R_{m}^{\text{UB}} = \log_2 \left[ 1 + \frac{p_d}{\sigma_d^2} \sum_{b=1}^{B} \alpha_{m,b} \right] \mathbb{I}_{m}^{\text{Delay}} \triangleq \Delta R_{m}^{\text{UB}}$$

where the term $\mathbb{I}_{m}^{\text{CEE}}$ reflects the impact of channel estimation errors, and the term $\mathbb{I}_{m}^{\text{Delay}}$ indicates the performance degradation caused by
For mathematical tractability, we consider a simple but fundamental
the MSE for both kinds of training. Then, we can understand how
B. Impact of Nonorthogonal Training on DL
under both orthogonal and nonorthogonal training.

It is worth noting that the analysis in [13] is for single-cell systems
local and cross channels, which is independent of the CSI delay. In the
when the MSs are located at
cell edge. In the
Remark 1: The rate loss caused by CSI delay depends on the sum of
channel gains of the local and cross channels. When
of its local channel exponentially increases. Although the path losses of
its cross channels exponentially reduce, the sum of average channel
exceeds that of a cell-edge MS. This implies that the impact of CSI delay on
transmission is larger for the MSs located at the cell center than for the MSs located at cell
edge.

It is shown from (9) that the impact of channel estimation on the
upper bound of rate loss depends on the sum of MSEs of the composite
local and cross channels, which is independent of the CSI delay. In the
following, we will analyze the MSE of the composite CSI estimates
under both orthogonal and nonorthogonal training.

B. Impact of Nonorthogonal Training on DL CoMP Transmission

Here, we strive to derive an explicit and unified expression of
the MSE for both kinds of training. Then, we can understand how
nonorthogonal training performs in the CoMP system by comparing it
with orthogonal training.

1) MSES of the MMSE and LS Estimators: The covariance matrix
channel estimation error vector vec(H[n − D]) = vec(H_ε[n − D] − H_ε[n − D])
can be derived from (4) as

\[
\begin{align*}
C_b & \triangleq \mathbb{E}\left\{ \begin{bmatrix} vec(H_ε[n − D]) & vec(H_ε[n − D]) \end{bmatrix}^H \right\} \\
& = (\mu \mathbf{R}_b^{-1} + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S})^{-1} \\
& = \left[ \mu (\mathbf{A}_b \otimes \mathbf{I}_{N_t}) + \frac{p_u}{\sigma_n^2} (\mathbf{S} \otimes \mathbf{I}_{N_t}) \right]^{-1} \\
& = \left[ \mu \mathbf{A}_b^{-1} \otimes \mathbf{I}_{N_t} + \frac{p_u}{\sigma_n^2} (\mathbf{S}^H \mathbf{S}) \otimes \mathbf{I}_{N_t} \right]^{-1} \\
& = \left( \mu \mathbf{A}_b^{-1} + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S} \right)^{-1} \otimes \mathbf{I}_{N_t}.
\end{align*}
\]

Then, the MSE of the estimated CSI between MS_m and the nth
antenna of BS_b, i.e., \( \varepsilon_{m,b,n}^2 \), is the \((m − 1)N_t + n\)th diagonal element of
\( C_b \). Define \( \mathbf{B}_b \triangleq (\mu \mathbf{A}_b^{-1} + \frac{p_u}{\sigma_n^2} \mathbf{S}^H \mathbf{S})^{-1} \). The MSE of
the estimated channel coefficient between any antenna of BS_m and MS_m
is the same, which is the \((m, n)\)th element of \( \mathbf{B}_b \), i.e., \( \varepsilon_{m,b,n}^2 = \cdots = \varepsilon_{m,b,1}^2 = \mathbf{B}_b(m, n) \). Due to the matrix inverse operation
in \( \mathbf{B}_b \), it is nontrivial to derive an explicit general expression of \( \varepsilon_{m,b}^2 \). For mathematical tractability, we consider a simple but fundamental
scenario, where BS BSs cooperatively serve two MSs.

Then, it is not hard to respectively derive the MSEs of the MMSE
and LS estimators, as follows:

\[
\begin{align*}
\varepsilon_{m,b}^2_{\text{MMSE}} &= \eta_{m,b} \frac{\sigma_n^2}{p_u} \frac{1}{K} \frac{1}{1 - \beta \lambda}, \quad m = 1, 2 \\
\varepsilon_{m,b}^2_{\text{LS}} &= \eta_{m,b} \frac{p_u}{p_u} \frac{1}{K} \frac{1}{1 - \lambda}, \quad m = 1, 2
\end{align*}
\]

where \( \eta_{m,b} = 1/(1 + (\sigma_n^2/\sigma_{B,b} p_u)1/K), \quad \beta = 1/(\prod_{b=1}^B (1 + \sigma_n^2/\sigma_{B,b} p_u)1/K), \quad \lambda = \| \mathbf{s}_m^H \mathbf{s}_m^H \|_2^2 = \| \mathbf{s}_m^H \mathbf{s}_m^H \|_2^2 / K^2 \) represents the cross correlation between the training sequences of MS_m and MS_b.

For the MMSE estimator, it is shown that the MSE of the composite
CSI depends on \( \alpha_{m,b}^2 \), which is the large-scale fading gain of \( h_{m,b} \). If
MS_m is in the same cell with BS_c, then \( h_{m,b} \) is the local composite
channel for MS_m, and \( h_{m,b} \) for \( b \neq c \) are its cross composite channels. Since, in general, \( \alpha_{c,b,c}^2 \geq \alpha_{m,b}^2, \ b \neq c \), we can observe that the MSEs of the weak cross channels are even less than that of the
strong local channel. For the LS estimator, the MSEs of local and cross
composite channels are identical.

At first glance, the results for both the LS and MMSE estimators
may appear inconsistent with intuition, where the MSEs of the estimates
of the cross channels should exceed that of the local channel.
However, this is true only for estimating small-scale fading channels
with unit average energy. To see this, we normalize the MSE of \( h_{m,b} \)
by \( \alpha_{m,b}^2 \) to obtain a normalized MSE (NMSE), which is actually the
MSE for estimating the small-scale fading channel with low SNR.

The NMSE for LS and MMSE estimators can be expressed as

\[
\begin{align*}
\text{NMSE}_{m,b}^{\text{MMSE}} & \triangleq \varepsilon_{m,b}^2_{\text{MMSE}} / \alpha_{m,b}^2 \\
& = 1 + \frac{1}{\gamma_{m,b}} \frac{1}{K} \frac{1}{1 - \lambda}, \quad m = 1, 2
\end{align*}
\]

where \( \gamma_{m,b} = \sigma_{B,b}^2 p_u / \sigma_n^2 \) is, in fact, the average UL receive signal-to-noise ratio (SNR) when estimating the small-scale fading channel vector \( \mathbf{g}_{m,b} \). Thus, the NMSE for estimating a small-scale fading channel with low SNR exceeds that with high SNR.

2) Impact of the Training Sequences: When the training se-
quences for the two MSs are orthogonal, we have \( \lambda = 0 \). Then,
both MMSE and LS estimators achieve their minimal MSEs, which
are \( \varepsilon_{m,b}^2_{\text{MMSE}} = (1 + (1/K)(1/\gamma_{m,b}))(\sigma_n^2/\sigma_{B,b}1/K) \) and \( \varepsilon_{m,b}^2_{\text{LS}} = (\sigma_n^2/\sigma_{B,b}1/K) \), respectively. In typical cellular OFDM systems,
the receive SNR of the link from MS_m to BS_b, i.e., \( \gamma_{m,b} \), is larger
than 0 dB, and the value of \( 1/K \) is usually small. Consequently,
\( \varepsilon_{m,b}^2_{\text{MMSE}} \approx \varepsilon_{m,b}^2_{\text{LS}} \).

When the training sequences of MSs in different cells are not
orthogonal, we have \( \lambda \neq 0 \).

For the LS estimator, if \( \lambda \) is close to 1, the value of \( \varepsilon_{m,b}^2_{\text{LS}} \) will be
extremely large, which means that the estimation performance will
severely degrade. Therefore, the LS estimator is quite sensitive to the
nonorthogonality of training sequences no matter where the MSs are
located.

For the MMSE estimator, \( \lambda \) is weighted by \( \beta \), in the expression
of MSE, where \( \beta_\gamma < 1 \) and its value is related to the large-scale fading
gains between the two MSs and the BSs. This indicates that the MSE
of MMSE estimator depends on the location of the MSs.

If the two MSs are within the same cell with BS_b, both \( \alpha_{1,b}^2 \) and
\( \alpha_{2,b}^2 \) are large; then, \( \beta_\gamma \approx 1 \) and \( \eta_{m,b}^{\text{MMSE}} \approx 1 \). As a result, \( \varepsilon_{m,b}^2_{\text{MMSE}} \approx \varepsilon_{m,b}^2_{\text{LS}} \).
This implies that, if the MSs in the same cell use nonorthogonal
training sequences, the MMSE estimator is also quite sensitive to the nonorthogonality of training sequences.

If the two MSs are in different cells, on the other hand, no matter where both MSs are located, $\beta_m$ will be small; hence, $\varepsilon_m^{\text{MMSE}} \approx (1/(1 + (1/\gamma_m))(1/K))(\sigma_n^2/p_u)/(1/K) \ll \varepsilon_{m,b}$. This is because, if both MSs are at the cell edge, their average channel energies are small; thereby, both values of $(1/\gamma_m)(1/K)$, $m = 1, 2$, are large. If at least one MS is at the cell center, due to the severe energy attenuation of the channels from this MS to its nonserving BSs, at least one of the $(1/\gamma_m)(1/K)$ is large. This indicates that the MMSE estimator is robust to intercell nonorthogonal training.

Substituting the expression of $\varepsilon_m^{\text{MMSE}}$ in (11) into (9), the upper bound of the rate loss becomes

$$
\Delta R_m^{\text{UB}} = \log_2 \left[ 1 + \left( M - 1 \right) \frac{P_d}{\sigma_z^2} \sum_{b=1}^{B} (1 - \rho^2) \sigma_{m,b}^2 \right] + \rho^2 (M - 1) \frac{P_d}{\sigma_z^2} \sum_{b=1}^{B} \left( \frac{1}{1 - \gamma_b} \right) \sigma_n^2 \frac{1}{K} \left( 1 - \beta_b \lambda \right)
$$

$$
\approx \log_2 \left[ 1 + (1 - \rho^2)(M - 1) \frac{P_d}{\sigma_z^2} \sum_{b=1}^{B} \sigma_{m,b}^2 \right] + \rho^2 B (M - 1) \frac{P_d \sigma_z^2}{\sigma_z^2 \sigma_n^2} \frac{1}{K} \left( 1 + \frac{1}{m} \right) + \rho^2 B (M - 1) \frac{P_d \sigma_z^2}{\sigma_z^2 \sigma_n^2} \frac{1}{K} \left( 1 + \frac{1}{m} \right)
$$

where the approximation of (15) is obtained when the values of both $(1/\gamma_m)(1/K)$ and $\beta_b \lambda$ approach 0. When the number of subcarriers $K$ is large, the value of $(1/\gamma_m)(1/K)$ will be close to 0. Regarding $\beta_b \lambda$, as we have analyzed, its value approaches 0 when the training signals are all orthogonal or when the training signals are nonorthogonal but the MSs are in different cells.

Remark 2: Since the MSEs of the weak cross composite channels are usually smaller than that of the local composite channel, it is shown from (14) that the channel estimation errors of cross channels have less of a contribution to the rate loss than that of local channel. Moreover, if the training signals for MSs within each cell are orthogonal, whereas the MSs in different cells are nonorthogonal, the average interference part $\Gamma_{m,c}^{\text{CIR}}$ caused by channel estimation errors in (15) will be a constant, no matter where the MSs are located.

IV. SIMULATION AND NUMERICAL RESULTS

In this section, we verify our previous analysis via simulations. We consider a CoMP system of $B$ BSs, each with four omnidirectional antennas. Considering that the DL transmit power is usually larger than the UL transmit power, we set $p_u = p_m + 5 \text{ dB}$. $K = 128$. Recall that, in the derivations, we assume flat-fading channels mainly for simplicity of notations and clarity of presentation. Similar derivations can likewise be made for frequency-selective channels, which are appropriate for a new generation of broadband cellular systems. Hence, the channels used in the simulations are frequency selective, the CIR of which is a tapped-delay line with independent Rayleigh fading coefficients and an exponential power delay profile with an attenuation factor of 1.4 and $L = 20$. These parameters are

the same as those in SCME channels [6]. Although a permissive time-varying channel model is employed in the previous derivation of rate loss for mathematical tractability, in simulations, we consider a more realistic channel mode, i.e., Jakes’ Model, which is a widely accepted channel model by various standardization organizations. Its temporal correlation function is $R_k(\tau) = J_0(2\pi f_\lambda \tau f_\lambda)$, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $f_\lambda$ is the Doppler spread, and $T_s$ is the symbol duration. The carrier frequency is 2 GHz, the OFDM symbol duration is 1 ms, and the speed of the MSs is 3 km/h.

The training sequences are constructed from the Constant-Amplitude Zero Autocorrelation Code (CAZAC) as $t(k) = e^{-j\pi k/N_{\text{CAZAC}}}k = 0, \ldots, K - 1$ [6], where $N_{\text{CAZAC}} = 127$ and $n_k = \text{mod}(k, N_{\text{CAZAC}})$. The training sequences for MSs may be orthogonal or nonorthogonal. For orthogonal training, the training sequences for all MSs are the cyclic shift version of one CAZAC. For nonorthogonal training, the CAZACs used by MSs are of different $c$’s.

A. Impact of the Training Sequence Orthogonality on the NMSEs of Different Estimators

To observe the robustness of different estimators to the nonorthogonal training, we consider a cooperative cluster of two BSs. Although our analytical results are derived for the case of two MSs, we consider four MSs in the simulation to demonstrate that the analysis is also valid for more general cases. We assume that the four MSs are symmetrically located, as shown in Fig. 1. Then, all MSs achieve the same performance, and we only need to show the performance of one MS.

To be consistent with the traditional understanding, we show the NMSE of the composite CIR, which is defined in (13) and is actually the MSE for estimating the small-scale fading channel. In Fig. 2, the NMSEs of the MMSE and LS estimators under orthogonal and nonorthogonal training versus the UL local SNR for both the local and cross channels are shown. The UL local SNR is the UL average receive SNR of the local link, which is $\gamma_{m,c,m} = (\alpha_{m,c,m}^2 p_u / \sigma_n^2)$ for MS $m$, where $\alpha_{m,c}$ and $\alpha_{c,m}$ are in the same cell. As we have analyzed, only when the training sequences for both MSs within a cell and MSs in different cells are orthogonal does the LS estimator achieve good performance, which is only slightly inferior to that of the MMSE estimator. On the other hand, when the MMSE estimator is used and when the training sequences for the MSs within a cell are orthogonal but those for MSs in different cells are nonorthogonal, the performance of both cross and local channel slightly degrades. By contrast, when the training sequences for the MSs within a cell and for the MSs in
different cells are both nonorthogonal, the performance of the local channel severely degrades, especially when the MSs are located at the cell center, whereas the performance of the cross channels only degrades a little since their channel estimation is noise limited. This agrees well with our previous analysis, i.e., the MMSE estimator is robust to the nonorthogonality of the training signals for the MSs in different cells, owing to the channel asymmetry.

B. Impact of Training Sequence Orthogonality and CSI Delay on the DL Average Per-User Rate

To observe the impact of different estimates and CSI delay on the performance of CoMP transmission, we first consider a case where the settings are the same as those in the previous section.

Fig. 3 shows the tightness of the per-user rate lower bound derived in Section III-A. The simulated per-user rate under perfect ZFBF and that with MMSE estimator under orthogonal training and different delays are plotted, together with the lower bound of the achievable rate derived from (15). It shows that the derived lower bound based on i.i.d. channel assumption is indeed a lower bound and is not very tight, which is due to the overestimated interference induced by the imperfect CSI for the CoMP channel. When no delay is introduced, i.e., $D = 0$, the performance degradation caused by only channel estimation errors are almost independent of the location of MS. By contrast, the per-user average rate loss of MS caused by the CSI delay is more severe for MS with high local SNR than that for MS with low local SNR. When the delay is large, the per-user average rate will arrive at a ceiling at high local SNR. These results agree well with our previous analysis.

The simulation results for the DL average rate of each MS with different estimators are shown in Fig. 4, where the average rate under non-CoMP transmission is also provided as a reference. To highlight the impact of different estimators and different training sequences, no CSI delay is considered. It is shown that, under orthogonal training, the per-user rates with different estimators are close. Moreover, for both the LS and MMSE estimators, the performance gaps from the perfect CSI-based ZFBF are almost independent of the location of MS. When the training sequences of MSs in different cells are not orthogonal, the performance degradation is minor when using the MMSE estimator. On the contrary, the performance degradation is severe when the LS estimator is used, and the rate is even lower than the non-CoMP transmission. We can also observe that the performance of the non-CoMP transmission reduces more by imperfect CSI than that of the non-CoMP transmission. This is no surprise because the rate loss of the CoMP transmission is induced by the sum of the estimation errors of multiple single-cell channel components, i.e., the channel estimation errors are larger in the CoMP system, given the same single-cell training resources with the non-CoMP system. Furthermore, the non-CoMP transmission is interference limited; hence, imperfect CSI leads to relatively less performance loss.
Finally, we simulate a more realistic setting, where $B = 3$ and each cell contains two MSs. The three BSs cooperatively serve all the six MSs without scheduling. With better scheduling algorithms such as those considering sum rate maximization or fairness among MSs, both CoMP and non-CoMP systems will perform better. All the MSs are randomly distributed in a “cell-edge region.” In particular, the ratio of the large-scale fading gain of the local channel to the sum of those of the cross channels of the MSs in this region, e.g., $\frac{\alpha_m c}{\sum_{b=1}^B \alpha_{m,b}}$ for $\text{MS}_m$, is less than a predefined value. The CSI delay is set as four symbols. In Fig. 5, we present the average cell-edge region throughput, which is the average sum rate of two MSs in the cell-edge region of each cell. We can see that the same conclusion can be drawn as before no matter how small the “cell-edge region” is, where the channels are not very asymmetric. When the MSs are randomly located in the whole cell region, CoMP only slightly outperforms non-CoMP. This result seems to be pessimistic. However, it is worth to note that only channel estimation is considered here. In practical systems, various channel prediction methods can be applied to alleviate the impact of CSI delay and enhance the performance of CoMP, which is out of the scope of this paper. We also simulate the throughputs when regularized ZFBF [14] is used for the DL CoMP transmission, but the results overlapped with those of ZFBF, which are not shown to make the figures clearer. This is because the regularized ZFBF outperforms ZFBF at low SNR, but in CoMP systems, the SNR is high. Furthermore, the analysis based on the simulated channels $ZFBF$ outperforms $ZFBF$ at low SNR, but in CoMP systems, the SNR is not shown to make the figures clearer. This is because the regularized CoMP, which is out of the scope of this paper. We also simulate the region to alleviate the impact of CSI delay and enhance the performance of practical systems, various channel prediction methods can be applied as a baseline, the results of non-CoMP transmission are also provided.

![Diagram of CoMP Performance](image)

**Fig. 5.** Average cell-edge region throughput when LS and MMSE channel estimators are used. The legend with (Orthogonal) means that the training sequences for all the six MSs are orthogonal. The legend with (Non-orthogonal) means that the training sequences for the MSs within the same cell are orthogonal but that those for MSs in different cells are nonorthogonal. For nonorthogonal training, the CAZACs used by the MSs in three cells are constructed from $c_1 = 1$, $c_2 = 7$, and $c_3 = 14$, respectively. The results are averaged over 1000 random drops. As a baseline, the results of non-CoMP transmission are also provided.

**V. CONCLUSION**

In this paper, we have analyzed the performance of LS and MMSE channel estimators under nonorthogonal training and the impact of both channel estimation errors and CSI delay on the CoMP transmission. Our analysis has shown that the LS estimator is quite sensitive to the nonorthogonal training, but the MMSE estimator is robust to intercell nonorthogonal training, owing to the unique feature of CoMP channels. When using the MMSE channel estimator, the estimation errors of the weak cross channels contribute even less to the average per-user rate loss for multicell precoding than those of local channels. The performance loss caused by channel estimation errors almost does not depend on the MS’s location, whereas the impact of CSI delay on CoMP transmission is larger for the MSs located at the cell center than for the MSs located at the cell edge. The simulation results show that CoMP transmission performs fairly well without intercell orthogonal training. This means that the cumbersome intercell signaling for coordinating the training resources may not be necessary.

**ACKNOWLEDGMENT**

The authors would like to thank J. Medbo and J. Furuskog of Ericsson Research, Stockholm, Sweden, for providing coherent multi-base-station measurement data that were used to verify the simulation analysis in the paper, and the anonymous reviewers for their comments, which helped to improve the paper.

**REFERENCES**


