Energy-Efficient MIMO-OFDMA Systems based on Switching Off RF Chains*

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***Abstract*—In this paper, both configuration of active *radio frequency* (RF) chains and resource allocation are investigated for improving energy efficiency of downlink *multiple-input-multiple-output* (MIMO) *orthogonal frequency division multiple access* (OFDMA) systems. We first formulate an optimization problem to minimize the total power consumed at the base station with the maximum transmit power constraint and ergodic capacity constraints from multiple users. Then a two-step suboptimal algorithm is proposed. Specifically, the continuous variable optimization problem is first solved, and then a discretization algorithm is presented to obtain the number of active RF chains and the number of subcarriers allocated to each user. Simulation results demonstrate that the proposed algorithm can provide significant power-saving gain over the all-on RF chain scheme and the adaptive subcarrier allocation helps to save more power.**

I. Introduction

Wireless networks are expected to be designed in an energy-efficient way since the explosive growth of wireless service is sharply increasing their contribution to the carbon footprint [1]. *Multiple-input-multiple-output* (MIMO) transmission has been widely applied in wireless networks nowadays owing to its high spectral efficiency. Although MIMO systems need less transmit power than *single-input-single-output* (SISO) systems for the same data rate, more circuit power is consumed since more active transmit or receive *radio frequency* (RF) chains are used, which has been shown comparable to the transmit power in practical networks, such as wireless sensor networks [2] and cellular networks [3]. After the circuit power consumption is taken into account, whether MIMO systems are energy efficient is not clear.

There have been some preliminary results on this topic. *Energy efficiency* (EE) of Alamouti diversity scheme has been discussed in [2]. It was shown that for short-range transmission, *multiple-input-multiple-output* (MISO) transmission decreases EE as compared with SISO transmission if adaptive modulation is not used. However, if modulation order is adaptively adjusted to balance the transmit and circuit power consumption, MISO systems will perform better. In [4], spatial multiplexing, space-time coding, and single antenna transmission have been adaptively selected based on the channel state information, and the EE improvement is up to 30% compared with non-adaptive systems. In [5], adaptive switching between MIMO and *single-input-multiple-output* (SIMO) modes has been addressed to save energy in uplink cellular networks. In [6], the number of active RF chains is optimized to maximize EE given the minimum data rate. These works only consider point-to-point transmission. For downlink cellular networks, RF chains at the *base station* (BS) are shared by multiple users and switching on or off RF chains will have different impact on the performance of different users. How to determine the number of active RF chains in this case is still unknown.

In this paper, we will study energy efficient configuration of active RF chains for downlink *orthogonal frequency division multiple access* (OFDMA) networks. Different from existing works, we will jointly optimize the number of active RF chains and resource allocation with capacity requirements for multiple users. A two-step algorithm is developed to find the suboptimal solution, which can reduce power consumption significantly.

The remainder of this paper is organized as follows. The system model and problem formulation are presented in Section II and III, respectively. In Section IV, we propose a two-step algorithm to find the suboptimal solution. Simulation results are provided in Section V and the paper is concluded in Section VI.

II. System Model

Consider a downlink MIMO-OFDMA cellular network with one BS and I users. It is assumed that M and N RF chains are respectively configured at the BS and each user and $M > N$. Each RF chain is connected with an antenna to transmit or receive signals. Overall K subcarriers are shared by different users with no overlap. The index set of subcarriers occupied by user i is denoted as $\mathcal{S}_i$ with size k_i.

Since most power is consumed at the BS during downlink transmission and a large portion of power is consumed by the RF chains including both transmit and circuit power, we

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consider adaptive RF chain configuration for saving energy. Specifically, both the number of active RF chains and the transmit power at the BS are adjusted to minimize the total power consumption with capacity constraints from multiple users. Denoting m as the number of active RF chains, then $m \leq M$ and $N \times m$ MIMO channel is built from BS to each user. Closed-loop *single-user MIMO* (SU-MIMO) schemes are considered and capacity-achieving precoder is assumed in this paper. Since the number of antennas is larger than that of active RF chains at the BS, transmit antenna selection can be applied to enhance channel capacity [7]. However, to highlight the impact of the number of active RF chains, the antenna selection will not be considered.

It is assumed that users undergo frequency-selective and spatial non-coherent block fading channels. Denote $\mathbf{H}_{ij}$ as the spatial channel matrix from BS to user i on subcarrier j. Elements in $\mathbf{H}_{ij}$ are *independent and identically distributed* (i.i.d) random variables with zero mean and variance μ_i, which is the large-scale channel gain from the BS to user i. The noise at the receiver of each user is assumed to be additive white Gaussian with zero mean and variance σ^2.

III. PROBLEM FORMULATION

In this section, we will formulate the optimization problem to minimize the overall power consumed at the BS with constraints on the upper bound of ergodic capacity for each user.

The overall power consumed at the BS, P_{tot}, includes the transmit power and the circuit power. Denoting ρ, P_t, and P_{rf} as the efficiency of power amplifier, the radiated power, and the circuit power consumed by each active RF chain, respectively, then the overall power consumption can be expressed as [5]

$$P_{tot} = P_t/\rho + mP_{rf}. \tag{1}$$

As in *Long Term Evolution* (LTE) systems, we consider that the transmit power is equally allocated over subcarriers and data streams. Then the transmit power on each subcarrier for each data stream is

$$P_{ea} = \frac{P_t}{KD}, \tag{2}$$

where $D \triangleq \min\{m, N\}$ is the number of data streams in an $N \times m$ MIMO system.

The instantaneous capacity of user i on subcarrier j can be expressed as

$$T_{ij} = \Delta f \log_2(\det(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{H}_{ij}\mathbf{Q}_x\mathbf{H}_{ij}^H)), \tag{3}$$

where Δf is the subcarrier spacing, $\mathbf{I}_N$ denotes an $N \times N$ identity matrix, and $\mathbf{Q}_x$ is the autocorrelation matrix of transmit signal $\mathbf{x}$.

When the capacity-achieving precoder and equal power allocation are applied, (3) can be simplified as

$$\begin{aligned} T_{ij} &= \Delta f \sum_{d=1}^{D} \log_2(1 + \frac{P_{ea}\lambda_{ijd}^2}{\sigma^2}) \\ &= \Delta f \sum_{d=1}^{D} \log_2(1 + \frac{P_t\lambda_{ijd}^2}{KD\sigma^2}), \end{aligned} \tag{4}$$

where λ_{ijd} denotes the dth singular value of $\mathbf{H}_{ij}$. The instantaneous capacity of user i is then obtained as

$$C_i = \sum_{j\in\mathcal{S}_i} T_{ij} = \Delta f \sum_{j\in\mathcal{S}_i}\sum_{d=1}^{D} \log_2(1 + \frac{P_t\lambda_{ijd}^2}{KD\sigma^2}). \tag{5}$$

Since RF chains cannot be switched between on and off statuses frequently due to hardware constraints, we consider ergodic capacity requirement of each user. Moreover, since the accurate expression of ergodic capacity is rather complicated and the channel distribution is hard to obtain in practical systems, we consider the constraints over the upper bound of ergodic capacity as follows,

$$\Delta f \sum_{j\in\mathcal{S}_i}\sum_{d=1}^{D} \log_2(1 + \frac{P_t\mathbb{E}[\lambda_{ijd}^2]}{KD\sigma^2}) \geq R_i, \tag{6}$$

where the upper bound is obtained by using Jensen's inequality on (5), R_i is the requirement of user i on the upper bound of ergodic capacity , and $\mathbb{E}[\cdot]$ is the expectation operation.

The variance of singular value λ_{ijd} is the same for different j and d [8] and can be obtained from the derivation in Appendix A as

$$\mathbb{E}[\lambda_{ijd}^2] = \mu_i mN/D. \tag{7}$$

Substituting (7) into (6), the capacity constraints can be finally expressed as

$$\Delta f k_i D \log_2(1 + \frac{\mu_i mNP_t}{KD^2\sigma^2}) \geq R_i. \tag{8}$$

It can be observed from (8) that the constraints only depend on the large-scale channel gains, $\{\mu_i\}_{i=1}^I$, which is easy to obtain in cellular networks. Note that the upper bound simplifies the analysis of the problem and such constraint is effective when the ergodic capacity scales in the same way with the upper bound.

We will optimize the number of active RF chains, m, the number of subcarriers for each user, k_i, and the radiated power, P_t, to minimize the total power consumption with the constraints on the upper bound of ergodic capacity for each user. The optimization problem is formulated as follows,

$$\min_{m,\{k_i\}_{i=1}^I,P_t} \quad P_t/\rho + mP_{rf} \tag{9}$$

$$\text{s. t.} \quad \Delta f k_i D \log_2(1 + \frac{\mu_i mNP_t}{KD^2\sigma^2}) \geq R_i, \tag{9a}$$

$$\sum_{i=1}^{I} k_i = K, \tag{9b}$$

$$0 < m \leq M, \tag{9c}$$

$$0 < P_t \leq P_{max}, \tag{9d}$$

$$k_i > 0, \qquad i = 1, 2, \cdots, I, \tag{9e}$$

where (9b) is the total subcarrier constraint and (9d) is the maximum transmit power constraint.

IV. Two-step Suboptimal Algorithm

In this section, we will develop an algorithm to find the solution of problem (9). Since the number of active RF chains, m, and the number of subcarrier, $\{k_i\}_{i=1}^{I}$, are both integer variables and the transmit power, P_t, is a real variable, (9) is a combinational optimization problem. Exhaustive searching is required to find the solution but its complexity is prohibitive. In the following, a two-step searching algorithm is proposed, which first solves the problem by treating k_i and m as continuous real variables and then discretizes them to find a suboptimal solution.

Since the number of data streams D is equal to the minimum of m and N, we divide the problem (9) into two cases depending on whether $m \geq N$ or not, which yield different solutions. When $m \geq N$, $D = N$ and the constraint (9a) is

$$\Delta f k_i N \log_2(1 + \frac{\mu_i m P_t}{K N \sigma^2}) \geq R_i. \quad (10)$$

When $m < N$, $D = m$ and the constraint (9a) is

$$\Delta f k_i m \log_2(1 + \frac{\mu_i N P_t}{K m \sigma^2}) \geq R_i. \quad (11)$$

Replacing (9a) in the problem (9) by (10) and (11), two optimization problems are formulated for the two cases and called problem (S1) and (S2), respectively. We will next solve these two problems for continuous m and $\{k_i\}_{i=1}^{I}$.

A. Solving Continuous Variable Problems

It is easy to find that the global optimum of problem (S1) is achieved when the equality in constraint (10) holds since both the objective function in (9) and the constraint in (10) increase monotonically with m and P_t. By solving the equation, we have

$$k_i = \frac{R_i}{\Delta f N \log_2(1 + \frac{\mu_i m P_t}{K N \sigma^2})}. \quad (12)$$

Substituting (12) into constraint (9b), we obtain

$$\sum_{i=1}^{I} \frac{R_i}{\Delta f N \log_2(1 + \frac{\mu_i m P_t}{K N \sigma^2})} = K. \quad (13)$$

Since the left hand side of this equation is a decreasing function with respect to mP_t, there exists a unique solution for mP_t, which can be found numerically by the bisection method. Denoting ψ as the solution of mP_t from (13), then problem (S1) can be rewritten as

$$\min_{m, P_t} \quad P_t/\rho + mP_{rf} \quad (14)$$

$$\text{s. t.} \quad mP_t = \psi \quad (14a)$$

$$N \leq m \leq M \quad (14b)$$

$$0 < P_t \leq P_{max}. \quad (14c)$$

We can derive that when $\psi > MP_{max}$, there is no intersection among (14a) $\sim$ (14c). Hence, the problem (14) is infeasible and outage occurs. When $\psi \leq MP_{max}$, according to the derivation in Appendix B, the solution of m to this problem can be obtained as follows,

$$m^o = \begin{cases} \sqrt{\frac{\psi}{\rho P_{rf}}}, \psi \leq MP_{max} \text{ and } N_{eq} \leq \sqrt{\frac{\psi}{\rho P_{rf}}} \leq M \\ M, \quad \psi \leq MP_{max} \text{ and } \sqrt{\frac{\psi}{\rho P_{rf}}} > M \\ N_{eq}, \quad \psi \leq MP_{max} \text{ and } \sqrt{\frac{\psi}{\rho P_{rf}}} < N_{eq}, \end{cases}$$

where $N_{eq} = \max\{\psi/P_{max}, N\}$, and the optimal P_t can be obtained as

$$P_t^o = \frac{\psi}{m^o}.$$

When $m < N$, we need to solve the problem (S2). It is readily shown that (11) is a convex constraint. In addition, the objective function and other constraints of the problem (S2) are linear. Therefore, the problem (S2) is convex, which can be solved by some efficient optimization methods, such as the interior-point method [9].

After the two problems are solved, we compare the total power consumption in these two cases and select the one with smaller power as the solution of m and P_t.

B. Variable Discretization

The solution of the continuous variable problem cannot be applied directly since in practice m and $\{k_i\}_{i=1}^{I}$ have integer values. In this subsection, we will discretize the optimized values. It is reasonable to set the optimal discrete number of active RF chains to be the smallest number larger than m^o, i.e., $m^\star = \lceil m^o \rceil$. The discrete number of k_i can be found by the following algorithm, whose basic idea is as follows. First, assign user i with the integer part of continuous k_i^o obtained from solving the continuous variable problem in last subsection. Then allocate the remaining subcarriers one by one to the user who has the largest capacity gap from the expected ergodic capacity. The detailed algorithm is summarized in Table I.

TABLE I

Algorithm for discretizing the number of subcarriers

Input: k_i^o, $m^\star$, and P_t^o.
Output: the number of subcarriers for user i, $k_i^\star$,.

1. Initialize $k_i = \lfloor k_i^o \rfloor$, and $\Delta K = K - \sum_{i=1}^{I} \lfloor k_i^o \rfloor$.
2. **while** $\Delta K > 0$
3. Calculate the capacity gaps from the expected ergodic capacity as follows,
$$\epsilon_i \triangleq R_i - \Delta f k_i D^\star \log_2(1 + \frac{m^\star N \mu_i P_t^o}{K D^{\star 2} \sigma^2}), \quad (15)$$
where $D^\star = \min\{m^\star, N\}$.
4. $i_{mx} = \arg\max\{\epsilon_1, \cdots, \epsilon_K\}$ and $k_{i_{mx}} = k_{i_{mx}} + 1$.
5. $\Delta K = \Delta K - 1$
6. **end**
7. **return** $k_i^\star = k_i$

After discretizing the number of active RF chains at the BS and the number of subcarriers for each user, the transmit power

TABLE II
LIST OF SIMULATION PARAMETERS

Subcarrier spacing, Δf	15 kHz
Number of subcarrier, K	1024
Number of users, I	10
Number of RF chains at BS, M	8
Number of RF chains at each user, N	2
Radius of a cell, R	500 m
User distribution	Uniformly distributed in the cell
Power spectral density of noise	-174 dBm/Hz
Noise amplifier gain	7 dBi
Minimum distance from BS to users	35 m
Path loss (dB)	35+38$\log_{10} d$
Standard variance of Shadowing	8 dB
Efficiency of power amplifier, ρ	38%
Maximum transmit power, P_{max}	40 W
Circuit Power of a RF chain, P_c	5-20 W

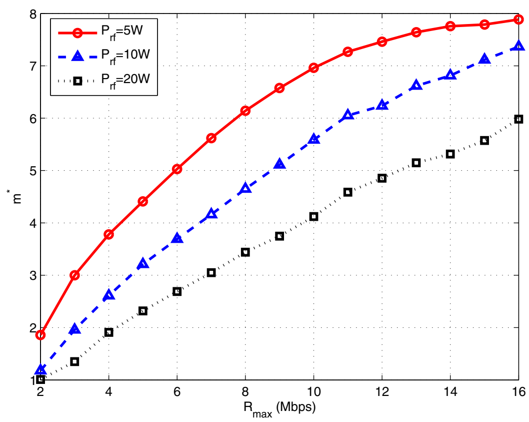

Fig. 1. Number of active RF chains at BS vs.capacity requirement

P_t^o may not satisfy the capacity requirement any more. We will find the suboptimal transmit power from (9a) as follows,

$$P_t^\star = \max\{P_{t,1}, \cdots, P_{t,I}\},$$

where $P_{t,i} = \frac{KD^{\star 2}\sigma^2(2^{\frac{R_i}{\Delta f k_i^\star D^\star}}-1)}{\mu_i m^\star N}$.

V. SIMULATION RESULTS

In this section, we will demonstrate the performance of the proposed energy-efficient algorithm and study the impact of the capacity requirement, circuit power consumption and subcarrier allocation. System parameters are listed in Table II. The maximum transmit power, the efficiency of power amplifier, and the range of circuit power are configured as in [3]. In the simulation, the capacity requirements from all users, $\{R_i\}_{i=1}^I$, are set to the same value and denoted as R_{max} in the figures.

Figure 1 shows the number of active RF chains found by the proposed algorithm versus the capacity requirement with different circuit powers. It can be observed that the number of active RF chains increases with the capacity requirement. With the increase of the circuit power, the number of active

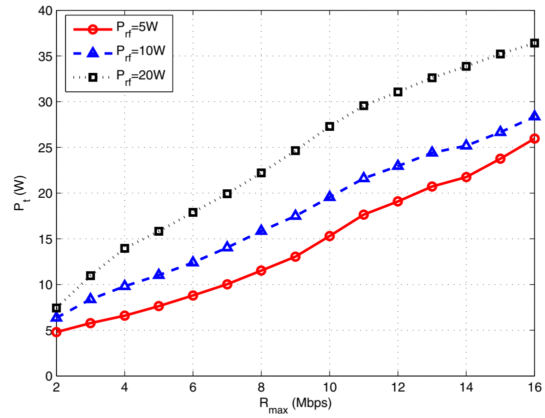

Fig. 2. Overall transmit power vs. capacity requirement

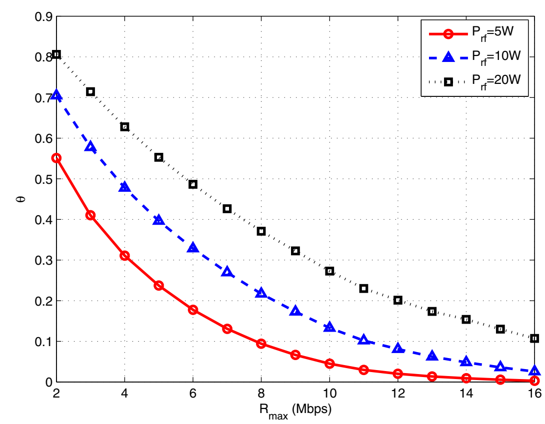

Fig. 3. Power saving gain vs. capacity requirement, where $\theta = \frac{P_{all}-P_t^\star}{P_{all}}$ and P_{all} is the power consumption when all RF chains are open.

RF chains decreases.

Figure 2 shows the overall transmit power versus the capacity requirement with different circuit powers. It is shown that the overall transmit power increases with the capacity requirement as well as the circuit power. The opposite trend with the circuit power implies that a tradeoff exists between the optimal number of active RF chains and the optimal overall transmit power.

To show the power saved by switching off RF chains, we compare the performance of the proposed scheme with that of all RF chains open at the BS. Fig. 3 shows the power saving gain versus the capacity requirement with different circuit power consumption. We can see that the power saving gain reduces with the capacity requirement and increases with the circuit power. For example, when $R_{max} = 4$ Mbps, the power saving gain is up to 60% when $P_{rf} = 20$ W.

Adaptively allocating the number of subcarriers to each user based on channel gains and capacity requirements helps to save more power. To show this, we compare the outage probability when adaptive subcarrier allocation is used with that when uniform subcarrier allocation, i.e. $k_i = K/I$, is used. The outage probability is the probability that there is no feasible solution for the problem (9). As shown in Fig.

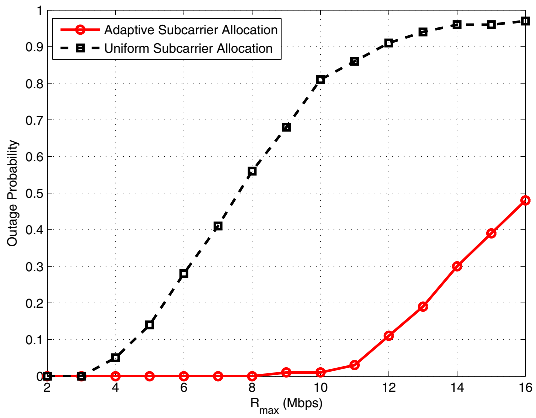

Fig. 4. Outage probability vs. capacity requirement

4, the outage probabilities in both cases increase with the capacity requirement. For the same outage probability, higher ergodic capacity requirement can be met by using the adaptive subcarrier allocation. In other words, less power is needed to achieve the same ergodic capacity requirement.

VI. Conclusion

In this paper, we studied the configuration of active RF chains and resource allocation from the perspective of EE for downlink MIMO-OFDMA systems. We first formulated the optimization problem to minimize the total power consumed at the BS with the constraints on the maximum transmit power and each user's upper bound of ergodic capacity. Then a two-step algorithm was proposed, which first found the continuous variable solutions and then discretized them to obtain the number of active RF chains and the number of subcarriers allocated to each user. Simulation results have shown that both the number of active RF chains and the overall transmit power increase with the capacity requirement. Moreover, there is a tradeoff between the number of active RF chains and the overall transmit power, which depends on the circuit power consumed by each RF chain. The proposed algorithm can save up to 60% power compared with the all-on RF chain scheme when the required capacity is 4 Mbps and the circuit power consumed by each RF chain is 20 W. The adaptive subcarrier allocation consumes less power than the uniform subcarrier allocation for a given ergodic capacity requirement and a given outage probability.

Appendix A
Derivation of the variance of λ_{ijd}

According to the property of singular value decomposition, we have

$$\sum_{d=1}^{D} \lambda_{ijd}^2 = \mathbf{Tr}(\mathbf{H}_{ij}\mathbf{H}_{ij}^H), \tag{16}$$

where $\mathbf{Tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$. After taking expectation over both sides of ths equation, we obtain

$$\mathbb{E}[\sum_{d=1}^{D} \lambda_{ijd}^2] = \mathbf{Tr}(\mathbb{E}[\mathbf{H}_{ij}\mathbf{H}_{ij}^H]). \tag{17}$$

Since all the singular values have the same variance due to the i.i.d property of all the elements in $\mathbf{H}_{ij}$, we have

$$\mathbb{E}[\lambda_{ijd}^2] = \mathbf{Tr}(\mathbb{E}[\mathbf{H}_{ij}\mathbf{H}_{ij}^H])/D = \mu_i mN/D. \tag{18}$$

Appendix B
Derivation of the solution for subproblem (S1)

From $mP_t = \psi$ in (14a), we know $P_t = \frac{\psi}{m}$. After substituting it into problem (S1), the problem turns into

$$\min_{m,P_t} \quad \frac{\psi}{\rho m} + mP_{rf} \tag{19}$$

$$\text{s. t.} \quad N \leq m \leq M \tag{19a}$$

$$m \geq \frac{\psi}{P_{max}}. \tag{19b}$$

When $\psi \leq MP_{max}$, this problem is feasible. Constraints (19a) and (19b) can be combined as follows,

$$N_{eq} \leq m \leq M, \tag{20}$$

where $N_{eq} = \max\{\frac{\psi}{P_{max}}, N\}$.

It is readily shown that the objective function is minimized when $m = \sqrt{\psi/\rho P_{RF}}$. When $m > \sqrt{\psi/\rho P_{RF}}$, it monotonically increases with m. When $m < \sqrt{\psi/\rho P_{RF}}$, it monotonically decreases with m.

Based on this property and the constraint (20), we can find the optimal m as follows,

$$m^o = \begin{cases} \sqrt{\frac{\psi}{\rho P_{rf}}}, \psi \leq MP_{max} \text{ and } N_{eq} \leq \sqrt{\frac{\psi}{\rho P_{rf}}} \leq M \\ M, \quad \psi \leq MP_{max} \text{ and } \sqrt{\frac{\psi}{\rho P_{rf}}} > M \\ N_{eq}, \quad \psi \leq MP_{max} \text{ and } \sqrt{\frac{\psi}{\rho P_{rf}}} < N_{eq}. \end{cases}$$

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