Secondary Transceiver Design in the Presence of Frequency Offset between Primary and Secondary Systems

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***Abstract*—When both primary and secondary systems are orthogonal frequency division multiplexing modulated and are non-cooperative, carrier frequency offset between the systems is inevitable to cause harmful interference. In this paper, we jointly optimize secondary transceivers assuming that the frequency offset between the secondary transmitter (ST) and the primary receiver (PR) and different channel information from the ST to the PR are known at the ST. We first derive unified interference constraints and obtain the secondary transceivers minimizing the mean square error through convex optimization techniques. We then derive closed-form transceivers for several special cases to reveal the impact of the frequency offset on the secondary transceivers. We show that when there is no frequency offset between the ST and the PR, the optimal processing at the ST is power allocation. Otherwise, both power allocation and precoding are necessary. The impact of the frequency offset on the performance of both systems increases as the interference constraints become tighter and the bandwidth of the primary system becomes smaller. When the proposed transceivers are used, the performance of the secondary system is robust to the frequency offset and the performance of the primary system degrades little due to the remanent frequency offset.**

***Index Terms*—Frequency offset, Orthogonal Frequency Division Multiplexing (OFDM), underlay, cognitive radio, channel state information (CSI).**

I. INTRODUCTION

COGNITIVE radio (CR) is a promising technology to meet the increasing demand of wireless communication services by reusing the allocated spectrum efficiently [1]. Among various spectrum sharing strategies, the underlay mode is an attractive strategy that secondary users can use the spectrum concurrently with the primary users provided that the secondary transmission does not cause performance degradation to the primary system [2].

Orthogonal Frequency Division Multiplexing (OFDM) is a competitive candidate of CR transmission schemes due to its high spectrum flexibility [3, 4]. On the other hand, OFDM techniques are applied in many existing and future wideband systems [5]. Since in general the primary users may not be OFDM modulated, most works in the literature investigate various issues in the design of overlay CR OFDM transceivers (see [3, 4, 6] and references therein), in which the CR users vacate the sub-bands where primary systems are active. The only *a priori* knowledge for the CR system design is the central frequency and bandwidth of primary system, and the interference to primary system is modeled as white noise. Nevertheless, in many practical scenarios much more information of primary systems is available, e.g., when CR systems coexist with commercial communication systems complying to some standards [4] or when CR systems have more advanced spectrum sensing abilities [7]. If we know that primary systems use OFDM transmission and we know their system parameters, it is unnecessary for OFDM CR systems to vacate the subcarriers occupied by primary systems. Instead, when some features of primary systems are known, we can exploit the interference structure to reduce the interference to primary systems and improve the performance of CR systems [8].

Recently, capacities of CR systems over flat fading channels with various interference constraints are analyzed in [9–11], which show the opportunities to enhance secondary systems due to the fluctuating interference channels. These results imply that a CR system can coexist with a primary system in the same spectrum band over flat fading channels through judicious design of power allocation. Moreover, the results can be extended to an OFDM secondary system coexisting with an OFDM primary system over frequency selective channels, when interference constraints are imposed on each subcarrier of the primary receiver (PR) and the primary and secondary systems are synchronous.

Most recent works [12–15] design the power allocation algorithms for CR systems operating in the sideband of primary systems assuming their modulation unknown. If we assume that the primary system is OFDM modulated, and the CR system has the same subcarrier spacing as the primary system and is perfectly synchronized to the primary system, we can optimize the power or subcarrier allocation for the OFDM secondary system that coexists with the OFDM primary system in an underlay way as did in [16]. However, although symbol timing synchronization may be possible in practice, carrier frequency synchronization is hard to achieve between the primary system and the secondary system when both of them are OFDM modulated and they are non-cooperative. It

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is well known that OFDM systems are very sensitive to the carrier frequency offset [5, 17]. Thereby it is critical to design the OFDM-based secondary system taking into account the frequency offset between primary and secondary transceivers. As far as the authors known, this has not been addressed in the literature.

In this paper, we consider that both primary and secondary systems are OFDM-based time division duplexing (TDD) systems, where the carrier frequency offset exists between their transceivers. We assume that the secondary transmitter (ST) knows the training sequences, the central frequency and the subcarrier spacing used by the primary system [4]. The ST can use the received training sequences that are transmitted by the PR to achieve the symbol timing synchronization to the PR and to estimate the frequency offset between the ST and the PR. This scenario may appear when the primary system is an orthogonal frequency division multiple access based cellular system, such as WiMAX and LTE [18, 19]. We consider two types of channel state information (CSI) of the interference channel from the ST to the PR, the instantaneous CSI and the statistical CSI, which can be also obtained at the ST by using the received training sequences.

Our basic idea is similar to that of the pre-whitening method in [8] which aims at sharing frequency spectrum between multi-antenna primary and secondary systems. The ST pre-compensates the intercarrier interference (ICI) induced by the frequency offset between the ST and the PR such that it does not cause harmful interference to the PR. The secondary receiver (SR) then adjusts itself based on the pre-processed transmit signal to improve its detection performance.

To achieve this goal, we design linear transceivers for the secondary system in the presence of the frequency offset between the ST and the PR using the minimum mean square error (MMSE) criterion under the transmission power constraint and the interference constraints. MMSE criterion is a useful alternative to that of maximizing capacity. When the specific signal constellation and coding schemes are given, it can optimize the combined effects of high data rate and low bit error rate (BER) [20, 21]. We first develop the interference constraints at the PR when both the frequency offset and two kinds of CSIs are taken into account. Then we formulate the linear transceiver design as a convex optimization problem, using the method proposed in [20]. Next, we derive closed-form pre-processors at the ST and post-processors at the SR in two special cases to analyze the impact of the frequency offset on the secondary transceiver structures and the secondary system performance. We show that the optimal pre-processor is power allocation when no frequency offset exists between the ST and the PR and the interference at the SR is white. Otherwise, the ST needs to use both power allocation and precoding. We use simulations to verify our analysis and demonstrate the impact of the frequency offset on the performance of both systems. The analysis shows that when the secondary system only uses power allocation, its performance degrades evidently with the increase of the frequency offset if its bandwidth is larger than that of the primary system. The performance of the primary system degrades as well. On the other hand, when the proposed transceivers are used, the frequency offset will cause neglectable performance degradation of both the primary and secondary systems.

TABLE I
LIST OF VARIABLES AND PARAMETERS

f_s	Subcarrier spacing in the OFDM network
δ_f	Frequency offset between the ST and the PR
$\mathbf{\Delta_f}$	Frequency offset matrix
ϵ_f	Residual frequency offset between the ST and the PR
N_s	Subcarrier number in the secondary system
N_p	Subcarrier number in the primary system
$\mathbf{\Gamma}$	Set of subcarrier positions for the primary system
$\mathbf{F}$	Fourier transform matrix
$\mathbf{B}$	Pre-processing matrix
$\mathbf{G}$	Post-processing matrix
$\mathbf{U}$	Transmit correlation matrix
$\mathbf{Q}$	Transmit precoding matrix
$\mathbf{\Lambda_p}$	Diagonal matrix with elements $\{P_i\}_{i=0}^{N_s-1}$
$\mathbf{d}$	Transmit data
$\mathbf{x_s}$	Transmit signal in time domain
$\mathbf{x_s^f}$	Transmit signal in frequency domain
$\mathbf{y_s}$	Receive signal in time domain
P_t	Total transmit power of the secondary system
P^{ICSI}	Interference threshold when instantaneous CSI is known
P^{SCSI}	Interference threshold when statistical CSI is known
P^{th}	Interference threshold for the primary system
$u_i^{s,f}$	Interference on the ith subcarrier of the primary system
$\mathbf{u_p}$, $\mathbf{u_s}$	Interference from the primary system or from the secondary system
$\mathbf{h_{pp}}$, $\mathbf{h_{ss}}$, $\mathbf{h_{sp}}$	Channel vectors from the PT to the PR, from the ST to the SR, or from the ST to the PR
$\mathbf{H_{ss}}$, $\mathbf{H_{sp}}$	Channel matrixes consisting of the channel coefficients of $\mathbf{h_{ss}}$ or $\mathbf{h_{sp}}$
$\lambda_\mathbf{k}^\mathbf{ss}$, $\lambda_\mathbf{k}^\mathbf{sp}$	Frequency response on the kth subcarrier of the channels $\mathbf{h_{ss}}$ or $\mathbf{h_{sp}}$
$\mathbf{\Lambda_{ss}}$, $\mathbf{\Lambda_{sp}}$	The eigenvalue diagonal matrix of $\mathbf{H_{ss}}$ or $\mathbf{H_{sp}}$
$\mathbf{R_{sp}}$	The covariance matrix of $\mathbf{h_{sp}}$
L_{pp}, L_{ss} L_{sp}	Channel length of $\mathbf{h_{pp}}$, $\mathbf{h_{ss}}$ or $\mathbf{h_{sp}}$
$\mathbf{n}$	Noise at the SR
$\tilde{\mathbf{n}}$	Total noise and interference at the SR
$\mathbf{R_{\tilde{n}}}$	Covariance matrix of $\tilde{\mathbf{n}}$

The rest of the paper is organized as follows. In Section II, we describe the system model and derive unified interference constraints based on different CSI. The optimization problem is formulated in Section III and the impact of the frequency offset between the ST and the PR on the secondary transceiver design is analyzed in Section IV. The interference to the PR due to the frequency offset is analyzed in Section V, and the simulation results are given in Section VI. Finally, we conclude the paper in Section VII.

Main parameters and variables used in this paper are listed in Table I.

II. SYSTEM MODEL

We consider an OFDM-based secondary system coexisting with an OFDM-based primary system with the underlay strategy, as shown in Fig. 1. We assume that the subcarrier spacings of the two systems are identical. The numbers of subcarriers used by the primary and secondary systems are N_p and N_s, respectively, where $N_p \leq N_s$. This indicates that

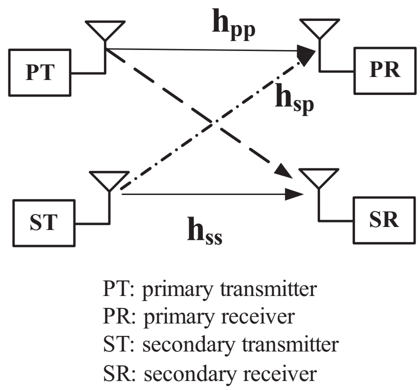

Fig. 1. Network structure consisting of a primary link and a secondary link.

we allow the secondary system to use wider bandwidth than the primary system. We assume that both systems are TDD systems. This means that the ST can operate as a receiver and the PR can operate as a transmitter, such that the ST can overhear the training sequences from the PR.

Carrier frequency offset is a major impairment of OFDM systems [5]. In an OFDM-based cognitive network, except for the frequency offset between secondary transceivers, the frequency offset between the ST and the PR also introduces ICI. Through judicious design of the training sequences, the primary and secondary systems can estimate and compensate the frequency offset between their own transceivers at their receivers [4]. However, the frequency synchronization between different systems is hard to achieve. Moreover, as we will show in *Theorem 2* of Section IV, ICI still remains even after the ST synchronizes to the PR in general channel conditions.

In this paper, we consider the frequency offset between the ST and the PR, δ_f. We assume that it can be estimated at the ST when the ST overhears the training sequence that the PR transmits toward the primary transmitter (PT). It can then be pre-compensated at the ST when the ST transmits to the SR. In order to highlight the impact of δ_f on the secondary system design, we assume that primary and secondary systems are perfectly synchronized between their own transceivers in both symbol timing and carrier frequency. We also assume that the time difference of the received signals from the ST and the PT at the PR is less than a cyclic prefix (CP) of the OFDM symbol[1].

Let $\mathbf{h_{pp}}$, $\mathbf{h_{ss}}$ and $\mathbf{h_{sp}}$ denote the frequency selective channels from the PT to the PR, from the ST to the SR and from the ST to the PR, with the numbers of resolvable paths being L_{pp}, L_{ss} and L_{sp}, respectively. We assume that $\mathbf{h_{pp}}$ is perfectly known by the PR and $\mathbf{h_{ss}}$ is perfectly known by secondary transceivers. We further assume that the instantaneous interference channel information, $\mathbf{h_{sp}}$, is known by the ST. This can be obtained when the ST overhears the training signals transmitted from the PR. We also consider the case when the statistical CSI, the covariance matrix $\mathbf{R_{sp}}$, is known at the ST, since it is easier to obtain in practice. Based on the known channel information, the secondary transceivers will be jointly designed to meet both the transmission power constraint at the ST and the interference constraints at the PR.

[1]The ST can synchronize to the PT in symbol timing when it overhears the transmitted training signals from the PT. Then the assumption will be valid when the propagation time difference between the PT-PR link and the ST-PR link is less than the duration of the CP, which is usually the case in LTE systems.

A. Signal Model of Secondary Transceivers

At the ST, the data symbols $d_0, d_1, \cdots, d_{N_s-1}$ are first serial-parallel converted and pre-processed by a matrix $\mathbf{B}$. After its output signal $\mathbf{x_s^f}$ passing an inverse discrete Fourier transform (DFT) and inserting a CP, an OFDM symbol is generated. An OFDM symbol without CP can be expressed as

$$\mathbf{x_s} = \mathbf{F}^H \mathbf{x_s^f} = \mathbf{F}^H \mathbf{B}\mathbf{d}, \tag{1}$$

where $\mathbf{d} \triangleq [d_0, d_1, \cdots, d_{N_s-1}]^T$, $\mathbf{F}$ is the DFT matrix with elements $[\mathbf{F}]_{mn} = \frac{1}{\sqrt{N_s}} e^{-j2\pi mn/N_s}$, $m, n = 0, 1, \cdots, N_s - 1$ and $\mathbf{F}^H$ denotes the Hermitian matrix of $\mathbf{F}$.

Assuming that $\mathbb{E}_\mathbf{d}[\mathbf{d}\mathbf{d}^H] = \mathbf{I}_{N_s}$, where $\mathbb{E}_\mathbf{d}[\cdot]$ denotes the expectation over $\mathbf{d}$, the transmission power constraint can be expressed as

$$\mathbb{E}_\mathbf{d}[\mathbf{Tr}(\mathbf{x_s}\mathbf{x_s}^H)] = \mathbf{Tr}(\mathbf{B}\mathbf{B}^H) \le P_t, \tag{2}$$

where $\mathbf{Tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$.

When the secondary transceivers are synchronous in both symbol timing and carrier frequency, the discrete received OFDM signal after removing CP is

$$\mathbf{y_s} = \mathbf{H_{ss}}\mathbf{x_s} + \mathbf{u_p} + \mathbf{n} = \mathbf{H_{ss}}\mathbf{F}^H\mathbf{B}\mathbf{d} + \tilde{\mathbf{n}}, \tag{3}$$

where $\mathbf{u_p}$ denotes the interference signal from the PT to the SR, $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2)$, i.e., $\mathbf{n}$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2, and $\tilde{\mathbf{n}} \triangleq \mathbf{u_p} + \mathbf{n}$ represents the total interference and noise at the SR. $\mathbf{H_{ss}}$ is an $N_s \times N_s$ circulant matrix whose first column is $[h_0^{ss}, \cdots, h_{L_{ss}-1}^{ss}, 0, \cdots, 0]^T$ and $\{h_i^{ss}\}_{i=0}^{L_{ss}-1}$ are the elements of $\mathbf{h_{ss}}$. $\mathbf{H_{ss}}$ can be decomposed as $\mathbf{H_{ss}} = \mathbf{F}^H \mathbf{\Lambda_{ss}} \mathbf{F}$ where the diagonal entries of diagonal matrix $\mathbf{\Lambda_{ss}}$ are the frequency responses of $\mathbf{h_{ss}}$ [22, Chap. 3].

The SR uses a linear post-processor in frequency domain to detect the transmitted data, i.e.,

$$\widehat{\mathbf{d}} = \mathbf{G}\mathbf{F}\mathbf{y_s},$$

where $\mathbf{G}$ is an $N_s \times N_s$ matrix.

B. Interference Constraints at the PR

To protect the primary system, the interference at the PR should be lower than a certain threshold. A reasonable constraint required by an OFDM-based primary system is to restrict the interference power on each subcarrier in use.

When the frequency offset exists between the ST and the PR, the discrete interference signal in time domain received by the PR can be expressed as [22, Chap. 4]

$$u_n^s = \frac{1}{\sqrt{N_s}} e^{j2\pi \frac{n\delta_f}{N_s f_s}} \sum_{k=0}^{N_s-1} \lambda_k^{sp} x_k^{s,f} e^{j2\pi \frac{nk}{N_s}}, \ n = 0, \cdots, N_s - 1,$$

where $x_k^{s,f}$ is the kth element of $\mathbf{x_s^f}$, λ_k^{sp} denotes the frequency response value of $\mathbf{h_{sp}}$ on the kth subcarrier, and f_s represents the subcarrier spacing.

Considering that $\mathbf{x_s^f} = \mathbf{Bd}$ and $\mathbf{\Lambda_{sp}} = \mathbf{FH_{sp}F}^H$, we can rewrite the interference signal as

$$\mathbf{u_s} = \mathbf{\Delta_f F}^H \mathbf{\Lambda_{sp} x_s^f} = \mathbf{\Delta_f H_{sp} F}^H \mathbf{Bd},$$

where $\mathbf{\Delta_f} = \mathrm{diag}\{1, e^{j2\pi\frac{\delta_f}{N_s f_s}}, \cdots, e^{j2\pi\frac{(N_s-1)\delta_f}{N_s f_s}}\}$, $\mathbf{\Lambda_{sp}} = \mathrm{diag}\{\lambda_0^{sp}, \lambda_1^{sp}, \cdots, \lambda_{N_s-1}^{sp}\}$ are diagonal matrices.

Thus, the interference imposed on the ith subcarrier of the primary system is

$$u_i^{s,f} = \mathbf{e}_i^H \mathbf{F u_s} = \mathbf{e}_i^H \mathbf{F \Delta_f H_{sp} F}^H \mathbf{Bd} \qquad \forall\, i \in \mathbf{\Gamma}, \tag{4}$$

where $\mathbf{\Gamma}$ denotes the set of subcarrier positions used by the primary system and $\mathbf{e}_i$ denotes a column vector with 1 in the ith position and 0 in other positions.

According to the different interference channel information that the ST can obtain, we consider the following two kinds of interference constraints at the PR:

1. When the instantaneous CSI, $\mathbf{h_{sp}}$, is available, the interference must satisfy

$$\mathbb{E}_{\mathbf{d}}[|u_i^{s,f}|^2] \leq P^{ICSI} \qquad \forall\, i \in \mathbf{\Gamma}. \tag{5}$$

Upon substituting (4) and after some manipulations, the interference constraints can be written as

$$\mathbf{Tr}(\mathbf{a}_i \mathbf{a}_i^H \mathbf{BB}^H) \leq P^{ICSI} \qquad \forall\, i \in \mathbf{\Gamma}, \tag{6}$$

where $\mathbf{a}_i = \mathbf{FH_{sp}^H \Delta_f^H F}^H \mathbf{e}_i$ and P^{ICSI} is the interference threshold when the instantaneous CSI is known.

2. When the statistical CSI, namely, the covariance matrix of $\mathbf{h_{sp}}$, $\mathbf{R_{sp}}$, is known, the interference constraints are

$$\mathbb{E}_{\mathbf{d},\mathbf{h_{sp}}}[|u_i^{s,f}|^2] \leq P^{SCSI} \qquad \forall\, i \in \mathbf{\Gamma}. \tag{7}$$

Upon substituting (4), we can derive the interference constraints as

$$\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{Tr}(\mathbf{a}_i \mathbf{a}_i^H \mathbf{BB}^H)] \leq P^{SCSI} \qquad \forall\, i \in \mathbf{\Gamma}. \tag{8}$$

After some manipulations (see Appendix A for details), the interference constraints are given by

$$\mathbf{Tr}(\mathbf{A}_i \mathbf{R_{sp}} \mathbf{A}_i^H \mathbf{BB}^H) \leq P^{SCSI} \qquad \forall\, i \in \mathbf{\Gamma}, \tag{9}$$

where the nth column of $N_s \times L_{sp}$ matrix $\mathbf{A}_i$ is $\mathbf{F\Pi}^{n-1,H} \mathbf{\Delta_f^H F}^H \mathbf{e}_i$, $\mathbf{\Pi} = [\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_{N_s-1}, \mathbf{e}_0]$ and P^{SCSI} is the interference threshold when the statistical CSI is known.

Define $\mathbf{\Psi}_i \triangleq \begin{cases} \mathbf{a}_i \mathbf{a}_i^H \\ \mathbf{A}_i \mathbf{R_{sp}} \mathbf{A}_i^H \end{cases}$ and $P^{th} \triangleq \begin{cases} P^{ICSI}, & \text{case 1} \\ P^{SCSI}, & \text{case 2} \end{cases}$, then (6) and (9) can be expressed in a unified form

$$\mathbf{Tr}(\mathbf{\Psi}_i \mathbf{BB}^H) \leq P^{th} \qquad \forall\, i \in \mathbf{\Gamma}. \tag{10}$$

III. Problem Formulation and the Optimal Solution

In this Section, we will jointly design the pre-processing matrix $\mathbf{B}$ and the post-processing matrix $\mathbf{G}$ for the ST and the SR. First, an optimization problem is formulated based on the MMSE criterion. Then we obtain the expression of $\mathbf{G}$ and transform the original optimization problem into a semidefinite programming problem only with argument $\mathbf{B}$, which can be solved by the interior point method [23].

The estimation error of the data symbol is

$$\mathbf{e} = \widehat{\mathbf{d}} - \mathbf{d} = (\mathbf{GFH_{ss}F}^H\mathbf{B} - \mathbf{I}_{N_s})\mathbf{d} + \mathbf{GF\tilde{n}}.$$

where $\mathbf{I}_{N_s}$ denotes an $N_s \times N_s$ identity matrix.

Then, the mean square error (MSE) is

$$\begin{aligned}\mathbf{Tr}(\mathbb{E}[\mathbf{ee}^H]) = \mathbf{Tr}(&\mathbf{GF}(\mathbf{H_{ss}F}^H\mathbf{BB}^H\mathbf{FH_{ss}^H} + \mathbf{R_{\tilde{n}}})\mathbf{F}^H\mathbf{G}^H \\ &- \mathbf{GFH_{ss}F}^H\mathbf{B} - (\mathbf{GFH_{ss}F}^H\mathbf{B})^H + \mathbf{I}_{N_s}),\end{aligned} \tag{11}$$

where $\mathbf{R_{\tilde{n}}} \triangleq \mathbb{E}[\mathbf{\tilde{n}\tilde{n}}^H]$ is the covariance matrix of the total interference and noise at the SR.

Considering the constraints (2) and (10), the problem to jointly design $\mathbf{B}$ and $\mathbf{G}$ based on the MMSE criterion can be formulated as

$$\begin{aligned}\min_{\mathbf{B},\mathbf{G}} \quad & \mathbf{Tr}(\mathbb{E}[\mathbf{ee}^H]) \\ \text{s.t.} \quad & \mathbf{Tr}(\mathbf{BB}^H) \leq P_t \\ & \mathbf{Tr}(\mathbf{\Psi}_i \mathbf{BB}^H) \leq P^{th} \qquad \forall\, i \in \mathbf{\Gamma}.\end{aligned} \tag{12}$$

By minimizing the objective function with respect to $\mathbf{G}$ when $\mathbf{B}$ is given , we can easily obtain

$$\mathbf{G} = \mathbf{B}^H\mathbf{FH_{ss}^H}(\mathbf{H_{ss}F}^H\mathbf{BB}^H\mathbf{FH_{ss}^H} + \mathbf{R_{\tilde{n}}})^{-1}\mathbf{F}^H, \tag{13}$$

where $\mathbf{A}^{-1}$ denotes the inverse of $\mathbf{A}$.

After substituting (13) into (11) and defining a transmit correlation matrix $\mathbf{U} \triangleq \mathbf{BB}^H$, the optimization problem (12) becomes

$$\begin{aligned}\min_{\mathbf{U}} \quad & \mathbf{Tr}(\mathbf{R_{\tilde{n}}}(\mathbf{H_{ss}F}^H\mathbf{UFH_{ss}^H} + \mathbf{R_{\tilde{n}}})^{-1}) \\ \text{s.t.} \quad & \mathbf{Tr}(\mathbf{U}) \leq P_t \\ & \mathbf{Tr}(\mathbf{\Psi}_i \mathbf{U}) \leq P^{th} \qquad \forall\, i \in \mathbf{\Gamma} \\ & \mathbf{U} \succeq 0.\end{aligned} \tag{14}$$

By using Schur's complement method introduced in [20], the nonconvex problem shown in (14) can be transformed into the following semidefinite programming problem

$$\begin{aligned}\min_{\mathbf{W},\mathbf{U}} \quad & \mathbf{Tr}(\mathbf{R_{\tilde{n}}W}) \\ \text{s.t.} \quad & \mathbf{Tr}(\mathbf{U}) \leq P_t \\ & \mathbf{Tr}(\mathbf{\Psi}_i \mathbf{U}) \leq P^{th} \qquad \forall\, i \in \mathbf{\Gamma} \\ & \begin{bmatrix} \mathbf{W} & \mathbf{I}_{N_s} \\ \mathbf{I}_{N_s} & \mathbf{H_{ss}F}^H\mathbf{UFH_{ss}^H} + \mathbf{R_{\tilde{n}}} \end{bmatrix} \succeq 0 \\ & \mathbf{U} \succeq 0.\end{aligned} \tag{15}$$

We can obtain the optimal $\mathbf{U}$ by solving the problem efficiently using the primal-dual interior point method. The computational complexity is approximately $O({N_s}^{6.5}\log(1/\varepsilon))$, where ε is the solution accuracy [23]. Let the eigenvalue decomposition of $\mathbf{U}$ be $\mathbf{Q\Lambda}_p\mathbf{Q}^H$, then we can obtain the optimal pre-processing matrix $\mathbf{B} = \mathbf{Q\Lambda}_p^{1/2}$, where $\mathbf{Q}$ is the precoding matrix and $\mathbf{\Lambda}_p$ is the power allocation matrix. The corresponding optimal post-processor matrix $\mathbf{G}$ can then be computed by substituting $\mathbf{B}$ into (13).

IV. The Impact of Frequency Offset on Secondary Transceiver Design

To gain more insight into the transceiver structures when the frequency offset between the ST and the PR exists, we will find the closed-form solutions of the problem (15) in some special cases. For comparison, we will first provide the optimal structure of the secondary transceivers when $\delta_f = 0$. We will then study the impact of δ_f on the structures and performance of the secondary system since we can only obtain numerical solutions in general cases.

A. No Frequency Offset Exists between the ST and the PR

When there is no frequency offset between the ST and the PR, the optimal structure of the secondary system is given by the following theorem.

Theorem 1: When $\delta_f = 0$ and the interference at the SR is white, the optimal transmit correlation matrix $\mathbf{U}$ is a diagonal matrix and the optimal precoder matrix $\mathbf{Q} = \mathbf{I}_{N_s}$.

Proof: The proof is shown in Appendix B. ■

Remark 1: Since the optimal precoder matrix $\mathbf{Q} = \mathbf{I}_{N_s}$, we can obtain the optimal linear pre-processor as $\mathbf{B} = \mathbf{\Lambda}_p^{1/2}$. It is not hard to derive the optimal linear post-processing matrix as $\mathbf{G} = \mathbf{\Lambda}_p^{1/2}\mathbf{\Lambda}_{ss}^H(\mathbf{\Lambda}_{ss}\mathbf{\Lambda}_p\mathbf{\Lambda}_{ss}^H + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}$ from (13), where $\sigma_{\tilde{n}}^2$ is the variance of the total interference at the SR. This indicates that when there is no frequency offset between the ST and the PR, the optimal processing of the secondary system is to allocate power on each subcarrier at the transmitter and to use one-tap MMSE equalization at the receiver, which is the same as the processing in traditional OFDM systems without interference constraints [20]. The optimal power allocation solution in this case is multi-level water filling, which is discussed in [24].

B. Frequency Offset Exists between the ST and the PR

When there exists frequency offset between the ST and the PR, the closed-form solutions of the problem (15) cannot be obtained in general cases. Here we consider two special cases.

Theorem 2: When both the channel from the ST to the SR and the channel from the ST to the PR are flat fading, and the interference at the SR is white, the optimal precoder matrix $\mathbf{Q} = \mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H$, and the MSE of the secondary system does not depend on δ_f.

Proof: The proof is shown in Appendix C. ■

Remark 2: We can further derive the optimal pre-processor as $\mathbf{B} = \mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{\Lambda}_p^{1/2}$ and the optimal post-processor as $\mathbf{G} = h_{ss}^*\mathbf{\Lambda}_p^{1/2}(|h_{ss}|^2\mathbf{\Lambda}_p + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H$, where h_{ss} is the coefficient of the flat fading channel from the ST to the SR. Consequently, the transmitted symbol is $\mathbf{x_s} = \mathbf{F}^H\mathbf{B}\mathbf{d} = \mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{\Lambda}_p^{1/2}\mathbf{d}$. It indicates that the ST allocates the power on each subcarrier, and then pre-corrects the frequency offset between the ST and the PR to meet the interference constraints, i.e., the ST adjusts its transmission signal to pre-synchronize its carrier frequency to the PR. Again, the power allocation is the multi-level water filling as in [24]. The data symbol estimated by the SR is $\widehat{\mathbf{d}} = \mathbf{\Lambda}_p^{1/2}(\mathbf{\Lambda}_p + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{y_s}$. Since we assume that no frequency offset exists between the oscillators of ST and SR, the SR only needs to correct the frequency offset caused by the pre-correction at the ST. Afterwards, a one-tap MMSE equalizer is applied. Comparing with the result in *Theorem 1*, we can observe that the ST needs to first pre-synchronize to the PR in its carrier frequency before transmission and then the SR needs to synchronize to the ST.

When either the channel from the ST to the SR or the channel from the ST to the PR is frequency selective fading, we can not come to the same conclusion as in *Theorem 2*. The optimal precoder matrix $\mathbf{Q}$ will be more complicated, and the performance of the secondary system will depend on δ_f. This can be observed from the results in another special scenario. Before providing the optimal precoder, we first introduce a lemma.

Lemma 1: When the interference threshold $P^{th} = 0$ or the large scale fading between the ST and the PR $\rho_{sp} \to \infty$, the precoder matrix $\mathbf{Q}$ lies in the null space of the interference space which is spanned by $\{\mathbf{a}_i\}_{i\in\Gamma}$ when the instantaneous CSI is known, or is spanned by $\{\mathbf{A}_i\}_{i\in\Gamma}$ when the statistical CSI is known, i.e., $\mathbf{a}_i^H\mathbf{Q} = 0$ or $\mathbf{A}_i^H\mathbf{Q} = \mathbf{0}$, $\forall\, i \in \mathbf{\Gamma}$.

Proof: The proof is shown in Appendix D. ■

For a special case where the number of subcarriers used by the primary system is $N_p = N_s - 1$ and the instantaneous CSI of $\mathbf{h_{sp}}$ is known, we can obtain the closed-form solution of $\mathbf{Q}$ and the MSE of the secondary system, which is shown in the following theorem.

Theorem 3: Assume that the instantaneous channel between the ST and the PR is known by the ST and the number of subcarriers used by the primary system $N_p = N_s - 1$. When the interference threshold $P^{th} = 0$ or the large scale fading between the ST and the PR $\rho_{sp} \to \infty$, the optimal precoder matrix

$$\mathbf{Q} = c\frac{\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}}{\|\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}\|}, \tag{16}$$

where i_0 is the index of the subcarrier not being used by the primary system and c is an arbitrary complex number with unit amplitude. If the interference at the SR is white, the MSE of the secondary system is

$$\mathbf{Tr}(\mathbb{E}[\mathbf{e}\mathbf{e}^H]) = N_s - 1 + \frac{1}{1 + \frac{P_t\|\mathbf{\Lambda_{ss}}\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}\|^2}{\sigma_{\tilde{n}}^2\|\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}\|^2}}. \tag{17}$$

Proof: The proof is shown in Appendix E. ■

Remark 3: It follows that the optimal precoder is no longer power allocation followed by frequency offset pre-compensation, and the MSE of the secondary system depends on δ_f. This is different from the impact of frequency offset between traditional OFDM transceivers on their performance[2].

V. The Interference to the Primary System Due to the Frequency Offset

In order to show the impact of the frequency offset between the ST and the PR on the primary system, we analyze the interference at the PR when $\delta_f \neq 0$ but the frequency offset is not considered during the secondary transceiver design.

[2]In a traditional OFDM system, after the frequency offset is perfectly estimated, its impact can be eliminated completely by the frequency correction at the receiver, thus the performance is independent of the frequency offset.

When the secondary system is designed as if there is no frequency offset, we know from *Theorem 1* that the optimal transmission scheme is power allocation, i.e., $\mathbf{U} = \mathbf{\Lambda_p}$. Then the average interference power on the ith subcarrier of the primary system with either instantaneous or statistical CSI known becomes

$$\begin{aligned}&\mathbb{E}_{\mathbf{d},\mathbf{h_{sp}}}[|u_i^{s,f}|^2] = \mathbb{E}_{\mathbf{h_{sp}}}\{\mathbb{E}_{\mathbf{d}}[|u_i^{s,f}|^2]\}\\ =&\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}\mathbf{H_{sp}}\mathbf{F}^H\mathbf{U}\mathbf{F}\mathbf{H_{sp}}^H\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i]\\ =&\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}\mathbf{F}^H\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{\Lambda_{sp}}\mathbf{\Lambda_p}\mathbf{\Lambda_{sp}}^H]\mathbf{F}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i\\ =&\frac{1}{N_s^2}\sum_{k=0}^{N_s-1}\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2P_k]\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))},\ i\in\mathbf{\Gamma}\end{aligned} \tag{18}$$

where $\eta_f = \delta_f/f_s$ is the normalized frequency offset and P_k is the kth diagonal element of $\mathbf{\Lambda_p}$.

When the secondary system treats δ_f as 0, the interference constraints used for its transceiver design under different CSI conditions can be expressed as follows.

When the instantaneous CSI of $\mathbf{h_{sp}}$ is known, the interference constraints (5) can be rewritten as

$$\begin{aligned}\mathbb{E}_{\mathbf{d}}[|u_i^{s,f}|^2]|_{\eta_f=0} &= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}|\lambda_k^{sp}|^2P_k\frac{\sin^2(\pi(k-i))}{\sin^2(\frac{\pi}{N_s}(k-i))}\\ &= |\lambda_i^{sp}|^2P_i \le P^{th},\ i\in\mathbf{\Gamma}.\end{aligned} \tag{19}$$

When the statistical CSI of $\mathbf{h_{sp}}$ is known, the interference constraints (7) can be rewritten as

$$\begin{aligned}\mathbb{E}_{\mathbf{d},\mathbf{h_{sp}}}[|u_i^{s,f}|^2]|_{\eta_f=0} &= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2]P_k\frac{\sin^2(\pi(k-i))}{\sin^2(\frac{\pi}{N_s}(k-i))}\\ &= \mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_i^{sp}|^2]P_i \le P^{th},\ i\in\mathbf{\Gamma}.\end{aligned} \tag{20}$$

Since the interference constraints are usually very tight, the equalities in (19) and (20) usually hold, i.e.

$$|\lambda_i^{sp}|^2P_i = P^{th}\ \text{ or }\ \mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_i^{sp}|^2]P_i = P^{th},\ i\in\mathbf{\Gamma}.$$

Further taking average over CSI on these two equalities with either the instantaneous or the statistic CSI known, we can obtain a unified expression as

$$\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_i^{sp}|^2P_i] = P^{th},\ i\in\mathbf{\Gamma}. \tag{21}$$

By using this expression in (18), we can analyze the impact of the bandwidths of the primary and secondary systems on the average interference power with different CSI known in a unified way.

When $N_p = N_s$, the average interference power on the ith subcarrier can be rewritten as

$$\begin{aligned}\mathbb{E}_{\mathbf{d},\mathbf{h_{sp}}}[|u_i^{s,f}|^2] &= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2P_k]\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}\\ &= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}P^{th}\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}\\ &= \mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}\mathbf{F}^HP^{th}\mathbf{I}_{N_s}\mathbf{F}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i = P^{th}.\end{aligned} \tag{22}$$

When $N_p < N_s$, the average interference power can be derived as shown in (23).

From Appendix B we know that when $\delta_f = 0$, the power allocated to the subcarriers that are not occupied by the primary system only depends on the total transmit power constraint P_t. Since P_t is much larger than P^{th}, the power allocated to the subcarriers not occupied by the primary system will be much larger than that on other subcarriers. This means that $\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_{k_1}^{sp}|^2P_{k_1}] > \mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_{k_2}^{sp}|^2P_{k_2}] = P^{th}$, when $k_1 \notin \mathbf{\Gamma}, k_2 \in \mathbf{\Gamma}$.

Since $\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2P_k] - P^{th} > 0$ when $k \notin \mathbf{\Gamma}$, and it is easy to show that $\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}$ is an increasing function of the normalized frequency offset η_f when it is small, we can see that the interference to the PR increases with η_f.

Now we come to the conclusion that when $N_p = N_s$ the interference power to the PR does not depend on the frequency offset, while when $N_p < N_s$ the interference power increases with the frequency offset.

Note that the problem formulation for the precoder design in this paper is similar to that for the case of the multi-antenna ST in [24]. Nonetheless, the interference patterns are very different. This can be observed from the interference constraints derived in Section II, as well as the interference energy shown in (23) which resembles the ICI in traditional OFDM systems [22]. Moreover, through the analysis in Section IV, we can find unique impact of the frequency offset on the structure and performance of the secondary system.

VI. Simulation Results

In this Section, we first simulate the performance of the primary system when the ST does not know δ_f to show the impact of the frequency offset on the primary system. We then evaluate the performance of the secondary system when the proposed optimal scheme from solving (15), the pre-correction scheme in *Theorem 2* and the power-allocation-only scheme in *Theorem 1* are used. Finally we show the effect of the residual frequency offset on the performance of the primary system when δ_f cannot be estimated at the ST perfectly.

In the simulations, we assume that the PT is far away from the SR and does not cause interference to the SR for simplicity.[3] In both primary and secondary systems, the received signal-to-noise-ratio (SNR) is set to be 20 dB, the noise variance is assumed to be identical, and BPSK modulation is employed. Because the complexity of solving problem (15) increases rapidly with the subcarrier numbers, we only simulate a primary system and a secondary system with small subcarrier numbers. Extensive simulation results show that the obtained conclusions do not change for the large subcarrier number case, which are omitted due to the lack of the space. To understand the impact of the bandwidth of the primary system on the performance, we consider two cases: $N_p = 4$, and $N_p = 16$. The subcarrier number in the secondary system, N_s, is set to 16 in all figures. We consider frequency selective channels with three resolvable paths, i.e., $L_{pp} = L_{ss} = L_{sp} = 3$. Specifically, the elements of $\mathbf{h_{pp}}$, $\mathbf{h_{ss}}$ and $\mathbf{h_{sp}}$ are independent and identically distributed (i.i.d) and subject to $\mathcal{CN}(0, \rho_{pp}/L_{pp})$, $\mathcal{CN}(0, \rho_{ss}/L_{ss})$ and

[3] If the interference at the SR exists and is not white, a whitening filter can be applied at the SR first and the interference can be transformed into an equivalent white noise. The filter can be incorporated into the channel matrix $\mathbf{H_{ss}}$ as did in [24].

$$\begin{aligned}
\mathbb{E}_{\mathbf{d},\mathbf{h_{sp}}}[|u_i^{s,f}|^2] &= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2 P_k]\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}\\
&= \frac{1}{N_s^2}\sum_{k\in\mathbf{\Gamma}}P^{th}\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}+\frac{1}{N_s^2}\sum_{k\notin\mathbf{\Gamma}}\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2 P_k]\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}\\
&= \frac{1}{N_s^2}\sum_{k=0}^{N_s-1}P^{th}\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}+\frac{1}{N_s^2}\sum_{k\notin\mathbf{\Gamma}}(\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2 P_k]-P^{th})\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}\\
&= P^{th}+\frac{1}{N_s^2}\sum_{k\notin\mathbf{\Gamma}}(\mathbb{E}_{\mathbf{h_{sp}}}[|\lambda_k^{sp}|^2 P_k]-P^{th})\frac{\sin^2(\pi(k-i+\eta_f))}{\sin^2(\frac{\pi}{N_s}(k-i+\eta_f))}. \qquad (23)
\end{aligned}$$

TABLE II
RELATION BETWEEN NIT AND DISTANCE

NIT(dB)	d_{sp}/d_{ss}
-10.0	2.51
-20.0	1.00
-30.0	0.40

$\mathcal{CN}(0, \rho_{sp}/L_{sp})$, respectively. ρ_{pp}, ρ_{ss} and ρ_{sp} are the large-scale fading gains of the corresponding channels. Without loss of generality, we assume $\rho_{pp} = \rho_{ss}$. The simulation results are averaged over 1000 Monte Carlo tests.

Once the SNR is given, the system performance only depends on the normalized interference threshold, which is defined as $NIT \triangleq \frac{N_s \rho_{ss} P^{th}}{\rho_{sp} P_t}$. In the simulations, it is assumed that the interference threshold P^{th} is equal to the variance of noise at the PR. Since the ratio ρ_{ss}/ρ_{sp} is related to NIT, we can observe the impact of the distance between the ST and the PR on the performance when the large-scale fading only comes from the path loss. Table II shows the relationship between NIT and the ratio of the distance between the ST and the PR, d_{sp} to the distance between the ST and the SR, d_{ss}, when the path loss factor is equal to 2.5.

We first analyze the performance of the primary system when δ_f is not pre-compensated by the ST and the instantaneous CSI is known at the ST. The result is similar when the statistical CSI is known and thus is omitted here. In this case, the secondary system is designed as if $\delta_f = 0$. We assume that the power is equally allocated to each subcarrier at the PT and the MMSE detector is used at the PR.

Figure 2 shows the BER of the primary system versus δ_f under different interference constraints. We also provide the result when there is no interference at the PR for reference, which is shown as *No Inf* in the legend. It is shown that the performance of the primary system degrades with the increase of δ_f when $N_p = 4$, whereas the performance is independent of δ_f when $N_p = 16$. This validates the interference power analysis in Section V. It is also shown that when NIT reduces and N_p becomes smaller, the BER will increase. The performance degradation with the increase of NIT can be explained as follows. When NIT becomes lower, i.e., the interference constraints get tighter, less power is allocated to the subcarriers occupied by the primary system and more power is allocated to other subcarriers under the sum power constraint. As a result, more interference is introduced to the

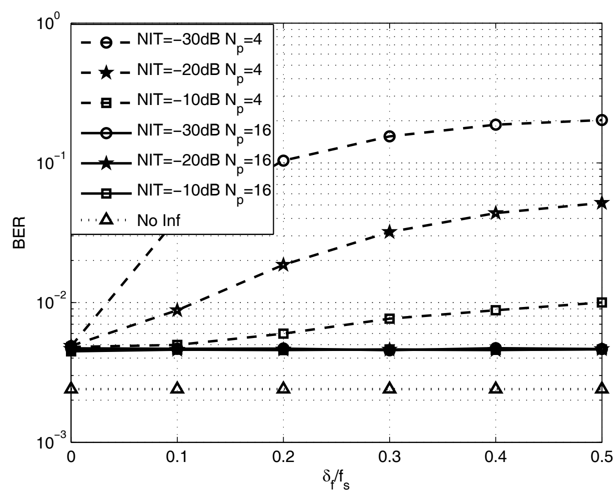

Fig. 2. BER of the primary system vs. δ_f/f_s when δ_f is not pre-compensated by the ST and the instantaneous CSI is known at the ST. The three curves with the condition $N_p = 16$ overlap.

PR when $\delta_f \neq 0$. The reason why the performance when $N_p = 16$ is better than that when $N_p = 4$ can also be found from the interference analysis in Section V. Comparing (22) with (23), we can see that the interference when $N_p < N_s$ is larger than that when $N_p = N_s$, which results in the performance degradation. The results imply that it is necessary to design a secondary system taking into account the frequency offset when the secondary system has larger bandwidth than the primary system.

We then analyze the impact of the frequency offset between the ST and the PR on the performance of the secondary system. The NIT is set to be -20dB. Fig. 3 shows the NMSE and BER of the secondary system when the instantaneous CSI is known. Here, we simulate the performance of the optimal scheme obtained from solving (15), the performance of the pre-correction scheme in *Theorem 2* and the performance of the power-allocation-only scheme in *Theorem 1*. The result is similar when the statistical CSI is known and thus is not shown here. We can see that when $\delta_f = 0$, the NMSE and BER of the optimal scheme are identical to those of the pre-correction scheme and the power-allocation-only scheme, which validates *Theorem 1*. When $\delta_f \neq 0$, the performance of the optimal scheme outperforms the other two schemes. The performance of the optimal scheme is almost immune to δ_f,

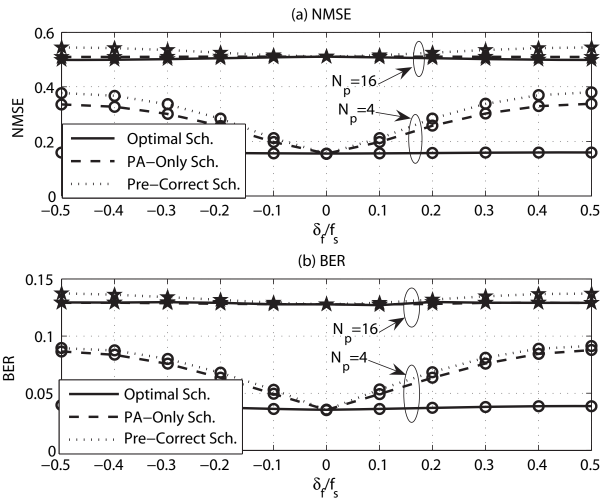

Fig. 3. Performance of the secondary system with different schemes when the instantaneous CSI is known and $NIT = -20dB$. *NMSE* means the normalized MSE. *Optimal Sch.*,*PA-Only Sch.* and *Pre-Correct Sch.* stand for the optimal scheme, power-allocation-only scheme and the pre-correction scheme, respectively. (a)NMSE vs. δ_f/f_s,(b)BER vs. δ_f/f_s.

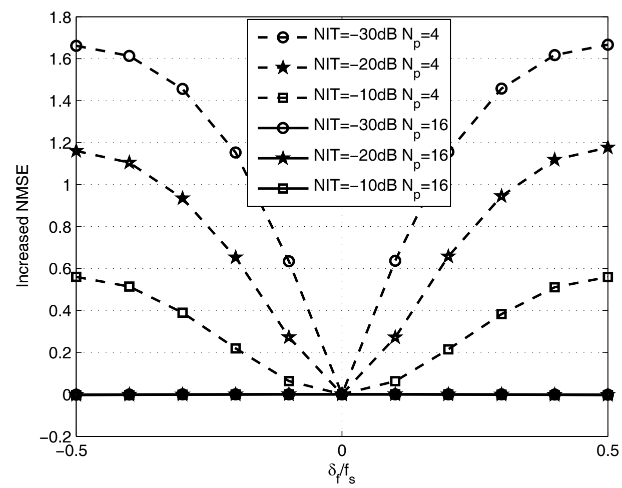

Fig. 5. Increased NMSE of the secondary system with the power-allocation-only scheme vs. δ_f/f_s for different values of NIT and N_p when the instantaneous CSI of $\mathbf{h_{sp}}$ is known. The three curves with the condition $N_p = 16$ overlap.

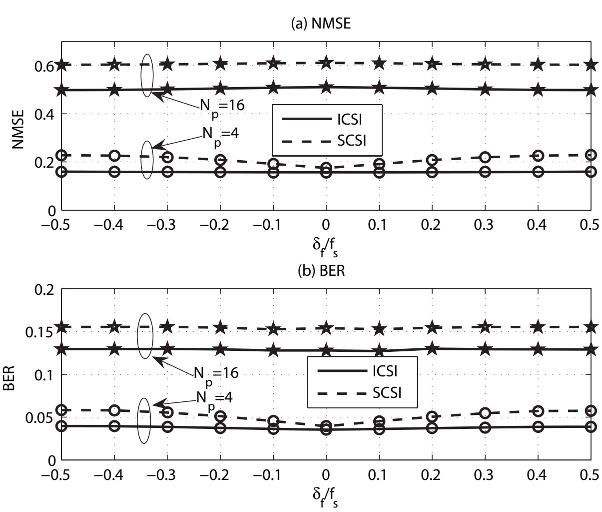

Fig. 4. Performance of the secondary system with the optimal schemes when different CSI of $\mathbf{h_{sp}}$ is known by the ST and $NIT = -20dB$. *ICSI* and *SCSI* respectively stand for the instantaneous CSI and statistical CSI. (a)NMSE vs. δ_f/f_s,(b)BER vs. δ_f/f_s.

while the performance of both the pre-correction scheme and the power-allocation-only scheme degrades with the increase of δ_f. As the bandwidth of the primary system N_p increases, the performance of the secondary system degrades since more interference constraints are introduced and the feasible region of problem (15) shrinks. We also compare the performance of the optimal scheme with different interference channel information known at the ST as shown in Fig. 4. We can see that the secondary system with the instantaneous CSI known outperforms that with the statistical CSI known in terms of NMSE and BER.

To further observe the performance degradation of the secondary systems when using the power-allocation-only scheme, we simulate the increased NMSE versus the frequency offset under different interference constraints as in Fig. 5. The increased NMSE is evaluated by $(\mathrm{NMSE}-\mathrm{NMSE}_0)/\mathrm{NMSE}_0$, where NMSE is the performance when $\delta_f \neq 0$ and NMSE_0 is the performance when $\delta_f = 0$. We can see that the increased NMSE becomes higher when the interference constraints are tighter and the bandwidth of the primary system is smaller. This is because when $\delta_f \neq 0$, the power allocated to more subcarriers of the secondary system will be limited by the interference constraints. Since the power-allocation-only scheme is only optimal when $\delta_f = 0$, its performance can reflect the impact of the frequency offset on the secondary system. This concludes that the secondary system is also sensitive to δ_f, if its transceiver design does not consider the frequency offset.

In practice, δ_f cannot be estimated perfectly and the residual frequency offset ϵ_f remains. Then the ST may cause interference to the PR even when the secondary system uses the proposed optimal transceivers since the interference constraints can no longer be met strictly. Now we simulate the impact of ϵ_f on the performance of the primary system when the secondary system uses the optimal precoder. The NIT is set to -30dB, since the primary system is very sensitive to δ_f in this scenario. As shown in Fig. 6, the performance of the primary system degrades with the increase of ϵ_f. We can also observe that the BER increases with the decrease of N_p, which is the same as the impact of δ_f on the performance of the primary system. However, since the residual frequency offset is usually kept at a very low level, say, $\epsilon_f/1000$, the performance loss is acceptable.

VII. Conclusion

In this paper, we have jointly designed the linear MMSE transceivers for an OFDM secondary system under both the transmission power constraint and the interference constraints, when the frequency offset between the ST and the PR exists and different types of interference channel information are known at the ST.

The optimal solutions in general cases can be obtained numerically by using convex optimization techniques. Closed-

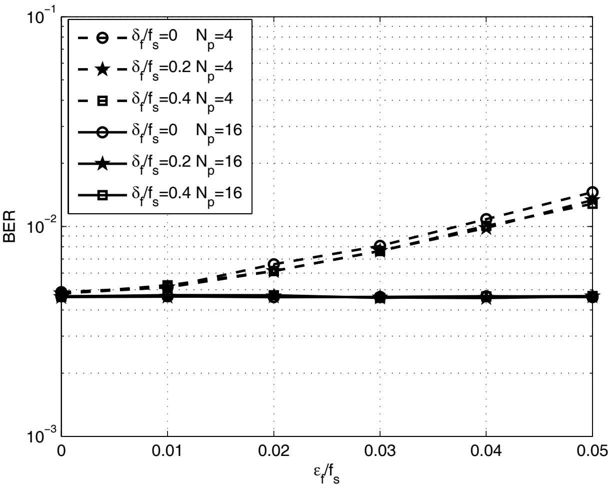

Fig. 6. BER of the primary system vs. ϵ_f/f_s with different δ_f when $NIT = -30dB$ and instantaneous CSI of $\mathbf{h_{sp}}$ is known.

form transceivers in several special cases are provided to reveal the impact of the frequency offset on the structure and performance of the secondary system. When there is no frequency offset between the ST and the PR, the optimal processers for the secondary system are multi-level water filling power allocation on each subcarrier at the ST and one-tap MMSE equalization at the SR. When the frequency offset between the ST and the PR exists, the secondary transmitter needs to use both power allocation and precoding.

It is shown that both the performance of the primary system and that of the secondary system degrade evidently with the increase of the frequency offset when the bandwidth of the primary system is smaller than that of the secondary system, if the frequency offset is not considered in the design of the secondary system. The sensitivity to the frequency offset increases as the interference constraints become tighter and the bandwidth of the primary system is smaller. Using the proposed transceivers, the performance of the primary system only degrades little by the frequency offset even when there is residual frequency offset due to imperfect estimation, and the secondary system is robust to the frequency offset. The performance of the secondary system with the instantaneous CSI known outperforms that with the statistical CSI known.

Appendix A
The Derivation of Expression (9)

Based on the structures of circulant matrices, the interference channel matrix can be expressed as

$$\mathbf{H_{sp}} = h_0^{sp}\mathbf{I}_{N_s} + h_1^{sp}\mathbf{\Pi} + ... + h_{L_{sp}-1}^{sp}\mathbf{\Pi}^{L_{sp}-1} = \sum_{n=0}^{L_{sp}-1} h_n^{sp}\mathbf{\Pi}^n, \tag{24}$$

where $\{h_i^{sp}\}_{i=0}^{L_{sp}-1}$ are the coefficients of $\mathbf{h_{sp}}$ and $\mathbf{\Pi} = [\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_{N_s-1}, \mathbf{e}_0]$.

When the statistical CSI is known, the left hand side of (8) can be rewritten as $\mathbf{Tr}(\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{a}_i\mathbf{a}_i^H]\mathbf{B}\mathbf{B}^H)$. Using (24), $\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{a}_i\mathbf{a}_i^H]$ can be derived as

$$\begin{aligned}
\mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{a}_i\mathbf{a}_i^H] &= \mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{F}\mathbf{H}_{\mathbf{sp}}^H\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}\mathbf{H_{sp}}\mathbf{F}^H] \\
&= \mathbb{E}_{\mathbf{h_{sp}}}[\mathbf{F}(\sum_{m=0}^{L_{sp}-1} h_m^{sp}\mathbf{\Pi}^m)^H\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}(\sum_{n=0}^{L_{sp}-1} h_n^{sp}\mathbf{\Pi}^n)\mathbf{F}^H] \\
&= \sum_{m=0}^{L_{sp}-1}\sum_{n=0}^{L_{sp}-1} [\mathbf{R_{sp}}]_{mn}\mathbf{F}\mathbf{\Pi}^{m,H}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta_f}\mathbf{\Pi}^n\mathbf{F}^H \\
&= \mathbf{A}_i\mathbf{R_{sp}}\mathbf{A}_i^H,
\end{aligned}$$

where $[\mathbf{R_{sp}}]_{mn} \triangleq \mathbb{E}[(h_m^{sp})^*(h_n^{sp})]$ and $\mathbf{A}_i$ is a $N_s \times L_{sp}$ matrix, whose nth column is $\mathbf{F}\mathbf{\Pi}^{n,H}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i$.

Then, the interference constraints when the statistical CSI is known can be expressed as

$$\mathbf{Tr}(\mathbf{A}_i\mathbf{R_{sp}}\mathbf{A}_i^H\mathbf{B}\mathbf{B}^H) \le P^{SCSI} \quad \forall\, i \in \mathbf{\Gamma}.$$

Appendix B
Proof of Theorem 1

Assume the covariance matrix of the total interference at the PR as $\mathbf{R_{\tilde{n}}} = \sigma_{\tilde{n}}^2\mathbf{I}_{N_s}$. By using $\mathbf{H_{ss}} = \mathbf{F}^H\mathbf{\Lambda_{ss}}\mathbf{F}$, the objective function in (14) can be simplified as $\mathbf{Tr}(\sigma_{\tilde{n}}^2(\mathbf{\Lambda_{ss}}\mathbf{U}\mathbf{\Lambda_{ss}}^H + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1})$.

Since $\mathbf{\Delta_f} = \mathbf{I}_{N_s}$ when $\delta_f = 0$ and $\mathbf{H_{sp}} = \mathbf{F}^H\mathbf{\Lambda_{sp}}\mathbf{F}$, the interference constraints with different types of CSI of $\mathbf{h_{sp}}$ known can be derived as follows:

1. When the instantaneous CSI of $\mathbf{h_{sp}}$ is known, $\mathbf{a}_i = \mathbf{F}\mathbf{H}_{\mathbf{sp}}^H\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_i = \mathbf{\Lambda}_{\mathbf{sp}}^H\mathbf{e}_i = (\lambda_i^{sp})^*\mathbf{e}_i$.
   Then, the interference constraints become
$$\begin{aligned}
\mathbf{Tr}(\mathbf{\Psi}_i\mathbf{U}) &= \mathbf{Tr}(\mathbf{a}_i\mathbf{a}_i^H\mathbf{U}) = \mathbf{Tr}(|\lambda_i^{sp}|^2\mathbf{e}_i\mathbf{e}_i^H\mathbf{U}) \\
&= |\lambda_i^{sp}|^2[\mathbf{U}]_{ii} \le P^{th} \qquad \forall\, i \in \mathbf{\Gamma}.
\end{aligned}$$
2. When the statistical CSI of $\mathbf{h_{sp}}$ is known,
$$\begin{aligned}
\mathbf{\Psi}_i &= \mathbf{A}_i\mathbf{R_{sp}}\mathbf{A}_i^H \\
&= \sum_{m=0}^{L_{sp}-1}\sum_{n=0}^{L_{sp}-1} [\mathbf{R_{sp}}]_{mn}\mathbf{F}\mathbf{\Pi}^{m,H}\mathbf{I}_{N_s}\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{I}_{N_s}\mathbf{\Pi}^n\mathbf{F}^H \\
&= \sum_{m=0}^{L_{sp}-1}\sum_{n=0}^{L_{sp}-1} [\mathbf{R_{sp}}]_{mn}e^{-j2\pi i(n-m)/N_s}\mathbf{e}_i\mathbf{e}_i^H \\
&= \psi_i\mathbf{e}_i\mathbf{e}_i^H,
\end{aligned}$$
   where $\psi_i \triangleq \sum_{m=0}^{L_{sp}-1}\sum_{n=0}^{L_{sp}-1}[\mathbf{R_{sp}}]_{mn}e^{-j2\pi i(n-m)/N_s}$.
   Then, the expressions of interference constraints are
$$\mathbf{Tr}(\mathbf{\Psi}_i\mathbf{U}) = \mathbf{Tr}(\psi_i\mathbf{e}_i\mathbf{e}_i^H\mathbf{U}) = \psi_i[\mathbf{U}]_{ii} \le P^{th}\ \forall\, i \in \mathbf{\Gamma}.$$

Define $c_i = \begin{cases} |\lambda_i^{sp}|^2, & \text{case 1} \\ \psi_i, & \text{case 2} \end{cases}$, then the interference constraints in the instantaneous and statistical CSI cases can be unified into $c_i[\mathbf{U}]_{ii} \le P^{th} \quad \forall\, i \in \mathbf{\Gamma}$.

Thus, when there is no frequency offset between the ST and the PR, the optimization problems in both CSI conditions can be formulated as

$$\begin{aligned}
\min_{\mathbf{U}} \quad & \mathbf{Tr}(\sigma_{\tilde{n}}^2(\mathbf{\Lambda_{ss}}\mathbf{U}\mathbf{\Lambda_{ss}}^H + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}) \\
\text{s.t.} \quad & \mathbf{Tr}(\mathbf{U}) \le P_t \\
& c_i[\mathbf{U}]_{ii} \le P^{th} \qquad \forall\, i \in \mathbf{\Gamma} \\
& \mathbf{U} \succeq 0.
\end{aligned}$$

It is not hard to show that the optimal $\mathbf{U}$ is diagonal by using Theorem (3.1) in [20]. Considering the eigenvalue decomposition $\mathbf{U} = \mathbf{Q}\mathbf{\Lambda}_p\mathbf{Q}^H$, we can obtain the optimal precoder matrix $\mathbf{Q} = \mathbf{I}_{N_s}$.

Appendix C
Proof of Theorem 2

Assume that $\mathbf{R}_{\tilde{\mathbf{n}}} = \sigma_{\tilde{n}}^2\mathbf{I}_{N_s}$, and the channels from the ST to the SR and from the ST to the PR are flat fading whose coefficients are h_{ss} and h_{sp}, respectively. Then we have $\mathbf{H_{ss}} = h_{ss}\mathbf{I}_{N_s}$, $\mathbf{H_{sp}} = h_{sp}\mathbf{I}_{N_s}$, $L_{ss} = 1$ and $L_{sp} = 1$. Using $\mathbf{H_{ss}} = h_{ss}\mathbf{I}_{N_s}$, the objective function in (14) can be simplified as $\mathbf{Tr}(\sigma_{\tilde{n}}^2(|h_{ss}|^2\mathbf{U} + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1})$.

Next, we give the expressions of interference constraints with different types of CSIs known.

1. When the instantaneous CSI of $\mathbf{h_{sp}}$ is known, we have $\mathbf{a}_i = \mathbf{F}\mathbf{H}_{\mathbf{sp}}^H\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i = h_{sp}^*\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i$.
   Then, the interference constraints become
$$\begin{aligned}\mathbf{Tr}(\mathbf{\Psi}_i\mathbf{U}) &= \mathbf{Tr}(\mathbf{a}_i\mathbf{a}_i^H\mathbf{U})\\ &= |h_{sp}|^2\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H\mathbf{U}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i\\ &\le P^{th} \quad \forall\, i \in \mathbf{\Gamma}.\end{aligned}$$
2. When the statistical CSI of $\mathbf{h_{sp}}$ is known, we have
$$\begin{aligned}&\mathbf{\Psi}_i = \mathbf{A}_i\mathbf{R_{sp}}\mathbf{A}_i^H\\ &= \sum_{m=0}^{L_{sp}-1}\sum_{n=0}^{L_{sp}-1}[\mathbf{R_{sp}}]_{mn}\mathbf{F}\mathbf{\Pi}^{m,H}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{\Pi}^n\mathbf{F}^H\\ &= \mathbb{E}[|h_{sp}|^2]\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H.\end{aligned}$$
   Then the interference constraints are given by
$$\begin{aligned}\mathbf{Tr}(\mathbf{\Psi}_i\mathbf{U}) &= \mathbb{E}[|h_{sp}|^2]\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H\mathbf{U}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i\\ &\le P^{th} \quad \forall\, i \in \mathbf{\Gamma}.\end{aligned}$$

Define $c_{sp} \triangleq \begin{cases} |h_{sp}|^2, & \text{case 1} \\ \mathbb{E}[|h_{sp}|^2], & \text{case 2}\end{cases}$, then the interference constraints in both instantaneous and statistical CSI cases can be unified into

$$c_{sp}\mathbf{e}_i^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H\mathbf{U}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i \le P^{th} \quad \forall\, i \in \mathbf{\Gamma}.$$

Further define $\tilde{\mathbf{U}} \triangleq \mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H\mathbf{U}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H$, the optimization problem considering different CSI can be formulated as

$$\begin{aligned}\min_{\tilde{\mathbf{U}}} \quad & \mathbf{Tr}(\sigma_{\tilde{n}}^2(|h_{ss}|^2\tilde{\mathbf{U}} + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1})\\ \text{s.t.} \quad & \mathbf{Tr}(\tilde{\mathbf{U}}) \le P_t\\ & c_{sp}[\tilde{\mathbf{U}}]_{ii} \le P^{th} \qquad \forall\, i \in \mathbf{\Gamma}\\ & \tilde{\mathbf{U}} \succeq 0. \end{aligned} \tag{25}$$

Using Theorem (3.1) in [20], we can obtain that the optimal $\tilde{\mathbf{U}}$ is diagonal. Assuming that $\tilde{\mathbf{U}} = \tilde{\mathbf{\Lambda}}_p$, then the optimal $\mathbf{U} = \mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\tilde{\mathbf{\Lambda}}_p\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}\mathbf{F}^H$. Therefore, the optimal precoder matrix $\mathbf{Q} = \mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H$. Because the problem (25) does not depend on δ_f, the MSE of secondary system does not depend on δ_f as well.

Appendix D
Proof of Lemma 1

First, we decompose the channel $\mathbf{h_{sp}}$ as $\mathbf{h_{sp}} = \rho_{sp}^{1/2}\tilde{\mathbf{h}}_{\mathbf{sp}}$, where ρ_{sp} is the large scale fading gain and the elements of $\tilde{\mathbf{h}}_{\mathbf{sp}}$ are the small scale fading coefficients. Then we can rewrite respectively the interference expressions (6) and (9) as

$$\mathbf{Tr}(\tilde{\mathbf{a}}_i\tilde{\mathbf{a}}_i^H\mathbf{U}) \le P^{th}/\rho_{sp}$$

and

$$\mathbf{Tr}(\mathbf{A}_i\tilde{\mathbf{R}}_{\mathbf{sp}}\mathbf{A}_i^H\mathbf{U}) \le P^{th}/\rho_{sp},$$

where $\tilde{\mathbf{a}}_i = \mathbf{a}_i/\rho_{sp}^{1/2}$ and $\tilde{\mathbf{R}} = \mathbf{R}/\rho_{sp}$.

When the interference threshold $P^{th} = 0$, or $\rho_{sp} \to \infty$, the interference expressions become

$$\mathbf{Tr}(\tilde{\mathbf{a}}_i\tilde{\mathbf{a}}_i^H\mathbf{U}) = \mathbf{Tr}(\tilde{\mathbf{a}}_i^H\mathbf{U}\tilde{\mathbf{a}}_i) = 0$$

and

$$\mathbf{Tr}(\mathbf{A}_i\tilde{\mathbf{R}}_{\mathbf{sp}}\mathbf{A}_i^H\mathbf{U}) = \mathbf{Tr}(\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2,H}\mathbf{A}_i^H\mathbf{U}\mathbf{A}_i\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2}) = 0.$$

Because $\tilde{\mathbf{a}}_i^H\mathbf{U}\tilde{\mathbf{a}}_i$ and $\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2,H}\mathbf{A}_i^H\mathbf{U}\mathbf{A}_i\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2}$ are positive semidefinite, the above results of zero trace imply that $\tilde{\mathbf{a}}_i^H\mathbf{U}\tilde{\mathbf{a}}_i = \mathbf{0}$ and $\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2,H}\mathbf{A}_i^H\mathbf{U}\mathbf{A}_i\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2} = \mathbf{0}$. Since $\tilde{\mathbf{R}}_{\mathbf{sp}}^{1/2}$ is positive definite, we can further obtain $\mathbf{A}_i^H\mathbf{U}\mathbf{A}_i = \mathbf{0}$. Using the decomposition $\mathbf{U} = \mathbf{Q}\mathbf{\Lambda}_p\mathbf{Q}^H$, we get the final results as follows

$$\tilde{\mathbf{a}}_i^H\mathbf{Q}\mathbf{\Lambda}_p\mathbf{Q}^H\tilde{\mathbf{a}}_i = 0 \quad \text{and} \quad \mathbf{A}_i^H\mathbf{Q}\mathbf{\Lambda}_p\mathbf{Q}^H\mathbf{A}_i = 0. \tag{26}$$

Assume that all the diagonal elements of $\mathbf{\Lambda}_p$ are positive[4]. Then (26) suggests that $\tilde{\mathbf{a}}_i^H\mathbf{Q} = 0$ and $\mathbf{A}_i^H\mathbf{Q} = 0$. This means that the precoder lies in the null space of the interference space which is spanned by $\{\tilde{\mathbf{a}}_i\}_{i\in\Gamma}$, i.e., $\{\mathbf{a}_i\}_{i\in\Gamma}$ when the instantaneous CSI is known, or spanned by $\{\mathbf{A}_i\}_{i\in\Gamma}$ when the statistical CSI is known.

Appendix E
Proof of Theorem 3

We consider a special case where the number of subcarriers used by primary system is $N_p = N_s - 1$ and the instantaneous CSI of $\mathbf{h_{sp}}$ is known by the ST. Assume that the covariance matrix of the total interference at the PR is $\mathbf{R}_{\tilde{\mathbf{n}}} = \sigma_{\tilde{n}}^2\mathbf{I}_{N_s}$.

Using $\mathbf{H_{sp}} = \mathbf{F}^H\mathbf{\Lambda}_{sp}\mathbf{F}$, we can obtain that

$$\mathbf{a}_i = \mathbf{F}\mathbf{H_{sp}}^H\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i = \mathbf{\Lambda}_{sp}^H\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_i.$$

It is readily to find that

$$\mathbf{a}_i^H\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0} = 0, \forall\, i \in \mathbf{\Gamma},$$

where i_0 is the index of the subcarrier not used by the primary system. Therefore, we can conclude that the vector $\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}$ lies in the null space of the interference space which is spanned by $\{\mathbf{a}_i\}_{i\in\Gamma}$.

Since we know from *Lemma 1* that $\mathbf{Q}$ also lies in the null space, and the dimension of the null space is one in the considered case, we can obtain the expression of $\mathbf{Q}$ as

$$\mathbf{Q} = c\frac{\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}}{\|\mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}\|} = c\frac{\mathbf{q}}{\|\mathbf{q}\|},$$

where c is an arbitrary complex number with unit amplitude, $\mathbf{q} \triangleq \mathbf{\Lambda}_{\mathbf{sp}}^{-1}\mathbf{F}\mathbf{\Delta}_{\mathbf{f}}^H\mathbf{F}^H\mathbf{e}_{i_0}$, and the power allocation matrix $\mathbf{\Lambda}_p$ degenerates to a scalar p whose optimal value is P_t.

[4]If some diagonal elements of $\mathbf{\Lambda}_p$ are zero, we can get a new $\mathbf{Q}$ by deleting columns of $\mathbf{Q}$ corresponding to zero power positions.

Consequently, we have $\mathbf{U} = \mathbf{Q}\mathbf{\Lambda}_p\mathbf{Q}^H = P_t\frac{\mathbf{q}\mathbf{q}^H}{\|\mathbf{q}\|^2}$, and the objective function in (14) can be derived as

$$\begin{aligned}\mathbf{Tr}(\mathbb{E}[\mathbf{e}\mathbf{e}^H]) &= \mathbf{Tr}(\sigma_{\tilde{n}}^2(\mathbf{\Lambda_{ss}}\mathbf{U}\mathbf{\Lambda_{ss}}^H + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}) \\ &= \mathbf{Tr}(\sigma_{\tilde{n}}^2(P_t\frac{\mathbf{\Lambda_{ss}}\mathbf{q}\mathbf{q}^H\mathbf{\Lambda_{ss}}^H}{\|\mathbf{q}\|^2} + \sigma_{\tilde{n}}^2\mathbf{I}_{N_s})^{-1}) \\ &= N_s - 1 + \frac{1}{1+\frac{P_t|\mathbf{\Lambda_{ss}}\mathbf{q}\|^2}{\sigma_{\tilde{n}}^2\|\mathbf{q}\|^2}} \\ &= N_s - 1 + \frac{1}{1+\frac{P_t\|\mathbf{\Lambda_{ss}}\mathbf{\Lambda_{sp}}^{-1}\mathbf{F}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_{i_0}\|^2}{\sigma_{\tilde{n}}^2\|\mathbf{\Lambda_{sp}}^{-1}\mathbf{F}\mathbf{\Delta_f}^H\mathbf{F}^H\mathbf{e}_{i_0}\|^2}},\end{aligned}$$

where the eigenvalue decomposition $\mathbf{H_{ss}} = \mathbf{F}^H\mathbf{\Lambda}_{ss}\mathbf{F}$ and the matrix inversion lemma are applied.

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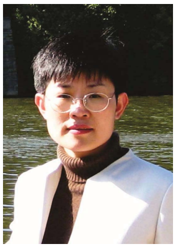

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