# Energy-Efficient Configuration of Spatial and Frequency Resources in MIMO-OFDMA Systems 

Zhikun Xu, Student Member, IEEE, Chenyang Yang, Senior Member, IEEE, Geoffrey Ye Li, Fellow, IEEE, Shunqing Zhang, Yan Chen, and Shugong Xu, Senior Member, IEEE


#### Abstract

In this paper, we investigate adaptive configuration of spatial and frequency resources to maximize energy efficiency (EE) and reveal the relationship between the $E E$ and the spectral efficiency (SE) in downlink multiple-input-multiple-output (MIMO) orthogonal frequency division multiple access (OFDMA) systems. We formulate the problem as minimizing the total power consumed at the base station under constraints on the ergodic capacities from multiple users, the total number of subcarriers, and the number of radio frequency (RF) chains. A three-step searching algorithm is developed to solve this problem. We then analyze the impact of spatial-frequency resources, overall SE requirement and user fairness on the SE-EE relationship. Analytical and simulation results show that increasing frequency resource is more efficient than increasing spatial resource to improve the SE-EE relationship as a whole. The EE increases with the $S E$ when the frequency resource is not constrained to the maximum value, otherwise a tradeoff between the $S E$ and the EE exists. Sacrificing the fairness among users in terms of ergodic capacities can enhance the $\mathrm{SE}-E E$ relationship. In general, the adaptive configuration of spatial and frequency resources outperforms the adaptive configuration of only spatial or frequency resource.


Index Terms-Energy efficiency (EE), spectral efficiency (SE), multiple-input-multiple-output (MIMO), orthogonal frequency division multiple access (OFDMA).

## I. Introduction

VARIOUS techniques have been developed during the last two decades to improve spectral efficiency (SE), such as multiple-input-multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM). Meanwhile, energy-efficient design of wireless communication systems is attracting more and more attention since explosive growth of wireless service results in its increasing contribution to the worldwide carbon footprint [1]. Therefore, future wireless systems are expected to be designed in an energy-efficient way while guaranteeing the SE.

More spatial and frequency resources enhance the SE, but also bring higher circuit power consumption. Although

[^0]MIMO needs less transmit power than single-input-singleoutput (SISO) to achieve the same channel capacity, it consumes more circuit power since more active transmit or receive radio frequency ( RF ) chains are used [2]. On the other hand, in MIMO-OFDM systems, spatial precoding and other baseband processing are carried out at each subcarrier and thus the circuit power consumption on processing increases with the total number of occupied subcarriers. Since signal processing becomes more complicated due to high requirement on the data rate and transmission reliability, we cannot neglect the circuit power consumed by the spatial and frequency resources when designing an energy-efficient MIMO-OFDM system.

There have been some preliminary results on saving energy by adaptively using the spatial and frequency resources. The energy efficiency (EE) of Alamouti diversity scheme has been discussed in [2]. It has been shown that for short-range transmission, multiple-input-single-output (MISO) transmission reduces the EE as compared with SISO transmission if adaptive modulation is not used. However, if modulation order is adaptively adjusted to balance the transmit and circuit power consumption, MISO systems always perform better. Spatial multiplexing, space-time coding, and single antenna transmission have been adaptively selected in [3] based on channel state information (CSI), and the EE improvement can be up to $30 \%$ compared with non-adaptive systems. Adaptive switching between MIMO and single-input-multipleoutput modes has been addressed in [4] to save the energy in uplink cellular networks. In [5], the number of active RF chains has been optimized to maximize the EE given the minimum data rate. Dynamic spectrum management has been discussed in [6]. It is shown that the energy can be saved significantly by allocating active frequency band and assigning users among different cells. The relationship between the EE and bandwidth has been investigated in [7] and [8]. The EE is shown to increase with bandwidth if the circuit power consumption either is independent of or linearly increases with the bandwidth. Energy-efficient link adaptation for MIMOOFDM systems has been studied in [9], where the number of active RF chains, the overall bandwidth, MIMO transmission modes are adjusted according to the data rate requirement and channel condition.

Priori work mainly focuses on single user systems. In downlink MIMO-orthogonal frequency division multiple access (OFDMA) networks, RF chains are shared by multiple users. In this scenario, switching on or off RF chains and allocating bandwidth are intertwined, which makes it complicated to investigate the EE. In this paper, we will study adaptive configuration of spatial and frequency resources to maximize
the EE in downlink MIMO-OFDMA systems, and we will reveal the relationship between the SE and the EE. Different from optimizing the system bandwidth in [9], we will fix the system bandwidth and only adjust the total number of active subcarriers, which can avoid the variation of the sampling rate and is more practical.

The rest of this paper is organized as follows. We first present the system model and formulate the EE maximization problem in Section II. In Section III, we propose a three-step algorithm to find a suboptimal solution. Then we investigate the impact of overall SE requirement and the user fairness in terms of ergodic capacities on the EE in Section IV. The simulation results are provided in Section V and the paper is concluded in Section VI.

## II. System Model and Problem Formulation

In this section, we first introduce the system model and then describe the power consumption at the base station (BS) based on the implementation structure. Next we provide the ergodic capacity for each user and formulate an optimization problem to maximize the EE .

## A. System Model

Consider a downlink MIMO-OFDMA system with $M$ users. $N_{t}$ and $N_{r}$ RF chains are respectively configured at the BS and each user. Overall $K$ subcarriers are shared by multiple users without overlap. Since a large portion of power is consumed by the BS during downlink transmission [7], we concern about how to save energy at the BS side. Assume that $n_{t}$ RF chains are active at the BS and $k_{i}$ subcarriers are employed for user $i$. Then $1 \leq n_{t} \leq N_{t}$ and $\sum_{i=1}^{M} k_{i} \leq K$. We will adjust $n_{t}$ and $\left\{k_{i}\right\}_{i=1}^{M}$ based on the channel fading gains and user's data rate requirements.

A typical implementation structure of MIMO-OFDMA systems is shown in Fig. 1. The data first pass the channel coding and modulation unit and are mapped into complex symbols. After spatial processing in the MIMO unit, the signals are fed to $n_{t}$ active RF chains. Each RF branch performs several OFDM operations, including series to parallel converting (S/P), inverse fast fourier transform (IFFT), and parallel to series converting ( $\mathrm{P} / \mathrm{S}$ ). After digital processing, the analog signals generated by the digital-to-analog converter (D/A) are filtered and up-converted to a high frequency band. Finally, the signals are transmitted after the power amplifiers (PAs).

We assume that users undergo frequency-selective and spatially uncorrelated block fading channels, where different OFDM symbols suffer from independent channel fading. Denote $\mathbf{H}_{i j}$ as the spatial channel matrix from the BS to user $i$ on subcarrier $j$. The elements of $\mathbf{H}_{i j}$ are independent and identically Gaussian distributed with zero mean and variance $\mu_{i}$, where $\mu_{i}$ is the average channel gain from the BS to user $i$. We assume that the instantaneous CSI is unavailable and the average channel gains, $\left\{\mu_{i}\right\}_{i=1}^{M}$, are known at the BS. Single-user MIMO (SU-MIMO) transceivers, which achieve the MIMO capacity without CSI at the transmitter (CSIT), are applied. The power for each user is equally distributed over multiple subcarriers and RF chains. The noise at the receiver of each user is additive white Gaussian with zero mean and variance $\sigma^{2}$.


Fig. 1. Implementation structure of a MIMO-OFDMA system
TABLE I
CIRCUIT POWER CONSUMPTIONS OF DIFFERENT COMPONENTS OF BS IN Fig. 1

| Unit | Expression | Description |
| :--- | :--- | :--- |
| P1 | $P_{c 1} \sum_{i=1}^{M} C_{i}$ | linearly increases with over- <br> all data rate [10], $C_{i}$ is the <br> data rate of user $i$ and $P_{c 1}$ is <br> a constant. |
| P2 | $\left(\alpha n_{t}^{2}+\beta n_{t}\right) P_{c 2} \sum_{i=1}^{M} k_{i}$ | linearly increases with <br> overall number of used <br> subcarriers. $\left(\alpha n_{t}^{2}+\beta n_{t}\right) P_{c 2}$ <br> is the power consumed by <br> matrix operations on each <br> subcarrier [9]. $\alpha, \beta$, and $P_{c 2}$ <br> are constant. |
| P3 | $n_{t} P_{c 3} \sum_{i=1}^{M} k_{i}$ | linearly increases with the <br> number of used subcarriers <br> and the number of active RF <br> chains [9]. $P_{c 3}$ is a constant. |
| P 4 | $n_{t} P_{c 4}$ | linearly increases with the <br> number of active RF chains <br> [2]. $P_{c 4}$ is a constant. |

## B. Power Consumption at the $B S$

The total power consumed by the BS consists of the transmit power and circuit power and can be expressed as

$$
\begin{equation*}
P_{t o t}=\frac{P_{t r}}{\rho}+P_{c c} \tag{1}
\end{equation*}
$$

where $P_{t r}$ is the overall transmit power radiated to the air, $P_{c c}$ is the circuit power, and $\rho$ denotes the efficiency of the PA which is defined as the ratio of the output power of a PA to its input power.

The transmit power that is radiated to the air is contributed by all users. Denote $P_{i}$ as the power per subcarrier at each RF chain for user $i$ and then $P_{t r}$ can be written as

$$
\begin{equation*}
P_{t r}=n_{t} \sum_{i=1}^{M} k_{i} P_{i} \tag{2}
\end{equation*}
$$

Besides a fixed circuit power consumption to maintain operations of the BS, circuit power consumptions from different components depend on different system parameters. For example, circuit power consumption from the channel coding and modulation mapping unit is proportional to the overall data rate [10]. The specific circuit power consumptions for different components are described in Table I. Based on the circuit power consumption models, the overall circuit power consumption at the BS can be written as follows,

$$
\begin{align*}
P_{c c}= & P_{c 1} \sum_{i=1}^{M} C_{i}+\left(\alpha n_{t}^{2}+\beta n_{t}\right) P_{c 2} \sum_{i=1}^{M} k_{i}+n_{t} P_{c 3} \sum_{i=1}^{M} k_{i} \\
& +n_{t} P_{c 4}+P_{c 5} \\
= & P_{c 1} \sum_{i=1}^{M} C_{i}+\sum_{i=1}^{M} k_{i} g\left(n_{t}\right)+n_{t} P_{c 4}+P_{c 5}, \tag{3}
\end{align*}
$$

where $g\left(n_{t}\right) \triangleq \alpha P_{c 2} n_{t}^{2}+\left(\beta P_{c 2}+P_{c 3}\right) n_{t}, C_{i}$ denotes the data rate of user $i$, and $P_{c 5}$ is the fixed circuit power.

Substituting (2) and (3) into (1), the total power consumption at the BS can be finally expressed as

$$
\begin{equation*}
P_{t o t}=\sum_{i=1}^{M} k_{i}\left[\frac{n_{t} P_{i}}{\rho}+g\left(n_{t}\right)\right]+P_{c 1} \sum_{i=1}^{M} C_{i}+n_{t} P_{c 4}+P_{c 5} \tag{4}
\end{equation*}
$$

## C. Ergodic Capacity for Each User

When the CSIT is not known at the BS and different OFDM symbols suffer from independent channel fading, the maximum achievable data date is equal to the ergodic capacity [11]. We assume that the transceiver that can achieve the channel capacity is applied and thus the data rate is the maximum value. Therefore, we will use the ergodic capacity instead of the data rate to avoid confusing from now on. For the SU-MIMO scheme, the ergodic capacity of user $i$ can be expressed as [12]

$$
\begin{align*}
C_{i} & =\Delta f \sum_{j=1}^{k_{i}} \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\frac{P_{i}}{\sigma^{2}} \mathbf{H}_{i j} \mathbf{H}_{i j}^{H}\right)\right] \\
& =\Delta f \sum_{j=1}^{k_{i}} \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\frac{\mu_{i} P_{i}}{\sigma^{2}} \frac{1}{\sqrt{\mu_{i}}} \mathbf{H}_{i j} \frac{1}{\sqrt{\mu_{i}}} \mathbf{H}_{i j}^{H}\right)\right] \\
& =\Delta f \sum_{j=1}^{k_{i}} \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\omega_{i} P_{i} \tilde{\mathbf{H}}_{i j} \tilde{\mathbf{H}}_{i j}^{H}\right)\right] \tag{5}
\end{align*}
$$

where $\Delta f$ denotes the subcarrier spacing, $\mathbf{I}_{N_{r}}$ denotes an $N_{r} \times$ $N_{r}$ identity matrix, $\mathbb{E}[\cdot]$ denotes expectation operation over small scale fading, $\omega_{i} \triangleq \frac{\mu_{i}}{\sigma^{2}}$ represents the average channel gain from the BS to user $i$ normalized by the noise power, and $\tilde{\mathbf{H}}_{i j} \triangleq \frac{1}{\sqrt{\mu_{i}}} \mathbf{H}_{i j}$ is the normalized channel matrix.

According to [12], the ergodic capacity in (5) can be calculated as follows,

$$
\begin{align*}
C_{i} & =\Delta f \sum_{j=1}^{k_{i}} m \int_{0}^{\infty} \log _{2}\left(1+\omega_{i} P_{i} x\right) p_{\mathbf{x}}(x) \mathrm{d} x \\
& =\Delta f k_{i} m \int_{0}^{\infty} \log _{2}\left(1+\omega_{i} P_{i} x\right) p_{\mathbf{x}}(x) \mathrm{d} x \tag{6}
\end{align*}
$$

where $m \triangleq \min \left\{n_{t}, N_{r}\right\}$ and $p_{\mathbf{x}}(x)$ is the probability density function of all nonzero eigenvalues of $\tilde{\mathbf{H}}_{i j} \tilde{\mathbf{H}}_{i j}^{H}$, whose expression can be found in [12].

Denote

$$
f\left(n_{t}, \omega_{i} P_{i}\right)=\Delta f m \int_{0}^{\infty} \log _{2}\left(1+\omega_{i} P_{i} x\right) p_{\mathbf{x}}(x) \mathrm{d} x
$$

then we rewrite the ergodic capacity as

$$
\begin{equation*}
C_{i}=k_{i} f\left(n_{t}, \omega_{i} P_{i}\right) . \tag{7}
\end{equation*}
$$

## D. Energy Efficiency Optimization

The EE in downlink transmission is defined as the overall average number of bits transmitted from the BS per unit energy
[13], and is equal to the sum of capacities of multiple users per unit power. From the total power consumption in (4), we can obtain the EE of the downlink MIMO-OFDMA network as

$$
\begin{equation*}
\eta=\frac{\sum_{i=1}^{M} C_{i}}{\sum_{i=1}^{M} k_{i}\left[\frac{n_{t} P_{i}}{\rho}+g\left(n_{t}\right)\right]+P_{c 1} \sum_{i=1}^{M} C_{i}+n_{t} P_{c 4}+P_{c 5}} \tag{8}
\end{equation*}
$$

The SE in downlink transmission, which is defined as the sum capacity per unit bandwidth, depends on the capacities of multiple users. To study the SE-EE relationship, we formulate a problem to maximize the EE under constraints on the capacities from multiple users.

When the capacities of multiple users are given, maximizing the EE is equivalent to minimizing the total power consumption. Considering the constraints on the total number of subcarriers and the number of active RF chains, the optimization problem can be formulated as follows,

$$
\begin{align*}
\min _{n_{t}, \mathbf{K}, \mathbf{P}} & \sum_{i=1}^{M} k_{i}\left[\frac{n_{t} P_{i}}{\rho}+g\left(n_{t}\right)\right]+P_{c 1} \sum_{i=1}^{M} C_{i}+n_{t} P_{c 4}+P_{c 5}  \tag{9}\\
\text { s. t. } & k_{i} f\left(n_{t}, \omega_{i} P_{i}\right)=C_{i}, \quad i=1,2, \cdots, M  \tag{9a}\\
& \sum_{i=1}^{M} k_{i} \leq K,  \tag{9b}\\
& 1 \leq n_{t} \leq N_{t},  \tag{9c}\\
& k_{i}>0, \quad P_{i}>0, \quad i=1,2, \cdots, M \tag{9d}
\end{align*}
$$

where $\mathbf{K} \triangleq\left\{k_{i}\right\}_{i=1}^{M}$ denotes the set of numbers of subcarriers and $\mathbf{P} \triangleq\left\{P_{i}\right\}_{i=1}^{M}$ denotes the set of transmit powers. We will optimize the number of active RF chains, $n_{t}$, the number of subcarriers used by each user, $\left\{k_{i}\right\}_{i=1}^{M}$, and the transmit power for each user, $\left\{P_{i}\right\}_{i=1}^{M}$, to find the maximum EE.

## III. Three-Step Searching Algorithm

Problem (9) is a mixed integer programming since both integer variables, $n_{t}$ and $\left\{k_{i}\right\}_{i=1}^{M}$, and continuous variables, $\left\{P_{i}\right\}_{i=1}^{M}$, are included. Because the maximum number of subcarriers, $K$, is usually very large in MIMO-OFDMA systems, the number of possible integer values for $\left\{k_{i}\right\}_{i=1}^{M}$ is huge and finding the optimal values by exhaustive searching is complexity-prohibitive. Moreover, $f\left(n_{t}, \omega_{i} P_{i}\right)$ is a very complicated function of $n_{t}$ [12], which makes it difficult to find the optimal $n_{t}$. In this section, we develop a threestep searching algorithm to obtain a solution with acceptable complexity. We first relax the numbers of subcarriers as continuous variables and find the optimal continuous solutions for $\left\{k_{i}\right\}_{i=1}^{M}$ and $\left\{P_{i}\right\}_{i=1}^{M}$ with a given $n_{t}$. Then we propose a searching algorithm to find the optimal $n_{t}$ with minimum power consumption. Finally, we discretize the number of subcarriers used by each user. Note that only the discretization step will cause the performance loss.

## A. Solution of Continuous Numbers of Subcarriers Given the Number of Active RF Chains

When the numbers of subcarriers occupied by multiple users, $\left\{k_{i}\right\}_{i=1}^{M}$, are relaxed as continuous variables, they can
be expressed as functions of $\left\{P_{i}\right\}_{i=1}^{M}$ from constraint $(9 a)$ as

$$
\begin{equation*}
k_{i}=C_{i} / f\left(n_{t}, \omega_{i} P_{i}\right), \quad i=1,2, \cdots, M \tag{10}
\end{equation*}
$$

Substituting (10) into problem (9) and considering that constraint $(9 c)$ can be discarded automatically for a given $n_{t}$, we can obtain a new optimization problem as follows,

$$
\begin{equation*}
\min _{\mathbf{P}} \sum_{i=1}^{M} \frac{C_{i}\left[\frac{n_{t} P_{i}}{\rho}+g\left(n_{t}\right)\right]}{f\left(n_{t}, \omega_{i} P_{i}\right)}+P_{c 1} \sum_{i=1}^{M} C_{i}+n_{t} P_{c 4}+P_{c 5} \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
\text { s. t. } \quad \sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i}\right)} \leq K \tag{11a}
\end{equation*}
$$

$$
\begin{equation*}
P_{i}>0, \quad i=1,2, \cdots, M . \tag{11b}
\end{equation*}
$$

The Lagrange function of this problem can be written as

$$
\begin{align*}
& L\left(\left\{P_{i}\right\}_{i=1}^{M}, \lambda,\left\{\xi_{i}\right\}_{i=1}^{M}\right) \\
& =\Phi\left(n_{t}, \mathbf{P}\right)+\lambda\left(\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i}\right)}-K\right)-\sum_{i=1}^{M} \xi_{i} P_{i} \tag{12}
\end{align*}
$$

where $\Phi\left(n_{t}, \mathbf{P}\right)$ denotes the objective function of problem (11), and $\lambda$ and $\left\{\xi_{i}\right\}_{i=1}^{M}$ represent Lagrange multipliers for constraints $(11 a)$ and (11b), respectively. The Karush-KuhnTucker (KKT) conditions of problem (11) can be expressed as follows,

$$
\begin{align*}
& n_{t} f\left(n_{t}, \omega_{i} P_{i}\right)-\omega_{i}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)=0 \\
& i=1,2, \cdots, M,  \tag{13a}\\
& \lambda \geq 0, \quad \sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i}\right)} \leq K,  \tag{13b}\\
& \lambda\left(\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i}\right)}-K\right)=0,  \tag{13c}\\
& P_{i}>0, \quad i=1,2, \cdots, M,  \tag{13d}\\
& \xi_{i} \geq 0, \quad \text { and } \xi_{i} P_{i}=0, \quad i=1,2, \cdots, M, \tag{13e}
\end{align*}
$$

where the left hand side of $(13 a)$ is the partial derivative of Lagrange function (12) with respect to $P_{i}$ and $f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right) \triangleq$ $\left.\frac{\partial f\left(n_{t}, \gamma\right)}{\partial \gamma}\right|_{\gamma=\omega_{i} P_{i}}$.

Because the equality is only included in constraint (11a), according to linear independence constraint qualification (LICQ) [14], the KKT conditions are necessary to achieve the global optimum of problem (11).

From complementary slackness condition, $\xi_{i} P_{i}=0$ and condition (13d), $P_{i}>0$, we can obtain that $\xi_{i}=0$. In the following, we find $\left\{P_{i}\right\}_{i=1}^{M}$ and $\lambda$ that satisfy KKT conditions in two steps. We first obtain the solutions for equations (13a) and ( $13 c$ ) and then judge whether they satisfy inequalities (13b) and (13d).

It is proved in Appendix A that $P_{i}$ which satisfies equation (13a) is a monotonically increasing function with $\lambda$. Denoting $P_{i}=\theta_{i}(\lambda)$ and substituting it into (13c), we have

$$
\begin{equation*}
\lambda\left(\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(\lambda)\right)}-K\right)=0 \tag{14}
\end{equation*}
$$

Then we can find two solutions of $\lambda$ that satisfy (13a) and (13c): $\lambda_{1}=0$ and $\lambda_{2}$ satisfies

$$
\begin{equation*}
\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(\lambda)\right)}=K \tag{15}
\end{equation*}
$$

Next we judge whether $\lambda_{1}$ and $\lambda_{2}$ satisfy (13b) and (13d). When $\lambda=0$, the term inside the bracket in (14) is not necessary to be equal to zero, which means the required total number of subcarriers does not need to be $K$. Based on this observation, we discuss the feasibility of $\lambda_{1}$ and $\lambda_{2}$ in the following two cases.
C 1. When $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)} \leq K$, i.e., the total number of used subcarriers when $\lambda=0$ is less than or equal to the maximum number, $\lambda_{1}=0$ satisfies (13b) obviously. It is readily derived that $\lambda=-g\left(n_{t}\right)$ when $P_{i}=0$. Therefore, $P_{i}>0$ holds automatically when $\lambda=0$ based on the monotonically increasing relationship between $\lambda$ and $P_{i}$. Therefore, KKT condition (13d) is satisfied. $\lambda_{1}=0$ is the solution of KKT conditions (13). Because $P_{i}=$ $\theta_{i}(\lambda)$ is a monotonically increasing function of $\lambda$ and $f\left(n_{t}, \omega_{i} P_{i}\right)$ is a monotonically increasing function of $P_{i}$, $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(\lambda)\right)}$ decreases with $\lambda$ due to the composition rule of monotonic function. Then, $\lambda_{2}$ that satisfies (15) is less than zero since $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)} \leq K$. Therefore, condition $\lambda \geq 0$ is not satisfied for $\lambda_{2}$ and $\lambda_{2}$ is not a solution of the KKT conditions.
C 2. When $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)}>K, \lambda_{1}=0$ is not a solution of KKT conditions (13) since the second inequality in KKT condition (13b) is not satisfied. From $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)}>K$ and $\lim _{\lambda \rightarrow \infty} \sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(\lambda)\right)}=0<$ $K$, we can conclude that $\lambda_{2}$ which satisfies (15) is greater than zero since $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(\lambda)\right)}$ decreases with $\lambda$. Therefore, KKT condition (13b) is satisfied. According to the monotonically increasing relationship between $P_{i}$ and $\lambda$, we have $P_{i}=\theta_{i}\left(\lambda_{2}\right)>0$ and thus KKT condition (13d) is satisfied. Consequently, $\lambda_{2}$ is the solution of the KKT conditions.
From the above analysis, we can observe that there exists only one solution of $\lambda$ in both cases. Since these two cases are disjoint, a unique solution of $\lambda$ exists for KKT conditions (13). Due to the monotonically increasing relationship between $P_{i}$ and $\lambda$, the solution of $P_{i}$ is also unique for KKT conditions. Consequently, we obtain an optimal solution for the problem (11).

Note that due to the complicated relationship between $\lambda$ and $P_{i}$ as shown in (13a), the closed-form expression of $P_{i}=\theta_{i}(\lambda)$ cannot be obtained. We use bisection searching algorithm to numerically find $P_{i}$ for a given $\lambda$ since $P_{i}$ monotonically increases with $\lambda$ [13], which is described in Table $\mathrm{II}^{*}$. In this algorithm, we first determine the interval that includes $P_{i}$ which satisfies $(13 a)$. Then we shorten the interval by half iteratively until the interval length is smaller than the required accuracy of $P_{i}$.

The specific procedure to find the optimal solution for problem (11), $\left\{P_{i}^{c}\right\}_{i=1}^{M}$, is summarized in Table III, which is

[^1]TABLE II
Bisection searching algorithm to solve $P_{i}=\theta_{i}(\lambda)$

```
Input: number of active RF chains at the BS and user sides,
    \(n_{t}\) and \(N_{r}, \lambda\), and constants \(\nu_{1}>0, \delta_{1}>1\), and \(\epsilon_{1}>0\)
Output: power values, \(\left\{P_{i}\right\}_{i=1}^{M}\), which satisfy (13a), i.e.,
    \(P_{i}=\theta_{i}(\lambda), \quad i=1, \cdots, M\)
    Set \(P_{i, l}=0\).
    Initialize \(P_{i, \text { temp }}=\nu_{1}\).
    while \(n_{t} f-\omega_{i}\left[n_{t} P_{i, \text { temp }}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f^{\prime} \leq 0\)
            \(P_{i, \text { temp }}=\delta_{1} P_{i, \text { temp }}\).
    end
    \(P_{i, r}=P_{i, \text { temp }}\).
    while \(\left(P_{i, r}-P_{i, l}\right)>\epsilon_{1}\)
            \(P_{i, \text { temp }}=\left(P_{i, l}+P_{i, r}\right) / 2\)
            if \(n_{t} f-\omega_{i}\left[n_{t} P_{i, \text { temp }}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f^{\prime}<0\),
                        \(P_{i, l}=P_{i, \text { temp }} ;\)
            else
                        \(P_{i, r}=P_{i, \text { temp }} ;\)
            end
        end
        \(P_{i}=P_{i, \text { temp }}\).
```

mainly from the discussion on the above two cases. We first check whether the solution when $\lambda=0$ satisfies the constraint on the total number of subcarriers in (11a). If constraint (11a) holds, it is the optimal solution for problem (11). Otherwise, we search the optimal nonzero $\lambda$ and the optimal $P_{i}$ by the bisection searching algorithm.

Substituting the optimal transmit power, $\left\{P_{i}^{c}\right\}_{i=1}^{M}$, into (10), we can obtain the optimal continuous values of the numbers of subcarriers for multiple users.

## B. Find the Optimal Number of Active RF Chains

When the maximum number of RF chains at the BS, $N_{t}$, is small, we can compute the EE for each possible value of $n_{t}$ in the interval $\left[1, N_{t}\right]$ and find the one with the maximum EE. When $N_{t}$ is large, this leads to high computational complexity, which may cause considerable extra energy consumption. In the sequel, we develop a searching algorithm to find the optimal number of active RF chains for the case with a large $N_{t}$.

Before introducing the method to optimize $n_{t}$, we first present the properties of the objective function of problem (11) in Theorem 1, which is proved in Appendix B.

Theorem 1. When the circuit power consumption is zero, the minimum value of the objective function of problem (11) does not depend on the number of active RF chains. When the circuit power consumption is not zero, the minimum value of the objective function of problem (11) increases with the number of active RF chains.

Denote the minimum value of the objective function of problem (11) for a given $n_{t}$ as $P_{t o t}^{\dagger}\left(n_{t}\right)$ and the optimal value of problem (11) for a given $n_{t}$ as $P_{\text {tot }}^{\star}\left(n_{t}\right)$, respectively. Since the optimal total power consumption, $P_{\text {tot }}^{\star}\left(n_{t}\right)$, is found by minimizing the objective function under some constraints, we know that

$$
\begin{equation*}
P_{t o t}^{\dagger}\left(n_{t}\right) \leq P_{t o t}^{\star}\left(n_{t}\right) . \tag{16}
\end{equation*}
$$

TABLE III
ALGORITHM FOR SOLVING KKT CONDITIONS (13)

Input: number of active RF chains at the BS and user sides, $n_{t}$ and $N_{r}$, and constants $\nu_{2}>0, \delta_{2}>1$, and $\epsilon_{2}>0$.
Output: optimal power values for multiple users, $\left\{P_{i}^{c}\right\}_{i=1}^{M}$.

1. Find $P_{i, 0}$ that satisfies $(13 a)$ when $\lambda=0$ by using the bisection searching algorithm in Table II.
if $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i, 0}\right)} \leq K$
else
Set $\lambda_{l}=0$ since $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i, 0}\right)}>K$
Initialize $\lambda_{t e m p}=\nu_{2}$.
Calculate $P_{i, \text { temp }}$ that satisfies (13a) when $\lambda=$ $\lambda_{t e m p}$ by the algorithm in Table II.
2. $\quad$ while $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i, \text { temp }}\right)} \geq K$
3. $\quad \lambda_{\text {temp }}=\delta_{2} \lambda_{\text {temp }}$ and calculate $P_{i, \text { temp }}$ that satisfies (13a) when $\lambda=\lambda_{\text {temp }}$ by using the algorithm in Table II.
4. end
$\lambda_{r}=\lambda_{\text {temp }}$.
while $\left(\lambda_{r}-\lambda_{l}\right)>\epsilon_{2}$
5. $\lambda_{\text {temp }}=\left(\lambda_{l}+\lambda_{r}\right) / 2$ and calculate $P_{i, \text { temp }}=$ $\theta_{i}\left(\lambda_{t e m p}\right)$ by using the algorithm in Table II.
if $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} P_{i, \text { temp }}\right)}>K$,
else $\lambda_{r}=\lambda_{t e m p} ;$
end
end
$P_{i}^{c}=P_{i, \text { temp }}$.
end

Moreover, if the equality in constraint (11a) does not hold at the optimal point when $n_{t}=n_{0}$, we have

$$
\begin{equation*}
P_{t o t}^{\star}\left(n_{0}\right)=P_{t o t}^{\dagger}\left(n_{0}\right) . \tag{17}
\end{equation*}
$$

According to Theorem 1, we know that for any $n_{t}>n_{0}$,

$$
\begin{equation*}
P_{t o t}^{\dagger}\left(n_{0}\right) \leq P_{t o t}^{\dagger}\left(n_{t}\right) \tag{18}
\end{equation*}
$$

From (16), (17), and (18), we can conclude that

$$
\begin{equation*}
P_{t o t}^{\star}\left(n_{0}\right) \leq P_{t o t}^{\star}\left(n_{t}\right) . \tag{19}
\end{equation*}
$$

This conclusion implies that once we find the value of $n_{0}$, the optimal number of active RF chains to achieve the minimum power consumption over all possible values of $n_{t}$ lies in the interval $\left[1, n_{0}\right]$. Consequently, we can first serially search $n_{0}$ from $n_{t}=1$, then compute the total power consumption for each $n_{t}$ in the interval $\left[1, n_{0}\right]$ and finally find the optimal number of active RF chains with minimal power consumption. When $n_{0}$ is much less than $N_{t}$, for example, when the capacity requirements of multiple users are very low, the complexity to find the optimal number can be dramatically reduced.

## C. Discretization of the Continuous Numbers of Subcarriers

The continuous values of the subcarrier's numbers for multiple users need to be discretized for practical application. We discretize the numbers in two steps, which is summarized in Table IV. We first ensure that the total number of subcarriers occupied by multiple users is an integer that is nearest to the sum of the continuous numbers of subcarriers and its value is shown in (23). Then, we discretize the number of subcarriers for each user. We initially assign user $i$ with the integer part of the continuous number of subcarriers as shown in line 2 of Table IV. Then the remaining subcarriers is allocated one by one to a user with the largest gap from its expected capacity as in lines 3-7 of this table.

After discretizing the number of subcarriers for each user, the transmit power $\left\{P_{i}^{c}\right\}_{i=1}^{M}$ may not satisfy the capacity requirements any more. According to (10), the final optimal transmit power values, $\left\{P_{i}^{o}\right\}_{i=1}^{M}$, should be

$$
\begin{equation*}
k_{i}^{o} f\left(n_{t}, \omega_{i} P_{i}^{o}\right)=C_{i}, \quad i=1,2, \cdots, M \tag{20}
\end{equation*}
$$

Since $f\left(n_{t}, \omega_{i} P_{i}\right)$ is a monotonically increasing function with respect to $P_{i}, P_{i}^{o}$ can be found numerically by the bisection searching algorithm [13].

## IV. Impact of SE REQUIREmENT and Capacity Fairness among Users on EE

In the downlink network, data streams are transmitted to multiple users and thus both the downlink SE and the fairness among users affect the EE. Since it is very hard to analyze the impact when the numbers of subcarriers occupied by multiple users, $\left\{k_{i}\right\}_{i=1}^{M}$, are discrete variables, we relax them as continuous variables as in Section III.A.

Consider the sum capacity and capacity ratios as follows,

$$
\begin{equation*}
C_{t o t} \triangleq \sum_{i=1}^{M} C_{i}, \quad \text { and } \quad \pi_{i} \triangleq \frac{C_{i}}{\sum_{i=1}^{M} C_{i}}, \quad i=1, \cdots, M \tag{21}
\end{equation*}
$$

Since the downlink SE is the ratio of the sum capacity to the system bandwidth, and the bandwidth is fixed, the impact of the SE requirement can be found by studying the relationship between $C_{t o t}$ and the EE. After substituting (21) into the objective function of problem (11), the optimal EE for a given $n_{t}$ can be expressed as

$$
\begin{align*}
& \eta^{\star}\left(n_{t}\right)= \\
& \frac{C_{t o t}}{\sum_{i=1}^{M} \frac{C_{t o t} \pi_{i}\left[n_{t} P_{i}^{\star}\left(n_{t}\right) / \rho+g\left(n_{t}\right)\right]}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}+P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5}}, \tag{22}
\end{align*}
$$

where $\left\{P_{i}^{\star}\left(n_{t}\right)\right\}_{i=1}^{M}$ is the optimal solution of problem (11) for a given $n_{t}$. The capacity ratios, $\left\{\pi_{i}\right\}_{i=1}^{M}$, reflect the fairness among users. In the following, we will investigate the impact of sum capacity and capacity ratios on the EE, respectively.

## A. Impact of the SE Requirement

Similar to finding the solution of Lagrange multiplier for KKT conditions (13) in Section III, we investigate the impact of the SE in two cases. To simplify the following description, we denote the solution of Lagrange multiplier as $\lambda^{\star}$.

TABLE IV
ALGORITHM FOR DISCRETIZING THE NUMBER OF SUBCARRIERS
Input: $\left\{k_{i}^{c}\right\}_{i=1}^{M}, n_{t}$, and $\left\{P_{i}^{c}\right\}_{i=1}^{M}$.
Output: the numbers of subcarriers for multiple users, $\left\{k_{i}^{\star}\right\}_{i=1}^{M}$.

1. Set the total number of used subcarriers to be

$$
\begin{equation*}
K_{d}=\left\lfloor\sum_{i=1}^{M} k_{i}^{c}\right\rfloor+\left\lfloor 2 *\left(\sum_{i=1}^{M} k_{i}^{c}-\left\lfloor\sum_{i=1}^{M} k_{i}^{c}\right\rfloor\right)\right\rfloor . \tag{23}
\end{equation*}
$$

2. Initialize $k_{i}=\left\lfloor k_{i}^{c}\right\rfloor$, and $\Delta K=K_{d}-\sum_{i=1}^{M}\left\lfloor k_{i}^{c}\right\rfloor$.
3. while $\Delta K>0$
4. 

Calculate the capacity gaps from the expected ergodic capacity as follows,

$$
\begin{equation*}
\chi_{i} \triangleq C_{i}-k_{i} f\left(n_{t}, \omega_{i} P_{i}^{c}\right) \tag{24}
\end{equation*}
$$

5. $i_{0}=\arg \max \left\{\chi_{1}, \cdots, \chi_{M}\right\}$ and $k_{i_{0}}=k_{i_{0}}+1$.
$\Delta K=\Delta K-1$.
end
return $k_{i}^{*}=k_{i}$.

C 1. When $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)} \leq K$, i.e. $\sum_{i=1}^{M} \frac{C_{t o t} \pi_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)} \leq K$, we know from Section III A that $\lambda^{\star}$ is zero. Then KKT condition (13a) becomes

$$
\begin{equation*}
n_{t} f\left(n_{t}, \omega_{i} P_{i}\right)-\omega_{i}\left[n_{t} P_{i}+\rho g\left(n_{t}\right)\right] f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)=0 \tag{25}
\end{equation*}
$$

It can be observed that the optimal power for user $i$, $P_{i}^{\star}\left(n_{t}\right)$, which is solved from (25) is independent of $C_{i}$ and thus is not changed with the total data rate $C_{t o t}$. When both the numerator and denominator of the EE expression in (22) are divided by $C_{t o t}$, we can easily obtain that the optimal EE increases with $C_{t o t}$.
C 2. When $\sum_{i=1}^{M} \frac{C_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)}>K$, the Lagrange multiplier, $\lambda^{\star}$, satisfies

$$
\begin{equation*}
\sum_{i=1}^{M} \frac{C_{t o t} \pi_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}\left(\lambda^{\star}\right)\right)}=K \tag{26}
\end{equation*}
$$

Due to the complicated relationship among $C_{t o t}, \lambda^{\star}$, and $P_{i}^{\star}\left(n_{t}\right)$ shown in (13a) and (26), it is difficult to study the impact of the SE on the EE in this case. Instead, we investigate an extreme scenario when the sum capacity goes to infinity. When $C_{t o t} \rightarrow \infty$, we can easily find from (26) that $\sum_{i=1}^{M} \frac{\pi_{i}}{f\left(n_{t}, \omega_{i} \theta_{i}\left(\lambda^{\star}\right)\right)} \rightarrow 0$. Therefore, $P_{i}^{\star}\left(n_{t}\right) \triangleq \theta_{i}\left(\lambda^{\star}\right)$ approaches infinity. Then the optimal EE in this extreme scenario can be derived as follows,

$$
\begin{aligned}
& \lim _{\text {tot } \rightarrow \infty} \eta^{\star}\left(n_{t}\right) \\
& =\lim _{C_{\text {tot }} \rightarrow \infty} \frac{1}{\sum_{i=1}^{M} \frac{\pi_{i}\left[n_{t} P_{i}^{\star}\left(n_{t}\right) / \rho+g\left(n_{t}\right)\right]}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}+P_{c 1}+\frac{n_{t} P_{c 4}+P_{c 5}}{C_{t o t}}}
\end{aligned}
$$

$$
\begin{equation*}
=\frac{1}{\sum_{i=1}^{M} \lim _{P_{i}^{\star}\left(n_{t}\right) \rightarrow \infty} \frac{\pi_{i}\left[n_{t} P_{i}^{\star}\left(n_{t}\right) / \rho+g\left(n_{t}\right)\right]}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}+P_{c 1}}=0 . \tag{27}
\end{equation*}
$$

From the result that the optimal EE is zero when the sum of data rates goes to infinity, it can be implied that a tradeoff between the SE and the EE exists when $C_{t o t}>$

$$
\frac{K}{\sum_{i=1}^{M} \pi_{i} / f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)} .
$$

In summary, the above discussion reveals that when the SE is low enough such that $C_{\text {tot }} \leq \frac{K}{\sum_{i=1}^{M} \pi_{i} / f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)}$, i.e.,
the total number of subcarriers used by all users is not constrained to the maximum number when $\lambda^{\star}=0$, the optimal EE increases with the SE. When the SE is so large that $C_{t o t}>\frac{K}{\sum_{i=1}^{M} \pi_{i} / f\left(n_{t}, \omega_{i} \theta_{i}(0)\right)}$, there exists a tradeoff between the SE and the EE.

## B. Impact of Capacity Fairness among Users

Substituting $\left\{C_{i}\right\}_{i=1}^{M}$ by $C_{\text {tot }}$ and $\left\{\pi_{i}\right\}_{i=1}^{M}$, problem (11) is formulated as a linear program with respect to $\left\{\pi_{i}\right\}_{i=1}^{M}$. According to the simplex method of linear programming [15], the optimal solution is achieved only when at most two elements of $\left\{\pi_{i}\right\}_{i=1}^{M}$ are nonzero. Without loss of generality, we assume that $\pi_{j}$ and $\pi_{k}$ are nonzero. Then problem (11) is reformulated as

$$
\begin{array}{ll}
\min _{\mathbf{P}} & C_{t o t}\left\{\frac{\pi_{j}\left[\frac{n_{t} P_{j}^{r}}{\rho \omega_{j}}+g\left(n_{t}\right)\right]}{f\left(n_{t}, P_{j}^{r}\right)}+\frac{\pi_{k}\left[\frac{n_{t} P_{k}^{r}}{\rho \omega_{k}}+g\left(n_{t}\right)\right]}{f\left(n_{t}, P_{k}^{r}\right)}\right\} \\
& +P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5} \\
\text { s. t. } & C_{t o t}\left[\frac{\pi_{j}}{f\left(n_{t}, P_{j}^{r}\right)}+\frac{\pi_{k}}{f\left(n_{t}, P_{k}^{r}\right)}\right] \leq K, \\
& P_{j}^{r}>0, \quad P_{k}^{r}>0, \tag{28b}
\end{array}
$$

where $P_{i}^{r} \triangleq \omega_{i} P_{i}$ denotes the received SNR of user $i$.
When the values of $\pi_{j}, \pi_{k}, P_{j}^{r}$, and $P_{k}^{r}$ are given, the value of the objective function of problem (28) decreases with $\omega_{j}$ and $\omega_{k}$. Therefore, choosing the maximum two values from $\left\{\omega_{i}\right\}_{i=1}^{M}$, i.e., selecting two users with maximum channel gains to transmit data, will minimize the total power consumption. Assume that $\omega_{1}$ and $\omega_{2}$ are the maximum two channel gains. We can further show that the maximal EE is achieved when $\pi_{1}=1$ in the following two extreme cases.
C 1. When $C_{t o t} \rightarrow 0$, the equality in constraint (28a) does not hold when the minimum value of problem (28) is achieved, we know from (13c) that the solution of Lagrange multiplier, $\lambda^{\star}$, is zero. Then according to KKT condition (13a), we can express the optimal values of transmit power, $P_{1}^{\star}\left(n_{t}\right)$ and $P_{2}^{\star}\left(n_{t}\right)$ as

$$
\begin{equation*}
P_{i}^{\star}\left(n_{t}\right)=\frac{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}{\omega_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}-\frac{\rho}{n_{t}} g\left(n_{t}\right), \quad i=1,2 \tag{29}
\end{equation*}
$$

After substituting it into (22), we have

$$
\eta^{\star}\left(n_{t}\right)=
$$

$$
\begin{equation*}
\frac{C_{t o t}}{\sum_{i=1}^{2} \frac{C_{t o t i} \pi_{t}}{\rho \omega_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}^{*}\left(n_{t}\right)\right)}+P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5}} \tag{30}
\end{equation*}
$$

It is proved in Appendix $C$ that the term $\omega_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)$ increases with $\omega_{i}$. Then we can obtain $\frac{n_{t}}{\rho \omega_{1} f^{\prime}\left(n_{t}, \omega_{1} P_{1}^{\star}\left(n_{t}\right)\right)}<\frac{n_{t}}{\rho \omega_{2} f^{\prime}\left(n_{t}, \omega_{2} P_{2}^{\star}\left(n_{t}\right)\right)}$. Furthermore,

$$
\begin{equation*}
\eta^{\star}\left(n_{t}\right) \leq \frac{C_{t o t}}{\frac{n_{t} C_{t o t}}{\rho \omega_{1} f^{\prime}\left(n_{t}, \omega_{1} P_{1}^{\star}\left(n_{t}\right)\right)}+P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5}} \tag{31}
\end{equation*}
$$

where the equality holds when $\pi_{1}=1$.
C 2. When $C_{t o t} \rightarrow \infty$, the equality in constraint (28a) holds when the minimum value of problem (28) is achieved. The optimal transmit power satisfies

$$
\begin{equation*}
C_{t o t}\left[\frac{\pi_{1}}{f\left(n_{t}, \omega_{1} P_{1}^{\star}\left(n_{t}\right)\right)}+\frac{\pi_{2}}{f\left(n_{t}, \omega_{2} P_{2}^{\star}\left(n_{t}\right)\right)}\right]=K \tag{32}
\end{equation*}
$$

Substituting (32) into (22), the optimal EE becomes

$$
\begin{align*}
& \eta^{\star}\left(n_{t}\right)= \\
& \frac{C_{t o t}}{\sum_{i=1}^{2} \frac{C_{t o t} n_{t} \pi_{i} P_{i}^{\star}\left(n_{t}\right)}{\rho f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}+K g\left(n_{t}\right)+P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5}} .
\end{align*}
$$

Since $P_{i}^{\star}\left(n_{t}\right) \rightarrow \infty$ when $C_{\text {tot }} \rightarrow \infty, f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)$ can be approximated as follows,

$$
\begin{align*}
& f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right) \approx \Delta f m \int_{0}^{\infty} \log _{2}\left(\omega_{i} P_{i}^{\star}\left(n_{t}\right) x\right) p_{\mathbf{x}}(x) \mathrm{d} x \\
& =\Delta f m\left(\log _{2}\left(\omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)+\int_{0}^{\infty} \log _{2} x p_{\mathbf{x}}(x) \mathrm{d} x\right) \tag{34}
\end{align*}
$$

Using this approximation, we prove in Appendix D that $\frac{\pi_{i} P_{i}^{\star}\left(n_{t}\right)}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}$ decreases with $\omega_{i}$. Consequently, the optimal EE satisfies

$$
\begin{aligned}
& \eta^{\star}\left(n_{t}\right) \leq \\
& \frac{C_{t o t}}{\frac{C_{t o t} n_{t} P_{1}^{\star}\left(n_{t}\right)}{\rho f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}+K g\left(n_{t}\right)+P_{c 1} C_{t o t}+n_{t} P_{c 4}+P_{c 5}}
\end{aligned}
$$

where the equality holds when $\pi_{1}=1$.
We have shown from the above analysis that the optimal EE is maximized when all data are transmitted by at most two users with the maximum channel gains in general cases. For two extreme cases, it is concluded that the optimal EE is maximized when all data are transmitted to one user with the maximum channel gain. These results imply that sacrificing the fairness among users can gain better performance.

## V. Simulation Results

In this section, we first study the impact of the SE, the fairness among users, and the spatial- frequency resources on the EE. Then we show the impact of the maximum amount of spatial and frequency resources on the relationship between the SE and the EE. Finally, we demonstrate the performance gain of jointly configuring spatial-frequency resources over those only adaptively configuring spatial or frequency resource. Main system parameters in the simulation are similar

TABLE V
List of Simulation Parameters

| Subcarrier spacing, $\Delta f$ | 15 kHz |
| :--- | :--- |
| Maximum number of subcarriers, $K$ | $512,768,1024$ |
| Number of RF chains at the BS, $N_{t}$ | $2,4,8$ |
| Number of RF chains at each user, $N_{r}$ | 4 |
| Power spectral density of noise | $-174 \mathrm{dBm} / \mathrm{Hz}$ |
| Noise amplifier gain | 7 dBi |
| Minimum distance from BS to users | 35 m |
| Path loss (dB) | $35+38 \log _{10} d$ |
| Efficiency of power amplifier, $\rho$ | $38 \%$ |
| $P_{c 1}$ | $0.01 \mu \mathrm{~W}$ |
| $\alpha P_{c 2}$ | 20 mW |
| $\beta P_{c 2}+P_{c 3}$ | 20 mW |
| $P_{c 4}$ | 1000 mW |
| $P_{c 5}$ | 10000 mW |



Fig. 2. EE vs. SE when the sum capacity is uniformly distributed among 8 users, $K=1024$, and $N_{t}=8$.
to those in [9] and are listed in Table V. Since the configuration of spatial and frequency resources depends on the channel gains of multiple users, we fix user locations to study the impact of the SE and the fairness among users. The distances of 8 users from the BS are set to $60 \mathrm{~m}, 80 \mathrm{~m}, 100 \mathrm{~m}, 120 \mathrm{~m}, 140$ $\mathrm{m}, 160 \mathrm{~m}, 180 \mathrm{~m}$, and 200 m , respectively.

Figure 2 shows the EEs achieved by the algorithm in Section IV.A versus the SE with different numbers of active RF chains at the BS. We can divide the SE region into two parts based on whether the total number of used subcarriers is constrained to the maximum value when the optimum of (11) is achieved. Solid curves represent the unconstrained regions while dash curves represent the constrained regions. We can see that the optimal EE increases with the SE in the unconstrained regions while there exists a tradeoff between the SE and the EE in the constrained regions. We can further observe that in the unconstrained regions, the EE decreases with the number of active RF chains, which is consistent with Theorem 1.

When we select the maximum value from all the EEs under different numbers of active RF chains, we can obtain the maximum EE by optimizing the number of active RF chains. We plot the optimal number of active RF chains and the sum of optimal numbers of used subcarriers in Fig. 3. We can see that both the amount of used spatial resource and that of


Fig. 3. Optimal number of active RF chains and optimal total number of subcarriers vs. SE when the sum capacity is uniformly distributed among 8 users, $K=1024$, and $N_{t}=8$.


Fig. 4. EE vs. SE under different capacity ratios when $K=1024$ and $N_{t}=8$.
frequency resource increase with the SE. We can also observe that the optimal number of active RF chains increases only when the the sum of optimal numbers of used subcarriers is constrained to the maximum value, $K$. This phenomenon implies that frequency resource is more beneficial to increase the EE than spatial resource in MIMO-OFDMA systems. We also plot $n_{0}$ which is mentioned in Section III.B versus the SE. We can see that the value of $n_{0}$ increases with the SE and is much smaller than $N_{t}$ in the low SE region. Therefore, the complexity to find the optimal number of active RF chains can be greatly reduced by the serial searching method in the low SE region.

In Fig. 4 and Fig. 5, we demonstrate the impact of fairness among users on the EE and the required optimal spatial and frequency resources, respectively. Because we have proved in Section IV.B that the optimal EE is maximized when all data are transmitted to at most two users with the maximum channel gains. We only consider the case when the sum capacity is only allocated to the users which are 60 m and 80 m away from the BS. Denote $\pi$ as the capacity ratio for the user 60 m away from the BS. Fig. 4 shows that the EE


Fig. 5. Optimal number of active RF chains and optimal total number of subcarriers vs. SE under different capacity ratios when $K=1024$ and $N_{t}=8$.
increases with $\pi$, which implies that transmitting more data by the user with highest channel gain improves the relationship between the SE and the EE as a whole. This allows us to extend the conclusion in the extreme two cases in Section IV.B to general cases. From Fig. 5, we can see that both the optimal number of active RF chains and the sum of the optimal numbers of subcarriers decrease with $\pi$, which means that transmitting more data by the user with higher channel gain can also reduce the required resources.
In the following simulation, we consider that 8 users are uniformly distributed within a circle with radius of 200 m and the required sum capacity is randomly allocated to multiple users. Fig. 6 shows the impact of the maximum amount of spatial and frequency resources on the SE-EE relationship. We can see that the relationship can be improved as a whole by increasing the overall frequency resource while increasing the overall spatial resource can only increase the relationship in the high SE region. This also infers that the frequency resource is more efficient than the spatial resource to improve the SEEE relationship.

Figure 7 shows the benefit of adaptive configuration of both spatial and frequency resources. We compare the EE of the proposed algorithm with spatial-only adaptation (SOA) and frequency-only adaptation (FOA). For the SOA, the total number of used subcarriers is set to be the maximum number, $K$. For the FOA, the number of active RF chains at the BS is set to be the maximum value, $N_{t}$. As shown in the figure, the spatial-frequency-adaptation outperforms both SOA and FOA and its performance is overlapped with its upper bound which is obtained by relaxing the numbers of subcarriers used by 8 users as continuous variables. This means that the proposed algorithm is near-optimal. In most of the SE region, the SOA outperforms the FOA.

## VI. Conclusion

In this paper, we have studied the configuration of spatial and frequency resources from the perspective of maximizing the EE for downlink MIMO-OFDMA systems when channel information is not available at the BS. We first formulated an optimization problem to minimize the total power including


Fig. 6. Impact of the maximum number of subcarrier, $K$, and that of RF chains, $N_{t}$, on the relationship between the EE and the SE.


Fig. 7. Comparison of the performance of different resource configuration schemes when $K=1024$ and $N_{t}=8$.
transmit power and circuit power consumption at the BS with ergodic capacity requirements from multiple users. Then we developed a three-step searching algorithm, which first found the continuous variable solution for the number of subcarriers occupied by each user based on the KKT conditions, then optimized the number of active RF chains, and finally discretized the number of subcarriers for each user with the optimal number of active RF chains. We investigated the impact of the SE and the user fairness on the optimal EE. It is shown that a tradeoff between the SE and the EE exists when the total number of active subcarriers is restricted to a maximum value. The optimal number of active RF chains increases only when the total number of used subcarriers cannot be increased, which means that frequency resource is more efficient than spatial resource on improving the EE. Moreover, increasing the amount of frequency resource can improve the SE-EE relationship as a whole while increasing the amount of spatial resource can only gain the enhancement in the high SE region. Transmitting more data by users with larger channel gains can obtain better SE-EE relationship, which implies a tradeoff between the capacity fairness among users and
the EE. The proposed spatial-frequency resource adaptive configuration outperforms both the spatial-only-adaptation and the frequency-only-adaptation.

## Appendix A

## Proof of Monotonically Increasing of the Optimal $P_{i}$ WITh $\lambda$

According to (13a), $\lambda$ can be expressed as

$$
\lambda=\frac{n_{t}}{\rho}\left(\frac{f\left(n_{t}, \omega_{i} P_{i}\right)}{\omega_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)}-P_{i}\right)-g\left(n_{t}\right)
$$

Then we can obtain the derivative of $\lambda$ on $P_{i}$ as

$$
\frac{d \lambda}{d P_{i}}=-\frac{n_{t} f\left(n_{t}, \omega_{i} P_{i}\right) f^{\prime \prime}\left(n_{t}, \omega_{i} P_{i}\right)}{\rho\left(f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)\right)^{2}}
$$

where $\left.f^{\prime \prime}\left(n_{t}, \omega_{i} P_{i}\right) \triangleq \frac{\partial^{2} f\left(n_{t}, \gamma\right)}{\partial \gamma^{2}}\right|_{\gamma=\omega_{i} P_{i}}$. Because $f\left(n_{t}, \omega_{i} P_{i}\right)$ is a concave function with respect to $P_{i}$ as shown in the expression under (7), we have $f^{\prime \prime}\left(n_{t}, \omega_{i} P_{i}\right)<0$. Consequently we can conclude $\frac{d \lambda}{d P_{i}}>0$ and thus $\lambda$ is a monotonically increasing function of $P_{i}$.

## Appendix B

## Proof of Theorem 1

When the circuit power is zero, the objective function of problem (11) becomes

$$
\begin{equation*}
P_{t o t}=\sum_{i=1}^{M} \frac{C_{i} n_{t} P_{i}}{\rho f\left(n_{t}, \omega_{i} P_{i}\right)} \tag{35}
\end{equation*}
$$

Differentiating $P_{\text {tot }}$ with respect to $P_{i}$, we can obtain

$$
\begin{equation*}
\frac{\partial P_{t o t}}{\partial P_{i}}=\frac{C_{i} n_{t}}{\rho} \frac{f\left(n_{t}, \omega_{i} P_{i}\right)-\omega_{i} P_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)}{f^{2}\left(n_{t}, \omega_{i} P_{i}\right)} \tag{36}
\end{equation*}
$$

Since $f\left(n_{t}, \omega_{i} P_{i}\right) \triangleq \Delta f m \int_{0}^{\infty} \log _{2}\left(1+\omega_{i} P_{i} x\right) p_{\mathbf{x}}(x) \mathrm{d} x$, which is a strict concave function with respect to $\omega_{i} P_{i}$ and $f\left(n_{t}, 0\right)=0$, we have

$$
\begin{equation*}
f\left(n_{t}, \omega_{i} P_{i}\right)-\omega_{i} P_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)>0 \tag{37}
\end{equation*}
$$

Substituting (37) into (36), we can obtain that $\frac{\partial P_{t o t}}{\partial P_{i}}>0$, which means that the total power consumption increases with $P_{i}$. Therefore, the minimum power consumption is achieved when $P_{i}=0$ and its value is

$$
\begin{equation*}
P_{t o t}^{\min }=\left.\sum_{i=1}^{M} \frac{C_{i} n_{t} P_{i}}{\rho f\left(n_{t}, \omega_{i} P_{i}\right)}\right|_{P_{i}=0} \tag{38}
\end{equation*}
$$

According to L'Hôpital's rule, we can further derive

$$
\begin{equation*}
P_{t o t}^{\min }=\sum_{i=1}^{M} \frac{C_{i} n_{t}}{\left.\rho \frac{\partial f\left(n_{t}, \omega_{i} P_{i}\right)}{\partial P_{i}}\right|_{P_{i}=0}} \tag{39}
\end{equation*}
$$

Based on the expression of $f\left(n_{t}, \omega_{i} P_{i}\right)$, we can easily find its derivative at $P_{i}=0$ as follows,

$$
\begin{equation*}
\left.\frac{\partial f\left(n_{t}, \omega_{i} P_{i}\right)}{\partial P_{i}}\right|_{P_{i}=0}=\frac{\Delta f m \omega_{i}}{\ln 2} \int_{0}^{\infty} x p_{\mathbf{x}}(x) \mathrm{d} x=\frac{\Delta f \omega_{i} N_{r} n_{t}}{\ln 2} \tag{40}
\end{equation*}
$$

Substituting (40) into (39), we can finally obtain the minimum power consumption as

$$
\begin{equation*}
P_{t o t}^{\min }=\frac{\ln 2}{\rho \Delta f N_{r}} \cdot \sum_{i=1}^{M} \frac{C_{i}}{\omega_{i}} \tag{41}
\end{equation*}
$$

We can see from (41) that when the circuit overhead is zero, the minimum power consumption is independent of the number of transmit antennas.

When the circuit power is not zero, we prove the theorem by first studying the property of $f\left(n_{t}, \omega_{i} P_{i}\right)$.

According to (5), the matrix form of $f\left(n_{t}, \omega_{i} P_{i}\right)$ is

$$
\begin{equation*}
f\left(n_{t}, \omega_{i} P_{i}\right)=\Delta f \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\omega_{i} P_{i} \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)\right] \tag{42}
\end{equation*}
$$

where $\mathbf{H}_{n_{t}}$ denotes an $N_{r} \times n_{t}$ matrix whose elements are subject to Gaussian distribution with zero mean and unit variance. To simplify the following expressions, we denote $\gamma=\omega_{i} P_{i}$. We first study some properties of $f\left(n_{t}, \gamma\right)$. From (42), we can derive $f\left(n_{t}+1, \gamma\right)$ as follows,

$$
\begin{aligned}
f\left(n_{t}+1, \gamma\right) & =\Delta f \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}+1} \mathbf{H}_{n_{t}+1}^{H}\right)\right] \\
& =\Delta f \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}+\gamma \mathbf{h} \mathbf{h}^{H}\right)\right]
\end{aligned}
$$

where the matrix partition, $\mathbf{H}_{n_{t}+1}=\left[\begin{array}{ll}\mathbf{H}_{n_{t}} & \mathbf{h}\end{array}\right]$, is used. Then we can obtain

$$
\begin{align*}
& \frac{f\left(n_{t}, \gamma\right)-f\left(n_{t}+1, \gamma\right)}{\Delta f} \\
& =\mathbb{E}\left[\log _{2} \operatorname{det}\left(\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)\right)\right]- \\
& \quad \mathbb{E}\left[\log _{2} \operatorname{det}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}+\gamma \mathbf{h} \mathbf{h}^{H}\right)\right] \\
& =-\mathbb{E}\left[\log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}\right)\right] \tag{43}
\end{align*}
$$

where the matrix inverse lemma is used. Similarly, we can derive

$$
\begin{aligned}
& \frac{f\left(n_{t}+1, \gamma\right)-f\left(n_{t}+2, \gamma\right)}{\Delta f} \\
= & -\mathbb{E}\left[\log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}+1} \mathbf{H}_{n_{t}+1}^{H}\right)^{-1} \mathbf{h}\right)\right] \\
= & -\mathbb{E}\left[\log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma\left(\mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}+\mathbf{h}_{1} \mathbf{h}_{1}^{H}\right)\right)^{-1} \mathbf{h}\right)\right] \\
= & -\mathbb{E}\left[\log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}-\gamma \mathbf{h}^{H} \mathbf{A} \mathbf{h}\right)\right],
\end{aligned}
$$

where $\mathbf{H}_{n_{t}+1}=\left[\begin{array}{ll}\mathbf{H}_{n_{t}} & \mathbf{h}_{1}\end{array}\right]$ is used and $\mathbf{A} \triangleq$ $\frac{\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}_{1} \mathbf{h}_{1}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1}}{\left(\frac{1}{\gamma}+\mathbf{h}_{1}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}_{1}\right)}$.

It is easy to show that $\mathbf{A}$ is a positive semi-definite matrix. Then we have

$$
\begin{aligned}
& \log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}\right) \geq \\
& \log _{2}\left(1+\gamma \mathbf{h}^{H}\left(\mathbf{I}_{N_{r}}+\gamma \mathbf{H}_{n_{t}} \mathbf{H}_{n_{t}}^{H}\right)^{-1} \mathbf{h}-\gamma \mathbf{h}^{H} \mathbf{A} \mathbf{h}\right)
\end{aligned}
$$

Since $\mathbf{H}_{n_{t}}, \mathbf{h}_{1}$ and $\mathbf{h}$ are mutually independent, the equality dose not hold for all the possible values of $\mathbf{H}_{n_{t}}, \mathbf{h}_{1}$ and $\mathbf{h}$. Consequently, we can obtain
$f\left(n_{t}+2, \gamma\right)-f\left(n_{t}+1, \gamma\right)<f\left(n_{t}+1, \gamma\right)-f\left(n_{t}, \gamma\right), n_{t}=1, \cdots$
In addition, we can easily prove from (43) that $f(2, \gamma)-$ $f(1, \gamma)<f(1, \gamma)$. Defining $f(0, \gamma)=0,(44)$ also holds when $n_{t}=0$. Based on this result, we derive the following inequality as

$$
\begin{aligned}
f\left(n_{t}+1, \gamma\right)-f\left(n_{t}, \gamma\right) & <\frac{1}{n_{t}} \sum_{i=1}^{n_{t}}(f(i, \gamma)-f(i-1, \gamma)) \\
& =\frac{f\left(n_{t}, \gamma\right)}{n_{t}}
\end{aligned}
$$

Finally, we have

$$
\begin{equation*}
\frac{f\left(n_{t}+1, \gamma\right)}{n_{t}+1}<\frac{f\left(n_{t}, \gamma\right)}{n_{t}} \tag{45}
\end{equation*}
$$

Meanwhile, the objective function of problem (11) can be rewritten as

$$
\begin{align*}
\Phi\left(n_{t}, \mathbf{P}\right)= & \sum_{i=1}^{M} \frac{C_{i}\left[P_{i} / \rho+g\left(n_{t}\right) / n_{t}\right]}{f\left(n_{t}, \omega_{i} P_{i}\right) / n_{t}}+P_{c 1} \sum_{i=1}^{M} C_{i} \\
& +n_{t} P_{c 4}+P_{c 5} . \tag{46}
\end{align*}
$$

We know from the expression of $g\left(n_{t}\right)$ that $g\left(n_{t}\right) / n_{t}$ increases with $n_{t}$ when $g\left(n_{t}\right) \neq 0$. Moreover, $f\left(n_{t}, \omega_{i} P_{i}\right) / n_{t}$ decreases with $n_{t}$ as shown in (45). Therefore, $\Phi\left(n_{t}, \mathbf{P}\right)$ increases with $n_{t}$, i.e.,

$$
\begin{equation*}
\Phi\left(n_{t}, \mathbf{P}\right)<\Phi\left(n_{t}+1, \mathbf{P}\right) \tag{47}
\end{equation*}
$$

Finally we have

$$
\begin{equation*}
\min _{\mathbf{P}}\left\{\Phi\left(n_{t}, \mathbf{P}\right)\right\}<\min _{\mathbf{P}}\left\{\Phi\left(n_{t}+1, \mathbf{P}\right)\right\} . \tag{48}
\end{equation*}
$$

Consequently, the minimum value of the objective function in problem (11) increases with $n_{t}$ when the circuit power related to RF chains is not zero.

## Appendix C

Proof of the Increasing of $\omega_{i} f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)$ WITh $\omega_{i}$

We can define an implicit function from KKT condition (13a) as follows,

$$
\begin{equation*}
F\left(\omega_{i}, P_{i}\right) \triangleq n_{t} f_{i}-\omega_{i}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime} \tag{49}
\end{equation*}
$$

where notations, $f_{i} \triangleq f\left(n_{t}, \omega_{i} P_{i}\right)$ and $f_{i}^{\prime} \triangleq f^{\prime}\left(n_{t}, \omega_{i} P_{i}\right)$ are used to simplify the expression.

According to the implicit function theorem, $P_{i}$ can be expressed as a function of $\omega_{i}$ from KKT condition, $F\left(\omega_{i}, P_{i}\right)=$ 0 , and its derivative with respect to $\omega_{i}$ can be calculated as

$$
\begin{align*}
\frac{\mathrm{d} P_{i}}{\mathrm{~d} \omega_{i}} & =-\frac{F_{\omega_{i}}^{\prime}}{F_{P_{i}}^{\prime}}  \tag{50}\\
& =-\frac{\rho\left(g\left(n_{t}\right)+\lambda\right) f_{i}^{\prime}+\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] \omega_{i} P_{i} f_{i}^{\prime \prime}}{\omega_{i}^{2}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime \prime}}
\end{align*}
$$

where $f_{i}^{\prime \prime} \triangleq f^{\prime \prime}\left(n_{t}, \omega_{i} P_{i}\right)$ is used to simplify the expression.
Then the derivative of $\omega_{i} f_{i}^{\prime}$ with respect to $\omega_{i}$ can be derived as follows,

$$
\begin{align*}
\frac{\mathrm{d}\left(\omega_{i} f_{i}^{\prime}\right)}{\mathrm{d} \omega_{i}} & =f_{i}^{\prime}+\omega_{i} f_{i}^{\prime \prime}\left(P_{i}+\omega_{i} \frac{d P_{i}}{d \omega_{i}}\right) \\
& =f_{i}^{\prime}-\omega_{i} f_{i}^{\prime \prime} \frac{\rho\left(g\left(n_{t}\right)+\lambda\right) f_{i}^{\prime}}{\omega_{i}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime \prime}} \\
& =\frac{n_{t} P_{i} f_{i}^{\prime}}{n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)} \tag{51}
\end{align*}
$$

where (50) is used. We know that $f_{i}^{\prime}>0$ because $f_{i}$ is a monotonically increasing function of $P_{i}$. Therefore, $\frac{\mathrm{d}\left(\omega_{i} f_{i}^{\prime}\right)}{\mathrm{d} \omega_{i}}>$ 0 , which means that $\omega_{i} f_{i}^{\prime}$ is a monotonically increasing function of $\omega_{i}$.

## Appendix D

PRoof of the Decreasing of $\frac{P_{i}^{\star}\left(n_{t}\right)}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}$ WITH $\omega_{i}$
From KKT condition (13a), $P_{i}$ can be expressed as a function of $\omega_{i}$ and its derivative is shown in (50). Denoting $y\left(\omega_{i}\right)=\frac{P_{i}}{f_{i}}$, we can find the derivative of $y\left(\omega_{i}\right)$ with respect to $\omega_{i}$ as shown on the top of this page.

Due to the concavity of $f_{i}$, we know $f_{i}^{\prime \prime}<0$. Therefore, we can obtain

$$
\begin{equation*}
\frac{\mathrm{d} y\left(\omega_{i}\right)}{\mathrm{d} \omega_{i}}<\frac{\rho\left(g\left(n_{t}\right)+\lambda\right)\left(\omega_{i} P_{i} f_{i}^{\prime 2}-f_{i}^{\prime} f_{i}-\omega_{i} P_{i} f_{i}^{\prime \prime} f_{i}\right)}{\omega_{i}^{2}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime \prime} f_{i}^{2}} \tag{52}
\end{equation*}
$$

Based on the approximation of $f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)$ in (34), we have

$$
\omega_{i} P_{i} f_{i}^{\prime 2}-f_{i}^{\prime} f_{i}-\omega_{i} P_{i} f_{i}^{\prime \prime} f_{i}=\omega_{i} P_{i} f_{i}^{\prime 2}
$$

Substituting it into (52), we have

$$
\begin{equation*}
\frac{\mathrm{d} y\left(\omega_{i}\right)}{\mathrm{d} \omega_{i}}<\frac{\rho\left(g\left(n_{t}\right)+\lambda\right) \omega_{i} P_{i} f_{i}^{\prime 2}}{\omega_{i}^{2}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime \prime} f_{i}^{2}} \tag{53}
\end{equation*}
$$

Because $f_{i}^{\prime \prime}<0$, we can conclude that $\frac{\mathrm{d} y\left(\omega_{i}\right)}{\mathrm{d} \omega_{i}}<0$ and $\frac{P_{i}^{\star}\left(n_{t}\right)}{f\left(n_{t}, \omega_{i} P_{i}^{\star}\left(n_{t}\right)\right)}$ decreases with $\omega_{i}$.

## REFERENCES

[1] G. Y. Li, Z. Xu, C. Xiong, C. Yang, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient wireless communications: tutorial, survey, and open issues," IEEE Wireless Commun. Mag., vol. 18, no. 6, pp. 28-35, Dec. 2011.
[2] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks," IEEE J. Sel. Areas Commun., vol. 22, no. 6, pp. 1089-1098, Aug. 2004.
[3] B. Bougard, G. Lenoir, A. Dejonghe, L. van Perre, F. Catthor, and W. Dehaene, "Smart MIMO: an energy-aware adaptive MIMO-OFDM radio link control for next generation wireless local area networks," EURASIP J. Wireless Commun. Networking, vol. 2007, no. 3, pp. 1-15, June 2007.
[4] H. Kim, C.-B. Chae, G. de Veciana, and J. R. W. Heath, "A cross-layer approach to energy efficiency for adaptive MIMO systems exploiting spare capacity," IEEE Trans. Wireless Commun., vol. 8, no. 8, pp. 42644275, Aug. 2009.
[5] H. Yu, L. Zhong, and A. Sabharwal, "Adaptive RF chain management for energy-efficient spatial multiplexing MIMO transmission," in Proc. 2009 Int. Sym. Low Power Electronics and Design, Aug. 2009.
[6] O. Holland, V. Friderikos, and A. H. Aghvami, "Green spectrum magagement for mobile operators," in Proc. 2010 IEEE GlobeCom.
[7] Y. Chen, S. Zhang, S. Xu, and G. Y. Li, "Fundamental tradeoffs on green wireless networks," IEEE Commun. Mag., vol. 49, no. 6, pp. 3037, June 2011.
[8] S. Zhang, Y. Chen, and S. Xu, "Improving energy efficiency through bandwidth, power, and adaptive modulation," in Proc. 2010 IEEE VTC - Fall.
[9] H. S. Kim and B. Daneshrad, "Energy-constrained link adaptation for MIMO OFDM wireless communication systems," IEEE Trans. Wireless Comтип., vol. 9, no. 9, pp. 2820-2832, Sep. 2010.
[10] C. Isheden and G. P. Fettweis, "Energy-efficient multi-carrier link adaptation with sum rate-dependent circuit power," in Proc. 2010 IEEE GlobeCom.
[11] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge University Press, 2005.
[12] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," Euro. Trans. Telecommun., vol. 10, no. 6, pp. 585-595, Nov.-Dec. 1999.
[13] G. Miao, N. Himayat, and G. Y. Li, "Energy-efficient link adaptation in frequency-selective channels," IEEE Trans. Commun., vol. 58, no. 2, pp. 545-554, Feb. 2010.
[14] M. S. Bazaraa and C. M. Shetty, Nonlinear Programming: Theory and Algorithms. Wiley, 1970.
[15] J. Nocedal and S. J. Wright, Numerical Optimization. Springer, 1999.

$$
\frac{\mathrm{d} y\left(\omega_{i}\right)}{\mathrm{d} \omega_{i}}=\frac{\frac{\mathrm{d} P_{i}}{\mathrm{~d} \omega_{i}} f_{i}-P_{i} f_{i}^{\prime}\left(P_{i}+\omega_{i} \frac{\mathrm{~d} P_{i}}{\mathrm{~d} \omega_{i}}\right)}{f_{i}^{2}}=\frac{\omega_{i} P_{i} \rho\left(g\left(n_{t}\right)+\lambda\right) f_{i}^{\prime 2}-\rho\left(g\left(n_{t}\right)+\lambda\right) f_{i}^{\prime} f_{i}-\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] \omega_{i} P_{i} f_{i}^{\prime \prime} f_{i}}{\omega_{i}^{2}\left[n_{t} P_{i}+\rho\left(g\left(n_{t}\right)+\lambda\right)\right] f_{i}^{\prime \prime} f_{i}^{2}} .
$$



Zhikun Xu received his B.S.E. degree in electronics engineering in 2007 and now is pursuing his Ph.D degree in signal and information processing, both in the School of Electronics and Information Engineering, Beihang University, Beijing, China. From September 2009 to September 2010, he worked as a visiting student in the School of Electrical and Computer Engineering, Georgia Institute of Technology. His research interests include cognitive radio, crosslayer resource allocation, and green radio.


Chenyang Yang received her M.S.E. and Ph.D. degrees in 1989 and 1997 in Electrical Engineering, from Beijing University of Aeronautics and Astronautics (BUAA, now renamed as Beihang University). She is now a full professor in the School of Electronics and Information Engineering, BUAA. She has published various papers and filed many patents in the fields of signal processing and wireless communications. She was nominated as an Outstanding Young Professor of Beijing in 1995 and was supported by the 1st Teaching and Research Award Program for Outstanding Young Teachers of Higher Education Institutions by Ministry of Education (P.R.C. "TRAPOYT") during 1999-2004. Currently, she serves as an associate editor for IEEE TRANSACTIONS ON WIRELeSs Communications, an associate editor-in-chief of Chinese Journal of Communications and an associate editor-in-chief of Chinese Journal of Signal Processing. She is the chair of Beijing chapter of IEEE Communications Society. She has ever served as TPC members for many IEEE conferences such as ICC and GLOBECOM. Her recent research interests include network MIMO, energy efficient transmission and interference management in multicell systems.


Geoffrey Ye Li received his B.S.E. and M.S.E. degrees in 1983 and 1986, respectively, from the Department of Wireless Engineering, Nanjing Institute of Technology, Nanjing, China, and his Ph.D. degree in 1994 from the Department of Electrical Engineering, Auburn University, Alabama. He was a Teaching Assistant and then a Lecturer with Southeast University, Nanjing, China, from 1986 to 1991, a Research and Teaching Assistant with Auburn University, Alabama, from 1991 to 1994, and a PostDoctoral Research Associate with the University of Maryland at College Park, Maryland, from 1994 to 1996. He was with AT\&T Labs - Research at Red Bank, New Jersey, as a Senior and then a Principal Technical Staff Member from 1996 to 2000. Since 2000, he has been with the School of Electrical and Computer Engineering at Georgia Institute of Technology as an Associate and then a Full Professor. He is also holding the Cheung Kong Scholar title at the University of Electronic Science and Technology of China since March 2006. His general research interests include statistical signal processing and telecommunications, with emphasis on crosslayer optimization for spectral- and energy-efficient networks, cognitive radios, and practical techniques in LTE systems. In these areas, he has published over 100 referred journal papers and two books in addition to many conference papers. These publications have been cited over 10,000 times. He also has over 20 granted patents. He once served or is currently serving as an editor, a member of editorial board, and a guest editor for over 10 technical journals. He organized and chaired many international conferences, including technical program vice-chair of IEEE ICC'03 and co-chair of IEEE SPARC'11. He has been awarded an IEEE Fellow for his contributions to signal processing for wireless communications since 2006, selected as a Distinguished Lecturer for 2009-2010 by IEEE Communications Society, and won 2010 IEEE Communications Society Stephen O. Rice Prize Paper Award in the field of communications theory.


Shunqing Zhang received his B.E. degree from Fudan University, Shanghai, China, in 2005 and his Ph.D. degree from HKUST in 2009, respectively. He joined the Green Radio Excellence in Architecture and Technology (GREAT) project at Huawei Technologies Co. Ltd. after the graduation. His current research interests include energy-efficient resource allocation and optimization in cellular networks, joint baseband and radio frequency optimization, fundamental tradeoffs on green wireless network design, and other green radio technologies for energy saving and emission reduction.


Yan Chen received her B.Sc. and the Ph.D. degree in Information and Communication Engineering from Zhejiang University, Hangzhou, China, in 2004 and 2009, respectively. She had been a visiting researcher at the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong, from 2008 to 2009. After graduation, she joined the Central Research Institute of Huawei Technologies Co., Ltd.. She is now the project manager and technical leader of GREAT (Green Radio Excellence in Architectures \& Technologies), an internal green research project of Huawei focusing on the fundamental design of energy efficient solutions for radio access networks. Her current research interests include fundamental tradeoffs on green wireless network design, green network information theory, energy-efficient network architecture and management, as well as the radio technologies, advanced processing and resource management algorithms therein.


Shugong Xu received a B.Sc. degree from Wuhan University, China, and his M.E. and Ph.D. from Huazhong University of Science and Technology (HUST), Wuhan, China, in 1990, 1993, and 1996, respectively. He is currently director of the Access Network Technology Research Department, principal scientist, and vice director of the Communication Laboratory, Huawei Corporate Research, and chief scientist on Wireless Access Technologies in the Nation al Key Laboratory on Wireless technologies at Huawei. He leads green research in Huawei including the GREAT (Green Radio Excellence in Architecture and Technologies) project, which focuses on power-efficient solutions for wireless radio access networks. Prior to joining Huawei Technologies in 2008, he was with Sharp Laboratories of America as senior research scientist for seven years. Before starting his career in industrial research, he worked at universities including Tsinghua University, China, Michigan State University, as well as City College of New York (CCNY). In his over 18+ years of research on cutting edge research on wireless/mobile networking and communication, home networking and multimedia communications, he published more than 30 peer-reviewed research papers as lead-author in top international conferences and journals, in which the most referenced one has over 900 Google Scholar citations. He holds more than 30 granted US patents or patent applications, of which technologies have been adopted in 802.11 and LTE standards, including the DRX protocol for power saving in LTE standard. Dr. Xu is a senior member of IEEE, a Concurrent Professor at HUST, and the Technical Committee co-chair of Green Touch consortium.


[^0]:    Paper approved by Y. Fang, the Editor for Wireless Networks of the IEEE Communications Society. Manuscript received November 10, 2011; revised June 22, 2012.

    The work is supported in part by the Research Gift from Huawei Technologies Co., Ltd., National Basic Research Program of China, 973 Program 2012CB316000, National Natural Science Foundation of China (NSFC) under Grant 61120106002, and the Innovation Foundation of BUAA for Ph.D. Graduates.
    Z. Xu and C . Yang are with the School of Electronics and Information Engineering, Beihang University, Beijing 100191, China (e-mail: xuzhikun@ee.buaa.edu.cn, cyyang@buaa.edu.cn).
    G. Y. Li is with the School of ECE, Georgia Institute of Technology, Atlanta, Georgia USA (e-mail: liye@ece.gatech.edu).
    S. Zhang, Y. Chen, and S. Xu are with Huawei University, Shanghai, China (e-mail: \{sqzhang, zhangshunqing, eeyanchen, shugong\} @huawei.com).

    Digital Object Identifier 10.1109/TCOMM.2012.100512.110760

[^1]:    * Variables in expressions $f$ and $f^{\prime}$ in this table are ignored for simplicity.

