Phase Ambiguity Quantization for Per-cell Codebook Based Limited Feedback Coordinated Multi-point Transmission Systems

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***Abstract*—Per-cell codebook based limited feedback strategy is desirable for coordinated multi-point (CoMP) transmission system due to its scalability. In this paper, we study phase ambiguity quantization for centralized CoMP system with per-cell codebook based global channel quantization. We first analyze the performance degradation led by the phase ambiguity when codeword is selected for each cell separately, and show that the loss can be effectively recovered by feeding back a few bits in each cell for the phase ambiguity information. We then analyze the impact of the inherent asymmetric feature of CoMP channel due to heterogeneous path loss on the performance of per-cell feedback scheme, and show that the quantization error of phase ambiguity will lead to significant performance degradation of the global channel quantization only for cell edge users. Simulation results are provided that validate our analysis.**

***Index Terms*—CoMP, Limited feedback, Phase ambiguity**

I. Introduction

Multi-cell cooperative transmission has drawn significant attention recently [1], [2]. Coherent cooperative transmission, also known as network MIMO in literature [1] and as coordinated multi-point transmission (CoMP) with joint processing (JP) in 3GPP long term evolution (LTE), can exploit the full benefits of base station (BS) cooperation to meet the ever-increasing demands for spectrum efficiency.

The performance gain of CoMP-JP largely depends on how much channel state information (CSI) can be acquired at the cooperative BSs. When limited feedback techniques [3] are applied for CoMP-JP to report the CSI from all served mobile station (MS) to all cooperative BSs, new challenging arises such as huge feedback overhead [4] and dynamic codebook size [5], which are introduced by the new feature of CoMP channels and new architecture of CoMP systems. Design a joint codebook with vector quantization theory [6] for multi-cell channels by treating the cooperative BSs as a "super-BS" is optimal, but in reality is not appropriate for CoMP systems as recognized in [5]. First, global channels in CoMP systems may be asymmetric due to the heterogeneous path loss of the per-cell channels, which depends on the MSs locations. Consequently, codebooks designed for single cell systems do not perform well in CoMP systems with non-identically and independent distribution (i.i.d.) channels, and the codebook needs frequent re-design when MSs move. Second, the number of cooperative BSs may be dynamic [7]. This means that the codebooks with pre-determined size are not scalable for the CoMP channels with dynamic dimensions [5]. Last but not the least, a large size codebook leads to a large searching space thereby induces a prohibitive complexity for codeword selection.

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As a result, it is more desirable to design per-cell codebook based limited feedback scheme for CoMP systems [5], [8], where single cell codebooks are applied for quantizing the channel between each BS and the MS. In the per-cell codebook feedback scheme, the codeword for each per-cell channel can be selected either jointly or separately [8]. Joint codeword selection is optimal, but is of unaffordable complexity [5]. By contrast, independent codeword selection is of low complexity, but the resulting phase ambiguity will degrade the quantization performance of the global channel. Nonetheless, if perfect phase ambiguity information can be acquired, its negative impact will be compensated, and the independent codeword selection will even outperform the joint codeword selection as shown in [8].

In this paper, we study phase ambiguity quantization of the independent codeword selection for per-cell codebook based CoMP-JP systems. By analyzing a normalized quantization gain, we will show that the performance loss led by the phase ambiguity can be effectively alleviated by a few bits quantization for each cell. We will also show that in asymmetric channels the quantization error of global channel led by the phase ambiguity will reduce.

II. System Modeling

Consider a CoMP-JP system [1] consisting of N_c BSs each equipped with N_t antennas, which cooperatively serve multi-

[1]For simplicity, we refer CoMP-JP as CoMP in the rest of the paper.

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ple single antenna MSs in a time division or frequency division manner. Both the data for the MSs and the CSI from the BSs to the MSs are forwarded to a central unit (CU) via ideal backhaul links. The CU selects one MS to serve in each time slot or frequency band, and employs a beamformer for downlink transmission.

We consider to use per-cell codebook since it is flexible and compatible to existing single cell systems, where each MS quantizes its per-cell channel direction information (CDI) with single cell codebook. We consider separate codeword selection, i.e., for the ith cell, each MS quantizes its per-cell CDIs by selecting codewords as follows

$$\hat{\mathbf{h}}_i = \arg\max_{\mathbf{c}\in\mathcal{B}_\mathbf{i}} \|\tilde{\mathbf{h}}_i^H \mathbf{c}\|, \tag{1}$$

where $\tilde{\mathbf{h}}_i = \mathbf{h}_i/\|\mathbf{h}_i\| \in \mathcal{C}^{N_t\times 1}$ is the CDI of the per-cell channel from ith BS to the MS $\mathbf{h}_i$, $\mathcal{B}_\mathbf{i}$ denotes the codebook used in the ith cell and $\mathbf{c}$ is its codeword, $\|\ \|$ is the norm operator, $i = 1, \cdots, N_c$.

Then the MS feeds back the indexes of N_c CDIs to its serving BS. After all cooperative BSs forward their received codewords to the CU, the global channel vector is reconstructed by aggregating all per-cell channel information, which is

$$\hat{\mathbf{h}} = [g_1\hat{\mathbf{h}}_1^T, \cdots, g_{N_c}\hat{\mathbf{h}}_{N_c}^T]^T, \tag{2}$$

where $g_i = \|\mathbf{h}_i\|$ is the per-cell channel norm, which includes both large scale fading factor and small scale channel fading norm. We assume that the per-cell channel norm can be obtained perfectly at each BS.

To measure the performance of the global channel quantization, we define a normalized quantization gain as follows

$$\mu \quad = \quad \frac{\|\mathbf{h}^H\hat{\mathbf{h}}\|^2}{\|\mathbf{h}^H\mathbf{h}\|^2}, \tag{3}$$

where $\mathbf{h} = [g_1\tilde{\mathbf{h}}_1^T, \cdots, g_{N_c}\tilde{\mathbf{h}}_{N_c}^T]^T$ is the global channel vector for each MS. $\mu \le 1$. A larger μ reflects more accuracy quantization. When the global channel is perfectly quantized, $\mu = 1$.

By defining $\tilde{g}_i = \frac{g_i}{\sqrt{\sum_{i=1}^{N_c} g_i^2}}$, the normalized quantization gain can be rewritten as

$$\mu = \|\sum_{i=1}^{N_c}\tilde{g}_i^2\tilde{\mathbf{h}}_i^H\hat{\mathbf{h}}_i\|^2 = \|\sum_{i=1}^{N_c}\tilde{g}_i^2\mu_i e^{j\xi_i}\|^2 = \|\mathbf{r}^H e^{j\boldsymbol{\xi}}\|^2, \tag{4}$$

where ξ_i is the phase ambiguity for the ith per-cell CDI, and $\mu_i = \|\tilde{\mathbf{h}}_i^H\hat{\mathbf{h}}_i\|$, which reflects the per-cell CDI quantization error, $\mathbf{r} = [\tilde{g}_1^2\mu_1, \ldots, \tilde{g}_{N_c}^2\mu_{N_c}]^T$, and $e^{j\boldsymbol{\xi}} = [e^{j\xi_1}, \ldots, e^{j\xi_{N_c}}]^T$.

It is shown from (4) that the normalized quantization gain is less than 1 even when the per-cell quantization is perfect, i.e., $\mu_i = 1$. This means that the phase ambiguity will lead to performance loss for quantizing the global channel of CoMP systems. In next section, we will analyze how much bits need to feed back to compensate the phase ambiguity.

III. Quantization of phase ambiguity

Denote the phase ambiguity vector as $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_{N_c}]^T$. It is quantized as $\hat{\boldsymbol{\xi}}$. Then the normalized quantization gain now becomes,

$$\mu = \|\sum_{i=1}^{N_c}\tilde{g}_i^2\mu_i e^{j(\xi_i-\hat{\xi}_i)}\|^2 = \|\mathbf{r}^H e^{j\epsilon_i}\|^2. \tag{5}$$

Define $\boldsymbol{\epsilon} = \boldsymbol{\xi} - \hat{\boldsymbol{\xi}} = [\epsilon_1, \ldots, \epsilon_{N_c}]^T$ as the quantization error of the phase ambiguity. It follows that μ depends both on the per-cell CDI quantization error, i.e., $\mathbf{r}$, and on the phase ambiguity quantization error $\boldsymbol{\epsilon}$. In order to investigate how much bits need to be fed back for the phase ambiguity, we remove the impact of per-cell CDI quantization error by taking average over it. Since large μ means high quantization accuracy, the phase ambiguity should be quantized aiming at maximizing

$$\mathcal{E}\{\mu\} = \mathcal{E}_\mathbf{r}\{e^{j\boldsymbol{\epsilon}^H}\mathbf{r}\mathbf{r}^H e^{j\boldsymbol{\epsilon}}\} = e^{j\boldsymbol{\epsilon}^H}\mathbf{A}_r e^{j\boldsymbol{\epsilon}}, \tag{6}$$

where the expectation operation $\mathcal{E}\{\ \}$ is taken over random vector $\mathbf{r}$, and $\mathbf{A}_r = \mathcal{E}_\mathbf{r}\{\mathbf{r}\mathbf{r}^H\}$.

Define $D(\boldsymbol{\epsilon}) = e^{j\boldsymbol{\epsilon}^H}A_r e^{j\boldsymbol{\epsilon}}$. When $\|\boldsymbol{\epsilon}\|$ is small, $D(\boldsymbol{\epsilon})$ can be accurately approximated by the second order Taylor expansions as (see Appendix A)

$$\begin{aligned} D(\boldsymbol{\epsilon}) &\approx D(\mathbf{0}) + D^{'T}(\mathbf{0})\boldsymbol{\epsilon} + 1/2\boldsymbol{\epsilon}^H D^{''}(\mathbf{0})\boldsymbol{\epsilon} \\ &= \underbrace{\mathbf{1}^T\mathbf{A_r}\mathbf{1}}_{Per-cell\ feedback} \quad - \underbrace{\boldsymbol{\epsilon}^H(diag(\mathbf{A}_r\mathbf{1}) - \mathbf{A}_r)\boldsymbol{\epsilon}}_{Phase\ ambiguity}, \end{aligned} \tag{7}$$

where $D^{'}(\mathbf{0})$ and $D^{''}(\mathbf{0})$ are the first and second order derivation of $D(\boldsymbol{\epsilon})$ with respect to $\boldsymbol{\epsilon}$, and $\mathbf{1}$ and $\mathbf{0}$ are respectively column vectors of size N_c with all ones and with all zeros.

It shows that $D(\boldsymbol{\epsilon})$ can be decomposed into two parts. The first part depends on the per-cell CDI quantization error without the impact of phase ambiguity, and the second part depends on phase ambiguity quantization error. The performance gap between those with and without phase ambiguity quantization error is bounded by the norm inequality, which is

$$\|D(\boldsymbol{\epsilon}) - D(\mathbf{0})\| \le \|\mathrm{diag}(\mathbf{A}_r\mathbf{1}) - \mathbf{A}_r\|\|\boldsymbol{\epsilon}\|^2. \tag{8}$$

Consider that the phase ambiguity is quantized by B_p bits in total. There are actually only $N_c - 1$ parameters in $\boldsymbol{\xi}$ required to be quantized, since $\|\mathbf{r}^H e^{j\boldsymbol{\xi}}\| = \|e^{j\theta}\mathbf{r}^H e^{j\boldsymbol{\xi}}\|$ holds for any θ. By applying the scalar quantization for each of the elements, asymptotically we have

$$\|D(\boldsymbol{\epsilon}) - D(\mathbf{0})\| \le c_0\|\mathrm{diag}(\mathbf{A}_r\mathbf{1}) - \mathbf{A}_r\|2^{\frac{-2B_p}{N_c-1}}, \tag{9}$$

where c_0 is a constant [6] depending on N_c.

It shows that the performance gap is upper bounded by a constant, which depends on the number of cooperated BSs.

IV. Impact of channel asymmetry on phase ambiguity

As shown in previous section, the matrix $\mathbf{A}_r$ plays an important role in the global channel quantization performance.

From (7), we can see that the phase ambiguity only affects the second item. If the per-cell CDI quantization is perfect, i.e., $u_i = 1$, we have $\mathbf{r}^H\mathbf{1} = 1$ and $\mathbf{1}^T\mathbf{A_r}\mathbf{1} = 1$. Then the average normalized channel gain only depends on the quantization error of the phase ambiguity. In the following, we will analyze the second term of (7).

Assume that per-cell channel vector $\mathbf{h}_i$ is i.i.d. complex Gaussian distributed with entries subjected to $\mathcal{CN}(0, \lambda_i^2/N_t)$, where $\lambda_i^2 = \mathcal{E}\{g_i^2\}$ is the large scale fading gain. Then, $\|\mathbf{h}_i\|$ and $\tilde{\mathbf{h}}_i$ is independent, and $\|\mathbf{h}_i\|$ as well as $\tilde{g}_i = \frac{g_i}{\sqrt{\sum_{i=1}^{N_c} g_i^2}} = \frac{\|\mathbf{h}_i\|}{\sqrt{\sum_{i=1}^{N_c}\|\mathbf{h}_i\|^2}}$ are independent with μ_i. Therefore, the (i, j) element of $\mathbf{A_r}$ can be obtained as

$$\mathbf{A_r}_{i,j} = \mathcal{E}\{\tilde{g}_i^2\tilde{g}_j^2\mu_i\mu_j\}, = \mathcal{E}\{\tilde{g}_i^2\tilde{g}_j^2\}\mathcal{E}\{\mu_i\mu_j\}. \quad (10)$$

When all per-cell channels are quantized by identical number of B bits, we have

$$\mathbf{A_r}_{i,j} = \begin{cases} \mathcal{E}\{\tilde{g}_i^4\}\beta(B, N_t), & \forall i = j \\ \mathcal{E}\{\tilde{g}_i^2\tilde{g}_j^2\}\eta(B, N_t), & \forall i \neq j \end{cases} \quad (11)$$

where $\beta(B, N_t) = \mathcal{E}\{\mu_i^2\}$ reflects the average per-cell CDI quantization error, and $\eta(B, N_t) = \mathcal{E}\{\mu_i\mu_j\}$, $i \neq j$.

Theorem 1: The matrix $\mathbf{F} = diag(\mathbf{A}_r\mathbf{1}) - \mathbf{A}_r$ is semi-positively definite.

Proof By using the well-known Gershgorin Circle Theorem [9], for any eigenvalue of $\mathbf{F}$, i.e., $\sigma_{\mathbf{F}}$, we have $\|\sigma_{\mathbf{F}} - \mathbf{F}_{i,i}\| \leq \sum_{j\neq i}^{N_c}\|\mathbf{F}_{i,j}\|$, $i = 1, \cdots, N_c$. Moreover, $\mathbf{F}_{i,j} = -\mathbf{A_r}_{i,j}$ are non-positive. Then we have $\sigma_{\mathbf{F}} \geq \mathbf{F}_{i,i} + \sum_{j\neq i}^{N_c}\mathbf{F}_{i,j} = \sum_{j\neq i}\mathbf{A_r}_{i,j} - \sum_{j\neq i}\mathbf{A_r}_{i,j} = 0$. Hence, the theorem is proved.

The theorem indicates that phase ambiguity quantization error will lead to performance loss of the global channel quantization, since the second term in (7) is non-negative as a result of the semi-positive definite nature of matrix $\mathbf{F}$.

To analyze the impact of the asymmetry of CoMP channels, now let us find the connection between $\mathbf{F}$ and the large scale fading gain λ_i. To this end, we need to derive $\mathcal{E}\{\tilde{g}_i^2\tilde{g}_j^2\}$ at first. Unfortunately, its closed-form expression is hard to derive if not impossible. Nonetheless, we can approximate the elements of $\mathbf{F}$ as (see Appendix B)

$$\mathbf{F}_{i,j} \approx \begin{cases} \eta(B, N_t)\frac{\sum_{k\neq i}^{N_c}\lambda_i^2\lambda_k^2}{(\sum_{i=1}^{N_c}\lambda_i^2)^2}, & \forall i = j \\ -\eta(B, N_t)\frac{\lambda_i^2\lambda_j^2}{(\sum_{i=1}^{N_c}\lambda_i^2)^2}, & \forall i \neq j \end{cases} \quad (12)$$

The MSs with different locations experience different extent of channel asymmetry. To quantitatively measure the channel asymmetry with a single parameter for multi-cell channels, we model the heterogeneous path loss as a geometric sequence, *i.e.*, $\boldsymbol{\lambda}^2 = [\lambda_1^2, \cdots, \lambda_{N_c}^2]^T = \frac{1-\rho}{1-\rho^{N_c}}[1, \rho, \cdots, \rho^{N_c-1}]^T$. In order to fairly compare two MSs with different locations, the large scale fading gain is normalized by $\frac{1-\rho}{1-\rho^{N_c}}$ so that $\sum_{i=1}^{N_c}\lambda_i^2 = 1$. When $\rho = 1$, $\boldsymbol{\lambda}^2 = [1/N_c, \cdots, 1/N_c]^T$, which represents the per-cell large scale gains for an exact cell-edge MS. Such a normalization will not affect the normalized gain μ. In this model, a parameter ρ provides a simple but effective characterization of the channel asymmetry, i.e., channels with a smaller ρ are more asymmetric.

To reflect the dependence on the parameter ρ, hereafter we express $\mathbf{F}$ as $\mathbf{F}(\rho)$.

Theorem 2: The impact of quantization error of phase ambiguity on per-cell feedback scheme reduces when ρ decreases.

Proof It is reasonable to assume that the phase ambiguity quantization errors ϵ_i are i.i.d. and with zero mean. When $||\boldsymbol{\epsilon}||$ is small, we can approximate the average value of the second term of (7) as follows

$$\mathcal{E}_{\boldsymbol{\epsilon}}\{\boldsymbol{\epsilon}^H\mathbf{F}(\rho)\boldsymbol{\epsilon}\} \approx \sum_{i=1}^{N_c}\mathbf{F}(\rho)_{i,i}\mathcal{E}_{\epsilon_i}\{\epsilon_i\epsilon_i\} \quad (13)$$

$$= \mathrm{Tr}(\mathbf{F}(\rho))\mathcal{E}_{\boldsymbol{\epsilon}}\{\|\boldsymbol{\epsilon}\|^2\}/N_c, \quad (14)$$

where (13) is a result of independence assumption, i.e., $\mathcal{E}\{\epsilon_i\epsilon_j\} = 0$ $(i \neq j)$, and (14) is from the assumption that $\mathcal{E}\{\epsilon_i^2\}$ are equal to $\mathcal{E}_{\boldsymbol{\epsilon}}\{\|\boldsymbol{\epsilon}\|^2\}/N_c$, $i = 1, \cdots, N_c$.

To observe the dependence of quantization error on ρ, now we analyze $\mathrm{Tr}(\mathbf{F}(\rho))$. For notation simplicity, define $\kappa = \frac{\eta(B,N_t)}{(\sum_{i=1}^{N_c}\lambda_i^2)^2}$, then we have

$$\mathrm{Tr}(\mathbf{F}_r(\rho) = \kappa\sum_{i=1}^{N_c}\sum_{k\neq i}^{N_c}\lambda_i^2\lambda_k^2 \quad (15)$$

$$= \kappa\mathbf{1}^T(\boldsymbol{\lambda}^2\boldsymbol{\lambda}^{2T} - \mathrm{diag}(\boldsymbol{\lambda}^2)\mathrm{diag}(\boldsymbol{\lambda}^2))\mathbf{1} \quad (16)$$

$$= \kappa - \kappa(\frac{1-\rho}{1-\rho^{N_c}})^2\frac{1-\rho^{2N_c}}{1-\rho^2} \quad (17)$$

$$= \kappa\big(1 - \frac{1-\rho}{1+\rho}\frac{1+\rho^{N_c}}{1-\rho^{N_c}}\big), \quad (18)$$

which is a non-decreasing function of ρ. Therefore, the theorem is proved.

Theorem 2 indicates that the asymmetric channels alleviate the impact of phase ambiguity quantization error. The worst case happens when all the per-cell channels have identical average channel power. This implies that only for cell-edge MSs the phase ambiguity needs to be quantized accurately.

V. Simulation Results

We verify our analysis in a CoMP system with N_c cooperative BSs, where each BS has $N_t = 4$ antennas. The per-cell channels subject to unequal average power Rayleigh fading. To reflect the channel asymmetry, in the first two simulations, we set $\lambda_i^2 = \frac{1-\rho}{1-\rho^{N_c}}\rho^i$ $(i = 1, \cdots, N_c)$. To highlight the impact of the phase ambiguity quantization error on the performance of global channel quantization, the number of per-cell feedback bits is assumed to be $B = 20$ bits, which provides a near perfect per-cell quantization [10]. Uniform quantization is used for quantizing the phase ambiguity. In the following simulations, the quantization performance of global channel with metric μ is investigated.

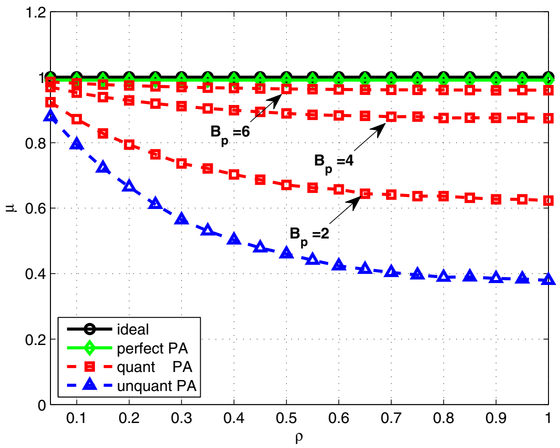

Fig. 1. Impact of phase ambiguity quantization error on global char quantization under asymmetric channel: per-cell CDI bits number B 20, $N_t = 4, N_c = 3$

We first show the impact of channel asymmetric on performance loss led by the phase ambiguity quantizati error. In Fig. 1, the normalized quantization gains of fo schemes versus ρ are plotted. The curve labeled with "ide denotes the result with perfect global CSI, the one w "perfect PA" means that the phase ambiguity is perfec compensated at the CU, the one with "quant PA" is the scheme with quantized phase ambiguity, and the c with "unquant PA" is for the scheme only with per-cell C feedback but not the phase ambiguity feedback. We can s that the performance gap between the scheme with perf CSI and that only with per-cell CDI feedback grows when channel asymmetric decreases. When $\rho = 1$, without phase ambiguity feedback, per-cell feedback only achieves 40% of the performance of perfect CSI. However, with a few bits for phase ambiguity quantization, the per-cell feedback scheme is significantly improved. When in total $B_p = 6$ bits are fed back for compensating the phase ambiguity, the performance loss is minor.

We then show how many bits are necessary for phase ambiguity quantization by considering the worst case, i.e., $\rho = 1$, which represents the case for cell-edge MSs. In Fig. 2, two settings are considered, $N_c = 3$ and $N_c = 6$. As our earlier analysis indicated, it is shown that the performance loss led by the phase ambiguity is effectively recovered by only a small number of bits for phase ambiguity quantization. When $N_c = 6$, i.e., six BSs cooperate, feeding back 12 bits in total for phase ambiguity can achieve 90 % of the performance with perfect CSI.

To validate our analysis which is based on the simple model to reflect the heterogenous path loss, we provide system level simulation to evaluate the performance of global channel under realistic channels. The cell radius is set as 250 m and path loss factor is 3.76. Again, two scenarios are considered, $N_c = 3$ and $N_c = 6$, and $N_t = 4$. Users are uniformly distributed in the cooperated cells. The cumulative distribution function (CDF) of μ is shown in Fig. 3. We can find that similar results are obtained as in previous simulations.

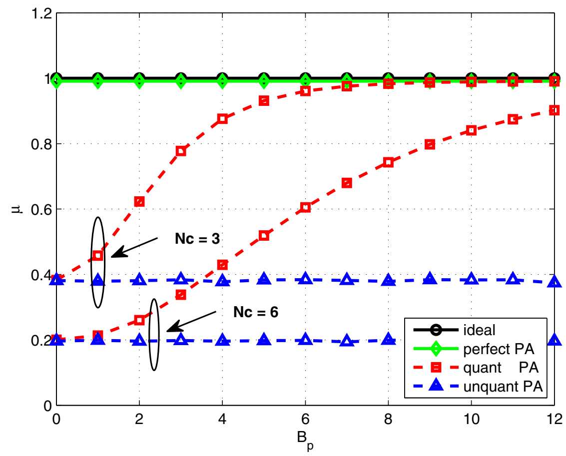

Fig. 2. Performance of global channel quantization versus the bits for phase ambiguity quantization: $B = 20, N_t = 4, \rho = 1$

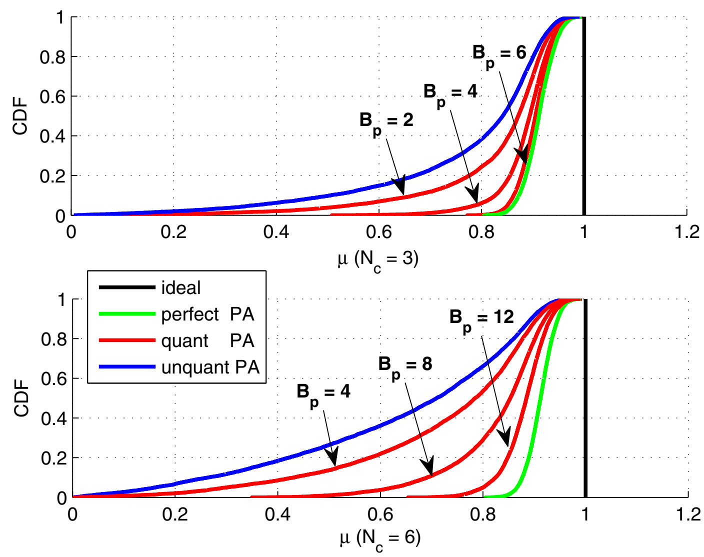

Fig. 3. CDF of normalized quantization gain for global channel, $B = 10$, $N_t = 4$).

VI. Conclusions

This paper studied the impact of phase ambiguity quantization on the quantization performance for global channels in per-cell codebook based limited feedback CoMP systems. We showed that the performance loss led by the phase ambiguity can be effectively recovered by feeding back a few bits in each cell, and the inherent asymmetric feature of CoMP channels will reduce the impact of phase ambiguity quantization error. Simulation results validate our theoretical analysis.

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APPENDIX A

Define $\mathbf{x} = e^{j\boldsymbol{\epsilon}}$. Then we have

$$\nabla_{\boldsymbol{\epsilon}}(\mathbf{x}^H \mathbf{A_r} \mathbf{x}) = \mathbf{x}^H \mathbf{A_r} \nabla_{\boldsymbol{\epsilon}} \mathbf{x} + \mathbf{x}^T \mathbf{A_r}^T \nabla_{\boldsymbol{\epsilon}} \mathbf{x}^*, \tag{19}$$

$$\nabla^2_{\boldsymbol{\epsilon}}(\mathbf{x}^H \mathbf{A_r} \mathbf{x}) = \sum_{i=1}^{N_c} (\mathbf{a}_i \nabla^T_{\boldsymbol{\epsilon}} x_i)^T \nabla_{\boldsymbol{\epsilon}} \mathbf{x}^* + \mathbf{x}^H \mathbf{a}_i \nabla^2_{\boldsymbol{\epsilon}} x_i + \mathbf{a}_i \nabla^T_{\boldsymbol{\epsilon}} x_i^* \nabla_{\boldsymbol{\epsilon}} x + \mathbf{x}^T \mathbf{a}_i \nabla^2_{\boldsymbol{\epsilon}} x_i^*, \tag{20}$$

where $\mathbf{a}_i$ is the ith column of $\mathbf{A}_r$, $()^*$, $()^T$ and $()^H$ respectively denote the conjugate, transpose and Hermitian operator, $\nabla_{\boldsymbol{\epsilon}}()$ and $\nabla^2_{\boldsymbol{\epsilon}}()$ are respectively a gradient vector and Hessian matrix with respect to $\boldsymbol{\epsilon}$.

When $\boldsymbol{\epsilon} = \mathbf{0}$, the vector $\mathbf{x}$ and its first-order and second order derivations become,

$$\mathbf{x}|_{\mathbf{0}} = \mathbf{1}, \tag{21}$$

$$\nabla_{\boldsymbol{\epsilon}} \mathbf{x}|_{\mathbf{0}} = -\nabla_{\boldsymbol{\epsilon}} \mathbf{x}^*|_{\mathbf{0}} = j \ \mathrm{diag}(\mathbf{x})|_{\mathbf{0}} = j \ \mathrm{diag}(\mathbf{1}), \tag{22}$$

$$\nabla^2_{\boldsymbol{\epsilon}} x_i|_{\mathbf{0}} = \nabla^2_{\boldsymbol{\epsilon}} {x_i}^*|_{\mathbf{0}} = -\triangle_i, \tag{23}$$

where $\triangle_i = \mathrm{diag}[\underbrace{0, \cdots, 0}_{i-1}, 1, \cdots, 0]$.

Substituting these expressions into (19) and (20), then we have

$$D^{'}(\mathbf{0}) = \nabla_{\boldsymbol{\epsilon}}(\mathbf{x}^H \mathbf{A_r} \mathbf{x})|_{\mathbf{0}} = \mathbf{0}, \tag{24}$$

$$D^{''}(\mathbf{0}) = \nabla^2_{\boldsymbol{\epsilon}}(\mathbf{x}^H \mathbf{A_r} \mathbf{x})|_{\mathbf{0}} = 2\mathbf{A_r} - 2\mathbf{diag}(\mathbf{A_r 1}). \tag{25}$$

APPENDIX B

Since $g_i^2 = \|\mathbf{h}_i\|^2 \sim \frac{\lambda_i^2}{2N_t}\chi^2(2N_t)$, which is a chi-square random variable with a freedom of $2N_t$, we know from [11] that

$$\mathcal{E}\{g_i^4\} = \mathcal{E}\{(\frac{\lambda_i^2}{2N_t}\chi_i^2)^2\} = (\frac{\lambda_i^2}{N_t})^2 \frac{\Gamma(N_t+2)}{\Gamma(N_t)} \tag{26}$$

$$= \lambda_i^4 \frac{N_t+1}{N_t}, \tag{27}$$

$$\mathcal{E}\{g_i^2 g_j^2\} = \mathcal{E}\{\frac{\lambda_i^2}{2N_t}\chi_i^2 \frac{\lambda_j^2}{2N_t}\chi_j^2\} = \frac{\lambda_i^2}{N_t}\frac{\lambda_j^2}{N_t}\frac{\Gamma^2(N_t+1)}{\Gamma^2(N_t)} \tag{28}$$

$$= \lambda_i^2 \lambda_j^2, \tag{29}$$

Using the facts that random variable g_i^2 is weakly correlated with $(\sum_{i=1}^{N_c} g_i^2)^2$, we have,

$$\mathcal{E}\{\tilde{g}_i^2 \tilde{g}_j^2\} = \mathcal{E}\{\frac{g_i^2 g_j^2}{(\sum_{i=1}^{N_c} g_i^2)^2}\} \tag{30}$$

$$\gtrsim \mathcal{E}\{g_i^2 g_j^2\} \frac{1}{\mathcal{E}\{(\sum_{i=1}^{N_c} g_i^2)^2\}}, \tag{31}$$

where (31) is obtained according to [12] (see pp. 661).

By approximating $\frac{N_t+1}{N_t} \approx 1$, and substituting (27) and (29) into (31), $\mathcal{E}\{\tilde{g}_i^2 \tilde{g}_j^2\}$ is simplified as

$$\mathcal{E}\{\tilde{g}_i^2 \tilde{g}_j^2\} \gtrsim \frac{\lambda_i^2 \lambda_j^2}{(\sum_{i=1}^{N_c} \lambda_i^2)^2}. \tag{32}$$

Substituting (32) into (11), it is not hard to get (12) from (11).

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