Energy Efficiency Comparison among Direct, One-way and Two-way Relay Transmission

Can Sun and Chenyang Yang
Beihang University, Beijing, China
Email: saga@ee.buaa.edu.cn, cyyang@buaa.edu.cn

***Abstract*— In this paper, we compare the energy efficiencies (EEs) of direct transmission (DT), one-way relay transmission (OWRT) and two-way relay transmission (TWRT) in a system where two source nodes transmit to each other and may be assisted by an amplify-and-forward relay node. We first find the maximum EEs of DT, OWRT and TWRT by jointly optimizing the transmission time and the transmit powers at each node. Then we compare the maximum EEs of the three strategies, and analyze the impact of circuit powers and bidirectional data amounts. Analytical and simulation results show that relaying is not always more energy efficient than DT. The EE of TWRT is higher than those of DT and OWRT when the bidirectional data amounts are equal. Otherwise, the advantage of TWRT diminishes, while either DT or OWRT may provide higher EE.**

I. Introduction

Since explosive growth of wireless services is sharply increasing their contribution to the carbon footprint and the operating costs, energy efficiency (EE) has drawn more and more attention recently as a new design goal for various systems [1]. A widely used metric for EE is the number of bits transmitted per unit of energy. In practical systems, not only the power for transmitting information bits but also those consumed by various signaling and circuits contribute to the system energy consumption (EC). Therefore, the optimization problem that minimizes the overall transmit power does not necessarily lead to an energy efficient design [1].

Relaying is viewed as an energy saving technique because it can reduce the transmit power by breaking one long range transmission into several short range transmissions. Considerable research efforts have been devoted to various relaying systems, see [2] and reference therein. One-way relay transmission (OWRT) suffers from the 1/2 spectral efficiency (SE) loss led by half-duplex constraint. To recover this SE loss, two-way relay transmission (TWRT) obtains significant interests [3], [4]. However, it is not well-understood whether these relay strategies are energy efficient, especially when various energy costs except for the transmit power are considered.

In [5], considering both the transmit power and receiver processing power, the EE of decode-and-forward (DF) OWRT systems is studied. In [6], after accounting for the energy cost of acquiring channel information, relay selection for an OWRT system with multiple DF relays is optimized to maximize the EE. In [7], the EEs of OWRT and base station cooperation transmission are compared, where the overall energy costs including those from deployment are considered. In [8], the EE of DF relaying is compared with that of direct transmission (DT), where it shows that OWRT is more energy efficient when the distance between source and destination is large, otherwise, DT is better. In [9], the EEs of TWRT, OWRT and DT are compared. It shows that with optimized relay position and transmit power, TWRT consumes less energy than OWRT and DT, where only transmit power is considered. In [10], the EEs of TWRT, OWRT and DT are compared by accounting for both the transmit and circuit power. The results show that when the circuit power is considered, TWRT is not always more energy efficient. Generally speaking, TWRT offers higher EE in large path loss attenuation and high SE region.

In this paper, we step further from the result of [10]. We analyze the EEs of TWRT and OWRT by studying an amplify-and-forward (AF) relay system where two nodes intend to exchange information, and compare with the EE of DT. Different from [10], we introduce idle mode for each node. To save energy, the system may not use the entire duration for transmission but can transmit with shorter duration. Correspondingly, we consider not only the circuit powers consumption (PC) in transmit and receive modes, but also that in idle mode. We maximize the EEs by jointly optimizing the transmission time and transmit powers for TWRT, OWRT and DT and compare the optimized EEs of them. We show that when all the three strategies operate with optimized transmission time and transmit power, TWRT is *not* always more energy efficient if the bidirectional data amounts are unequal.

II. System Model

Consider a system consisting of two source nodes $\mathbb{A}$ and $\mathbb{B}$, and a half-duplex AF relay $\mathbb{R}$, each equipped with a single antenna. The bandwidth is W. We assume frequency-flat block fading channels, where h_{ab}, h_{ar} and h_{br} denote the channels between nodes $\mathbb{A}$ and $\mathbb{B}$, between nodes $\mathbb{A}$ and $\mathbb{R}$, and between nodes $\mathbb{B}$ and $\mathbb{R}$, respectively. We assume perfect channel knowledge at each node. The noise power N_0 is assumed to be identical at each node. The AF gain at relay is chosen with the aid of instantaneous channel gain, i.e., the channel-assisted AF gain [11].

Consider a delay-constrained scenario, where B_{ab} and B_{ba} bits need to be transmitted with a hard deadline T respectively on the $\mathbb{A} \rightarrow \mathbb{B}$ and $\mathbb{B} \rightarrow \mathbb{A}$ directions during each block. To reduce the EC, the system may not use the entire duration T for transmission in each block. After the B_{ab} and B_{ba}

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bits have been transmitted, the nodes can operate at an idle status until next block. Therefore, each node has three modes: transmission, reception and idle. The PCs in these modes are respectively denoted as $P^t/\epsilon+P^{ct}$, P^{cr} and P^{ci}, where P^t is the transmit power, subscripts $(\cdot)_a$, $(\cdot)_b$ and $(\cdot)_r$ will be used to denote the transmit power at different nodes, $\epsilon \in (0,1]$ denotes the power amplifier efficiency, P^{ct}, P^{cr} and P^{ci} are respectively the circuit PCs in transmission, reception and idle modes.

We assume that P^{ct}, P^{cr} and P^{ci} are dominated by the circuit PC of radio-frequency circuit, which can be modeled as constants independent of data rate [12]. In practical systems, an idle node is not shut down although it does not transmit or receive. The circuit PC in idle mode is modeled as a constant, and $0 < P^{ci} \le P^{ct}$, $0 < P^{ci} \le P^{cr}$.

III. Energy Efficiency Optimization

In this section, we respectively optimize the EEs for DT, OWRT and TWRT. The EE is defined as the number of bits transmitted per unit of energy, i.e., $\eta_{EE} = (B_{ab}+B_{ba})/E$, where E is the EC per block.

To guarantee a fair comparison, we optimize the EEs for different strategies with the same data amounts B_{ab} and B_{ba}. From the definition of EE, we see that EE maximization is equivalent to EC minimization for a given pair of B_{ab} and B_{ba}. Then we will minimize the EC per block for different strategies by optimizing the transmission time and transmit power of each node. The transmission time should not exceed the entire duration of one block, T. The transmit powers should not exceed the maximal values, otherwise outage occurs. Considering the maximum transmit power constraints renders very complicated derivation, but the conclusions are almost the same no matter if they are considered [13]. Therefore, we do not consider the transmit power constraints here for concise.

A. Direct Transmission

In DT, nodes $\mathbb{A}$ and $\mathbb{B}$ transmit to each other without the assistance of relay. During each block, the system respectively allocates duration T_{ab} and T_{ba} for the transmissions from $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$, and then remains idle during $T - T_{ab} - T_{ba}$.

During T_{ab}, nodes $\mathbb{A}$ and $\mathbb{B}$ are respectively in transmission and reception mode, and the total PC is $P_a^t/\epsilon + P^{ct} + P^{cr}$. Similarly we can obtain the total PC during T_{ba}. In idle duration, both nodes $\mathbb{A}$ and $\mathbb{B}$ are idle. Therefore, the EC of DT per block is given by

$$\begin{aligned} E_D &= T_{ab}(P_a^t/\epsilon + P^{ct} + P^{cr}) + T_{ba}(P_b^t/\epsilon + P^{ct} + P^{cr}) \\ &\quad + (T - T_{ab} - T_{ba})(2P^{ci}) \\ &\triangleq T_{ab}(\frac{P_a^t}{\epsilon} + P_D^c - P_D^{ci}) + T_{ba}(\frac{P_b^t}{\epsilon} + P_D^c - P_D^{ci}) + TP_D^{ci}, \end{aligned} \quad (1)$$

where $P_D^c \triangleq P^{ct} + P^{cr}$ denotes the total circuit PC during T_{ab} and T_{ba}, and $P_D^{ci} \triangleq 2P^{ci}$ denotes that in idle duration.

The EC is a function of both the transmission time and transmit powers, which need to be jointly optimized to minimize the EC by solving the following problem,

$$\begin{aligned} \min_{T_{ab},T_{ba},P_a^t,P_b^t} \; & T_{ab}\big(\frac{P_a^t}{\epsilon} + P_D^c - P_D^{ci}\big) + T_{ba}\big(\frac{P_b^t}{\epsilon} + P_D^c - P_D^{ci}\big) \\ & + TP_D^{ci} \\ \text{s.t.} \quad & T_{ab} + T_{ba} \le T. \end{aligned} \quad (2)$$

Since the transmit powers P_a^t and P_b^t depend on the transmission time T_{ab} and T_{ba}, we can first express P_a^t and P_b^t as functions of T_{ab} and T_{ba}, then the problem can be converted to a problem that is only related to T_{ab} and T_{ba}. Given T_{ab} and T_{ba}, nodes $\mathbb{A}$ and $\mathbb{B}$ should respectively transmit with data rates B_{ab}/T_{ab} and B_{ba}/T_{ba}. Then using Shannon capacity formula, the transmit power can be obtained as follows,

$$P_a^t = \frac{N_0}{|h_{ab}|^2}(2^{\frac{B_{ab}}{WT_{ab}}} - 1), \; P_b^t = \frac{N_0}{|h_{ab}|^2}(2^{\frac{B_{ba}}{WT_{ba}}} - 1). \quad (3)$$

The transmit powers obtained via the capacity formula are the minimum powers to support the required data rates. As a result, we can analyze the maximal EE with a given SE. We will also use the capacity formula to express the transmit powers in OWRT and TWRT cases.

By substituting (3) into the objective function of (2), the problem (2) can be reformulated as follows,

$$\begin{aligned} \min_{T_{ab},T_{ba}} \quad & T_{ab}\big(\frac{N_0(2^{\frac{B_{ab}}{WT_{ab}}} - 1)}{\epsilon|h_{ab}|^2} + P_D^c - P_D^{ci}\big) + \\ & T_{ba}\big(\frac{N_0(2^{\frac{B_{ba}}{WT_{ba}}} - 1)}{\epsilon|h_{ab}|^2} + P_D^c - P_D^{ci}\big) + TP_D^{ci} \\ \text{s.t.} \quad & T_{ab} + T_{ba} \le T. \end{aligned} \quad (4)$$

where only the transmission time need to be optimized now. It is easy to show that (4) is a convex problem with respect to T_{ab} and T_{ba}, which can be solved by using efficient convex optimization techniques [14].

B. One-way Relay Transmission

During each block, the system allocates duration T_{ab} for $\mathbb{A} \to \mathbb{B}$ transmission, which is completed in two phases. In the first phase, node $\mathbb{A}$ transmits to relay, while node $\mathbb{B}$ is idle. Then the total PC is $P_a^t/\epsilon + P^{ct} + P^{cr} + P^{ci}$. In the second phase, relay forwards its received signal to node $\mathbb{B}$, while node $\mathbb{A}$ is idle. Then the total PC is $P_{r1}^t/\epsilon + P^{ct} + P^{cr} + P^{ci}$, where P_{r1}^t is the relay transmit power in $\mathbb{A} \to \mathbb{B}$ link. The two phases employ identical time duration $T_{ab}/2$ with AF relay protocol.

Consider that the system allocates time duration T_{ba} for $\mathbb{B} \to \mathbb{A}$ transmission, and then remains idle during $T - T_{ab} - T_{ba}$. Similarly we can obtain the PC during the two phases of T_{ba}. In idle duration, all the three nodes are idle. Finally, the EC of OWRT per block is given by

$$\begin{aligned} E_O =& \frac{T_{ab}}{2}\Big[\frac{P_a^t + P_{r1}^t}{\epsilon} + 2(P^{ct} + P^{cr} + P^{ci})\Big] + \frac{T_{ba}}{2}\Big[\frac{P_b^t + P_{r2}^t}{\epsilon} \\ & + 2(P^{ct} + P^{cr} + P^{ci})\Big] + (T - T_{ab} - T_{ba})(3P^{ci}) \\ \triangleq& T_{ab}\Big(\frac{P_a^t + P_{r1}^t}{2\epsilon} + P_O^c - P_O^{ci}\Big) + T_{ba}\Big(\frac{P_b^t + P_{r2}^t}{2\epsilon} \\ & + P_O^c - P_O^{ci}\Big) + TP_O^{ci}, \end{aligned} \quad (5)$$

where P^t_{r2} is the relay transmit power in $\mathbb{B} \to \mathbb{A}$ link, $P^c_O \triangleq P^{ct} + P^{cr} + P^{ci}$ denotes the circuit PC during T_{ab} and T_{ba}, and $P^{ci}_O \triangleq 3P^{ci}$ denotes that in the idle duration.

The relationships between transmit powers and required data rates can be obtained through capacity formula and the signal-to-noise ratio (SNR) derived in [11], which are

$$\frac{B_{ab}}{T_{ab}} = \frac{W}{2} \log_2 \big(1 + \frac{P^t_a P^t_{r1} |h_{ar}|^2 |h_{br}|^2 / N_0}{|h_{ar}|^2 P^t_a + |h_{br}|^2 P^t_{r1} + N_0}\big), \quad (6)$$

$$\frac{B_{ba}}{T_{ba}} = \frac{W}{2} \log_2 \big(1 + \frac{P^t_b P^t_{r2} |h_{br}|^2 |h_{ar}|^2 / N_0}{|h_{br}|^2 P^t_b + |h_{ar}|^2 P^t_{r2} + N_0}\big), \quad (7)$$

where the factor $1/2$ is due to the two-phase transmission.

Similar to the DT case, we first express the transmit powers as functions of the transmission time using (6) and (7). Then the joint optimization of transmit power and transmission time can be solved by only optimizing the transmission time.

For a given T_{ab}, both P^t_a and P^t_{r1} can be obtained from (6), where multiple feasible solutions exist. In order to minimize the EC, we find the transmit powers that minimize their summation.

$$\begin{aligned} \min_{P^t_a, P^t_{r1}} \quad & P^t_a + P^t_{r1} \\ \text{s.t.} \quad & (6), \end{aligned} \quad (8)$$

from which the minimum value of $P^t_a + P^t_{r1}$ can be solved as a function of T_{ab} as [10],

$$P_{min1}(T_{ab}) = \frac{C_1 N_0}{|h_{br}|^2} + \frac{C_1 N_0}{|h_{ar}|^2} + \frac{2\sqrt{C_1^2 + C_1} N_0}{|h_{ar} h_{br}|}, \quad (9)$$

where $C_1 \triangleq 2^{2B_{ab}/(T_{ab}W)} - 1$.

For a given T_{ba}, we can also find P^t_b and P^t_{r2} that minimize their sum. Following an analogous procedure, the minimum $P^t_b + P^t_{r2}$ can be derived as a function of T_{ba}, i.e., $P_{min2}(T_{ba})$, which has a similar form as $P_{min1}(T_{ab})$, and can be obtained by replacing C_1 in (9) by $C_2 \triangleq 2^{2B_{ba}/(T_{ba}W)} - 1$.

Then the EC can be minimized by optimizing T_{ab} and T_{ba},

$$\begin{aligned} \min_{T_{ab}, T_{ba}} \quad & T_{ab}\big(\frac{P_{min1}(T_{ab})}{2\epsilon} + P^c_O - P^{ci}_O\big) + \\ & T_{ba}\big(\frac{P_{min2}(T_{ba})}{2\epsilon} + P^c_O - P^{ci}_O\big) + TP^{ci}_O \\ \text{s.t.} \quad & T_{ab} + T_{ba} \leq T. \end{aligned} \quad (10)$$

By taking the first and second order derivatives of the objective function in (10), we can prove that it is quasi-convex with respect to T_{ab} and T_{ba} (detailed proof is provided in [13]), thus efficient quasi-convex optimization techniques [14] can be applied to solve the problem (10).

C. Two-way Relay Transmission

During each block, the system allocates duration T_{twr} for bidirectional transmission, which is completed in two phases. First, both nodes $\mathbb{A}$ and $\mathbb{B}$ transmit to relay, and the total PC is $P^t_a/\epsilon + P^t_b/\epsilon + 2P^{ct} + P^{cr}_r$. Then relay broadcasts its received signal to nodes $\mathbb{A}$ and $\mathbb{B}$, and the total PC is $P^t_r/\epsilon + P^{ct} + 2P^{cr}$. After receiving the superimposed signal, each of nodes $\mathbb{A}$ and $\mathbb{B}$ removes its own transmitted signal via self-interference cancelation, and obtains its desired signal [3]. The two phases employ identical time duration $T_{twr}/2$ with AF relay protocol.

After the bidirectional transmission, the system remains idle during $T - T_{twr}$, where all the three nodes are in idle mode. The EC of TWRT per block is given by

$$\begin{aligned} E_T = & \frac{T_{twr}}{2}(P^t_a/\epsilon + P^t_b/\epsilon + P^t_r/\epsilon + 3P^{ct} + 3P^{cr}) + \\ & (T - T_{twr})(3P^{ci}) \\ \triangleq & T_{twr}\big(\frac{P^t_a + P^t_b + P^t_r}{2\epsilon} + P^c_T - P^{ci}_T\big) + TP^{ci}_T, \end{aligned} \quad (11)$$

where $P^c_T \triangleq 1.5(P^{ct} + P^{cr})$ and $P^{ci}_T \triangleq 3P^{ci}$ denotes circuit PCs during T_{twr} and idle duration, respectively.

The relationships between transmit powers and required data rates can be obtained through capacity formula and the SNR in TWRT derived by [11], which are

$$\frac{B_{ab}}{T_{twr}} = \frac{W}{2} \log_2(1 + \frac{P^t_a P^t_r |h_{ar}|^2 |h_{br}|^2 / N_0}{|h_{ar}|^2 P^t_a + |h_{br}|^2 P^t_b + |h_{br}|^2 P^t_r + N_0}), \quad (12)$$

$$\frac{B_{ba}}{T_{twr}} = \frac{W}{2} \log_2(1 + \frac{P^t_b P^t_r |h_{br}|^2 |h_{ar}|^2 / N_0}{|h_{ar}|^2 P^t_a + |h_{br}|^2 P^t_b + |h_{ar}|^2 P^t_r + N_0}), \quad (13)$$

where the factor $1/2$ is due to the two-phase transmission.

Again, we first derive the transmit powers as function of the transmission time. For a given T_{twr}, we can find P^t_a, P^t_b and P^t_r from (12) and (13), where multiple feasible solutions exist. To minimize the EC, we find P^t_a, P^t_b and P^t_r that minimizes their sum, which can be obtained as a function of the transmission time T_{twr} as [10],

$$\begin{aligned} P_{min}(T_{twr}) = & \frac{N_0}{2}\big(\frac{C_1 + C2}{|h_{ar}|^2} + \frac{C_1 + C_2}{|h_{br}|^2}\big) + \\ & \frac{N_0}{|h_{ar}||h_{br}|}\sqrt{(C_1 + C_2 + 1)(C_1 + C_2)}, \end{aligned} \quad (14)$$

where $C_1 \triangleq 2^{\frac{2B_{ab}}{WT_{twr}}} - 1$ and $C_2 \triangleq 2^{\frac{2B_{ba}}{WT_{twr}}} - 1$.

Then we minimize EC by optimizing T_{twr} as follows,

$$\begin{aligned} \min_{T_{twr}} \quad & T_{twr}\big(\frac{P_{min}(T_{twr})}{2\epsilon} + P^c_T - P^{ci}_T\big) + TP^{ci}_T \\ \text{s.t.} \quad & T_{twr} \leq T. \end{aligned} \quad (15)$$

Similarly as that in OWRT optimization, we can prove that the objective function in (15) is a quasi-convex function of T_{twr}. Therefore, efficient quasi-convex optimization techniques [14] can be applied to solve the problem.

IV. Energy Efficiency Analysis

The expressions of $P_{min1}(T_{ab})$, $P_{min2}(T_{ba})$ and $P_{min}(T_{twr})$ are too complex, which render the analysis for OWRT and TWRT very difficult. To gain some useful insight, we consider the following approximations,

$$2^{\frac{2B_{ab}}{WT_{ab}}} - 1 \approx 2^{\frac{2B_{ab}}{WT_{ab}}}, \; 2^{\frac{2B_{ba}}{WT_{ba}}} - 1 \approx 2^{\frac{2B_{ba}}{WT_{ba}}}, \quad (16a)$$

$$2^{\frac{2B_{ab}}{WT_{twr}}} + 2^{\frac{2B_{ba}}{WT_{twr}}} - 2 \approx 2^{\frac{2B_{ab}}{WT_{twr}}} + 2^{\frac{2B_{ba}}{WT_{twr}}} - 1. \quad (16b)$$

These approximations affects the values of transmit powers $P_{min1}(T_{ab})$, $P_{min2}(T_{ba})$ and $P_{min}(T_{twr})$. When the SEs i.e., $B_{ab}/(WT_{ab})$, $B_{ba}/(WT_{ba})$, $B_{ab}/(WT_{twr})$ and $B_{ba}/(WT_{twr})$ are high, it is easy to see that these approximations are accurate. When the SEs are low, the transmit powers are much lower than the circuit PC. Then the approximations have little impact on the analysis of EC.

By applying (16), the ECs of OWRT and TWRT in the objective functions of (10) and (15) can be simplified as

$$\begin{aligned} E_O \approx & T_{ab}\Big[\tfrac{N_0(2^{\frac{2B_{ab}}{WT_{ab}}}-1)}{2\epsilon|h_{eff}|^2} + P_O^c - P_O^{ci}\Big] + \\ & T_{ba}\Big[\tfrac{N_0(2^{\frac{2B_{ba}}{WT_{ba}}}-1)}{2\epsilon|h_{eff}|^2} + P_O^c - P_O^{ci}\Big] + TP_O^{ci}, \end{aligned} \tag{17}$$

$$\begin{aligned} E_T \approx & T_{twr}\Big[\tfrac{N_0}{2\epsilon|h_{eff}|^2}(2^{\frac{2B_{ab}}{WT_{twr}}} + 2^{\frac{2B_{ba}}{WT_{twr}}} - 2) \\ & + P_T^c - P_T^{ci}\Big] + TP_T^{ci}, \end{aligned} \tag{18}$$

where $|h_{eff}| \triangleq 1/(\frac{1}{|h_{ar}|} + \frac{1}{|h_{br}|})$ can be viewed as an equivalent channel gain between the two source nodes due to the usage of relay.

A. Impact of Circuit Power Consumption

Here we consider symmetric traffic, i.e., $B_{ab} = B_{ba} \triangleq B$.

We take TWRT as an example to analyze the optimal EE. The optimal transmission time in TWRT can be obtained by taking the derivative of E_T in (18) with respect to T_{twr} and setting it to be zero, which is

$$\frac{\mathrm{d}}{\mathrm{d}T_{twr}}\{T_{twr}[\frac{N_0(2^{\frac{2B}{WT_{twr}}}-1)}{\epsilon|h_{eff}|^2} + P_T^c - P_T^{ci}] + TP_T^{ci}\} \tag{19a}$$

$$= [\frac{N_0(2^{\frac{2B}{WT_{twr}}}-1)}{\epsilon|h_{eff}|^2} + P_T^c - P_T^{ci}] - \frac{2BN_0\ln 2}{WT_{twr}\epsilon|h_{eff}|^2}2^{\frac{2B}{WT_{twr}}} \tag{19b}$$

$$\begin{aligned} &\triangleq [\frac{N_0(2^{\eta_{SE}}-1)}{\epsilon|h_{eff}|^2} + P_T^c - P_T^{ci}] - \frac{\eta_{SE}N_0\ln 2}{\epsilon|h_{eff}|^2}2^{\eta_{SE}} \\ &= 0|_{\eta_{SE}=\eta_{SE}^{opt}}, \end{aligned} \tag{19c}$$

where $\eta_{SE} \triangleq 2B/(WT_{twr})$ is the bidirectional SE of TWRT.

Although it is difficult to obtain a closed form solution of the optimal T_{twr}, we can obtain some observations from (19). The optimal SE that minimizes the EC should satisfy (19c), from which we can see that η_{SE}^{opt} does not depend on the data amount B. Therefore, the optimal transmission time $T_{twr}^{opt} = 2B/(W\eta_{SE}^{opt})$ increases linearly with B. Considering that $T_{twr} \le T$, we have the following observation.

Observation 1: In high traffic region, $T_{twr}^{opt} = T$. In low traffic region where $2B/(W\eta_{SE}^{opt}) \le T$, the optimal transmission time T_{twr}^{opt} increases linearly with the data amount B.

In high traffic region, the system transmits with entire duration T. The bidirectional SE increases linearly with data amount B, thus the transmit EC increases exponentially with B according to the capacity formula. In this case, the transmit EC is much larger than the circuit EC, thus circuit PC has little impact on EE in this region.

In low traffic region where $2B/(W\eta_{SE}^{opt}) \le T$, when the system transmits with the optimal transmission time, the equation in (19b) equals to zero. Then we have

$$\begin{aligned} T_{twr}^{opt}[\frac{N_0(2^{\frac{2B}{WT_{twr}^{opt}}}-1)}{\epsilon|h_{eff}|^2} + P_T^c - P_T^{ci}] &= \frac{2BN_0\ln 2}{W\epsilon|h_{eff}|^2}2^{\frac{2B}{WT_{twr}^{opt}}} \\ &= \frac{2BN_0\ln 2}{W\epsilon|h_{eff}|^2}2^{\eta_{SE}^{opt}}, \end{aligned} \tag{20}$$

where the first equality is because (19b) equals to zero, and the second equality is because $T_{twr}^{opt} = 2B/(W\eta_{SE}^{opt})$.

By substituting $B_{ab} = B_{ba} = B$ and $T_{twr} = T_{twr}^{opt}$ into the EC of TWRT in (18), and then substituting (20), the minimum EC of TWRT can be obtained as

$$E_T^{min} = \frac{2BN_0\ln 2}{W\epsilon|h_{eff}|^2}2^{\eta_{SE}^{opt}} + TP_T^{ci}, \tag{21}$$

and the optimal EE of TWRT is given by

$$\eta_{EE}^{opt} = \frac{2B}{\frac{2BN_0\ln 2}{W\epsilon|h_{eff}|^2}2^{\eta_{SE}^{opt}} + TP_T^{ci}}, \tag{22}$$

from which we can obtain the following observation.

Observation 2: In low traffic region where $2B/(W\eta_{SE}^{opt}) \le T$, if the circuit PC in idle mode $P_T^{ci} = 0$, we have $\eta_{EE}^{opt} = W\epsilon|h_{eff}|^2/[N_0(\ln 2)2^{\eta_{SE}^{opt}}]$. Since we have shown that η_{SE}^{opt} does not depend on the data amount B, η_{EE}^{opt} does not depend on it, neither. If $P_T^{ci} \neq 0$, $\lim_{B\to 0}\eta_{EE}^{opt} = 0$, since a large portion of energy is consumed in the idle duration.

From observation 2, if $P_T^{ci} = 0$, η_{EE}^{opt} does not change with B when $2B/(W\eta_{SE}^{opt}) \le T$, i.e., $B \le TW\eta_{SE}^{opt}/2$. By taking derivative with respect to P_T^c at both side of (19c), it is not difficult to obtain that $\mathrm{d}\eta_{SE}^{opt}/\mathrm{d}P_T^c > 0$. Therefore, as the circuit power P_T^c increases, η_{SE}^{opt} increases, and then the region $(0, TW\eta_{SE}^{opt}/2)$ where the EE does not change with data amount becomes wider.

Following analogous procedure, we can obtain the same observations 1 and 2 for DT and OWRT.

B. Impact of Unequal Data Amounts in Two Directions

Define $B_{ab} = \beta B_s$ and $B_{ba} = (1-\beta)B_S$, where B_s is the overall bidirectional sum data amount, and β is a factor that reflects the asymmetry of the data amounts.

First we show that once B_s is given, the optimal EE of OWRT is independent of β.

By solving the corresponding Karush-Kuhn-Tucker (KKT) conditions, we can show that in order to minimize the EC in (17), the optimal transmission time T_{ab}^{opt} and T_{ba}^{opt} in OWRT should satisfy (detailed proof is provided in [13]),

$$\beta B_s/T_{ab}^{opt} = (1-\beta)B_s/T_{ba}^{opt} \triangleq R_O, \tag{23}$$

i.e., the data rates on $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ directions are identical, where R_O is not a function of β. Then the minimum

EC of OWRT can be derived as follows by substituting $B_{ab} = \beta B_s$, $B_{ba} = (1-\beta)B_S$ and (23) into (17),

$$E_O^{min} = \frac{B_s}{R_O}\left[\frac{N_0(2^{\frac{2R_O}{W}}-1)}{2\epsilon|h_{eff}|^2} + P_O^c - P_O^{ci}\right] + TP_O^{ci}, \quad (24)$$

which is not a function of β, thus the optimal EE is independent of β.

This conlusion is easy to understand. With the optimized transmission time, OWRT system transmits with the same data rate on each direction. Each bit is transmitted with identical data rate R_O and thus with identical time duration $1/R_O$. Therefore, the energy consumed by each bit is identical no matter in which direction it is transmitted. Then the minimum EC only depends on the overall data amount B_s. We can draw the same conclusion for DT.

Then we show that the optimal EE of TWRT is reduced when $\beta \neq 0.5$, i.e., the difference between the bidirectional data amounts degrades the EE of TWRT.

The EC of TWRT in (18) can be rewritten as,

$$E_T = T_{twr}\left[\frac{N_0(2^{\frac{2\beta B_s}{WT_{twr}}}-1)}{2\epsilon|h_{eff}|^2} + \frac{P_T^c - P_T^{ci}}{2}\right] + T_{twr}\left[\frac{N_0(2^{\frac{2(1-\beta)B_s}{WT_{twr}}}-1)}{2\epsilon|h_{eff}|^2} + \frac{P_T^c - P_T^{ci}}{2}\right] + TP_T^{ci}. \quad (25)$$

The first two terms can be viewed as the energy consumed by the transmissions in two directions. Since the transmission time T_{twr} must be identical in the two terms, it is easy to see that only when $\beta = 0.5$, we can find a T_{twr} to simultaneously minimize the first two terms. Otherwise, the system can not simultaneously minimize the energy consumed by transmissions in two directions. It suggests that only when $\beta = 0.5$, the EC of TWRT achieves its minimum value. When the bidirectional data amounts are unequal, its EC increases and EE reduces. More rigorous proof is provided in [13].

V. Simulation Results

In this section, we evaluate the EEs of DT, OWRT and TWRT, and validate our analysis via simulations.

We consider that three nodes are located on a straight line. The distance between nodes $\mathbb{A}$ and $\mathbb{B}$ is denoted as D, and the relay is on the midpoint of nodes $\mathbb{A}$ and $\mathbb{B}$. The path loss attenuation is $PL = 30 + 10\log_{10}(\text{distance}^{\alpha})$, where α is the attenuation factor. We assume that all the small scale fading channels are i.i.d. Rayleigh fading channels, which do not change during one block, but are independent from one block to another. All the results are averaged over 500 Monte-carlo trails of fading channels.

Since the increase of distance D, noise power N_0, and attenuation factor α all result in higher transmit power, their impacts are similar, therefore we only analyze the impact of α, and fix $D = 100$ m and $N_0 = -94$ dBm, considering -174 dBm/Hz noise spectral density, 10 MHz bandwidth and 10 dB receiver noise figure. Because the increase of block duration T is equivalent to a reduction of the data amount in unit time, we only change the values of B_{ab} and B_{ba}, and fix $T = 5$ ms. The power amplifier efficiency is set as $\epsilon = 0.35$.

In Fig. 1, we compare the EEs of different strategies for a baseline case where the circuit PCs are zero, and the data amounts $B_{ab} = B_{ba}$. To show the EEs in different channel conditions, we set the attenuation factor α as 2 or 4. Since we are more interested in comparing the EEs rather than showing their absolute values, we normalize the EEs by the maximum EE of DT system for each α. The x-axis is the data amount normalized by block duration and bandwidth, i.e., $(B_{ab} + B_{ba})/(TW)$, which can be seen as averaged bidirectional SE per block[1].

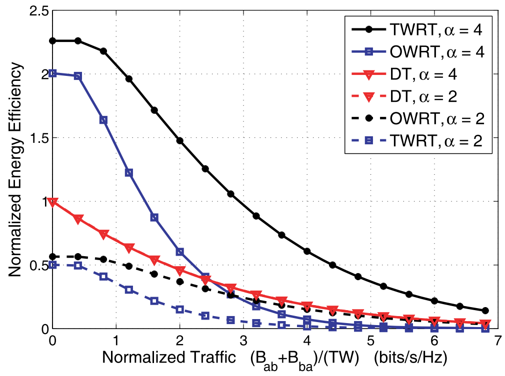

Fig. 1. EE comparison with zero circuit power and equal bidirectional traffics.

Because of the normalization, the EE curves of DT under different α overlap. It shows that the more spectral efficient strategy TWRT is also more energy efficient than OWRT in this case. When the attenuation factor is large, i.e., $\alpha = 4$, the EE of TWRT is higher than that of DT, while when $\alpha = 2$ the result is just the opposite. The comparison between DT and OWRT depends both on the data amount and the channel condition. When $\alpha = 2$, DT always outperforms OWRT. When $\alpha = 4$, OWRT is superior to DT in low traffic region, but is inferior to DT in high traffic region.

In Fig. 2, we take TWRT as an example to show the impact of circuit powers. In the top four curves, the EEs are obtained with the optimized transmission time and there may be idle duration in each block. To show the impact of the idle duration, in the lowest curve we provide the EE for a system without transmission time optimization (i.e., the system transmits with the entire duration and there is no idle duration).

As expected, higher circuit PC reduces the EE. It shows that the circuit PC only affects the EE in low traffic region, i.e., in low SE region. In the low SE region, if the circuit PC in idle mode $P^{ci} = 0$, the EE does not change with SE. As the circuit PCs in transmit and receive modes P^{ct} and P^{cr} increase, this region becomes wider.

If $P^{ci} \neq 0$, the EE reduces to zero as the data amount decreases. This does not mean that the idle duration is unnecessary. Comparing the lowest two curves, we can see that the EE will decrease if we do not consider the idle duration.

[1]The averaged bidirectional SE per block takes into account the entire block duration, including not only the transmission time, but also the idle duration.

Moreover, it is shown that when $P^{ci} \neq 0$, there is a non-zero optimal data amount that maximizes the optimal EE.

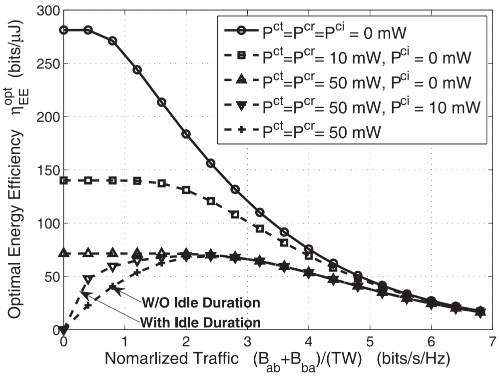

Fig. 2. EE of TWRT with equal bidirectional traffics, $\alpha = 4$.

All these results agree with our earlier analytical analysis. We do not show the corresponding results of OWRT and DT, which are similar as those of TWRT.

In Fig. 3, we compare the EEs of different strategies with non-zero circuit PCs and equal bidirectional data amounts. We see that the comparison results among DT, OWRT and TWRT are the same as those in the zero circuit PC scenario.

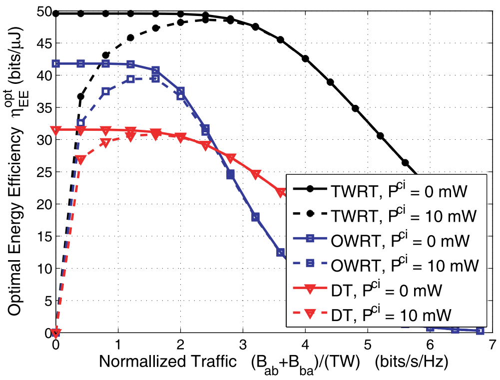

Fig. 3. EE comparison with non-zero circuit powers and equal bidirectional traffics, $\alpha = 4$, $P^{ct} = P^{cr} = 100$ mW, $P^{ci} = 0, 10$ mW.

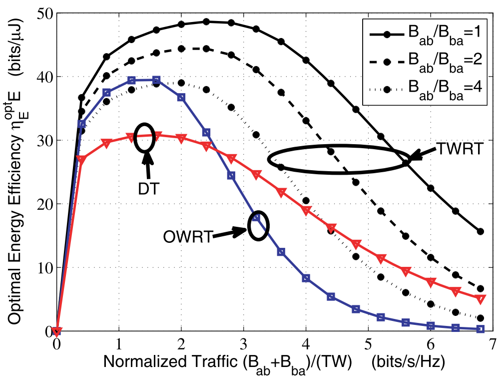

Fig. 4. EE comparison with unequal bidirectional traffics, $\alpha = 4$, $P^{ct} = P^{cr} = 100$ mW, $P^{ci} = 10$ mW.

Finally in Fig. 4, we compare the EEs with unequal bidirectional data amounts. It shows that the EEs of DT and OWRT do not depend on the ratio B_{ab}/B_{ab}, but the EE of TWRT is reduced with the ratio and may even become lower than those of OWRT and DT.

These results are quite different from those provided in [10]. This is because we jointly optimize the transmission time and transmit power to maximize the EE in this work while only the transmit power is optimized in [10].

VI. Conclusion

In this paper, we maximized the energy efficiencies of direct transmission, one-way and two-way relaying by jointly optimizing the transmission time and transmit power, and then compared their maximal energy efficiencies. We showed that circuit power has significant impact on the energy efficiency in low spectral efficiency region, and bidirectional data amounts have large impact on the comparison results of the three transmit strategies. With optimized transmission time and transmit power, the spectral efficient two-way relaying is also more energy efficient when the data amounts in two directions are equal. Otherwise, either one-way relaying or direct transmission may offer higher energy efficiency, depending on the channel condition and data amount.

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