Energy Efficient Relaying Design for OFDM Systems

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Abstract— **In this paper, we design energy efficient relay transmission strategy for an orthogonal frequency division multiplexing (OFDM) system, where two source nodes exchange information with each other via an amplify-and-forward relay node. To maximize the energy efficiency (EE), which is defined as the number of transmitted bits per unit of energy, we jointly optimize the bidirectional bit allocation, which leads to a hybrid one- and two-way relay scheme. In particular, we propose a bit allocation algorithm to balance the transmit and circuit power consumptions of the hybrid relay in order to minimize the overall power consumption. Simulation results show that the proposed algorithm achieves much higher EE than the approach that only minimizes the transmit power, and the hybrid relay is more energy efficient than the pure one- and two-way relay.**

I. Introduction

Due to the sharp increasing of the carbon emission and operating cost of wireless communication systems, energy efficiency (EE) has become a new design goal recently [1]–[4], which is usually defined as the number of transmitted bits per unit of energy. In practical systems, the energy consumption not only comes from transmitting information bits, but also from various circuits and signaling. This indicates that only minimizing the transmit power does not necessarily lead to an energy efficient design [1].

Relay transmission is a promising technique to extend the coverage and enhance the reliability of wireless systems. With network coding, two-way relay (TWR) is able to provide higher spectral efficiency (SE) than one-way relay (OWR) under half-duplex constraint [5]. Recently, the EEs of single-carrier OWR and TWR systems have been studied in various scenarios and different power models. In [2], the EE of decode-and-forward (DF) OWR was analyzed, where both the transmit power and receiver processing power are considered. In [3], relay selection for an OWR system with multiple DF relays was optimized to maximize the EE, where the energy cost of acquiring channel information is considered. In [4], the EEs of OWR and TWR were compared, considering both the transmit and circuit powers. It shows that the spectral efficient TWR is not always more energy efficient than OWR when the bidirectional transmitted bit numbers are unequal.

For the orthogonal frequency division multiplexing (OFDM) relay systems, most existing works focus on the SE, where only transmit power is considered [6]. In this paper, we optimize the EE for an OFDM amplify-and-forward (AF) relay system, taking into consideration not only the transmit power, but also the power consumed by radio-frequency (RF) and baseband processing circuits [7].

We consider a simple three-node system, where two source nodes exchange information via a half-duplex relay node. More complicated systems, e.g., cellular orthogonal frequency division multi-access (OFDMA) systems where a base-station and multiple users exchange uplink and downlink messages via a relay node, can be decoupled into multiple three-node systems when the multi-user resource allocation is fixed. From this point of view, the results derived in three-node system model can be directly used to improve the EE of such complicated multi-user systems. Furthermore, the work on three-node system can act as a good start, based on which we can further optimize the EE of OFDMA relay systems in future work.

Specifically, we jointly optimize the bidirectional bit allocation at the two source nodes to minimize the overall power consumption, under the constraints of data rate requirements. The analysis shows that when the data amounts in two directions are unequal, the optimal transmission strategy is hybrid relay, which employs OWR on some subcarriers and uses TWR on other subcarriers. To improve the EE, we propose an algorithm to balance the transmit and circuit powers of the hybrid relay. Simulation results show that the proposed transmission strategy provides higher EE than the pure TWR and pure OWR as well as the hybrid relay which only minimizes the transmit power.

II. System and Power Consumption Model

Consider an OFDM system consisting of two source nodes $\mathbb{A}$ and $\mathbb{B}$, and a half-duplex AF relay $\mathbb{R}$, each equipped with a single antenna. Assume that the signal attenuates severely between the source nodes, and thus the direct link is not considered for transmission. The whole bandwidth W is divided into K subcarriers, where $h_{\rm ar}^i$ and $h_{\rm br}^i$ respectively denote the channels from nodes $\mathbb{A}$ and $\mathbb{B}$ to node $\mathbb{R}$ on the ith subcarrier, as shown in Fig. 1. We assume perfect channel knowledge at each node. Quasi-static channel is considered, where the channel coefficients keep constant during one OFDM symbol, and may vary from one symbol to another. The noise power N_0 is assumed to be identical on all the subcarriers at each node.

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The AF gain at the relay is chosen with the aid of instantaneous channel gain.

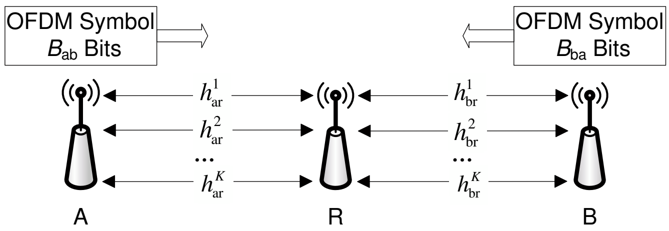

Fig. 1. Three-node OFDM AF relay system model

Consider that $B_{\rm ab}$ and $B_{\rm ba}$ bits need to be transmitted respectively on the $\mathbb{A} \rightarrow \mathbb{B}$ and $\mathbb{B} \rightarrow \mathbb{A}$ directions in each OFDM symbol. The system needs two phases to exchange the messages between the two source nodes. First, both nodes $\mathbb{A}$ and $\mathbb{B}$ transmit their own OFDM symbols to the relay node. Then the relay node forwards its received signals to nodes $\mathbb{A}$ and $\mathbb{B}$. In OWR, the two source nodes modulate their messages on different subcarriers. In TWR, the two source nodes modulate their messages on the same set of subcarriers.

The circuit power at each node is modeled as [7]

$$P_{\rm c} = P_0 + kP_{\rm sc}, \qquad (1)$$

where P_0 denotes the power consumed by the RF circuit, which can be modeled as a constant independent of the data rate [8], $P_{\rm sc}$ is the power consumed by the baseband processing on each subcarrier, and k is the number of employed subcarriers. We assume identical P_0 and identical $P_{\rm sc}$ at each node for simplicity, while the analysis can be easily extended to the case with different P_0 and $P_{\rm sc}$ at each node.

Then the total power consumption of each node is given by

$$P = P_{\rm c} + \sum_{i=1}^{K} P_{\rm t}^i/\epsilon, \qquad (2)$$

where $P_{\rm t}^i$ is the transmit power at the ith subcarrier, and ϵ denotes the power amplifier efficiency, which is assumed as a constant [7], [8]. In the following, subscriber $(\cdot)_{\rm a}$, $(\cdot)_{\rm b}$ and $(\cdot)_{\rm r}$ will be used to distinguish the power consumptions at each node. Note that when a node is receiving, its transmit power is zero, but it still consumes the circuit power $P_{\rm c}$.

III. Energy Efficient Relay Optimization

The EE is defined as the number of transmitted bits over the consumed energy in each OFDM symbol, i.e.,

$$\eta_{\rm EE} = \frac{B_{\rm ab} + B_{\rm ba}}{2T(P_{\rm a} + P_{\rm b} + P_{\rm r})}, \qquad (3)$$

where T is the duration of an OFDM symbol, the factor of 2 is because the system needs two-phase transmission.

For the given values of $B_{\rm ab}$ and $B_{\rm ba}$, i.e., the bidirectional data rates are given, maximizing the EE is equivalent to minimizing the total power consumption, i.e., $P_{\rm a} + P_{\rm b} + P_{\rm r}$. Different from existing works which minimize the transmit power under data rate constraints such as [9] where point-to-point OFDM system is considered, we consider both the transmit and circuit powers in the EE. In the sequel, we optimize the relay transmission strategy to minimize the total power consumption under the constraint of data rate requirements in the two directions.

A. Transmit Power

First, we derive the required transmit powers at each node for exchanging the $B_{\rm ab}$ and $B_{\rm ba}$ bits in an OFDM symbol. For a given data rate requirement, the transmit power obtained from Shannon capacity formula is the required minimal transmit power, based on which we can analyze the maximal EE. In the following, we derive the transmit power at the ith subcarrier respectively for the cases where the system uses TWR or OWR on the ith subcarrier.

If the system transmits $B_{\rm ab}^i$ bits from node $\mathbb{A}$ to node $\mathbb{B}$ using OWR on the ith subcarrier, node $\mathbb{A}$ transmits to node $\mathbb{R}$ in the first phase, and node $\mathbb{R}$ transmits to node $\mathbb{B}$ in the second phase. Using the Shannon capacity formula and the expression of signal-to-noise ratio (SNR) in OWR system [10], the minimal total transmit power of nodes $\mathbb{A}$ and $\mathbb{R}$ on the ith subcarrier can be obtained as [4],

$$(P_{\rm ta}^i + P_{\rm tr}^i)_{\min} = \frac{1}{2}\Big(\frac{C_1 N_0}{|h_{\rm br}^i|^2} + \frac{C_1 N_0}{|h_{\rm ar}^i|^2} + \frac{2\sqrt{C_1^2 + C_1}N_0}{|h_{\rm ar}^i h_{\rm br}^i|}\Big), \qquad (4)$$

where $C_1 \triangleq 2^{\frac{B_{\rm ab}^i}{TW/K}} - 1$, W/K is the bandwidth of a single subcarrier, and the factor $1/2$ is because node $\mathbb{A}$ only transmits in the first phase and node $\mathbb{R}$ only transmits in the second phase.

To simplify the analysis, we consider the same approximations as in [4], which is

$$2^{\frac{B_{\rm ab}^i}{TW/K}} - 1 \approx 2^{\frac{B_{\rm ab}^i}{TW/K}}, \qquad (5)$$

which is accurate when the SE on the ith subcarrier, $\frac{B_{\rm ab}^i}{TW/K}$, is high. When the SE is low, the circuit power will become dominant, the approximation on the transmit power will not affect our analysis on the total power consumption.

After substituting (5), the total transmit power of nodes $\mathbb{A}$ and $\mathbb{R}$ can be approximated as

$$(P_{\rm ta}^i + P_{\rm tr}^i)_{\min} \approx \frac{N_0}{2|h_{\rm eff}^i|^2}\big(2^{\frac{B_{\rm ab}^i}{TW/K}} - 1\big), \qquad (6)$$

where $|h_{\rm eff}^i| \triangleq 1/(\frac{1}{|h_{\rm ar}^i|} + \frac{1}{|h_{\rm br}^i|})$ can be viewed as an effective channel gain between the two source nodes due to the usage of the relay.

If the system transmits $B_{\rm ba}^i$ bits from node $\mathbb{B}$ to node $\mathbb{A}$ with OWR on the ith subcarrier, node $\mathbb{B}$ transmits to node $\mathbb{R}$ in the first phase, and node $\mathbb{R}$ transmits to node $\mathbb{A}$ in the second phase. We can similarly approximate the minimum total transmit power of nodes $\mathbb{B}$ and $\mathbb{R}$ as [4]

$$(P_{\rm tb}^i + P_{\rm tr}^i)_{\min} \approx \frac{N_0}{2|h_{\rm eff}^i|^2}\big(2^{\frac{B_{\rm ba}^i}{TW/K}} - 1\big). \qquad (7)$$

If the system transmits $B_{\rm ab}^i$ and $B_{\rm ba}^i$ bits bidirectionally with TWR on the ith subcarrier, both nodes $\mathbb{A}$ and $\mathbb{B}$ transmit to node $\mathbb{R}$ in the first phase, and node $\mathbb{R}$ broadcasts its received signal to both source nodes in the second phase. Using the similar approach, we can approximate the minimum total transmit power of the three nodes as [4]

$$(P_{\rm ta}^i + P_{\rm tb}^i + P_{\rm tr}^i)_{\min} \approx \frac{N_0[(2^{\frac{B_{\rm ab}^i}{TW/K}} - 1) + (2^{\frac{B_{\rm ba}^i}{TW/K}} - 1)]}{2|h_{\rm eff}^i|^2}. \quad (8)$$

From (6), (7) and (8), we can see that the total transmit power on the ith subcarrier can be expressed in a unified form as follows no matter whether OWR or TWR is used on the ith subcarrier,

$$P_{\rm t}^i \approx \frac{N_0}{2|h_{\rm eff}^i|^2}[(2^{\frac{B_{\rm ab}^i}{TW/K}} - 1) + (2^{\frac{B_{\rm ba}^i}{TW/K}} - 1)]. \quad (9)$$

When the system uses TWR on the ith subcarrier, (9) is the same as (8). When the system uses OWR, $B_{\rm ba}^i = 0$ if the system transmits in $\mathbb{A} \to \mathbb{B}$ direction, or $B_{\rm ab}^i = 0$ if the system transmits in $\mathbb{B} \to \mathbb{A}$ direction. Then, (9) will degenerate into (6) or (7), respectively. When the system does not transmit on the ith subcarrier, i.e., $B_{\rm ab}^i = B_{\rm ba}^i = 0$, (9) equals to zero.

B. Circuit Power

Since all the three nodes always transmit or receive on some subcarriers during the bidirectional transmission, the RF circuit at the three nodes always operates. The total RF circuit power consumption is $3P_0$.

If the system does not transmit any message on the ith subcarrier, there will be no baseband circuit power consumption on this subcarrier. If the system employs OWR on the ith subcarrier, in the first phase one of the source node transmits to the relay node, and in the second phase the relay node transmits to another node. Therefore, there are always a pair of transmitting and receiving nodes on this subcarrier, and the corresponding baseband circuit power is $2P_{\rm sc}$. If the system employs TWR on the ith subcarrier, all the three nodes are transmitting or receiving, and the corresponding baseband circuit power consumption is $3P_{\rm sc}$.

Therefore, the total circuit power of the system is given by

$$P_{\rm c} = 3P_0 + \sum_{i=1}^{K} n_i P_{\rm sc}, \quad (10)$$

where

$$n_i = \begin{cases} 0, & B_{\rm ab}^i = B_{\rm ba}^i = 0, \\ 2, & B_{\rm ab}^i = 0, \text{ or } B_{\rm ba}^i = 0, \\ 3, & B_{\rm ab}^i \neq 0, \text{ and } B_{\rm ba}^i \neq 0. \end{cases} \quad (11)$$

C. Total Power Consumption Minimization

Based on previous analysis, the optimization problem of bidirectional bit allocation on each subcarrier that minimizes the total power consumption under the constraints of the bidirectional data rates can be formulated as follows

$$\min_{B_{\rm ab}^i, B_{\rm ba}^i} 3P_0 + \sum_{i=1}^{K} \{ \frac{N_0[(2^{\frac{B_{\rm ab}^i}{TW/K}} - 1) + (2^{\frac{B_{\rm ba}^i}{TW/K}} - 1)]}{2|h_{\rm eff}^i|^2} + n_i P_{\rm sc} \} \quad (12a)$$

$$\text{s.t.} \sum_{i=1}^{K} B_{\rm ab}^i = B_{\rm ab}, \sum_{i=1}^{K} B_{\rm ba}^i = B_{\rm ba}, \quad (12b)$$

$$B_{\rm ab}^i \geq 0, B_{\rm ba}^i \geq 0, \text{ and (11)}, \quad (12c)$$

where the constraint (12b) ensures the required data rates in two directions. To observe the performance limit under both low and high spectral efficiency, here we do not consider the maximal transmit power constraint.

Note that if the system only uses OWR to complete the bidirectional transmission, i.e., $B_{\rm ab}^i$ and $B_{\rm ba}^i$ can not simultaneously be non-zero for any subcarrier, we need to add the following constraint into problem (12),

$$\begin{aligned} B_{\rm ab}^i = 0, \text{ if } B_{\rm ba}^i \neq 0, \\ B_{\rm ba}^i = 0, \text{ if } B_{\rm ab}^i \neq 0. \end{aligned} \quad (13)$$

If the system only uses TWR to complete the bidirectional transmission, i.e., both $B_{\rm ab}^i$ and $B_{\rm ba}^i$ should be non-zero on the employed subcarrier i, we need to add the following constraint into problem (12),

$$\begin{aligned} B_{\rm ab}^i \neq 0, \text{ if } B_{\rm ba}^i \neq 0, \\ B_{\rm ba}^i \neq 0, \text{ if } B_{\rm ab}^i \neq 0. \end{aligned} \quad (14)$$

Without the OWR-constraint (13) and TWR-constraint (14), the optimized strategy by solving problem (12) naturally yields a hybrid relay scheme, where the system uses OWR for some subcarriers and employs TWR for other subcarriers. Since the OWR-constraint or TWR-constraint are extra constraints on the optimization problem (12), the hybrid relay can achieve lower power consumption than the pure OWR and the pure TWR. On the other hand, when considering the OWR-constraint (13) or the TWR-constraint (14), the optimization problem becomes very difficult. For example, when considering constraint (13), the optimization problem turns into a similar form as that of multi-user multi-carrier resource allocation, which is known as a combinatorial optimization problem [9].

Since the hybrid relay transmission is more energy efficient, in the following, we solve the problem (12).

Directly solving problem (12) is not easy because of the discrete variable n_i. Therefore, we first study the case where $P_{\rm sc} = 0$, then the last term of $n_i P_{\rm sc}$ in the objective function of (12) can be omitted. Since the first term $3P_0$ in the objective function is a constant which can also be omitted, the problem can be simplified as

$$\begin{aligned}
&\min_{B^i_{\rm ab},B^i_{\rm ba}} \sum_{i=1}^{K} \frac{N_0(2^{\frac{B^i_{\rm ab}}{TW/K}}-1)}{2|h^i_{\rm eff}|^2} + \frac{N_0(2^{\frac{B^i_{\rm ba}}{TW/K}}-1)}{2|h^i_{\rm eff}|^2} \\
&\text{s.t. } \sum_{i=1}^{K} B^i_{\rm ab} = B_{\rm ab}, \sum_{i=1}^{K} B^i_{\rm ba} = B_{\rm ba}, \\
&\quad B^i_{\rm ab} \geq 0, B^i_{\rm ba} \geq 0. \qquad (15)
\end{aligned}$$

Note that the objective function is a sum of two functions, each of which is only associated with $B^i_{\rm ab}$ or $B^i_{\rm ba}$. Moreover, all the constraints on $B^i_{\rm ab}$ and $B^i_{\rm ba}$ are also decoupled. Therefore, problem (15) can be decoupled into two subproblems as follows,

$$\begin{aligned}
&\min_{B^i_{\rm ab}} \sum_{i=1}^{K} \frac{N_0(2^{\frac{B^i_{\rm ab}}{TW/K}}-1)}{2|h^i_{\rm eff}|^2} \\
&\text{s.t. } \sum_{i=1}^{K} B^i_{\rm ab} = B_{\rm ab}, B^i_{\rm ab} \geq 0, \qquad (16)
\end{aligned}$$

and

$$\begin{aligned}
&\min_{B^i_{\rm ba}} \sum_{i=1}^{K} \frac{N_0(2^{\frac{B^i_{\rm ba}}{TW/K}}-1)}{2|h^i_{\rm eff}|^2} \\
&\text{s.t. } \sum_{i=1}^{K} B^i_{\rm ba} = B_{\rm ba}, B^i_{\rm ba} \geq 0. \qquad (17)
\end{aligned}$$

These two subproblems respectively optimize the bit allocation on $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ directions to minimize their corresponding transmit power consumptions. Both problems (16) and (17) have the same form as the problem that minimizes the total transmit power of a point-to-point OFDM system with given data rate constraint [9], and thus can be solved using the well-known water-filling algorithm [11]. Since the system allocates the bit numbers with water-filling for both $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ directions, we refer to it as bidirectional water-filling bit allocation, with which we can obtain the optimal values of $B^i_{\rm ab}$ and $B^i_{\rm ba}$, as well as the minimal transmit powers on each subcarrier for the $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ transmissions.

Sorting all the subcarriers by the effective channel gain $|h^i_{\rm eff}|$ in a descending order, then the first subcarrier is the best subcarrier with highest channel gain. With the bidirectional water-filling bit allocation, the system will employ the subcarriers with high effective channel gains for transmission. The subcarriers that are not allocated any bits will not be used for transmission. Assume that the system employs the best $K_{\rm ab}$ subcarriers for $\mathbb{A} \to \mathbb{B}$ transmission, i.e., $\mathcal{S}_{\rm ab} = \{1, 2, \cdots, K_{\rm ab}\}$, and the best $K_{\rm ba}$ subcarriers for $\mathbb{B} \to \mathbb{A}$ transmission, i.e., $\mathcal{S}_{\rm ba} = \{1, 2, \cdots, K_{\rm ba}\}$, as shown in Fig. 2.

Without loss of generality, we assume that $B_{\rm ab} \geq B_{\rm ba}$. As a result, the $\mathbb{A} \to \mathbb{B}$ link will employ more subcarriers than the $\mathbb{B} \to \mathbb{A}$ link, i.e., $K_{\rm ab} \geq K_{\rm ba}$. On the subcarriers $\{1, 2, \cdots, K_{\rm ba}\}$, both $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ links allocate non-zero bits for transmission, i.e., $B^i_{\rm ab} \neq 0$ and $B^i_{\rm ba} \neq 0$, where the system uses TWR to exchange information bidirectionally. On the subcarriers $\{K_{\rm ba}+1, \cdots, K_{\rm ab}\}$, only $\mathbb{A} \to \mathbb{B}$ link allocates non-zero bits for transmission, i.e., $B^i_{\rm ab} \neq 0$ and $B^i_{\rm ba} = 0$, where the system uses OWR to transmit from node $\mathbb{A}$ to node $\mathbb{B}$.

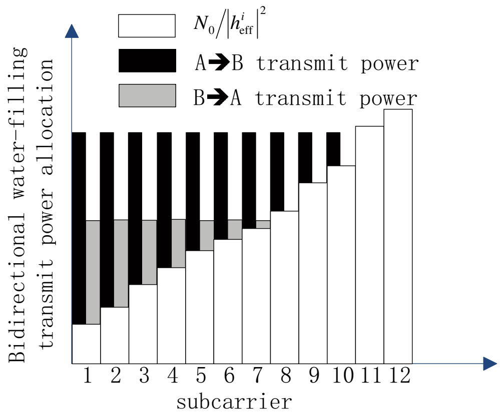

Fig. 2. The transmit power allocation after the bidirectional water-filling bit allocation, where the subcarriers have been sorted by the effective channel gain $|h^i_{\rm eff}|$ in a descending order. In this example, $K = 12$, $\mathbb{A} \to \mathbb{B}$ link employs $K_{\rm ab} = 10$ subcarriers, and $\mathbb{B} \to \mathbb{A}$ link employs $K_{\rm ba} = 7$ subcarriers.

Then we study the case with non-zero $P_{\rm sc}$. In this case, the total power consumption in the objective function of problem (12) includes not only the transmit power, but also the circuit power. We have shown that by respectively employing subcarrier sets $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$ obtained from bidirectional water-filling for the bidirectional transmission, the transmit power can be minimized. Using either more or less subcarriers will lead to higher transmit power consumption. On the other hand, it is straightforward to see that using more subcarriers will result in higher circuit power consumption. Therefore, using more subcarriers than $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$ will increase both the transmit and circuit power. By using a part of subcarriers in $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$, the system can reduce the total power consumption by balancing the transmit and circuit power consumptions. This suggests that the optimal subcarrier sets employed in two directions should satisfy that $\mathcal{S}^{\rm opt}_{\rm ab} \in \mathcal{S}_{\rm ab}$ and $\mathcal{S}^{\rm opt}_{\rm ba} \in \mathcal{S}_{\rm ba}$, respectively.

Now our task is to find the optimal subsets of subcarriers in $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$ that minimizes the total power consumption.

Define the cardinality of a set as the number of its elements, and define the prime subset with cardinality of $k_{\rm ab} \leq K_{\rm ab}$ in $\mathcal{S}_{\rm ab}$ as $\{1, 2, \cdots, k_{\rm ab}\}$, which consists of $k_{\rm ab}$ subcarriers in the set $\mathcal{S}_{\rm ab}$ with the best channel quality. Two systems using the subcarrier subsets with the same cardinality will consume identical circuit powers, because they employ identical number of subcarriers. However, the system that uses the prime subset needs the minimal transmit power due to the usage of the subcarriers with better channel conditions. Therefore, the optimal subcarrier subset in two directions must be the prime subsets of $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$.

The optimal subcarriers subset can be found as follows. For a given pair of subcarrier numbers used in the two directions $(k_{\rm ab}, k_{\rm ba})$, we choose the prime subset in $\mathcal{S}_{\rm ab}$ with cardinality $k_{\rm ab}$ and the prime subset in $\mathcal{S}_{\rm ba}$ with cardinality $k_{\rm ba}$. Then we compute the required minimal transmit power to complete the bidirectional transmission via the water-filling bit allocation from solving the problems (16) and (17), where the subcarrier pool $\{1, 2, \cdots, K\}$ is replaced as $\{1, 2, \cdots, k_{\rm ab}\}$ and $\{1, 2, \cdots, k_{\rm ba}\}$, respectively. We also compute the circuit power consumed by these subcarriers. Denote the obtained total power consumption as $P(k_{\rm ab}, k_{\rm ba})$. By analyzing the first order derivative of $P(k_{\rm ab}, k_{\rm ba})$ with respect to $k_{\rm ab}$ and $k_{\rm ba}$, we can prove that $P(k_{\rm ab}, k_{\rm ba})$ is a convex function when $1 \le k_{\rm ab} \le K_{\rm ab}$ and $1 \le k_{\rm ba} \le K_{\rm ba}$ (Detailed derivation is omitted here due to the lack of space, which is presented in our journal paper in preparation). Therefore, we can use some efficient searching algorithm such as bisection method to obtain the optimal $(k_{\rm ab}^{\rm opt}, k_{\rm ba}^{\rm opt})$ and the corresponding minimum total power consumption.

In summary, in practical systems where the baseband precessing power consumption is not zero, the system should first use bidirectional water-filling bit allocation to find the subcarrier set $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$ which minimizes the total transmit power. Then the system finds the optimal prime subsets of subcarriers in $\mathcal{S}_{\rm ab}$ and $\mathcal{S}_{\rm ba}$, as well as the bit allocations on these chosen subcarriers. By using less subcarriers, the system can balance the transmit and circuit power and improve the EE. We refer this algorithm as bidirectional water-filling with subcarrier reduction. Since the subcarrier reduction procedure only needs a bisection searching, the computation complexity of the proposed algorithm is comparable to that of classical water-filling algorithm.

IV. Simulation Results

In this section, we evaluate the EE of the proposed algorithm, and show the impact of circuit power on the subcarrier allocation and the EE.

We consider that three nodes are located on a straight line. The distance between nodes $\mathbb{A}$ and $\mathbb{B}$ is denoted as D, and the relay is on the midpoint of nodes $\mathbb{A}$ and $\mathbb{B}$. The path loss attenuation is $PL = 30 + 10\log_{10}(\text{distance}^{\alpha})$, where α is the attenuation factor. The noise spectral density is -174dBm/Hz. The subcarrier spacing is 15kHz, and the total number of subcarriers is 1024. We assume that all the small scale fading channels on each subcarrier are i.i.d. Rayleigh fading channels, which do not change during each OFDM symbol but are independent from one OFDM symbol to another. All the results are averaged over 500 Monte-carlo trails of fading channels.

The RF circuit power of a relay node in practical system usually ranges from dozens to hundreds of mWs [12]. Here we set the RF circuit power as $P_0 = 100$ mW. From [7], the ratio of the baseband circuit power on each subcarrier (i.e., $P_{\rm sc}$) over P_0 ranges from 10^{-3} to 10^{-2}. Here we use the same ratio of $P_{\rm sc}/P_0$ as in [7], and thus set $P_{\rm sc} = 0.4$ and 1 mWs, respectively. We assume that the source nodes have the same values of P_0 and $P_{\rm sc}$ as those in the relay node for simplicity.

Fig. 3 compares the EEs of the bidirectional water-filling bit allocation with and without the subcarrier reduction, which are respectively denoted as "B-WF-SR" and "B-WF", where $D = 100$m, $\alpha = 4$, and the ratio between the bit numbers in two directions is set as $B_{\rm ab}/B_{\rm ba} = 4$. When only applying the bidirectional water filling bit allocation, the system only minimizes the transmit power consumption. By combining the bit allocation with the subcarrier reduction, the system can balance the transmit and circuit powers to minimize the overall power consumption, and thus achieves higher EE. The EE gain of "B-WF-SR" over "B-WF" increases with the value of $P_{\rm sc}$. When the SE is high, the transmit power will be dominant, while the circuit power can be neglected. In that case, the EEs with and without the subcarrier reduction overlap.

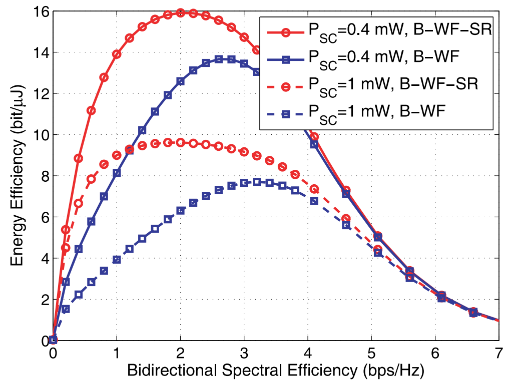

Fig. 3. EEs of the bidirectional water-filling bit allocation with and without the subcarrier reduction

Fig. 4 provides the corresponding numbers of employed subcarriers by $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ links when $P_{\rm sc} = 0.4$ mW. We can see that the algorithm with subcarrier reduction employs fewer subcarriers in both directions than that without the subcarrier reduction, and thus can reduce the circuit power consumption. Since $B_{\rm ab}/B_{\rm ba} = 4$, the $\mathbb{A} \to \mathbb{B}$ link employs more subcarriers than the $\mathbb{B} \to \mathbb{A}$ link. On those subcarriers employed by both links, the system uses TWR, while on those only employed by the $\mathbb{A} \to \mathbb{B}$ link, the system uses OWR.

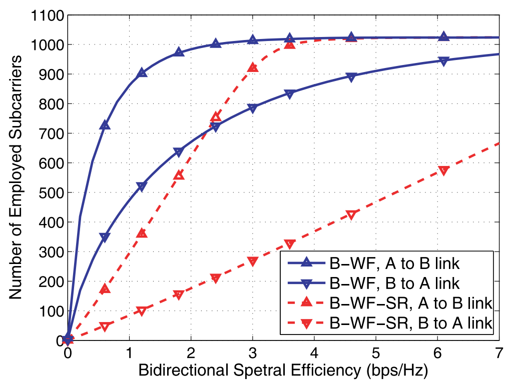

Fig. 4. Numbers of employed subcarriers in two directions with and without the subcarrier reduction

Fig. 5 compares the EEs of the optimized hybrid relay transmission strategy with the pure OWR and TWR transmission. The minimal power consumption of the pure TWR should be obtained by solving problem (12) with the TWR-constraint (14). The optimized TWR transmit strategy we developed is almost the same as that of the hybrid relay scheme. The only difference is that on the subcarriers where the hybrid relay scheme uses OWR to transmit from node $\mathbb{A}$ to node $\mathbb{B}$, the optimized TWR not only transmits on $\mathbb{A} \to \mathbb{B}$ direction, but also transmits on $\mathbb{B} \to \mathbb{A}$ direction with a near-zero data rate in order to satisfy the TWR-constraint (14). The minimal power consumption of the pure OWR should be obtained by solving problem (12) with the OWR-constraint (13). This is a very hard combinatorial optimization problem. Inspired by the multi-user downlink OFDMA resource allocation, where the bit allocation is difficult with individual data rate constraint for each user but is easy with a sum rate constraint, here we relax the OWR optimization by replacing the separate data rate constraints of two directions (12b) as a bidirectional sum rate constraint, i.e., $\sum_{i=1}^{K}(B_{\rm ab}^{i} + B_{\rm ba}^{i}) = B_{\rm ab} + B_{\rm ba}$. Then the relaxed problem can be solved easily via water-filling bit allocation combined with the subcarrier reduction. Due to such a relaxing, the optimized power will be a lower bound of the power consumption that can be achieved by the pure OWR, and thus the obtained EE is an upper bound of that can be achieved by the OWR. Due to the lack of space, here we do not give the detailed optimization procedure of the pure OWR and TWR.

From the figure we can see that the EE of the proposed hybrid relay strategy is always higher than that of the pure TWR and the EE upper bound of the pure OWR. When the SE is high, the hybrid relay transmission has the same EE as the pure TWR. When the required SE is low, the EE of the hybrid relay transmission is identical to the EE upper bound of the pure OWR.

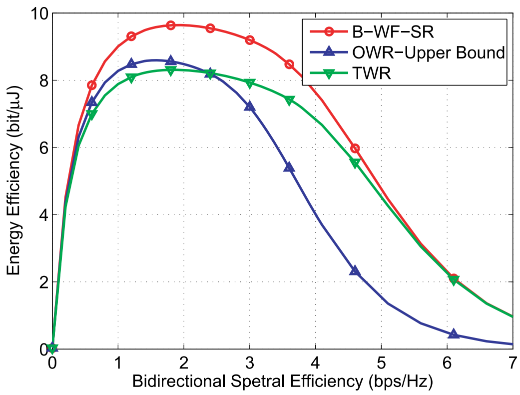

Fig. 5. EE comparison among the hybrid, one-way and two-way relay transmission, where $D = 100$ m, $P_{\rm sc} = 1$ mW, and $B_{\rm ab}/B_{\rm ba}$ = 4.

Due to the lack of space, we do not provide the similar results with different attenuation factor α and circuit powers.

V. Conclusion

In this paper, we studied energy efficient OFDM relay systems. We jointly optimized the bit allocation from two source nodes to maximize the energy efficiency under the constraint of the bidirectional data rate requirements. The analysis showed that the optimal relay strategy is hybrid relay transmission when the bidirectional data amounts are unequal. We proposed an efficient algorithm for the hybrid relay strategy, which employs bidirectional water-filling to allocation the number of bits for the two directions and employs subcarrier reduction to balance the transmit and circuit power consumptions. Simulation results demonstrated that the proposed algorithm outperforms the pure one-way and the pure two-way relay transmission in terms of energy efficiency.

References

[1] G. Y. Li, Z. Xu, C. Xiong, C. Yang, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient wireless communications: tutorial, survey, and open issues," *IEEE Wireless Commun. Mag.*, vol. 18, no. 6, pp. 28–35, Dec. 2011.

[2] C. Bae and W. E. Stark, "End-to-end energy–bandwidth tradeoff in multihop wireless networks," *IEEE Trans. Inform. Theory*, vol. 55, no. 9, pp. 4051–4066, Sept. 2009.

[3] R. Madan, N. Mehta, A. Molisch, and J. Zhang, "Energy-efficient cooperative relaying over fading channels with simple relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3013–3025, Aug. 2008.

[4] C. Sun and C. Yang, "Energy efficiency analysis of one-way and two-way relay systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2012, no. 46, pp. 1–46, Feb. 2012.

[5] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[6] D. Wang, Z. Li, and W. X., "Joint optimal subcarrier and power allocation for wireless cooperative networks over OFDM fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 1, pp. 249–257, Jan. 2012.

[7] H. S. Kim and B. Daneshrad, "Energy-constrained link adaption for MIMO OFDM wireless communication systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2820–2832, Sept. 2010.

[8] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2349–2360, Sept. 2005.

[9] S. Chieochan and E. Hossain, "Adaptive radio resource allocation in OFDMA systems: a survey of the state-of-the-art approaches," *Wireless Communications and Mobile Computing*, vol. 9, no. 4, pp. 513–527, Apr. 2009.

[10] R. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: performance analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 764 –777, Feb. 2010.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York: Cambridge University Press, 2004.

[12] M. Dohler and Y. Li, *Cooperative communications, hardware, channel & PHY*. UK: Wiley, 2010.