# Hybrid Cooperative Transmission in Heterogeneous Networks 

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#### Abstract

Coordinated multi-point (CoMP) transmission through joint processing (JP) and coordinated beamforming (CB) is promising to provide high spectral efficiency by avoiding intercell interference in downlink heterogeneous cellular networks. Pseudo-inverse based zero-forcing beamforming (P-ZFBF) is a popular low-complexity multi-user precoder that can achieve maximal sum rate under sum power constraint. The P-ZFBF under pure CoMP-JP transmission mode is able to achieve high multiplexing gain and array gain, which however leads to very inefficient usage of transmit power in heterogeneous networks. The P-ZFBF under pure CoMP-CB mode can fully use transmit power but achieves low multiplexing and array gains. This paper investigates hybrid cooperative transmission strategies to exploit the advantages of CoMP-JP and CoMP-CB. Both the optimal and closed-form suboptimal hybrid strategies are proposed. Simulation results demonstrate their evident performance gain over the pure CoMP-JP and CoMP-CB transmission modes.


## I. INTRODUCTION

Cellular systems are evolving toward heterogeneous networks (HetNets) with universal frequency reuse due to the increasing demand of high quality and capacity mobile wireless services [1]. HetNet consists of different sizes of cells covered by different types of base stations (BSs), including the traditional macro BSs and low-power nodes such as micro, pico, and remote radio head (RRH). For simplicity, we refer to the low-power nodes as pico BSs hereinafter. Compared with homogeneous networks with only macro BSs, HetNet provides "cell-splitting" gains and hence higher network throughput. The performance gain, however, is largely limited by the intercell interference (ICI) caused by the universal frequency reuse. Efficient ICI coordination mechanisms are therefore critical to support the HetNet deployment [2].

Among various interference mitigation techniques, multicell cooperative transmission, also known as coordinated multipoint (CoMP) transmission, is promising and has attracted much attention recently $[3,4]$. Depending on the types of information shared among the BSs, CoMP can be divided into CoMP joint processing (JP) and coordinated beamforming (CB). CoMP-JP requires full data and channel sharing, with which multicell precoders can be jointly designed to convert ICI into desired signals. By contrast, CoMP-CB requires only channel sharing, with which single-cell precoders are

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respectively designed to avoid the ICI. Theoretically, if optimal transmission strategies are applied, CoMP-JP is superior to CoMP-CB under perfect backhaul links. In practice, however, it is often not the case, e.g., when backhaul capacity is limited [5] or suboptimal precoders are applied.

Pseudo-inverse based zero-forcing beamforming (P-ZFBF) is a popular downlink multiuser multi-input multi-output (MUMIMO) transmission strategy with closed-form expressions [6,7]. In single-cell systems, P-ZFBF was shown to be asymptotically optimal in terms of sum rate for large number of users [6], and in CoMP systems it provides a significant gain over Non-CoMP systems [7]. However, P-ZFBF is not the only ZFBF subject to the zero-forcing principle [8]. Under the sum power constraint (SPC), it was shown in [8] that P-ZFBF is the optimal ZFBF in the sense of maximizing sum rate, which is the case when a single-cell ZFBF is designed for a single BS, e.g., under CoMP-CB or Non-CoMP transmission. Yet in CoMP-JP mode, multicell ZFBF is designed for multiple cooperative BSs under per-BS power constraints (PBPC). In this scenario, P-ZFBF no longer achieves the maximal sum rate, and the optimal ZFBF was designed in [8, 9].

As will be clear later in Section III, the non-optimality of P-ZFBF under the CoMP-JP mode comes from the inefficient usage of the transmit powers of multiple BSs, which leads to severe performance degradation especially for HetNet where the maximal transmit powers of various types of BSs largely differ. P-ZFBF under CoMP-CB mode can fully use the transmit power of every BS. However, the single-cell ZFBF achieves much lower multiplexing gain and array gain than the multicell P-ZFBF under CoMP-JP mode. Therefore, when the low complexity P-ZFBF is applied for HetNet, pure CoMP-JP or pure CoMP-CB transmission cannot always achieve high spectrum efficiency.

In this paper, we investigate hybrid cooperative transmission strategies to exploit the advantages of both the CoMP-JP and CoMP-CB modes. The optimal hybrid strategy is proposed, which performs very close to the optimal ZFBF but with much lower complexity. Inspired by the precoding structure of the optimal hybrid strategy, a suboptimal hybrid strategy is then proposed, which is with closed-form expression but at a minor performance loss. Simulation results demonstrate a noticeable performance gain of the proposed hybrid strategies over the P-ZFBF under the pure CoMP-JP and CoMP-CB modes.


Fig. 1. Illustration of the considered two-cell HetNet.

## II. System Model

## A. System Model

We consider a HetNet system consisting of one macro BS and one pico $\mathrm{BS}, \mathrm{BS}_{1}$ and $\mathrm{BS}_{2}$, respectively equipped with $N_{1}$ and $N_{2}$ antennas, who communicates to two single-antenna users, $\mathrm{MS}_{1}$ and $\mathrm{MS}_{2}$, where $N_{b} \geq 2$ for $b=1,2$. This model is illustrated in Fig. 1.

Let $\mathbf{h}_{k b} \in \mathbb{C}^{N_{b} \times 1}$ denote the composite downlink channel from $\mathrm{BS}_{b}$ to $\mathrm{MS}_{k}$ including both large-scale and small-scale fading. Then we can respectively express the global channel of $\mathrm{MS}_{k}$ from two BSs, $\mathbf{h}_{k}$, the channel matrix of $\mathrm{BS}_{b}$ to two users, $\mathbf{H}_{b}$, and the global channel matrix, $\mathbf{H}$, as follows

$$
\begin{equation*}
\mathbf{h}_{k}=\left[\mathbf{h}_{k 1}^{T} \mathbf{h}_{k 2}^{T}\right]^{T}, \mathbf{H}_{b}=\left[\mathbf{h}_{1 b} \mathbf{h}_{2 b}\right]^{H}, \text { and } \mathbf{H}=\left[\mathbf{h}_{1} \mathbf{h}_{2}\right]^{H} . \tag{1}
\end{equation*}
$$

Let $\mathbf{w}_{k b}$ denote the ZFBF vector of $\mathrm{MS}_{k}$ at $\mathrm{BS}_{b}$, then the entire beamforming vector of $\mathrm{MS}_{k}$ at two BSs can be written as $\mathbf{w}_{k}=\left[\begin{array}{ll}\mathbf{w}_{k 1}^{T} & \mathbf{w}_{k 2}^{T}\end{array}\right]^{T}$. Let $\mathbf{x}_{b}$ denote the transmit signal of $\mathrm{BS}_{b}$, whose average power is constrained by

$$
\begin{equation*}
\mathbb{E}\left\{\left\|\mathbf{x}_{b}\right\|^{2}\right\} \leq P_{b}, \tag{2}
\end{equation*}
$$

where $\mathbb{E}\{\cdot\}$ is expectation operator, $\|\cdot\|$ is Euclidian norm, and $P_{b}$ is the maximal transmit power of $\mathrm{BS}_{b}$. Then the signal received by $M S_{k}$ can be expressed as

$$
\begin{equation*}
y_{k}=\sum_{b=1}^{2} \mathbf{h}_{k b}^{H} \mathbf{x}_{b}+n_{k}, \tag{3}
\end{equation*}
$$

where $n_{k}$ is the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^{2}$.

## B. P-ZFBF under CoMP-CB Mode

Let $\mathbf{w}_{k b}^{C B}$ denote the P-ZFBF vector of $\mathrm{MS}_{k}$ at $\mathrm{BS}_{b}$ in CoMP-CB mode. In this case, the data of each MS is transmitted from only a single BS. Without loss of generality, we assume that $\mathrm{MS}_{k}$ is served by $\mathrm{BS}_{b}$. For notational simplicity, we use $\mathrm{MS}_{\bar{k}}$ and $\mathrm{BS}_{\bar{b}}$ to denote the other user and its serving BS . Then the transmit signal at $\mathrm{BS}_{b}$ is $\mathbf{x}_{b}=\mathbf{w}_{k b}^{C B} s_{k}$, and from (2) the SPC for $\mathbf{w}_{k b}^{C B}$ is

$$
\begin{equation*}
\left\|\mathbf{w}_{k b}^{C B}\right\|^{2} \leq P_{b} \tag{4}
\end{equation*}
$$

Herein, $s_{k}$ is the data symbol of $\mathrm{MS}_{k}$ with $\mathbb{E}\left\{\left\|s_{k}\right\|^{2}\right\}=1$.
The zero-forcing constraints require that $\mathrm{MS}_{\bar{k}}$ does not receive the interfering signal $\mathbf{x}_{k}$, i.e., $\mathbf{h}_{\bar{k} b}^{H} \mathbf{w}_{k b}^{C B}=0$. In this
case, P-ZFBF is the optimal ZFBF precoder [8], which can be expressed as

$$
\begin{equation*}
\mathbf{w}_{k b}^{C B}=p_{k b}^{C B} \cdot \mathbf{g}_{k b}^{C B} /\left\|\mathbf{g}_{k b}^{C B}\right\|, \quad \mathbf{w}_{k \bar{b}}^{C B}=\mathbf{0}, \tag{5}
\end{equation*}
$$

where $\mathbf{g}_{k b}^{C B}$ is the $k$-th column of the matrix $\mathbf{H}_{b}^{\dagger}, p_{k b}^{C B}$ is the power allocated to $\mathrm{MS}_{k}$ by $\mathrm{BS}_{b}$, and (. $)^{\dagger}$ denotes MoorePenrose inverse. It is easy to find that $p_{k b}^{C B}=\sqrt{P_{b}}$ since $\mathrm{MS}_{k}$ is the only user served by $\mathrm{BS}_{b}$. Therefore, the full power of each BS can be used.

## C. P-ZFBF under CoMP-JP Mode

Let $\mathbf{w}_{k b}^{J P}$ and $\mathbf{w}_{k}^{J P}$ denote the P-ZFBF vector at $\mathrm{BS}_{b}$ and the global P-ZFBF vector for $\mathrm{MS}_{k}$ in CoMP-JP mode. In this case, each user receives the desired signal from both BSs. Then the transmit signal at $\mathrm{BS}_{b}$ is $\mathbf{x}_{b}=\sum_{k=1}^{2} \mathbf{w}_{k b}^{J P} s_{k}$ and the PBPC for all precoders is

$$
\begin{equation*}
\sum_{k=1}^{2}\left\|\mathbf{w}_{k b}^{J P}\right\|^{2} \leq P_{b} . \tag{6}
\end{equation*}
$$

To null the inter-user interference with zero-forcing principle, $\mathbf{w}_{k}^{J P}$ needs to ensure $\mathbf{h}_{\bar{k}}^{H} \mathbf{w}_{k}^{J P}=0$. The P-ZFBF vector can be expressed as

$$
\begin{equation*}
\mathbf{w}_{k}^{J P}=p_{k}^{J P} \cdot \mathbf{g}_{k}^{J P} /\left\|\mathbf{g}_{k}^{J P}\right\|, \tag{7}
\end{equation*}
$$

where $\mathbf{g}_{k}^{J P}$ is the $k$-th column of the matrix $\mathbf{H}^{\dagger}, p_{k}^{J P}$ is the power allocated to $\mathrm{MS}_{k}$ by two BSs. Since $p_{k}^{J P}$ and $p_{\bar{k}}^{J P}$ are coupled by the PBPC as shown in (6), the power allocation to maximize sum rate should be jointly optimized. This problem was shown to be convex in [8] and can be numerically solved efficiently. On the other hand, since $p_{k}^{J P}$ is constrained by the maximal power of two BSs simultaneously, the low power of the pico BS generally leads to a very inefficient usage of the transmit power of the macro BS.

## III. Hybrid Cooperative Transmission Strategies

In this section, we first illustrate the advantages and disadvantages of P-ZFBF in pure CoMP-JP and CoMP-CB modes in terms of the transmit power usage and array gain. Then we propose the optimal and low-complexity suboptimal hybrid cooperative transmission strategies by exploiting the benefits of both CoMP-JP and CoMP-CB modes.

## A. Transmit Power Usage and Array Gain

Fig. 2 depicts the transmit power of macro BS as a function of the maximal transmit power of pico BS when using P-ZFBF under pure CoMP-JP and CoMP-CB modes, which is obtained via simulations with the parameters configured in Section IV. The achieved array gain for the pico user under the two modes is also shown, which depends on the antenna number at each BS and channel distribution, but is independent of the transmit powers.
It can be observed that in CoMP-CB mode the macro BS always transmits with the maximal power (i.e., 46 dBm ) as analyzed before. However, in CoMP-JP mode the transmit power of the macro BS is very low when the maximal transmit power of the pico BS is low. For instance, when the typical maximal transmit power of 30 dBm is considered for the pico


Fig. 2. Transmit power of macro BS and achieved array gain of pico user in pure CoMP-JP and CoMP-CB modes.

BS, only about 42 dBm power can be transmitted by the macro BS even though its maximal transmit power is 46 dBm . As expected, the transmit power of the macro BS increases with the growth of the maximal transmit power of the pico BS. On the other hand, since CoMP-CB only exploits the antennas at each single BS, much lower array gain can be achieved than CoMP-JP.

In the following, we propose hybrid cooperative transmission strategies by judiciously combining the P-ZFBF in CoMP-JP and CoMP-CB modes in order to efficiently use the transmit power as well as achieve high array gain.

## B. Optimal Hybrid Cooperative Transmission Strategy

The proposed hybrid strategy in the following combines CoMP-JP and CoMP-CB, which implies that both data and channels should be shared among the coordinated BSs. We assume perfect channel information at the BSs and ideal backhaul links for data and channel sharing between the BSs. In this subsection, therefore, the difference between the socalled CoMP-JP and CoMP-CB does not lie in whether each user is served by one or two BSs, but in the forms of PZFBF. Specifically, if $\mathrm{MS}_{k}$ is served in CoMP-JP mode, it can receive desired signal from both BSs, and the P-ZFBF vector is expressed by (7). If $\mathrm{MS}_{k}$ is served in CoMP-CB mode, it can also receive desired signals from both BSs, but the P-ZFBF vector is expressed as

$$
\begin{equation*}
\mathbf{w}_{k b}^{C B}=\frac{\mathbf{g}_{k b}^{C B}}{\left\|\mathbf{g}_{k b}^{C B}\right\|} p_{k b}^{C B}, \quad b=1,2 \tag{8}
\end{equation*}
$$

which is different from (5), since it does not require that at least one $\mathbf{w}_{k b}^{C B}$ is zero.

The proposed hybrid strategy is a weighted sum of the P ZFBF vectors in CoMP-JP and CoMP-CB modes. Denote the complex number $\lambda_{k}^{J P}, \lambda_{k}^{C B, 1}$ and $\lambda_{k}^{C B, 2}$ as the weight factors for the P-ZFBF vector in CoMP-JP mode, and for the P-ZFBF vectors at $\mathrm{BS}_{1}$ and $\mathrm{BS}_{2}$ in CoMP-CB mode, respectively. Then the proposed hybrid global precoding vector can be expressed as follows

$$
\begin{align*}
\mathbf{w}_{k}^{H Y}= & \lambda_{k}^{J P} \frac{\mathbf{g}_{k}^{J P}}{\left\|\mathbf{g}_{k}^{J P}\right\|} p_{k}^{J P}+ \\
& {\left[\lambda_{k}^{C B, 1} \frac{\left(\mathbf{g}_{k 1}^{C B}\right)^{T}}{\left\|\mathbf{g}_{k 1}^{C B}\right\|} p_{k 1}^{C B} \quad \lambda_{k}^{C B, 2} \frac{\left(\mathbf{g}_{k 2}^{C B}\right)^{T}}{\left\|\mathbf{g}_{k 2}^{C B}\right\|} p_{k 2}^{C B}\right]^{T} . } \tag{9}
\end{align*}
$$

It is easy to verify that with the hybrid precoder (9) the inter-user interference is thoroughly eliminated. Therefore, the hybrid precoder is essentially a ZF precoder.

For simplicity, we define $\lambda_{k, 1}=\lambda_{k}^{J P} p_{k}^{J P}, \lambda_{k, 2}=$ $\lambda_{k}^{C B, 1} p_{k 1}^{C B}, \lambda_{k, 3}=\lambda_{k}^{C B, 2} p_{k 2}^{C B}, \tilde{\mathbf{g}}_{k, 1}=\mathbf{g}_{k}^{J P} /\left\|\mathbf{g}_{k}^{J P}\right\|, \tilde{\mathbf{g}}_{k, 2}=$ $\mathbf{g}_{k 1}^{C B} /\left\|\mathbf{g}_{k 1}^{C B}\right\|$, and $\tilde{\mathbf{g}}_{k, 3}=\mathbf{g}_{k 2}^{C B} /\left\|\mathbf{g}_{k 2}^{C B}\right\|$. Then (9) can be simplified as

$$
\mathbf{w}_{k}^{H Y}=\lambda_{k, 1} \tilde{\mathbf{g}}_{k, 1}+\left[\begin{array}{ll}
\lambda_{k, 2} & \tilde{\mathbf{g}}_{k, 2}^{T} \tag{10}
\end{array} \lambda_{k, 3} \tilde{\mathbf{g}}_{k, 3}^{T}\right]^{T} .
$$

By defining $\tilde{\mathbf{g}}_{k, 1}=\left[\tilde{\mathbf{g}}_{k 1,1}^{T} \tilde{\mathbf{g}}_{k 2,1}^{T}\right]^{T}$ with $\tilde{\mathbf{g}}_{k b, 1} \in \mathbb{C}^{N_{b} \times 1}$, the transmit power of $\mathrm{BS}_{1}$ and $\mathrm{BS}_{2}$ can be obtained as

$$
\begin{align*}
& P_{1}^{t x}=\sum_{k=1}^{2}\left\|\lambda_{k, 1} \tilde{\mathbf{g}}_{k 1,1}+\lambda_{k, 2} \tilde{\mathbf{g}}_{k, 2}\right\|^{2},  \tag{11}\\
& P_{2}^{t x}=\sum_{k=1}^{2}\left\|\lambda_{k, 1} \tilde{\mathbf{g}}_{k 2,1}+\lambda_{k, 3} \tilde{\mathbf{g}}_{k, 3}\right\|^{2} . \tag{12}
\end{align*}
$$

Proposition 1: With the proposed hybrid cooperative transmission strategy, both the macro BS and the pico BS will transmit with the maximal power to maximize the sum rate.

Proof: One can see from (11) and (12) that the transmit power of $\mathrm{BS}_{1}$ and $\mathrm{BS}_{2}$ is independent of $\lambda_{k, 3}$ and $\lambda_{k, 2}$, respectively. This means that if the transmit power of $\mathrm{BS}_{1}$ or $\mathrm{BS}_{2}$ has not reached the maximum, then the sum rate can be always improved by increasing the magnitude of $\lambda_{k, 2}$ or $\lambda_{k, 3}$, respectively, while not affecting the transmit power of the other BS.

With the hybrid precoder, the signal-to-noise ratio (SNR) of $\mathrm{MS}_{k}$ can be obtained as

$$
\begin{equation*}
\operatorname{SNR}_{k}=\frac{\left|\lambda_{k, 1} \mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1}+\lambda_{k, 2} \mathbf{h}_{k 1}^{H} \tilde{\mathbf{g}}_{k, 2}+\lambda_{k, 3} \mathbf{h}_{k 2}^{H} \tilde{\mathbf{g}}_{k, 3}\right|^{2}}{\sigma^{2}} \tag{13}
\end{equation*}
$$

To express both the SNR and PBPC in a compact form, we define $\boldsymbol{\lambda}_{k}=\left[\begin{array}{lll}\lambda_{k, 1} & \lambda_{k, 2} & \lambda_{k, 3}\end{array}\right]^{T}, \quad \mathbf{q}_{k}=$ $\frac{1}{\sigma}\left[\begin{array}{lll}\mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1} & \mathbf{h}_{k 1}^{H} \tilde{\mathbf{g}}_{k, 2} & \mathbf{h}_{k 2}^{H} \tilde{\mathbf{g}}_{k, 3}\end{array}\right]^{T}, \mathbf{G}_{k 1}=\left[\begin{array}{cc}\tilde{\mathbf{g}}_{k 1,1} & \tilde{\mathbf{g}}_{k, 2} \\ \mathbf{0}\end{array}\right]$, and $\mathbf{G}_{k 2}=\left[\begin{array}{lll}\tilde{\mathbf{g}}_{k 2,1} & \mathbf{0} & \tilde{\mathbf{g}}_{k, 3}\end{array}\right]$. Then we can formulate the hybrid precoder optimization problem aimed at maximizing the sum rate of the macro and pico users subject to PBPC as follows

$$
\begin{align*}
\max _{\boldsymbol{\lambda}} & \sum_{k=1}^{2} \log \left(1+\left|\mathbf{q}_{k}^{H} \boldsymbol{\lambda}_{k}\right|^{2}\right)  \tag{14a}\\
\text { s.t. } & \sum_{k=1}^{2}\left\|\mathbf{G}_{k b} \boldsymbol{\lambda}_{k}\right\|^{2} \leq P_{b}, \quad b=1,2 . \tag{14b}
\end{align*}
$$

This problem is not convex due to the non-convexity of the objective function. However, it can be efficiently solved by using the semi-definite relaxation method. In particular, by defining $\boldsymbol{\Lambda}_{k}=\boldsymbol{\lambda}_{k} \boldsymbol{\lambda}_{k}^{H}$, problem (14) can be equivalently written as

$$
\begin{align*}
\max _{\boldsymbol{\Lambda}_{k}} & \sum_{k=1}^{2} \log \left(1+\operatorname{tr}\left(\mathbf{q}_{k} \mathbf{q}_{k}^{H} \boldsymbol{\Lambda}_{k}\right)\right)  \tag{15a}\\
\text { s.t. } & \sum_{k=1}^{2} \operatorname{tr}\left(\mathbf{G}_{k b}^{H} \mathbf{G}_{k b} \boldsymbol{\Lambda}_{k}\right) \leq P_{b}, \quad b=1,2  \tag{15b}\\
& \boldsymbol{\Lambda}_{k} \succeq \mathbf{0}, \quad k=1,2  \tag{15c}\\
& \operatorname{rank}\left(\boldsymbol{\Lambda}_{k}\right)=1, \quad k=1,2 \tag{15d}
\end{align*}
$$

where $\operatorname{tr}(\cdot)$ and $\operatorname{rank}(\cdot)$ denote the trace and the rank of a matrix, respectively.

By omitting the non-convex constraints (15d), problem (15) can be relaxed to be a convex problem. Its optimal solution $\boldsymbol{\Lambda}_{k}^{\star}$ can be numerically obtained with efficient convex optimization methods [10]. If the matrix $\boldsymbol{\Lambda}_{k}^{\star}$ is of rank-one, then we can easily recover the optimal solution to problem (14) by performing eigenvalue decomposition to $\Lambda_{k}^{\star}$. More importantly, with the method proposed in [8], the optimal solution to problem (14) can be always found from $\boldsymbol{\Lambda}_{k}^{\star}$ even when $\boldsymbol{\Lambda}_{k}^{\star}$ is of high rank.
As shown in [8], the optimal ZFBF precoder subject to PBPC can be computed similarly with the semi-definite relaxation method. For a general scenario where $L$ BSs jointly serve $K$ single-antenna users and $\mathrm{BS}_{b}$ has $N_{b}$ antennas, the number of variables to be optimized is $K \sum_{b=1}^{L} N_{b}$. By contrast, with the hybrid precoder, only $K(L+1)$ variables need to be optimized, which is independent of the number of antennas at BSs. This leads to a significant reduction of computational complexity.

## C. Suboptimal Hybrid Cooperative Transmission Strategy

From the viewpoint of implementation in practical systems, it is highly desirable to develop a suboptimal transmission strategy with closed-form expression. This is the focus of this subsection.

To reduce the complexity, we restrict the variables in $\boldsymbol{\lambda}_{k}$ as real-valued numbers. However, this may lead to destructive combination of the desired signals from multiple BSs. To avoid this, we perform phase rotation on the P-ZFBF vectors in CoMP-JP and CoMP-CB modes so that the terms $\mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1}$, $\mathbf{h}_{k 1}^{H} \tilde{\mathbf{g}}_{k, 2}$, and $\mathbf{h}_{k 2}^{H} \tilde{\mathbf{g}}_{k, 3}$ in (13) are positive numbers.

The proposed suboptimal hybrid precoder is a two-step water-filling power allocation. In the first step, by assuming $\lambda_{k, 2}=\lambda_{k, 3}=0$ for $k=1,2$, the optimization with respect to $\lambda_{k, 1}$ is formulated as follows

$$
\begin{align*}
\max _{\lambda_{k, 1}} & \sum_{k=1}^{2} \log \left(1+\frac{\lambda_{k, 1}^{2}\left|\mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1}\right|^{2}}{\sigma^{2}}\right)  \tag{16a}\\
\text { s.t. } & \sum_{k=1}^{2} \lambda_{k, 1}^{2}\left\|\tilde{\mathbf{g}}_{k b, 1}\right\|^{2} \leq P_{b}, \quad b=1,2 \tag{16b}
\end{align*}
$$

However, due to the PBPC, it is still hard to obtain a closedform solution to problem (16). Instead, we first consider the relaxed SPC

$$
\begin{equation*}
\sum_{k=1}^{2} \lambda_{k, 1}^{2}\left(\left\|\tilde{\mathbf{g}}_{k 1,1}\right\|^{2}+\left\|\tilde{\mathbf{g}}_{k 2,1}\right\|^{2}\right) \leq P_{1}+P_{2} \tag{17}
\end{equation*}
$$

With the single SPC and taking $\lambda_{k, 1}^{2}$ as a new variable, the optimal $\lambda_{k, 1}$ can be obtained as

$$
\begin{equation*}
\lambda_{k, 1}=\sqrt{\left(\frac{\mu_{1}}{\left\|\tilde{\mathbf{g}}_{k 1,1}\right\|^{2}+\left\|\tilde{\mathbf{g}}_{k 2,1}\right\|^{2}}-\frac{\sigma^{2}}{\left|\mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1}\right|^{2}}\right)^{+}} \tag{18}
\end{equation*}
$$

where $\mu_{1}$ is the water-level variable for power allocation and $(x)^{+}=\max (x, 0)$. Since the sum rate increases with $\lambda_{k, 1}$ and
hence $\mu_{1}$, the maximal value of $\mu_{1}$ is then found subject to the PBPC (16b).

It is easy to understand that after the first-step power allocation, at least one BS should transmit with its maximal power. Without loss of generality, we assume that $\mathrm{BS}_{2}$ has transmitted with its maximal power after the first step. Then (12) implies that $\lambda_{k, 3}$ should be zero for $k=1,2$. In the following, we find the value of $\lambda_{k, 2}$ to fully use the transmit power of $\mathrm{BS}_{1}$ for $k=1,2$.

In the second step, given $\lambda_{k, 1}$, the optimal value of $\lambda_{k, 2}$ maximizing the sum rate can be obtained by solving the following problem

$$
\begin{align*}
\max _{\lambda_{k, 2}} & \sum_{k=1}^{2} \log \left(1+\frac{\left|\lambda_{k, 1} \mathbf{h}_{k}^{H} \tilde{\mathbf{g}}_{k, 1}+\lambda_{k, 2} \mathbf{h}_{k 1}^{H} \tilde{\mathbf{g}}_{k, 2}\right|^{2}}{\sigma^{2}}\right)  \tag{19a}\\
\text { s.t. } & \sum_{k=1}^{2}\left\|\lambda_{k, 1} \tilde{\mathbf{g}}_{k 1,1}+\lambda_{k, 2} \tilde{\mathbf{g}}_{k, 2}\right\|^{2} \leq P_{1} \tag{19b}
\end{align*}
$$

However, finding the closed-form expression of $\lambda_{k, 2}$ directly from problem (19) is difficult. To circumvent this problem, we simply regard the value of $\lambda_{k, 1}$ in problem (19) as zero for $k=1,2$. Then the optimal value of $\lambda_{k, 2}$ can be obtained as follows

$$
\begin{equation*}
\lambda_{k, 2}=\sqrt{\left(\frac{\mu_{2}}{\left\|\tilde{\mathbf{g}}_{k, 2}\right\|^{2}}-\frac{\sigma^{2}}{\left|\mathbf{h}_{k 1}^{H} \tilde{\mathbf{g}}_{k, 2}\right|^{2}}\right)^{+}} \tag{20}
\end{equation*}
$$

where $\mu_{2}$ is the water-level variable, which can be efficiently selected by, e.g., bisection method to make (19b) hold with equality.
With $\lambda_{k, 1}, \lambda_{k, 2}, \lambda_{k, 3}$ and the phase-rotated $\tilde{\mathbf{g}}_{k, 1}, \tilde{\mathbf{g}}_{k, 2}$ and $\tilde{\mathbf{g}}_{k, 3}$ for $k=1,2$, the suboptimal hybrid precoders can be obtained from (10).

## IV. Simulation Results

In this section, we evaluate the performance of the proposed hybrid cooperative transmission strategies. The HetNet illustrated in Fig. 1 is considered. In the simulations, the macro BS has 4 antennas with the maximal transmit power of 46 dBm , and the pico BS has 2 antennas with the maximal transmit power of 30 dBm [11]. The path loss models from the macro and pico BSs to the users are set as $128.1+37.6 \log _{10} d$ and $140.7+36.7 \log _{10} d$, respectively, where $d$ is the distance between the BS and the user in $k m$ [11]. The shadowing is lognormal distribution with a standard deviation of 8 dB , and the small-scale fading is flat Rayleigh fading. The distance between the two BSs is 150 m , and the radius is 250 m and 40 m for the macro and pico cells, respectively. The average receive SNR of the users located at the boundary of the macro cell is set to 10 dB . Unless otherwise specified, these parameters are used in all simulations.

Figure 3 shows the cumulative distribution function (CDF) of the per-user data rate achieved by five relevant precoders. It can be seen that the proposed optimal hybrid precoder achieves almost the same performance as the optimal ZFBF precoder [8]. The suboptimal hybrid precoder with closed-form expression pays a price of minor performance degradation, which


Fig. 3. The CDF of the data rate per user for the five relevant precoders.


Fig. 4. Average sum rate vs. the maximal transmit power of the pico BS.
however still provides evident performance gain over the P ZFBF under the pure CoMP-JP and CoMP-CB modes. In CoMP-CB mode each BS uses full transmit power to serve one user, while in CoMP-JP mode the powers of the two BSs are jointly allocated aimed at maximizing the sum rate, which often leads to an unfair power allocation between the users. Therefore, CoMP-CB achieves higher cell-edge data rate.
Figure 4 shows the average sum rate, taken over both largescale and small-scale fading channels, as a function of the maximal transmit power of the pico BS. With the typical maximal transmit power of 30 dBm for the pico BS, the pure CoMP-JP is even inferior to the pure CoMP-CB due to the inefficient usage of transmit power of the macro BS. This agrees to the results shown in Fig. 2. With the growth of the maximal transmit power of the pico BS, more transmit power of the macro BS can be used by the pure CoMP-JP, and hence it achieves much better performance than the pure CoMP-CB. Nevertheless, the performance gap between the pure CoMP-JP and the proposed hybrid strategies is obvious especially when the transmit power of the pico BS is low.

Figure 5 shows the average sum rate versus different celledge SNRs. Again, the proposed optimal and suboptimal hybrid precoders provide substantial performance gain over the P-ZFBF under the pure CoMP-JP and CoMP-CB modes.


Fig. 5. Average sum rate vs. cell-edge SNR.

## V. Conclusions

In this paper, we investigated hybrid cooperative transmission strategies for downlink heterogeneous networks. Both the optimal and suboptimal hybrid precoders were proposed to exploit the advantages of the pseudo-inverse based ZFBF in pure CoMP-JP and CoMP-CB modes. The optimal hybrid precoder achieves almost the same performance as the optimal ZFBF precoder but requiring much lower complexity due to reduced number of variables to be optimized. The suboptimal hybrid precoder has a closed-form expression as in the pseudoinverse based ZFBF, which however provides a significant performance gain over the pseudo-inverse based ZFBF in the pure CoMP-JP and CoMP-CB modes.

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