A GRAPH-BASED INTERFERENCE TOPOLOGY CONTROL FOR ULTRA-DENSE NETWORKS

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ABSTRACT

In densely deployed cellular networks, we can proactively avoid interference by controlling interference topology rather than passively mitigating the interference after it has been generated. While there are many ways to control the interference topology, we employ resource allocation as an example to illustrate its performance gain. Traditional graph-based resource allocation methods are often developed from the minimal coloring problem, which can improve the performance of cell-edge users, but suffer from severe loss of network throughput. In this paper, we propose a graph-based resource allocation method to maximize the resource usage efficiency (RUE) under interference-free constraint, which is related to the maximal independent sets of the interference topology graph. Considering that the Max RUE method is inclined to allocate almost all resources to a few cells, causing unfairness among the users, we further propose another graph-based resource allocation method to achieve a good trade-off between the RUE and user fairness. Simulation results show that the cell-edge throughput is significantly boosted with minor degradation in average throughput of the network.

1. INTRODUCTION

To support the exponentially increasing demand on wireless data services, it is necessary for 5G wireless networks to boost network throughput by more than 1000 times beyond 2020. By densely deploying low-cost and low-power small cells, ultra-dense networks (UDNs) are expected to increase both the spectrum efficiency and energy efficiency of the network [1].

However, the ultra-dense reuse of spectrum resources may cause a mass of inter-cell interference (ICI) and limit the throughput especially for the cell-edge users. Moreover, small cells are usually customer-deployed so that their locations are out of control of the operator. Therefore, the interference environment of UDNs is much more complicated than traditional cellular networks. As a result, the existing interference management technologies that passively mitigate the ICIs, such as interference randomization, interference alignment [2] and interference cancellation are not able to provide satisfactory performance. To improve the network throughput, we can proactively avoid the possibly generated interference by controlling interference topology.

A simple way to control topology is to allocate orthogonal resource blocks (RBs) for different links, e.g., allocating orthogonal time slots or subcarriers to avoid generating ICI to the neighboring cells. Among various resource allocation methods investigated for interference networks in the literature, graph-theory has been widely applied as a useful tool for design [3-6].

The methods in [3-5] formulate the resource allocation problem as a graph theoretical coloring problem that finds the minimum number of required colors (i.e. RBs). These methods can improve the performance of cell-edge users, but suffer from severe loss of average throughput. To improve user experience without compromising network throughput, we need to promote the resource usage efficiency (RUE) under the interference-free constraint.

In this paper, we propose a graph-based interference topology control method through resource allocation. The proposed method aims to maximize the RUE under the interference-free constraint. Different from the existing heuristic method to improve the RUE in [6], our method is optimal to maximize the RUE. By changing the variables of optimization problem, the optimal solution is obtained through searching the maximal independent sets of the interference topology graph. When the total number of RBs is large enough, the optimal solution is inclined to allocate almost all RBs to a few cells. Consequently, the increase in RUE is achieved at the cost of sacrificing the fairness among the users. To balance the RUE and fairness, we proceed to maximize the RUE by minimizing required colors (i.e. RBs) and uniformly assigning the rest of the RBs into different cells. Simulations show that this method can improve the rate of cell-edge users without causing significant decrease in the network throughput.

The remainder of the paper is organized as follows. Section 2 describes the system model. Traditional graph-based resource allocation method and the proposed methods are introduced in Section 3. The simulation results are presented in Section 4. Finally, Section 5 draws the conclusions.

2. SYSTEM MODEL

2.1. Signal Model

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We consider a randomly deployed UDN as shown in Fig.1, which consists of multiple small base stations (SBSs) with index set of $\mathcal{L} = \{1,\cdots,L\}$ and multiple mobile stations (MSs) with index set of $\mathcal{K} = \{1,\cdots,K\}$ in a given area, where $L \geq K$. Each SBS serves only one user and each user accesses the SBS with the best channel quality, i.e.,

$$l(k) = \arg\min_{j\in\mathcal{L}} \{g_{j,k}\} \quad (1)$$

where $l(k)$ is the index of the SBS who serves the kth MS, $g_{j,k}$ is the channel gain from the jth small BS to the kth MS, which consists of path loss, shadowing and small scale fading.

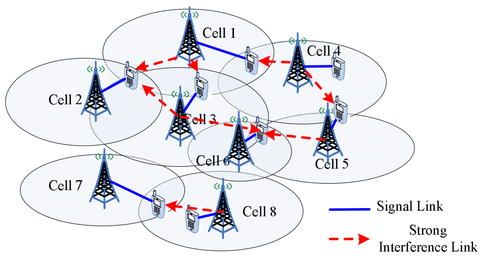

Fig 1. An example of UDN, where $L = K = 8$.

We assume N mutually orthogonal RBs (e.g., orthogonal subcarriers or time slots) with index set $\mathcal{N} = \{1,\cdots,N\}$ can be used for coordinating ICI. Let $\mathcal{Q}_k \subseteq \mathcal{N}$ denote the index set of RBs allocated to the kth cell. If $n \in \mathcal{Q}_k$, the $l(k)$ th BS will use the nth RB to serve the kth MS. Assume that each cell uses at least one RB, therefore $\mathcal{Q}_k$ will not be an empty set, i.e., $|\mathcal{Q}_k| \geq 1, \forall k \in \mathcal{K}$, where $|\mathcal{Q}_k|$ is the cardinality of set $\mathcal{Q}_k$.

For the kth MS, the average data rate is

$$R_k = \frac{1}{N}\sum_{n\in\mathcal{Q}_k} \log\left(1+\gamma_k^n\right) \quad (2)$$

where γ_k^n is the signal-to-interference-plus-noise ratio (SINR) at the nth RB of the kth MS, which is

$$\gamma_k^n = \frac{P_{l(k)}^n g_{l(k),k}}{\sum_{j\neq l(k)} P_j^n g_{j,k} + \sigma^2} \quad (3)$$

and P_j^n denotes the transmit power of the jth BS at the nth RB. If $\forall n \in \mathcal{Q}_j$, then $P_{l(j)}^n = P$, otherwise, $P_{l(j)}^n = 0$. P is the maximal transmit power and σ^2 is the noise power.

2.2. Interference Topology Graph Construction

To show which ICIs need to be coordinated in the network, we first construct an interference topology graph, where each node corresponds to a small cell and the edge that connects two nodes represents the ICI between the two nodes.

In the UDN, many weak ICIs exist. It is inefficient to use orthogonal resource to avoid the weak interference. Therefore, we need to distinguish strong ICIs from weak ICIs and introduce a neighbor list $\mathcal{J}_k$ to record the SBSs who generate strong ICIs to the user in the kth cell. Considering the fact that a user close to its SBS can tolerate stronger ICIs than a user far from its SBS, we choose the signal to interference ratio (SIR) as a metric to identify the strong ICI. By comparing the SIR with a predetermined threshold η_{th}, we obtain a neighbor list of the kth cell denoted as the following set

$$\mathcal{J}_k = \{i \mid \forall \gamma(i,k) < \eta_{th}, \forall i \neq k\} \quad (4)$$

where $\gamma(i,k) \triangleq P_{l(k)} g_{l(k),k} / (P_{l(i)} g_{l(i),k})$ is the ratio of the signal received at the kth user to the ICI generated by the ith SBS.

Assume that there exists a central controller to coordinate the ICIs. When the controller collects the neighbor lists of all cells, it can judge which cells interfere with each other, i.e., which nodes are connected in the interference topology graph. For the kth cell, the index set of strong interfering cells is

$$\mathcal{I}_k = \mathcal{J}_k \bigcup \{i \mid \forall k \in \mathcal{J}_i, i \in \mathcal{K}\} \quad (5)$$

where $\mathcal{J}_k$ is the index set of the cells that generate ICIs to cell k, and $\{i \mid \forall k \in \mathcal{J}_i, i \in \mathcal{K}\}$ is index set of the cells that suffer ICIs from cell k. Thus, $\mathcal{I}_k$ contains all interfering cells. Since $|\mathcal{I}_k|$ is the number of the strong interfering cells for the kth cell, we called $|\mathcal{I}_k|$ as the *degree of interference*.

In Fig. 1, all the strong interference links are shown in red dash lines, from which we can obtain the set of interfering cells as $\mathcal{I}_1 = \{2,3,4\}$, $\mathcal{I}_2 = \{1,3\}$, $\mathcal{I}_3 = \{1,2,6\}$, $\mathcal{I}_4 = \{1,5\}$, $\mathcal{I}_5 = \{4,6\}$, $\mathcal{I}_6 = \{3,5\}$, $\mathcal{I}_7 = \{8\}$ and $\mathcal{I}_8 = \{7\}$, respectively. The corresponding degrees of interference are 3, 2, 3, 2, 2, 2, 1, and 1, and the interference topology graph is shown in Fig.2.

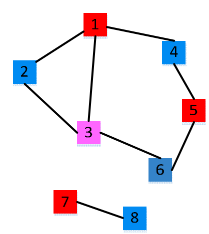

Fig 2. The interference topology graph of the example in Fig. 1.

3. GRAPH-BASED INTERFERENCE TOPOLOGY CONTROL

In this section, we first formulate the resource allocation problem, then briefly review a traditional graph-based resource allocation method, and finally introduce our method to achieve the maximal RUE for the UDN.

3.1. Problem Formulation

We strive to find optimal policies of reusing (under a close-to-unity reuse factor) resources that maximize the network spectral efficiency meanwhile control the interference each user experienced at a tolerable level.

To avoid the ICIs in the network, it is necessary to allocate different RBs for the cells that interfere with each other, i.e., to allocate different RBs for the connected nodes in the interference graph. Therefore, the allocated set of RBs should satisfy the following constraint:

$$\begin{cases} |\mathcal{Q}_k| \geq 1 \\ \mathcal{Q}_i \cap \mathcal{Q}_k = \varnothing, \forall i \in \mathcal{I}_k \end{cases}, \forall k \in \mathcal{K} \tag{6}$$

In this way, the SINR of each user can be controlled to a reasonable level. Then, we can improve the spectral efficiency by increasing the RUE, which is defined as

$$\lambda \triangleq \frac{1}{N} \sum_{k=1}^{K} |\mathcal{Q}_k| \tag{7}$$

Since the RUE is more mathematically tractable than the spectral efficiency, we will optimize the RUE in (7) under the interference-free constraint in (6).

3.2. Resource Allocation to Minimize Resource Cost

In traditional graph-based methods, the optimization problem of resource allocation in interference networks is formulated as a minimum coloring problem, which is

$$\begin{aligned} &\min_{\{\mathcal{Q}_k\}} \left\{ \left| \cup_{k=1}^{K} \mathcal{Q}_k \right| \right\} \\ &s.t. \quad (6) \end{aligned} \tag{8}$$

Since the optimal solution minimizes the required resource cost to coordinate ICI, i.e., minimizes the number of required RBs, we call it "Min cost" in the sequel.

Considering that the optimal solution of (8) is NP-hard, many low-complexity methods have been developed to find near-optimal solutions [6-8]. Using the method proposed in [8], it is not difficult to obtain the near-optimal solution for the interference topology graph in Fig. 2 as $\mathcal{Q}_1 = \{1\}$, $\mathcal{Q}_2 = \{2\}$, $\mathcal{Q}_3 = \{3\}$, $\mathcal{Q}_4 = \{2\}$, $\mathcal{Q}_5 = \{1\}$, $\mathcal{Q}_6 = \{2\}$, $\mathcal{Q}_7 = \{1\}$, $\mathcal{Q}_8 = \{2\}$, which allocates only one RB to each cell. In this case, the ICIs among eight cells are completely removed by three orthogonal RBs, hence the achievable RUE is $\lambda = 8/3$.

In the case of $|\mathcal{Q}_k| = 1$, the RUE in (7) can be expressed as $\lambda = K/N$. That is to say, Minimize Resource Cost achieves max RUE on the condition that $|\mathcal{Q}_k| = 1$, . In fact, in general using $|\mathcal{Q}_k| = 1$ for each cell is not a necessary condition to satisfy the constraint in (6). For example, when cell 7 uses RBs 1 and 2 simultaneously, the RUE is improved without generating interference. Therefore, Minimize Resource Cost method does not achieve maximal RUE.

3.3. Resource Allocation to Maximize RUE

In the sequel, we propose resource allocation methods to maximize the RUE.

1) Max RUE

To obtain the maximal RUE, we formulate the optimization problem as follows

$$\begin{aligned} &\max_{\{\mathcal{Q}_k\}} \left\{ \frac{1}{N} \sum_{k=1}^{K} |\mathcal{Q}_k| \right\} \\ &s.t. \quad (6) \end{aligned} \tag{9}$$

Before solving this problem, we first observe the feature of the interference topology of UDN. Given that the small cells are not uniformly located due to the random deployment, some groups of cells never interfere with other groups of cells, e.g., cells 1~6 never interfere with cells 7 and 8 in Fig. 2. Therefore, it is possible to divide all cells into multiple non-overlapping clusters. Assume that there are G non-overlapping clusters, where the cells in the gth cluster are denoted by set $\mathcal{K}_g$. For the example UDN in Fig.2, there are two clusters, the cells in the first and second clusters are $\mathcal{K}_1 = \{1,2,3,4,5,6\}$ and $\mathcal{K}_2 = \{7,8\}$, respectively.

Then, the RUE can be expressed as

$$\lambda = \frac{1}{N} \sum_{k=1}^{K} |\mathcal{Q}_k| = \sum_{g=1}^{G} \lambda_g \tag{10}$$

where $\lambda_g = \sum_{k \in \mathcal{K}_g} |\mathcal{Q}_k| / N$ is the RUE of the gth cluster.

Then, the optimization problem in (8) can be decomposed into G independent sub-problems without any performance loss. Specifically, we only need to maximize λ_g under the constraint in (6). This is a typical combination optimization problem, which is also NP-hard. In other words, there is no polynomial-time algorithm to find the optimal solution. In what follows, we transform the problem into another equivalent problem.

First, the objective function can be expressed as

$$\lambda_g = \frac{1}{N} \sum_{k \in \mathcal{K}_g} |\mathcal{Q}_k| = \frac{1}{N} \sum_{n=1}^{N} |\mathcal{W}_g^n| \tag{11}$$

where $\mathcal{W}_g^n = \{k \mid \forall n \in \mathcal{Q}_k, k \in \mathcal{K}_g\}$ denotes the index set of the cells using the nth RB in the gth cluster.

Then, the constraint in (6) can be rewritten as $\cup_{n=1}^{N} \mathcal{W}_g^n = \mathcal{K}_g$ and $i \notin \mathcal{I}_k, \forall i \neq k, i, k \in \mathcal{W}_g^n$, which ensures that each cell is allocated at least one RB and any two cells using the same RB do not interfere with each other. Therefore, the optimization problem of (9) is equivalent to

$$\begin{aligned} &\max_{\{\mathcal{W}_n\}} \left\{ \frac{1}{N} \sum_{n=1}^{N} |\mathcal{W}_g^n| \right\} \\ &s.t. \quad \cup_{n=1}^{N} \mathcal{W}_g^n = \mathcal{K}_g \\ &\qquad i \notin \mathcal{I}_k, \forall i \neq k, i, k \in \mathcal{W}_g^n \end{aligned} \tag{12}$$

The optimal solution achieves the maximal RUE so that it is called "Max RUE" in the following.

In graph theory, the independent set is a set of nodes, where no two of which are adjacent. A maximal independent set is an independent set and adding any other node to the set forces the set to contain the adjacent nodes. The optimal solution of problem (12) is comprised of maximal independent sets. This suggests that we need to first obtain all maximal independent sets, and then choose a group of independent sets to maximize λ_g under the constraint of $\cup_{n=1}^{N} \mathcal{W}_g^n = \mathcal{K}_g$. The algorithm in [9] can be applied to find all maximal independent sets with acceptable complexity. Then, we can obtain the optimal solution of (12) by exhaustingly searching.

We take the first cluster in Fig. 2 as an example to illustrate the optimal solution. By using the algorithm in [9], we can obtain all maximal independent set as

$$\{1,5\},\{1,6\},\{2,4,6\},\{2,5\},\{3,4\},\{3,5\}$$

By exhausting searching, we can obtain the optimal solution of problem (12) with different number of RBs. The results are summarized in Table I. Note that, only when $N \ge N_g$, there exist feasible solutions for problem (12), where N_g is the minimal number of required RBs in the gth cluster satisfying the constraint of (12). In this case, $N_g = 3$, $g = 1$， only when $N \ge 3$, there exist feasible solutions.

Table I. Opitmal solution of problem (12) and RUE in the 1st cluseter

N	Optimal solution in the 1st cluseter $\mathcal{W}_g^n$	λ_1
3	$\{1,5\}$, $\{3,4\}$, $\{2,4,6\}$	$\frac{7}{3} = 2.33$
4	$\{1,5\}$, $\{3,4\}$, $\{2,4,6\}$, $\{2,4,6\}$	$\frac{10}{4} = 2.5$
5	$\{1,5\}$, $\{3,4\}$, $\{2,4,6\}$, $\{2,4,6\}$, $\{2,4,6\}$	$\frac{13}{5} = 2.6$
6	$\{1,5\}$, $\{3,4\}$, $\{2,4,6\}$, $\{2,4,6\}$, $\{2,4,6\}$, $\{2,4,6\}$	$\frac{16}{6} \approx 2.67$

It is not hard to obtain that $\lambda_2 = 1$. Therefore, when $N = 6$, the achieved RUE is $\lambda = \lambda_1 + \lambda_2 = 11/3 \approx 3.67$. Compared with the "Min cost" resource allocation, the "Max RUE" method improves the RUE significantly.

From Table I, we can see that the "Max RUE" method will allocate almost all RBs to the maximal independent set that have the most elements, e.g., $\{2,4,6\}$. Consequently, the cells in this set employ most RBs, which will affect the fairness among users considerably.

***Remark 1*:** It is not hard to prove that the RUE achieved by the Max RUE method is a monotonically increasing function of the number of RBs, *N*. As *N* goes to infinity, λ_g approaches $\max\{|\mathcal{W}_g^n|\}$, which is the maximal cardinality of all maximal independent sets. However, the fairness is worst.

2) Max RUE with fairness

From the case of $N = 6$ in Table I, if we allocate the RBs to these three maximal independent sets uniformly, i.e.,

$$\mathcal{W}_g^n : \underbrace{\{1,5\},\{3,4\},\{2,4,6\}}_{N_g \text{ RBs}}, \underbrace{\{1,5\},\{3,4\},\{2,4,6\}}_{N_g \text{ RBs}}$$

the RBs will no longer be allocated to a few cells. From the above example, if we repeat the allocation result of the first N_g RBs over the remaining RBs, it can ensure that RBs are uniformly allocated among maximal independent sets and each cell reuses at least $\lfloor N / N_g \rfloor$ RBs, which improves user fairness.

Consequently, we repeat the allocation result over the N_g + 1th to the Nth RBs uniformly, i.e.,

$$\mathcal{W}_g^l = \mathcal{W}_g^{(l-1)\% N_g + 1}, l = N_g + 1, \cdots N \qquad (13)$$

where % is the modulus operator. We improve fairness of the Max RUE method with uniformly allocating the rest RBs.

Therefore, we only need to find the maximal independent sets under the constraint of the minimal number of required RBs and obtain the result of allocating the first N_g RBs. The optimal problem is changed into

$$\max_{\{\mathcal{Q}_k\}} \left\{ \frac{1}{N_g} \sum_{n=1}^{N_g} |\mathcal{W}_g^n| \right\}$$
$$s.t. \quad \bigcup_{n=1}^{N_g} \mathcal{W}_g^n = \mathcal{K}_g \qquad (14)$$
$$i \notin \mathcal{I}_k, \forall i \ne k, i, k \in \mathcal{W}_g^n$$

By solving problem (14), we can obtain the result of allocating the first N_g RBs.

Compared with the "Max RUE" method, this resource allocation method takes the user fairness into account, which is called "Max RUE with fairness".

Since (14) is a NP-hard problem, we provide a low-complexity suboptimal algorithm for practical systems summarized in Algorithm 1.

Algorithm 1:

Step1: determine the minimal resource cost N_g

1) Initialization:
   $\mathcal{Q}_k = \varnothing, \forall k \in \mathcal{K}_g$, $f(n) = 0, \forall n \in \mathcal{N}$, $\mathcal{U}_g = \varnothing$, $\mathcal{N}_g = \varnothing$
2) Allocate RBs:
   a) Select the cell to allocate RB firstly,
      $$k = \arg\max_{i \in \mathcal{K}_g \setminus \mathcal{U}_g} \{|\mathcal{I}_i|\}$$
   b) If $\mathcal{P}_k \triangleq \bigcup_{i\in\mathcal{U}_g} \mathcal{Q}_i \setminus \bigcup_{j\in\mathcal{I}_k} \mathcal{Q}_j = \varnothing$,
      then $\mathcal{N}_g \leftarrow \mathcal{N}_g \cup \{|\mathcal{N}_g| + 1\}$, $\mathcal{P}_k = |\mathcal{N}_g| + 1$
   c) Allocate the resource with less usage frequency:
      $$n = \arg\min_{\mathcal{P}_k} \{f(m)\}, \mathcal{Q}_k \leftarrow \{n\}$$
   d) Update the set of cells who have been allocated RBs and the RB counter $f(n)$ to record how many cells have used nth RB.
      $$\mathcal{U}_g \leftarrow \mathcal{U}_g \cup \{k\},\ f(n) \leftarrow f(n) + 1$$
   e) Go to step a) until $\mathcal{U}_g = \mathcal{K}_g$
3) Obtain the minimal resource cost: $N_g = |\mathcal{N}_g|$

Step 2: Search the cells that can reuse the nth RB.

For $n = 1 : N_g$

1) Initialization: $\mathcal{W}_g^n = \{k \mid \forall n \in \mathcal{Q}_k, k \in \mathcal{K}_g\}$, $\mathcal{V}_g^n = \mathcal{K}_g \setminus \mathcal{W}_g^n$
2) Reuse the nth RB:
   a) Determine which cell should be considered firstly
      $$k = \arg\min_{i \in \mathcal{V}_g^n} \{|\mathcal{I}_i|\}$$
   b) If $k \notin \mathcal{I}_j$, $\forall j \in \mathcal{W}_g^n$, then $\mathcal{Q}_k \leftarrow \mathcal{Q}_k \cup \{n\}$
   c) Update the set of cells who are allocated the nth RB,
      $$\mathcal{W}_g^n \leftarrow \mathcal{W}_g^n \cup \{k\} \quad \mathcal{V}_g^n \leftarrow \mathcal{V}_g^n \setminus \{k\}$$
   d) Go to step a) until $\mathcal{V}_g^n = \varnothing$

where $\mathcal{U}_g$ is the set of cells that have been allocated RBs, $\mathcal{N}_g$ is the set of allocated RBs, $\mathcal{V}_g^n$ is the set of cells that have not been allocated with the nth RB.

In step 1, $\mathcal{P}_k$ is the index set of RBs available for the kth cell. If $\mathcal{P}_k$ is empty, it is necessary to allocate the kth cell a new RB whose index is $|\mathcal{N}_g| + 1$. If $\mathcal{P}_k$ is nonempty, we can choose one RB with the least usage frequency from $\mathcal{P}_k$ for the kth cell. Now, the obtained $\mathcal{W}_g^n$ is an independent set rather than the maximal independent set. To maximize the RUE, we need to check whether each cell in the set of $\mathcal{V}_g^n$ can reuse the nth RB or not, until $\mathcal{W}_g^n = \{k \mid \forall n \in \mathcal{Q}_k, k \in \mathcal{K}_g\}$ becomes the maximal independent set. Thus, the cardinality of set $\mathcal{W}_g^n$ reaches its maximal value, i.e., the RUE of RB n cannot be increasing any more.

***Remark 2*:** In Step 1, we first allocate RBs to the cell with the maximal degree of interference to reduce the upper-bound of the resource cost [7]. In Step 2, we first check the cell with the minimal degree of interference to make sure that each RB can be allocated to as many cells as possible.

***Remark 3*:** Since the accumulated weak interference on each RB depends on its usage frequency, we allocate the RB used least times to mitigate the impact of accumulated weak interference.

For the example in Fig. 2, when there are $N = 6$ RBs, by applying Algorithm 1 and (13), we can obtain the final allocation result as

$$\mathcal{W}_1^1 = \mathcal{W}_1^4 = \{1,5\}, \mathcal{W}_1^2 = \mathcal{W}_1^5 = \{2,4,5\}, \mathcal{W}_1^3 = \mathcal{W}_1^6 = \{3,4\}$$
$$\mathcal{W}_2^1 = \mathcal{W}_2^3 = \mathcal{W}_2^5 = \{7\}, \mathcal{W}_2^2 = \mathcal{W}_2^4 = \mathcal{W}_2^6 = \{8\}$$

or equivalently as

$$\mathcal{Q}_1 = \{1,4\}, \mathcal{Q}_2 = \{2,5\}, \mathcal{Q}_3 = \{3,6\}, \mathcal{Q}_4 = \{2,3,5,6\},$$
$$\mathcal{Q}_5 = \{1,4\}, \mathcal{Q}_6 = \{2,5\},\ \mathcal{Q}_7 = \{1,3,5\},\ \mathcal{Q}_8 = \{2,4,6\}$$

Consequently, each cell can be allocated at least two RBs and the total RUE is $\lambda = 10/3$. As a result, the Max RUE with fairness method can obtain higher RUE than the Min cost method and higher fairness than the Max RUE method. This suggests that the proposed method can achieve a trade-off between network throughput and user fairness.

4. SIMULATION RESULTS

In this section, we evaluate the proposed interference topology control methods through simulation, where the parameters are based on 3GPP Long Term Evolution (LTE) [10] and summarized in Table II.

Table II Parameters used in simulations

Parameters	Value
Traffic model	Full buffer
Network deployment	7 macro cells where each macro cell's radius is 250m
Num of small cells in each macro cell	$K = 20 \sim 140$
Minimal distance between the BSs	20 m
Minimal distance between BS and MS	5 m
Num of transmit antennas	2
Num of orthogonal RBs	$N = 10$
Transmit power	30 dBm
Noise power	-95 dBm
Path loss	36.8+36.7log10(d), d is the distance between BS and MS

We compare the case without resource allocation to coordinate ICIs (with legend "Full reuse") with three resource allocation methods: *Min cost, Max RUE and Max RUE with fairness*.

To compare the RUE and data rate of different methods, we provide the cumulative distribution function (CDF) of RUE and data rate per user in each RB in Figs. 3 and 4, respectively.

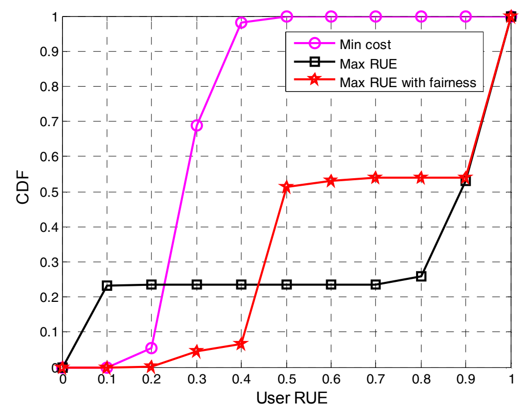

Fig 3. CDF of the RUE per user, η_{th}=4dB, K=40

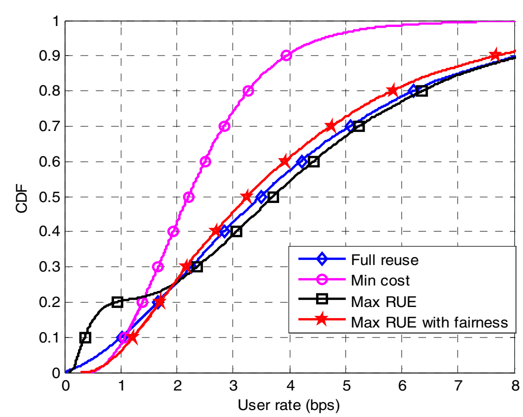

Fig 4. CDF of the data rate per user, η_{th}=4dB, K=40

From the results, we see that in the network with full reuse, the system can achieve a good average sum rate but low cell-edge users' rate. When using the Min cost method, the cell-edge rate is improved but the average throughput is reduced considerably due to its low RUE. Both the methods of Max RUE and Max RUE with fairness can achieve a good average sum rate since they maximize the RUE. However, the fairness of these methods is very different. As shown in Fig. 3, there are 20% cells whose RUE is lower than 0.1 by using the Max RUE method. By contrast, there are only 10% cells whose RUE is lower than 0.4 by using the Max RUE with fairness method. As a result, the cell-edge users' rate of the Max RUE method is not improved but reduced, whereas the cell-edge users' rate of the Max RUE with fairness method is enhanced significantly. The proposed method of Max RUE with fairness can improve the cell-edge users' rate without a sharp decrease of the system throughput.

Since the interference topology depends on the SIR threshold η_{th}, we show the impact of the threshold on these resource allocation methods in Fig. 5. We can observe that as the increase of the threshold, the cell-edge users' rate is improved slightly but the average sum rate is reduced obviously. Therefore, by choosing a proper threshold, a good trade-off between cell-edge users' rate and average rate can be achieved. For example, when η_{th} = 4 dB, the Max RUE with fairness method can enhance cell-edge users' rate by 56.7% with only 4.3% loss in the average sum rate. In the following simulation, we show the result when the threshold is 4 dB.

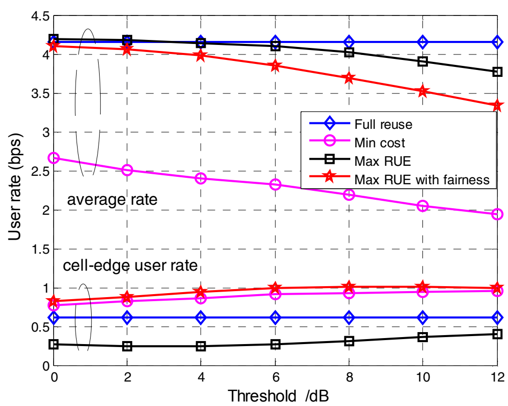

Fig 5. Data rate per-user versus interference threshold, K =40

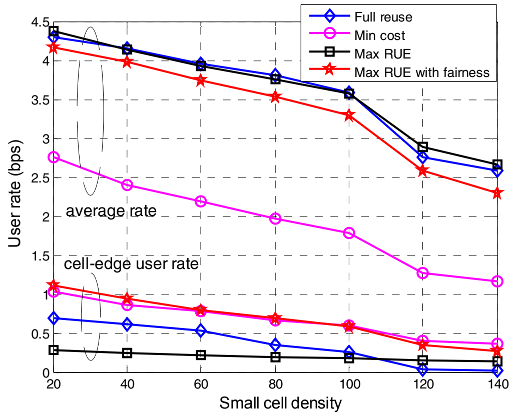

Fig 6. Data rate per-user versus small cell densities, η_{th}=4dB

To show the impact of cell density, we show the average rate per-user and the cell-edge users' rate versus the number of cells in the considered area in Fig. 6. For an arbitrary cell density, the Max RUE with fairness method can improve cell-edge users' rate with a minor loss of average rate. Especially, when the number of cells is 100, compared with the method without ICI coordination, the Max RUE with fairness method can improve cell-edge user rate by 126.9% at the expense of a decrease of average user rate by 8%.

5. CONCLUSION

In this paper, we have presented graph-based topology control methods for ultra-dense networks. Different from traditional graph-based method, we aim to improve the resource usage efficiency under interference-free constraint. Simulation results show that the proposed topology control method based on the maximal independent set can achieve high average throughput by sacrificing the cell-edge users' rate. The proposed method which further considering user fairness can improve the performance of cell-edge users with minor loss in network throughput.

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