Low Complexity Channel Estimation in TDD Coordinated Multi-point Transmission Systems

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***Abstract*—Coordinated multi-point transmission (CoMP) is a promising strategy to provide high spectral efficiency for cellular systems. To facilitate multicell precoding, downlink channel is estimated via uplink training in time division duplexing systems by exploiting channel reciprocity. Virtual subcarriers in practical orthogonal frequency division multiplexing (OFDM) systems degrade the channel estimation performance severely when discrete Fourier transform (DFT) based channel estimator is applied. Minimum mean square error (MMSE) channel estimator is able to provide superior performance, but at the cost of high complexity and more *a priori* information. In this paper, we propose a low complexity channel estimator for CoMP multi-antenna OFDM systems. We employ series expansion to approximate the matrix inversion in MMSE estimator as matrix multiplications. By exploiting the feature of frequency domain training sequences in prevalent systems, we show that the proposed estimator can be implemented by DFT. To reduce the required channel statistical information, we use the average channel gains instead of channel correlation matrix, which leads to minor performance loss in CoMP systems. Simulation results show that the proposed channel estimator performs closely to the MMSE estimator.**

I. Introduction

Coordinated multipoint transmission with joint processing (CoMP-JP) can provide high spectral efficiency for cellular systems when both data and channel state information (CSI) are available at the base station (BS) [1, 2].

To facilitate downlink precoding in CoMP systems, the channels from the cooperative BSs to the mobile stations (MSs) need to be estimated through uplink training by exploiting channel reciprocity in time-division duplexing (TDD) systems [3]. To achieve the optimal performance of channel estimation, the training sequences of multiple MSs in the whole cooperative cluster should be orthogonal [4].

The orthogonality of the training sequences can be provided by code-division multiplexing (CDM), time-division multiplexing, or frequency-division multiplexing. In prevalent cellular systems, such as those complying with the Long Term Evolution (LTE) standard [5], the CDM orthogonal training sequences are provided via using cyclic shifts of the Zadoff-Chu (ZC) sequences [6]. It was shown in [7] that the phase-shifted training sequences are orthogonal when they are transmitted at equal-power pilot tones that are equally spaced over all subcarriers. However, in practical orthogonal frequency division multiplexing (OFDM) systems, a certain number of subcarriers at the band edge are un-modulated in order to prevent the interference to other systems and to ease the filter requirements [8]. As a result, the pilot tones can not be equally spaced over the overall band and the phase-shifted training sequences are no longer orthogonal, which leads to low quality of channel estimation [9, 10]. The performance degradation is especially severe for CoMP-JP systems, where more users are expected to be served to provide high multiplexing gain but fewer orthogonal training sequences are available due to the longer propagation delay caused by the coverage extension.

Linear minimum mean square error (MMSE) estimator can provide superior performance when the channel impulse responses (CIRs) of multiple MSs in multiple cells are jointly estimated [11]. However, the matrix inverse operation is required, whose complexity is too high to afford in practice.

In this paper, we propose a low complexity channel estimator for CoMP-JP to facilitate multi-cell precoding. Specifically, we use a popular way of series expansion to reduce the complexity of computing matrix inverse in the MMSE estimator [12]. To improve the performance of the channel estimator, a scaling factor will be introduced to the series expansion and then optimized, where a closed-form solution will be derived. We will show that after accounting for the unique feature of the frequency domain training sequence used in prevalent cellular systems, the proposed channel estimator can be implemented by discrete Fourier transform (DFT) operations, which is desirable for practical purpose. To reduce the required statistical information, we use the average channel gain instead of the channel correlation matrix in channel estimation. Simulation results show that the proposed low complexity channel estimator performs close to the MMSE estimator in CoMP-JP systems.

Notations: $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote the transpose, conjugate and Hermitian transpose, respectively. $[\mathbf{Z}]_{m,n}$ and $\mathbf{I}_m$

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represent the (m,n) entry of $\mathbf{Z}$ and the $m\times m$ identity matrix. $|\cdot|$ is the cardinality of a set and $\|\cdot\|$ is the two norm. $\mathbb{E}\{\cdot\}$ is the expectation operator, $\mathrm{tr}\{\cdot\}$ is the trace operator, $\mathrm{diag}\{\cdot\}$ denotes diagonalization and $(\cdot)!$ is the factorial operator.

II. System Model

Consider a CoMP-JP OFDM system, where N_B BSs each equipped with N_t antennas cooperatively serve K single-antenna MSs. There are overall N subcarriers in the system, where N_v virtual subcarriers are not employed for transmission. In the employed subcarriers, N_d carriers are used for data and N_p carriers are used as pilots for channel estimation. Let $\mathcal{F}_p$ and $\mathcal{F}_d$ represent the index sets of pilot and data positions, respectively. $|\mathcal{F}_p| = N_p$ and $|\mathcal{F}_d| = N_d$. For simplicity, we refer CoMP-JP as CoMP in the following.

We consider TDD systems, where the CSI for downlink transmission is estimated via uplink training by exploiting channel reciprocity. We consider quasi-static frequency selective fading channels, i.e., the channel remains constant during each round of uplink training and downlink transmission. In the uplink training phase, all MSs send training sequences and each BS estimates the CSI from all MSs to itself. Then a central unit (CU) collects the estimated CSI and computes the precoders. Finally, the CU sends the downlink data and precoders to each BS and all BSs serve the MSs cooperatively.

Consider that the training sequences of the K MSs transmitted on the pilots are multiplexed by phase shifting of one base sequence, as defined in LTE [13]. The maximum number of the training sequences is denoted as M, which is limited and is determined by the length of the CIR and the bandwidth of the system. Denote the uplink transmit power of each MS as P_0, and the base sequence as $\mathbf{s}_0 = [s(0),\ldots,s(N_p-1)]^T$, whose elements are of constant amplitude [13], i.e., $(s(i))^* s(i) = P_0$, $i = 0,\ldots,N_p-1$. The phase shifted training sequence for MS_k is $\mathbf{s}_k = \mathbf{\Phi}_k \mathbf{s}_0$, where $\mathbf{\Phi}_k = \mathrm{diag}\{1 \;\; e^{-j\theta(k-1)} \;\; \cdots \;\; e^{-j\theta(k-1)(N_p-1)}\}$, $\theta = \frac{2\pi}{M}$.

Denote the CIR from MS_k to the ath antenna of BS_b as $\mathbf{h}^t_{b,a,k} \in \mathbb{C}^{L\times 1}$, where L is the number of resolvable paths. $\mathbb{E}\{\|\mathbf{h}^t_{b,a,k}\|^2\} = \alpha^2_{b,k}$, where $\alpha_{b,k}$ is the large scale fading gain including path loss, shadowing and sector antenna gain.

In the uplink training phase, after the removal of cyclic-prefix and FFT operation, the received signal vector of the N_p pilot subcarriers at the ath antenna of BS_b can be expressed as

$$\mathbf{y}_{b,a} = \sum_{k=1}^{K} \mathbf{S}_0 \mathbf{\Phi}_k \mathbf{F}_p \mathbf{h}^t_{b,a,k} + \mathbf{n}_{b,a} \triangleq \mathbf{S}_0 \mathbf{G}_p \mathbf{h}^t_{b,a} + \mathbf{n}_{b,a}, \quad (1)$$

where $\mathbf{S}_0 = \mathrm{diag}\{s(0),\ldots,s(N_p-1)\}$, $\mathbf{F}_p \in \mathbb{C}^{N_p\times L}$ is a partial DFT matrix with entries $[\mathbf{F}_p]_{n,l} = e^{-j2\pi(l-1)[p(n)-1]/N}$, $n = 1,\ldots,N_p$, $l = 1,\ldots,L$, $p(n) \in \mathcal{F}_p$, $\mathbf{G}_p \triangleq [\mathbf{\Phi}_1\mathbf{F}_p,\ldots,\mathbf{\Phi}_K\mathbf{F}_p] \in \mathbb{C}^{N_p\times KL}$, $\mathbf{h}^t_{b,a} = [(\mathbf{h}^t_{b,a,1})^T,\ldots,(\mathbf{h}^t_{b,a,K})^T]^T \in \mathbb{C}^{KL\times 1}$ represents the aggregated CIRs of all MSs, and $\mathbf{n}_{b,a}$ is the additive white Gaussian noise vector with zero mean and variance σ^2_n elements.

To support CoMP transmission, BS_b needs to estimate the aggregated CIRs $\mathbf{h}^t_{b,a}$, $a = 1,\ldots,N_t$. Since the base sequence is of constant amplitude, i.e., $\mathbf{S}_0^H\mathbf{S}_0 = P_0\mathbf{I}_{N_p}$, the optimal linear estimator under MMSE criterion can be obtained as

$$\hat{\mathbf{h}}^{t,\mathrm{MMSE}}_{b,a} = \frac{1}{P_0}[\mathbf{G}_p^H\mathbf{G}_p + \bar{\sigma}^2_n \mathbf{R}^{-1}_{b,a}]^{-1}\mathbf{G}_p^H\mathbf{S}_0^H\mathbf{y}_{b,a}, \quad (2)$$

where $\mathbf{R}_{b,a} = \mathbb{E}\{\mathbf{h}^t_{b,a}(\mathbf{h}^t_{b,a})^H\}$ is the correlation matrix of the channels from all MSs to the ath antenna of BS_b, and $\bar{\sigma}^2_n = \sigma^2_n/P_0$.

The required statistical information of the correlation matrix and the high complexity to compute the inverse of a $KL\times KL$ matrix hinder the application of the MMSE estimator in practical systems. In the next section, we will propose a low-complexity estimator with less *a priori* information.

III. Low-Complexity Channel Estimation Method Based on Series Expansion

To develop a channel estimator with low complexity, we apply the series expansion to approximate the matrix inverse in the MMSE estimator. By adjusting a scaling factor, the series expansion is optimized for a fixed order. Furthermore, we show that after the series expansion, the channel estimator can be implemented with DFT operations.

A. Low Complexity Channel Estimator

The complexity of the MMSE estimator lies in computing the inverse of the matrix $\mathbf{X} \triangleq \mathbf{G}_p^H\mathbf{G}_p + \bar{\sigma}^2_n\mathbf{R}^{-1}_{b,a}$ in (2), whose dimension increases linearly with the number of MSs. Using the series expansion, the inverse of the matrix can be expressed as

$$\mathbf{X}^{-1} = \rho\sum\nolimits_{n=0}^{\infty}(\mathbf{I}-\rho\mathbf{X})^n, \quad (3)$$

where ρ is a scaling factor, it should satisfy $0 < \rho < 2/\lambda_{\max}(\mathbf{X})$ to ensure the convergence of the series expansion [12], and $\lambda_{\max}(\mathbf{X})$ is the largest eigenvalue of matrix $\mathbf{X}$. To reduce the complexity, we use Pth order expansion to approximate the matrix inverse, i.e.,

$$\mathbf{X}^{-1} \approx \rho\sum\nolimits_{n=0}^{P}(\mathbf{I}-\rho\mathbf{X})^n = \sum\nolimits_{n=0}^{P} a_n\rho^{n+1}\mathbf{X}^n, \quad (4)$$

where the last step is obtained by the expansion of the power function, and $a_n = (-1)^n(P+1-n)(P+2-n)\cdots(P+1)/(n+1)!$.

By substituting (4) into (2), the aggregated CIR can be estimated as

$$\begin{aligned}\hat{\mathbf{h}}^t_{b,a} &= \frac{1}{P_0}\sum_{n=0}^{P} a_n\rho^{n+1}[\mathbf{G}_p^H\mathbf{G}_p + \bar{\sigma}^2_n\mathbf{R}^{-1}_{b,a}]^n\mathbf{G}_p^H\mathbf{S}_0^H\mathbf{y}_{b,a} \\ &= \frac{1}{P_0}a_0\rho\mathbf{v}_{b,a}(0) + \frac{1}{P_0}\sum\nolimits_{n=1}^{P} a_n\rho^{n+1}\mathbf{v}_{b,a}(n), \quad (5)\end{aligned}$$

where $\mathbf{v}_{b,a}(0) \triangleq \mathbf{G}_p^H\mathbf{S}_0^H\mathbf{y}_{b,a}$, and $\mathbf{v}_{b,a}(n) \triangleq [\mathbf{G}_p^H\mathbf{G}_p + \bar{\sigma}^2_n\mathbf{R}^{-1}_{b,a}]\mathbf{v}_{b,a}(n-1)$, $n = 1,\ldots,P$.

Now the matrix inverse in the MMSE estimator is approximated by the matrix multiplication operations, which can be implemented with DFT by exploiting the special structure of the frequency domain training sequences, as shown in the sequel. As a result, the channel estimator can be realized with fast FFT/IFFT algorithms.

The channel estimator in (5) can be obtained by computing two components. The first component is $\mathbf{v}_{b,a}(0)$, which can be further expressed as $\mathbf{v}_{b,a}(0) = [(\mathbf{F}_p^H\mathbf{\Phi}_1^H\mathbf{S}_0^H\mathbf{y}_{b,a})^T, \ldots, (\mathbf{F}_p^H\mathbf{\Phi}_K^H\mathbf{S}_0^H\mathbf{y}_{b,a})^T]^T$ by substituting $\mathbf{G}_p$ defined after (1). Recall that both $\mathbf{\Phi}_k$ and $\mathbf{S}_0$ are diagonal matrices, and $\mathbf{F}_p$ is a partial DFT matrix extracted from N-points DFT matrix. To compute each term $\mathbf{v}_{b,a,k}(0) \triangleq \mathbf{F}_p^H\mathbf{\Phi}_k^H\mathbf{S}_0^H\mathbf{y}_{b,a}$, we can first obtain the vector $\mathbf{\Phi}_k^H\mathbf{S}_0^H\mathbf{y}_{b,a}$ by N_p complex multiplications to the received signal vector with negligible complexity, then take a N-points DFT to the obtained vector. Overall K DFTs are required to compute $\mathbf{v}_{b,a}(0)$.

The second component is $\mathbf{v}_{b,a}(n)$, which can be further expressed as $\mathbf{v}_{b,a}(n) = \mathbf{G}_p^H\mathbf{G}_p\mathbf{v}_{b,a}(n-1) + \bar{\sigma}_n^2\mathbf{R}_{b,a}^{-1}\mathbf{v}_{b,a}(n-1)$. Since $\mathbf{v}_{b,a}(n)$ depends on $\mathbf{v}_{b,a}(n-1)$, we can calculate the value of $\mathbf{v}_{b,a}(n)$ with different n in a serial manner with the increasing order of n. When we obtain $\mathbf{v}_{b,a}(n-1)$, the value of $\mathbf{v}_{b,a}(n)$ can also be calculated by DFT. Specifically, to obtain the first term in the expression of $\mathbf{v}_{b,a}(n)$, we can obtain $\mathbf{G}_p\mathbf{v}_{b,a}(n-1)$ similarly as in calculating $\mathbf{v}_{b,a}(0)$, then take K DFTs to the obtained vector to compute the first term of $\mathbf{v}_{b,a}(n)$. In the second term, the correlation matrix $\mathbf{R}_{b,a}$ is diagonal when the CIRs of different MSs and the resolvable paths in the CIR of each MS are uncorrelated. Then, only N_p complex multiplications to the received signal vector are required to obtain the second term.

When the pilot settings in LTE systems are used, the complexity can be further reduced. To be specific, when the intervals between any two adjacent pilots are equal to Δ, and the value of Δ is a power of two [13], the N-points DFT in the estimator can be realized by $\frac{N}{\Delta}$-points DFT. When both the pilot interval Δ and the maximum number of orthogonal training sequences generated by one base sequence M are powers of two, the matrix $\mathbf{G}_p$ in (5) can be transformed into a partial DFT matrix that extracted from a $\frac{N}{\Delta}$-points DFT matrix multiplied with a diagonal phase matrix. This indicates that the complexity of the channel estimator does not depend on the number of MSs.

B. Robust Channel Estimator

The channel estimator in (5) requires the correlation matrix $\mathbf{R}_{b,a}$ as *a priori* information. Considering that the channels among different BS-MS links are uncorrelated, then $\mathbf{R}_{b,a} = \mathrm{diag}\{\mathbf{R}_{b,a,1}, \ldots, \mathbf{R}_{b,a,K}\}$, where $\mathbf{R}_{b,a,k} = \mathbb{E}\{\mathbf{h}_{b,a,k}^t(\mathbf{h}_{b,a,k}^t)^H\}$ is the correlation matrix of the CIR between MS_k and the ath antenna of BS_b and satisfies $\mathrm{tr}\{\mathbf{R}_{b,a,k}\} = \alpha_{b,k}^2$. In practice, accurate correlation matrices of the CIRs are hard to obtain but the the large scale fading gains can be obtained easily by averaging the received signal over a certain period. In the following, we present a robust low-complexity estimator, which only requires the large scale fading gains and is not sensitive to the accuracy of the correlation matrices.

To avoid the use of correlation matrix, we approximate the power delay profile (PDP) as uniform. Then, the estimator in (5) reduces to

$$\hat{\mathbf{h}}_{b,a}^{t,\mathrm{Rb}} = \frac{1}{P_0}\sum_{n=0}^{P} a_n\rho^{n+1}[\mathbf{G}_p^H\mathbf{G}_p + \bar{\sigma}_n^2\mathbf{D}_b^{-1}]^n\mathbf{G}_p^H\mathbf{S}_0^H\mathbf{y}_{b,a}, \quad (6)$$

where $\mathbf{D}_b = \mathrm{diag}\{\frac{\alpha_{b,1}^2}{L}\mathbf{I}_L, \ldots, \frac{\alpha_{b,K}^2}{L}\mathbf{I}_L\}$.

The estimated CFR of all MSs at both data and pilots subcarriers can be obtained as

$$\hat{\mathbf{h}}_{b,a}^{f,\mathrm{Rb}} = \mathbf{G}_{pd}\hat{\mathbf{h}}_{b,a}^{t,\mathrm{Rb}}, \quad (7)$$

where $\hat{\mathbf{h}}_{b,a}^{f,\mathrm{Rb}} = [(\hat{\mathbf{h}}_{b,a,1}^{f,\mathrm{Rb}})^T, \ldots, (\hat{\mathbf{h}}_{b,a,K}^{f,\mathrm{Rb}})^T]^T$, $\hat{\mathbf{h}}_{b,a,k}^{f,\mathrm{Rb}}$ represents the estimated CFR from MS_k to the ath antenna of BS_b, $\mathbf{G}_{pd} = \mathrm{diag}\{\mathbf{F}_{pd}, \ldots, \mathbf{F}_{pd}\} \in \mathbb{C}^{K(N_p+N_d)\times KL}$, $\mathbf{F}_{pd} \in \mathbb{C}^{(N_p+N_d)\times L}$ is a partial DFT matrix with entries $[\mathbf{F}_{pd}]_{m,n} = e^{-j2\pi(n-1)[g(m)-1]/N}$, $m = 1, \ldots, N_p+N_d$, $n = 1, \ldots, L$, and $g(m) \in (\mathcal{F}_p \bigcup \mathcal{F}_d)$.

Through the simulations in Section IV we will show that the performance degradation of the robust channel estimator is negligible.

C. Optimization of the Scaling Factor

Although any value of ρ satisfying $0 < \rho < \frac{2}{\lambda_{\max}(\mathbf{X})}$ can ensure the convergence of the series expansion in (3), the convergence rate with different ρ differs. In the following, we optimize the value of ρ such that the mean square error (MSE) of the channel estimation is minimized for a fixed order P.

Since the CFRs at both data and pilot subcarriers of all MSs in the uplink are applied for precoding in the downlink, we optimize ρ by minimizing the sum MSE of the estimated CFRs. Denote $\mathbf{h}_{b,a,k}^f = \mathbf{F}_{pd}\mathbf{h}_{b,a,k}^t$ as the CFR from MS_k to the ath antenna of BS_b, then the aggregated CFRs of all MSs can be expressed as

$$\mathbf{h}_{b,a}^f = [(\mathbf{h}_{b,a,1}^f)^T, \ldots, (\mathbf{h}_{b,a,K}^f)^T]^T = \mathbf{G}_{pd}\mathbf{h}_{b,a}^t. \quad (8)$$

Then the optimization problem can be formulated as

$$\min_{\rho} \quad \mathbb{E}\{\|\hat{\mathbf{h}}_{b,a}^{f,\mathrm{Rb}} - \mathbf{h}_{b,a}^f\|^2\} \quad (9)$$

$$s.t. \quad 0 < \rho < 2/\lambda_{\max}(\mathbf{X}). \quad (10)$$

By substituting (7) and (8) into (9), the objective function can be derived as

$$\begin{aligned} f(\rho) &\triangleq \mathbb{E}\{\|\hat{\mathbf{h}}_{b,a}^{f,\mathrm{Rb}} - \mathbf{h}_{b,a}^f\|^2\} \\ &= \mathbb{E}\{[\mathbf{G}_{pd}(\hat{\mathbf{h}}_{b,a}^{t,\mathrm{Rb}} - \mathbf{h}_{b,a}^t)]^H\mathbf{G}_{pd}(\hat{\mathbf{h}}_{b,a}^{t,\mathrm{Rb}} - \mathbf{h}_{b,a}^t)\} \quad (11) \\ &= \sum_{i,j=0}^{P} a_i a_j \eta(i,j)\rho^{i+j+2} - \sum_{i=0}^{P} a_i\nu(i)\rho^{i+1} + c_0, \quad (12) \end{aligned}$$

where (12) is derived by substituting the expression of $\hat{\mathbf{h}}_{b,a}^{t,\mathrm{Rb}}$ in (6) into (11) and by taking the expectation with respect to the small scale fading channels and the noise, $c_0 = \mathrm{tr}\{\mathbf{R}_{b,a}\mathbf{V}\}$,

$$\eta(i,j) = \mathrm{tr}\{(\mathbf{U} + \bar{\sigma}_n^2\mathbf{D}_b^{-1})^i\mathbf{U}\mathbf{R}_{b,a}\mathbf{U}(\mathbf{U} + \bar{\sigma}_n^2\mathbf{D}_b^{-1})^j\mathbf{V}\},$$

$$\nu(i) = \mathrm{tr}\{(\mathbf{U} + \bar{\sigma}_n^2\mathbf{D}_b^{-1})^i(\mathbf{V}\mathbf{R}_{b,a}\mathbf{U} + \mathbf{U}\mathbf{R}_{b,a}\mathbf{V})\},$$

with $\mathbf{U} = \mathbf{G}_p^H\mathbf{G}_p$ and $\mathbf{V} = \mathbf{G}_{pd}^H\mathbf{G}_{pd}$.

The optimal solution of problem (9) can be found by exhaustive searching, but this will lead to high complexity for the channel estimation. In the sequel, we derive a sub-optimal solution with closed form by introducing approximations to the objective function.

The values of $\eta(i,j)$ and $\nu(i)$ depend on the noise to signal power ratio $\bar{\sigma}_n^2$ and the large scale fading gains of all users. To obtain a practical solution that does not rely on these parameters, we assume that the SNR is high, i.e., $\bar{\sigma}_n^2 \approx 0$, and we consider that all the K MSs are located at the exact cell edge, i.e., $\alpha_{b,k} = \alpha_{\text{edge}}$, $b = 1, \ldots, N_B$, $k = 1, \ldots, K$, and the PDP of all channels are uniform. Then, $\mathbf{D}_b = \mathbf{R}_{b,a} \approx \frac{\alpha_{\text{edge}}^2}{L}\mathbf{I}_{KL}$. Under these assumptions, the values $\eta(i,j)$, $\nu(i)$ and c_0 in (12) can be approximated as

$$\begin{aligned}\eta(i,j) &\approx \frac{\alpha_{\text{edge}}^2}{L}\text{tr}\{\mathbf{U}^{i+j+2}\mathbf{V}\} \overset{(a)}{\leq} \frac{\alpha_{\text{edge}}^2}{L}\text{tr}\{\mathbf{U}^{i+j+2}\}\text{tr}\{\mathbf{V}\} \\ &\overset{(b)}{<} (N_p+N_d)LK^2\alpha_{\text{edge}}^2(\lambda_{\max}(\mathbf{U}))^{i+j+2}, \end{aligned} \tag{13}$$

$$\begin{aligned}\nu(i) &\approx 2\frac{\alpha_{\text{edge}}^2}{L}\text{tr}\{\mathbf{U}^{i+1}\mathbf{V}\} \overset{(a)}{\leq} 2\frac{\alpha_{\text{edge}}^2}{L}\text{tr}\{\mathbf{U}^{i+1}\}\text{tr}\{\mathbf{V}\} \\ &\overset{(b)}{<} 2(N_p+N_d)LK^2\alpha_{\text{edge}}^2(\lambda_{\max}(\mathbf{U}))^{i+1}, \end{aligned} \tag{14}$$

$$c_0 \approx \frac{\alpha_{\text{edge}}^2}{L}\text{tr}\{\mathbf{V}\} = \alpha_{\text{edge}}^2 K(N_p+N_d), \tag{15}$$

where the step (a) in both (13) and (14) are derived from the fact that $\text{tr}\{\mathbf{AB}\} \leq \text{tr}\{\mathbf{A}\}\text{tr}\{\mathbf{B}\}$ for arbitrary Hermitian matrices $\mathbf{A}$ and $\mathbf{B}$, the step (b) in (13) and (14) come from the fact that $\text{tr}\{\mathbf{V}\} = \text{tr}\{\mathbf{G}_{pd}^H\mathbf{G}_{pd}\} = (N_p+N_d)KL$ and $\text{tr}\{\mathbf{U}^{i+1}\} = \sum_{l=1}^{KL}(\lambda_l(\mathbf{U}))^{i+1} \leq KL(\lambda_{\max}(\mathbf{U}))^{i+1}$, and $\lambda_l(\mathbf{U})$ and $\lambda_{\max}(\mathbf{U})$ are the lth and the largest eigenvalues of $\mathbf{U}$, respectively.

By substituting (13), (14) and (15) into (12), the objective function can be approximated as

$$\begin{aligned} f(\rho) \approx &(N_p+N_d)LK^2\alpha_{\text{edge}}^2\Big[\Big(\sum_{i=0}^{P}a_i(\lambda_{\max}(\mathbf{U})\rho)^{i+1}-1\Big)^2 \\ &+\frac{1}{KL}-1\Big] \triangleq \tilde{f}(\rho). \end{aligned} \tag{16}$$

Observing the expression of $\tilde{f}(\rho)$, we can see that its value is lower bounded as

$$\tilde{f}(\rho) \geq (N_p+N_d)LK^2\alpha_{\text{edge}}^2\Big(\frac{1}{KL}-1\Big), \tag{17}$$

where the equality holds when

$$\sum\nolimits_{i=0}^{P} a_i(\lambda_{\max}(\mathbf{U})\rho)^{i+1} = 1. \tag{18}$$

This indicates that to find the value of ρ that minimizes $\tilde{f}(\rho)$, we only need to find ρ from this equation. To find the solution of the equation in (18), we further derive the left-hand side of the equation as

$$\begin{aligned}\sum_{i=0}^{P}a_i(\lambda_{\max}(\mathbf{U})\rho)^{i+1} &\overset{(a)}{=} \sum_{i=0}^{P}\lambda_{\max}(\mathbf{U})\rho(1-\lambda_{\max}(\mathbf{U})\rho)^i \\ &= 1-(1-\lambda_{\max}(\mathbf{U})\rho)^{P+1}, \end{aligned} \tag{19}$$

where (a) comes by substituting the value of a_i defined after (4) and with some regular manipulations.

Then, the equation becomes $1-(1-\lambda_{\max}(\mathbf{U})\rho)^{P+1} = 1$, and the solution can be found as

$$\rho = 1/\lambda_{\max}(\mathbf{U}) = 1/\lambda_{\max}(\mathbf{G}_p^H\mathbf{G}_p). \tag{20}$$

It is worthy to notice that the value of ρ should satisfy the convergence condition shown in (10). When the SNR is high, we have $\mathbf{X} \approx \mathbf{G}_p^H\mathbf{G}_p$, and the convergence condition becomes $0 < \rho < 2/\lambda_{\max}(\mathbf{G}_p^H\mathbf{G}_p)$. It is clear that the closed-form solution of ρ shown in (20) satisfies the condition.

It is non-trival to theoretically analyze the performance of the robust estimator with sub-optimal solution in (20). We will verify by simulation in Section IV that the sub-optimal solution in (20) performs closely to the optimal solution of problem (9) even when the SNR is not very high.

D. Complexity Comparison

In the sequel we compare the complexity of the proposed channel estimator with the MMSE channel estimator in (2), the DFT channel estimator [9] and the leakage suppression estimator proposed in [10].

The complexity of the MMSE channel estimator is dominated by the inverse of the matrix $\mathbf{X} \in \mathbb{C}^{KL\times KL}$, whose complexity is on the order of $\mathcal{O}((KL)^3)$.

For the proposed channel estimator, we only consider the complexity of the DFT operations, because other operations are negligible compared with DFT. According to previous analysis, the proposed estimator with Pth order series expansion can be implemented by $2PK$ DFT operations of size N under general parameter settings, whose complexity is on the order of $\mathcal{O}(PKN\log(N))$. When the pilots are equally placed and the pilot interval Δ is a power of two, the size of DFT can be reduced to $\frac{N}{\Delta}$, and then the complexity of the estimator is reduced to $\mathcal{O}(PK\frac{N}{\Delta}\log(\frac{N}{\Delta}))$. Furthermore, when both the pilot interval Δ and the maximum number of orthogonal training sequences generated by one base sequence M are powers of two, the proposed estimator can be implemented by P DFT operations each of size $\frac{N}{\Delta}$. Therefore, the complexity of the proposed channel estimator is on the order of $\mathcal{O}(P\frac{N}{\Delta}\log(\frac{N}{\Delta}))$ when the pilots in LTE systems are considered.

The complexities of the DFT estimator under the above three parameter settings are $\mathcal{O}(K\frac{N}{2}\log(N))$, $\mathcal{O}(K\frac{N}{2\Delta}\log(\frac{N}{\Delta}))$ and $\mathcal{O}(\frac{N}{2\Delta}\log(\frac{N}{\Delta}))$, respectively. The complexities of the estimator in [10] are all $\mathcal{O}(KL^3)$ under the above three parameter settings.

For ease of understanding, we give an example, where the system parameters are set in accordance with the LTE systems [13]: the total number of subcarriers $N = 1024$, the pilot interval $\Delta = 2$, the maximum number of orthogonal training sequences $M = 8$, the number of resolvable paths $L = 36$. For the proposed channel estimator, the series expansion order is set as $P = 5$. Then, the number of complex multiplications required by the MMSE method and the method in [10] is over 2.4×10^7 and 3.7×10^5, respectively. By contrast, the number of complex multiplications required by the proposed channel

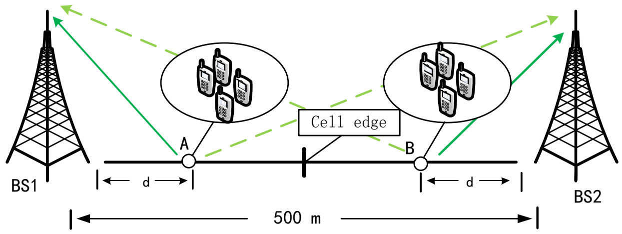

Fig. 1. An example of CoMP system. The inter-site distance is 500 m. The user-groups in different cells are at the same distance from their local BSs. The solid and dash lines represent the local channels and cross channels of the users, respectively.

estimator and the DFT channel estimator are about 2.3×10^4 and 2.3×10^3, respectively.

IV. Simulation Results

In this section, the performance of different channel estimators will be compared via simulations.

Unless otherwise specified, we consider a CoMP system with two cooperative BSs each with four antennas cooperatively serve eight single-antenna MSs. The system parameters are in accordance with the LTE standard [13]: we consider 10 MHz bandwidth with 1024 FFT size, in which 600 subcarriers are employed. The training sequence generated by phase shift of a root Zadoff-Chu sequence is transmitted on 300 equally placed pilots for each MS.

The path-loss model is $\text{PL}^{\text{dB}} = 35.3 + 37.6\log_{10}(d_{k,b})$ [13], where $d_{k,b}$ (in meter) is the distance between MS_k and BS_b. The downlink receive SNR of the cell-edge MS is 15 dB and the uplink receive SNR is set as 5 dB lower than the downlink SNR to incorporate the difference of the transmit power and the interference between uplink and downlink. The CIR in the simulations follows a tapped-delay line model with independent Rayleigh fading coefficients. Although we assume uniform PDP when deriving the robust estimator, we consider an exponential PDP with attenuation factor 1.4. Considering the longer propagation delays in CoMP systems, the length of PDP is set as $L = 36$. All simulation results are obtained by averaging over 1000 realizations of the small scale fading channels. To clearly show the channel estimation performance of different MSs, we consider a scenario with the MS locations shown in Fig. 1. Specifically, the four MSs in the same cell are located in the same place and the MS-groups in different cells are at the same distance from their local BSs. In this way, we only need to show the performance of one MS.

In Fig. 2, the normalized MSEs (NMSEs) of the local channels versus the series expansion order P are compared between the proposed channel estimators with the optimal scaling factor and the sub-optimal scaling factor. The NMSE is defined as $\mathbb{E}\{\|\hat{\mathbf{h}}_{b,a,k}^{f,\text{Rb}} - \mathbf{h}_{b,a,k}^{f}\|^2\}/\alpha_{b,k}^2$. As a performance baseline for comparison, the performance of the MMSE estimator is provided. We can observe that the convergence rate is fast when the user is located at the cell edge, and the performance gap between the proposed channel estimators with the optimal scaling factor and the sub-optimal scaling factor is minor.

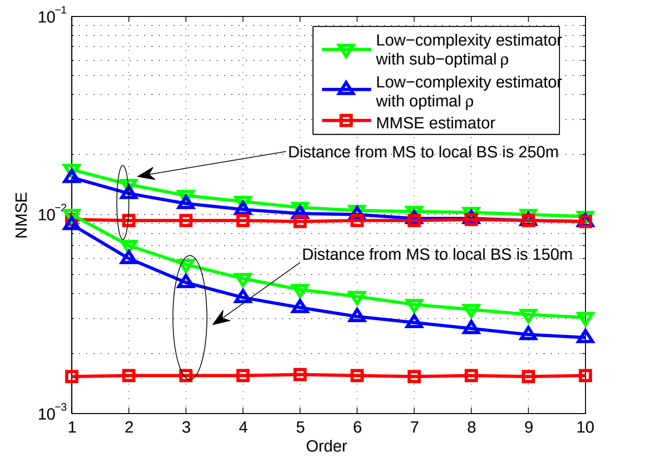

Fig. 2. NMSE of local channels versus the series expansion order P.

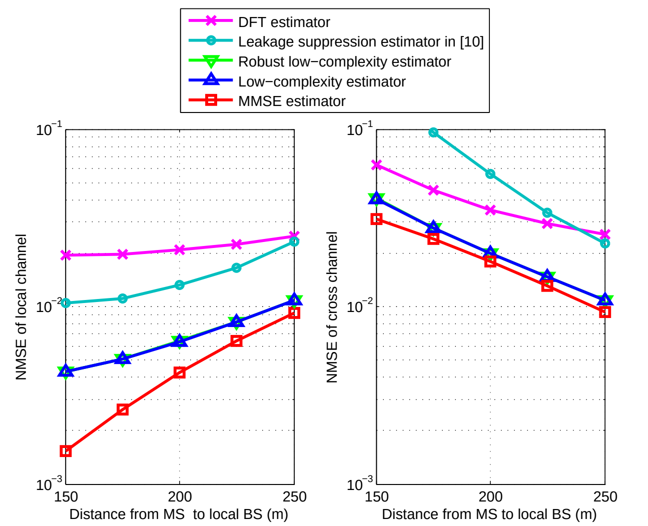

Fig. 3. NMSE of local and cross channels of different channel estimators versus the distance between MS and its local BS.

In Fig. 3, the NMSE of the local channels and the cross channels estimated by different channel estimators versus the distance between the MSs and their local BS are compared. The five considered estimators are: the MMSE estimator, the DFT estimator [9], the leakage suppression estimator in [10], the proposed estimator with perfect channel correlation matrix and the proposed robust estimator with only large scale fading gains. For the proposed methods, the series expansion order is set as five, and the scaling factor ρ is set as $1/512$, which is the sub-optimal value obtained from (20). We can observe that the performance of the robust channel estimator with only large scale fading gains overlaps with that of the proposed channel estimator with perfect channel correlation matrix, and both perform close to the MMSE estimator.

In Fig. 4, the NMSE of the CFR of different estimators are compared. The performance of the exact cell-edge users are provided. The scaling factor and the order of series expansion are set the same as in Fig. 3. We can observe that the

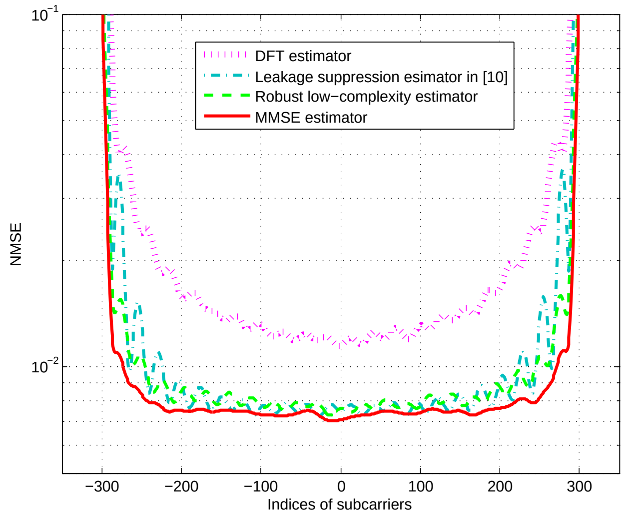

Fig. 4. NMSE of CFR with different channel estimators versus the indices of subcarriers.

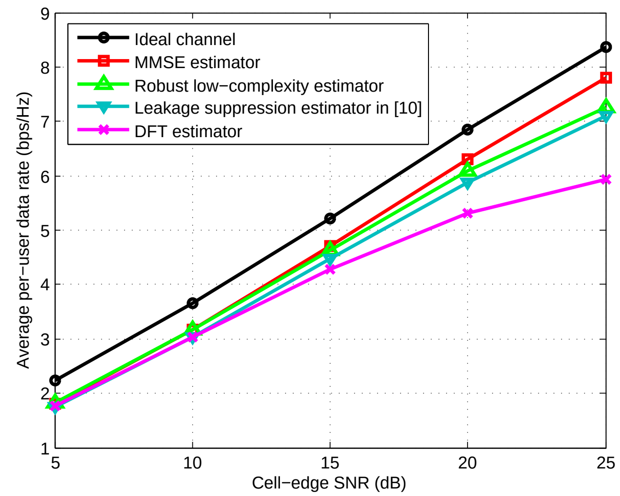

Fig. 5. Average per-user data rate achieved by different channel estimators under different cell-edge SNRs.

performance of the proposed estimator is close to the MMSE estimator on the whole used band. By contrast, the estimated channels at the subcarriers adjacent to the band-edge have large NMSE.

To show the impact of different channel estimators on the performance of CoMP transmission, we compare the downlink average per-user data rate when the channels for precoding are estimated by different channel estimators. In the simulation, we consider three BSs cooperatively serve eight users. The users are randomly distributed in a "cell-edge region", where $\min_{l \neq b_k} \frac{\alpha^2_{b_k,k}}{\alpha^2_{l,k}}$ for any MS_k is less than 15 dB, and $\alpha^2_{b_k,k}$ is the large scale fading gain of the local channel of MS_k. Zero-forcing beamforming with equal power allocation among MSs are applied. The scaling factor and the order of the series expansion applied in the proposed channel estimators are set the same as in Fig. 3. The results are obtained by averaging over 50 random locations of the users. In Fig. 5, the average per-user data rate achieved by different channel estimators versus the cell-edge SNRs are shown. The performance achieved by perfect CSI is also provided for comparison. We can observe that the data rate achieved by the proposed channel estimator is very close to that by the MMSE estimator and is higher than that by the DFT estimator, especially for high cell-edge SNR where CoMP is more beneficial.

V. Conclusions

In this paper, a low complexity channel estimator was proposed for TDD CoMP-JP multi-antenna multi-carrier systems, which can be implemented by FFT. The optimal scaling factor in the channel estimator was optimized and a closed-form suboptimal solution was provided. By using the average channel gains, less statistical information is required compared with the MMSE channel estimator. Analysis and simulation results showed that the proposed channel estimator yields minor per-user rate loss of the downlink CoMP-JP system from that using the linear MMSE channel estimator, but with much lower complexity and much less *a priori* information.

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