Cross-tier Interference Mitigation in Wideband HetNets with Full Duplex

Shengqian Han†, Chenyang Yang*, Andreas F. Molisch‡, and Gang Wang‡

*School of Electronics and Information Engineering, Beihang University, Beijing 100191, China
†Department of Electrical Engineering, University of Southern California, Los Angeles, California 90089
‡NEC Laboratories, China

Email: sqhan@buaa.edu.cn, cyyang@buaa.edu.cn, molisch@usc.edu, wang_gang@nec.cn

Abstract—Full duplex assisted inter-cell interference coordination (fICIC) has been shown effective to mitigate the cross-tier inter-cell interference (ICI) generated by the macro-cell base station (MBS) to the small-cell user equipment (SUE) in downlink narrowband heterogeneous networks. The fICIC applies full duplex technique at the small-cell BS (SBS) such that the SBS can forwarded the overheard ICI to neutralize the ICI directly received at the SUE at the same time as sending the desired signal. In this paper, we develop the framework for the design of wideband fICIC. Different from the narrowband system where only a single-tap forwarding filter is required, in the wideband system the multi-tap time-domain finite impulse response (FIR) forwarding filter needs to be jointly designed with multi-subcarrier frequency-domain power allocation for desired signals. A suboptimal wideband fICIC scheme is proposed in closed form. Simulation results demonstrated the gain of the proposed scheme.

Index Terms—Full duplex, ICIC, HetNets, OFDM.

I. INTRODUCTION

Effective cross-tier inter-cell interference coordination (ICIC) mechanisms are critical for realizing the promised benefits of heterogeneous networks (HetNets) [1]. Various ICIC methods for HetNets have been proposed in the literature, such as enhanced ICIC techniques in the time-frequency domain for Long-term Evolution (LTE) systems [2] and coordinated beamforming in the spatial domain [3]. Different from enhanced ICIC and coordinated beamforming techniques that control the transmission of the macro-cell base station (MBS) in time, frequency, or spatial domain in order to generate an ICI-free environment for the transmission in small cell, a full duplex (FD) assisted ICIC (fICIC) scheme was proposed in [4, 5] that does not rely on the participation of the MBS. With the fICIC, the small-cell BS (SBS) transmits not only the information-bearing signal it wants to send to the small-cell user equipment (SUE), but also sends a signal to cancel the cross-tier ICI. In order to obtain this signal, it needs to listen to the signal transmitted by the MBS at the same time as it is transmitting; this is achieved by applying FD techniques. In order to coherently combine the forwarded ICI with the ICI directly received at the SUE, the SBS needs to implement sample-by-sample ICI forwarding in the time domain to reduce the processing delay, as considered in FD relay systems to achieve co-phasing combining gain at the destination node [6]. In other words, the SBS cannot first decode and then forward the ICI in frequency domain because this will lead to symbol-level processing delay.

In a narrowband system the fICIC only needs a single-tap forwarding filter as considered in [4,5], where the processing delay at the SBS can be easily compensated with a phase shift so that the forwarded ICI can effectively neutralize the directly received ICI at the SUE. In contrast, in orthogonal frequency division multiplexing (OFDM) system the multi-tap time-domain finite impulse response (FIR) forwarding filter needs to be designed, where the order of the FIR filter should be selected to ensure that the total delay of the forwarded ICI does not exceed the cyclic prefix of the OFDM system in order to maintain orthogonality between subcarriers. Given the constraint on the order of the FIR forwarding filter, we cannot obtain it by simply first applying the existing narrowband design in frequency domain and then convert the frequency responses into time domain. Instead, we need to jointly design the multi-tap time-domain FIR forwarding filter and multi-subcarrier frequency-domain power allocation for the desired signals.

In this paper we develop a framework for optimizing the wideband fICIC in a single-user OFDM system, aimed at maximizing the sum rate over all subcarriers subject to the constraints on both maximal transmit power and the order of the FIR forwarding filter. The resulting optimization problem is shown to be non-convex, and we propose a suboptimal solution in closed form. Simulation results demonstrate the effectiveness of the proposed wideband fICIC scheme even with a small order of the FIR forwarding filter.

II. SYSTEM MODEL

Consider downlink transmission of an OFDM-based HetNet consisting of one MBS and B SBSs, where the MBS serves a macro-cell UE (MUE) and each SBS serves a single SUE. Due to the large difference in transmit power between the MBS and SBSs, in this paper we focus on the suppression of the cross-tier ICI generated by the MBS to SUEs, which is commonly recognized as a bottleneck to improve the spectral efficiency of HetNets [2]. The interference from other low-power interfering SBSs is treated as noise. With fICIC, the ICI is mitigated by every SBS individually, which is transparent to the MBS and other SBSs in the sense that no changes are needed for the
transmission of the MBS and other SBSs. Therefore, in the sequel we only consider a reference SBS and focus on the performance of the SUE served by the reference SBS. The system model is demonstrated in Fig. 1.

Suppose that the MBS has one transmit antenna, which is applicable to the multi-antenna MBS with single-stream beamforming, the FD SBS has one single transmit antenna and a single receive antenna, and each UE has a single receive antenna. Let $h_M(t)$ and $h_P(t)$ denote the time-domain channel impulse responses from the MBS and the SBS to the SUE, $h_{MP}(t)$ denote the time-domain channel impulse response from the MBS to the SBS, and $h_{PP}(t)$ denote the time-domain impulse response of the self-interference channel of the FD SBS. The corresponding frequency responses on the $n$-th subcarrier of the four chains are denoted by $g_{Mn}$, $g_{Pn}$, $g_{MPn}$, and $g_{PPn}$, respectively.

A. Signals at the FD SBS

By applying the FD technique, the SBS can transmit and receive signals simultaneously. Considering the hardware impairments of transmitter chains on self-interference cancellation, we can express the received signal at the SBS before self-interference cancellation based on [7] as

$$y_p(t) = h_{MP}(t) \circledast x_M(t) + h_{PP}(t) \circledast (x_p(t) + z_x(t)) + n_p(t),$$

where $x_M(t) = \sum_{n \in N} w_{Mn}s_{Mn}e^{j2\pi nt/T}$ is the OFDM signal sent from the MBS to the MUE with $T$ and $N$ denoting the symbol duration and the index set of data subcarriers, $w_{Mn}^2$ and $s_{Mn} \sim C\mathcal{N}(0,1)$ are the power allocation and desired data on the $n$-th subcarrier for the MUE, the term $h_{PP}(t) \circledast (x_p(t) + z_x(t))$ is the self-interference, $x_p(t)$ is the transmitted signal of the FD SBS, $z_x(t)$ is the transmitter noise due to hardware impairments such as noise and non-linearities in power amplifier with power spectral density $\mu_x \mathbb{E}\{x_{pn}\}$ on the $n$-th subcarrier, $x_{pn}$ denotes the frequency response of $x_p(t)$ on the $n$-th subcarrier, $\mu_x \ll 1$ is a scaling constant, and $n_p(t)$ is the additive white Gaussian noise (AWGN) with power spectral density $\sigma^2$, which takes into account both thermal noises and the ICI from other SBSs, and the operator $\circledast$ denotes convolution product.

Further considering the hardware impairments of receiver chains [7], the distorted received signal can expressed as

$$y_p'(t) = y_p(t) + z_y(t),$$

where $z_y(t)$ is the additive Gaussian distortion of received signal caused by, e.g., non-linearities in analog-to-digital converter, with power spectral density $\mu_y \mathbb{E}\{y_{pn}\}$ with $\mu_y \ll 1$ denoting a scaling constant and $y_{pn}$ denoting the frequency response of $y_p(t)$ on the $n$-th subcarrier.

Since the transmitted signals $x_p(t)$ is known at the FD SBS, the self-interference $h_{PP}(t) \circ x_p(t)$ in (1) can be cancelled if the self-interference channel $h_{PP}(t)$ is estimated. The channel response in frequency domain can be estimated as in a narrowband system, which can be expressed as $\hat{g}_{PPn} = g_{PPn} + c_{PPn}$ [5], where $\hat{g}_{PPn}$ and $c_{PPn}$ denote the channel estimate and estimation error, respectively, $c_{PPn}$ follows the distribution $CN(0, \sigma_{cn}^2)$ with $\sigma_{cn}^2 = (\mu_x + \mu_y + \mu_x \mu_y)|g_{PPn}|^2 + (1 + \mu_y)\sigma_{PP}^2$ [5], and $P_{tr}$ is the power of training signal.

By taking the inverse discrete fourier transform (IDFT) over $\{\hat{g}_{PPn}\}$, we can obtain the time-domain self-interference channel estimation, denoted by $h_{PP}(t) = h_{PP}(t) + c_{PP}(t)$, where $c_{PP}(t)$ denotes the time-domain channel estimation error. Then, the self-interference can be cancelled as

$$y_p(t) = y_p(t) - \hat{h}_{PP}(t) \circ x_p(t) = h_{MP}(t) \circ x_M(t) - c_{PP}(t) \circ x_p(t) + h_{PP}(t) \circ z_x(t) + n_p(t) + z_y(t).$$

Suppose that the SBS employs a $L$-tap FIR forwarding filter $u_{1n}(t) = \sum_{l=0}^{L-1} u_{1l} \delta(t - lT_s)$ to forward the overhead ICI, where the order $L$ is selected to ensure the delay of the forwarded ICI shorter than cyclic prefix, and $T_s$ is the sampling interval. The transmitted signal of the SBS can be expressed as

$$x_p(t) = u_{1n}(t) \circledast y_p(t - \tau) + x_d(t),$$

where $\tau$ is the processing delay at the SBS, $x_d(t) = \sum_{n \in N} w_{Dn}s_{Dn}e^{j2\pi nt/T}$ is the desired OFDM signal of the SUE, $w_{Dn}^2$ is the power allocated to the desired signal $s_{Dn}$ on the $n$-th subcarrier, and $s_{Dn} \sim C\mathcal{N}(0,1)$.

The transmit power of $x_p(t)$ on the $n$-th subcarrier can be obtained as $P_n = \mathbb{E}\{|x_{pn}|^2\}$, where the expectation is taken over the transmitted data $s_{Mn}$ and $s_{Dn}$, noises $n_p$, transmitter and receiver distortions $z_x$ and $z_y$, channel estimation errors $c_{PP}$, and self-interference channel $h_{PP}$. Further assuming that the frequency-domain self-interference channel $g_{PPn}$ follows a complex Gaussian distribution $C\mathcal{N}(0, \alpha P_{tr})$ with $\alpha P_{tr}$ denoting the average channel gain,

$$P_n = w_{Dn}^2 + (|\tilde{g}_{MPn}|^2 + \sigma^2 P_n + \sigma^2)|u_{1n}|^2,$$

where $\tilde{g}_{MPn} \triangleq g_{MPn}/w_{Mn}$ and $u_{1n}$ are the effective MBS-SBS channel and the frequency response of the forwarding filter $u_{1n}(t)$ on the $n$-th subcarrier, respectively, and $\sigma^2 = \sigma^2/P_{tr} + 2\alpha P_{tr} \mu_x \mu_y$ reflects the residual self-interference, in which $\sigma^2/P_{tr}$ comes from imperfect estimation of self-interference channel and $2\alpha P_{tr} \mu_x \mu_y$ comes from hardware impairments. The approximation in (5) follows from $\mu_x \ll 1$ and $\mu_y \ll 1$, which is accurate as discussed in [7]. Thus, in the following we simply consider that (5) holds with equality.

From (5), we can obtain the transmit power of the FD SBS on the $n$-th subcarrier as

$$P_n = (|\tilde{g}_{MPn}|^2 + \sigma^2)|u_{1n}|^2 + w_{Dn}^2 / (1 - \sigma^2|u_{1n}|^2).$$

\[1\] As discussed in [5], the received antenna of the SBS can be mounted far away from the transmit antenna to reduce the self-interference, which justifies this assumption.
For null subcarriers with indices in $\bar{N}$ on which data are not transmitted, we have $w_{\text{Dn}} = 0$, $w_{\text{Mn}} = 0$ and $\bar{g}_{\text{MPn}} = 0$. Then, (6) becomes

$$P_n = \sigma^2|w_{\text{fIn}}|^2/(1 - \sigma^2|w_{\text{fIn}}|^2), \text{ n} \in \bar{N}. \quad (7)$$

Then, the transmit power constraint of the SBS can be expressed as $\sum_{n=1}^{N} P_n \leq P_0$, where $N = |N| + |\bar{N}|$ is the total number of subcarriers, $P_0$ is the maximal transmit power of the SBS, and $|\cdot|$ denote cardinality of a set.

**B. Signal at the SUE**

The received signal of the SUE can be expressed as

$$y_u(t) = h_V(t) \odot x_p(t) + h_M(t) \odot x_M(t) + n_u(t), \quad (8)$$

where for the half duplex (HD) SUE, the impact of hardware impairments is negligible and hence ignored as commonly considered in the literature, and $n_u(t)$ is the AWGN at the SUE with power spectral density $\sigma^2$, which includes the received ICI from interfering SBSs and thermal noise.

With (3), (4) and (8), we can obtain the received signal on the $n$-th subcarrier as

$$y_{\text{un}} = g_{\text{un}} w_{\text{Dn}} s_{\text{dn}} + (g_{\text{Mn}} + g_{\text{pn}} w_{\text{Mn}} g_{\text{MPn}} e^{-jd_n}) s_{\text{mn}} + e^{-jd_n} g_{\text{pn}} w_{\text{fIn}} (\bar{y}_x + \bar{y}_s + n_p + n_n), \quad (9)$$

where $e^{-jd_n}$ is the phase shift caused by processing delay at the SBS with $d = 2\pi \Delta f \tau$ and $\Delta f$ denoting subcarrier spacing. $\bar{g}_{\text{Mn}} = g_{\text{Mn}} w_{\text{Mn}}$ is the effective channel from the MBS to the SUE on the $n$-th subcarrier, and $\bar{y}_x, n_p, \bar{y}_s, n_n$ denote the frequency responses of $\bar{y}_x(t), n_p(t), \bar{y}_s(t)$, and $n_n(t)$ on the $n$-th subcarrier, respectively.

Similar to the derivations for (5), we can obtain from (9) the signal-to-interference plus noise ratio (SINR) of the SUE on the $n$-th subcarrier as

$$\text{SINR}_n = \frac{|g_{\text{pn}}|^2 w_{\text{Dn}}^2}{|g_{\text{Mn}} + g_{\text{pn}} w_{\text{Mn}} g_{\text{MPn}} e^{-jd_n} w_{\text{fIn}}|^2 + |g_{\text{pn}} w_{\text{fIn}}|^2 (P_n \sigma_1^2 + \sigma^2) + \sigma^2}. \quad (10)$$

**III. WIDEBAND FIICIC OPTIMIZATION**

The optimization problem for the wideband FIICIC scheme, aimed at maximizing the sum rate of the SUE over all subcarriers, can be formulated as

$$\begin{align*}
\max_{\{w_{\text{fIn}}\}, \{P_n\}} \sum_{n \in N} & \log(1 + \text{SINR}_n) \\
\text{s.t.} \quad & w_{\text{fIn}} = t^T_{\text{fIn}} w_{\text{fIn}}, \quad \forall n \\
& P_n = \frac{(|g_{\text{Mn}}|^2 + \sigma^2)|w_{\text{fIn}}|^2 + w_{\text{Dn}}^2}{1 - \sigma^2 |w_{\text{fIn}}|^2}, \quad \forall n \\
& \sum_{n=1}^{N} P_n \leq P_0 \\
& |w_{\text{fIn}}|^2 \leq 1/\sigma^2, \quad \forall n, \quad (11 \text{a}-11 \text{e})
\end{align*}$$

where the constraint on the order of the FIR forwarding filter is included in (11b), $w_{\text{fIn}} = [w_{\text{fIn}}, \ldots, w_{\text{fIn}}] \in \mathbb{C}^{1 \times L}$ consisting of the $L$ coefficients of the FIR forwarding filter, $t^T_{\text{fIn}} \in \mathbb{C}^{1 \times L}$ is the vector containing the first $L$ elements of the $n$-th row of the $N \times N$ DFT matrix, and constraint (11e) limits the power of FIR forwarding filter in case that $P_n$ becomes negative in (11c) due to self-oscillations at the FD PBS [8].

Problem (11) is non-convex, whose global optimal solution is difficult to find. We can obtain a local optimal solution to the problem by using, e.g., a gradient-based method, which however has high complexity and is not applicable for large $L$ and $N$. In the following, we strive to find a suboptimal solution to problem (11) in closed form. The basic idea is to relax (11) by omitting constraint (11b) on the order of the FIR filter, then the frequency responses $\{w_{\text{fIn}}\}$ are optimized, and finally we approximate $w_{\text{fIn}}$ with the obtained $\{w_{\text{fIn}}\}$, given which the optimal $\{w_{\text{Dn}}\}$ is computed.

**A. Optimization of $w_{\text{fIn}}$**

By omitting constraint (11b), given $\{P_n\}$, the optimization of $w_{\text{fIn}}$ and $w_{\text{Dn}}$ can be solved for each subcarrier separately, which thus falls into the narrowband case considered in [5]. For $n \in N$, the optimal $w_{\text{fIn}}$ and $w_{\text{Dn}}$ are given by [5]

$$\begin{align*}
\hat{w}_{\text{fIn}} & = -\bar{g}_{\text{Mn}} \bar{g}_{\text{pn}} e^{-jd_n} \beta_n, \quad (12 \text{a}) \\
\bar{w}_{\text{Dn}} & = P_n (P_n \sigma_1^2 + |\bar{g}_{\text{Mn}}|^2 + \sigma^2)|w_{\text{fIn}}|^2, \quad (12 \text{b})
\end{align*}$$

where $\beta_n = \frac{\sqrt{2}}{C_n} (A_n + D_n - \sqrt{(A_n + D_n)^2 - \frac{A_n C_n}{B_n^2}})$, $A_n = P_n |\bar{g}_{\text{Mn}}|^2$, $B_n = |g_{\text{Mn}}|^2 (P_n \sigma_1^2 + |\bar{g}_{\text{Mn}}|^2 + \sigma^2)$, $C_n = 2|\bar{g}_{\text{Mn}}||\bar{g}_{\text{pn}}|$, and $D_n = |\bar{g}_{\text{Mn}}|^2 + \sigma^2$. Upon substituting (12a) and (12b) into (10), the optimal SINR can be obtained as

$$\text{SINR}_n = \frac{2B_n \beta_n}{1 - 2B_n \beta_n}. \quad (13 \text{a})$$

where both $B_n$ and $\beta_n$ are functions of $P_n$.

Unfortunately, problem (13) is again non-convex because the objective function is non-convex. We can use, e.g., a gradient-based method, to find a local optimal solution, but the complexity is affordable only for small $N$. Alternatively, we can also consider some suboptimal power allocation strategies, e.g., equal power allocation or water-filling power allocation under HD mode (i.e., the SBS does not forward ICI).

Given the frequency responses $\{w_{\text{fIn}}\}$ in (12a), if without the length constraint of the FIR filter (11b) and the transmit power constraints (11c)~(11e), then one can easily find the optimal $w_{\text{fIn}}$ by taking the IDFT over $\{w_{\text{fIn}}\}$. When these constraints are taken into account, however, we cannot find the $w_{\text{fIn}}$ with exactly the same frequency responses as $\{w_{\text{fIn}}\}$. Instead, in the following we find $w_{\text{fIn}}$ by approximating its frequency responses to $\{w_{\text{fIn}}\}$. In order to obtain a closed-form solution, we first omit the power constraints (11c)~(11e).
and find a $w_{ft}$ subject to the length constraint of the FIR filter (11b), and then adjust the obtained $w_{ft}$ to ensure the power constraints are satisfied.

Considering that the performance of the SUE is directly determined by its received signal, we therefore propose to optimize $w_{ft}$ by minimizing the sum mean square errors (MSE) of the two received signals when the SBS forwards the ICI with $\{w_{Hn}\}$ and $w_{ft}$, denoted by $\bar{y}_{un}$ and $\hat{y}_{un}$, respectively.

The MSE between $\bar{y}_{un}$ and $\hat{y}_{un}$ can be expressed based on (9) as

$$\epsilon_n = \mathbb{E}\{|\bar{y}_{un} - \hat{y}_{un}|^2\} = \mathbb{E}\{|g_{pn}\bar{y}_{Mn} + \epsilon_n \cdot e^{-jdn}(w_{Hn} - f_n^Hw_{ft})s_{Mn}|^2 + |g_{pn}(w_{Hn} - f_n^Hw_{ft})(-e_{ppn}x_{pn} + g_{ppn}\bar{y}_{zn} + n_{pn} + z_{yn})|^2\}.$$  

Similar to the expression in the denominator of the SNIR in (10), we can obtain the MSE for $n \in N'$ as

$$\epsilon_n = |g_{pn}|^2(|\bar{y}_{Mn}|^2 + P_n\sigma_t^2 + \sigma^2)|w_{Hn} - f_n^Hw_{ft}|^2.$$  

For $n \in N'$, considering that $\bar{y}_{Mn} = 0$, $w_{Hn} = 0$ and $P_n = 0$ on all subcarriers, we can obtain the MSE as

$$\epsilon_n = \sigma^2|g_{pn}|^2|f_n^Hw_{ft}|^2.$$  

The MSE $\epsilon_n$ for $n \in N'$ reflects the effectiveness of $w_{ft}$ in forwarding ICI on data subcarriers, and the MSE $\epsilon_n$ for $n \in N'$ reflects the waste of transmit power of the SBS that is used to forward noise on null subcarriers. To combine the two effects of $w_{ft}$, we minimize the weighted sum MSE, i.e.,

$$\min_{w_{ft}} \sum_{n \in N'} \epsilon_n + \kappa \sum_{n \in N} \epsilon_n,$$  

where $\kappa$ is the weight to balance the impact of $\epsilon_n$ and $\epsilon_n$.

Problem (17) can be solved in closed form as

$$w_{ft} = \left(\sum_{n \in N} |g_{pn}|^2(|\bar{y}_{Mn}|^2 + P_n\sigma_t^2 + \sigma^2)f_n^H + \sum_{n \in N} \kappa \sigma_n^2|g_{pn}|^2|f_n^Hw_{ft}|^2\right)^{-1}\left(\sum_{n \in N} |g_{pn}|^2(|\bar{y}_{Mn}|^2 + P_n\sigma_t^2 + \sigma^2)f_n^Hw_{HN}f_n\right).$$  

We can find from (18) that the power of $w_{ft}$ decreases with $\kappa$. Further noting that the power constraints (11c)~(11e) will be satisfied if $w_{ft}$ is small, therefore a large value of $\kappa$ should be selected for problem (17). On the other hand, we can see that $w_{ft}$ becomes zero as $\kappa$ approaches infinity, in which case the IJCIC reduces to the HD mode without forwarding ICI.

B. Optimization of $w_{Dn}$

Given $w_{ft}$, we can update $\{w_{Dn}\}$ to maximize the sum rate by solving (11), which can be rewritten as

$$\max_{\{w_{Dn}\}} \sum_{n \in N} \log \left(1 + \frac{w_{Dn}}{\epsilon_n + w_{Dn} + D_n}\right)$$  

subject to $\sum_{n = 1}^N \hat{A}_n w_{Dn}^2 + \hat{B}_n \leq P_0,$  

where $\hat{A}_n = 1 - \frac{1}{\sigma_n^2|f_n^Hw_{ft}|^2}A_n$, $\hat{B}_n = \frac{|f_n^Hw_{ft}|^2}{\sigma_n^2}$, $\hat{C}_n = |f_n^Hw_{ft}|^2\sigma_n^2\hat{A}_n$, and $\hat{D}_n = -\frac{1}{\sigma_n^2|f_n^Hw_{ft}|^2}\left(\frac{|\bar{y}_{Mn}|^2}{\sigma_n^2} + g_{pn}\bar{y}_{Mn}e^{-jdn}|f_n^Hw_{ft}|^2 + |g_{pn}|^2|f_n^Hw_{ft}|^2(\sigma_t^2\hat{B}_n + \sigma^2)\right).$

Based on the Karush-Kuhn-Tucker (KKT) conditions, we can obtain the optimal $w_{Dn}$ for $n \in N'$ as

$$w_{Dn} = \frac{1}{2C_n(C_n + 1)}\left(-2C_n\hat{D}_n + \hat{D}_n\right) + \sqrt{(2C_n\hat{D}_n - \hat{D}_n)^2 - 4C_n(C_n + 1)\hat{D}_n(D_n - \frac{1}{\lambda\hat{A}_n})},$$  

where $\lambda$ and $\{\nu_n\}$ are non-negative lagrangian variables, which satisfy the complementary conditions $\lambda\left(\sum_{n = 1}^N \hat{A}_n w_{Dn}^2 + \hat{B}_n - P_0\right) = 0$ and $\nu_n w_{Dn}^2 = 0$. For $n \in N'$, we have $w_{Dn} = 0$ on null subcarriers.

We can observe from (20) that when $\hat{D}_n - \frac{1}{\lambda\hat{A}_n} > 0$, then if $\nu_n = 0$, we have $w_{Dn}^2 < 0$, which is infeasible. Thus, in this case $\nu_n > 0$ must hold, which leads to $w_{Dn}^2 = 0$ from the complementary condition $\nu_n w_{Dn}^2 = 0$. When $\hat{D}_n - \frac{1}{\lambda\hat{A}_n} = 0$, then $\nu_n = 0$ and $w_{Dn}^2$ can be solved as

$$w_{Dn}^2 = \frac{-(2C_n + 1)\hat{D}_n + \sqrt{\hat{D}_n^2 + 4C_n(C_n + 1)\hat{D}_n(\hat{D}_n - \frac{1}{\lambda\hat{A}_n})}}{2C_n(C_n + 1)}.$$  

It is clear that $w_{Dn}^2$ decreases with $\lambda$. Then, the optimal $\lambda$ can be easily found by a bisection method to make the power constraint (19b) hold with equality.

C. Algorithm Summary

1) **Initial power allocation** $\{P_n\}$: choose suboptimal closed-form power allocation, e.g., equal power allocation or water-filling power allocation in HD mode. Alternatively, one can find a local optimal power allocation by solving problem (13) for small $N$.

2) **Relaxed** $\{w_{Hn}, w_{Dn}\}$: given $\{P_n\}$, compute $\{w_{Hn}\}$ and $\{w_{Dn}\}$ with (12a) and (12b).

3) **FIR forwarding filter** $w_{ft}$: given $\{w_{Hn}\}$, compute $w_{ft}$ with (18).

4) **Update power allocation** $\{w_{Dn}\}$ for desired signal: given $w_{ft}$. If $\hat{D}_n - \frac{1}{\lambda\hat{A}_n} > 0$, $w_{Dn} = 0$; otherwise, compute $w_{Dn}$ with (21), where $\lambda$ is found by bisection.

IV. SIMULATION RESULTS

For the simulations in this section, the HetNet layout in Fig. 1 is considered, where the MBS is located at the center of a macro cell with radius $R_M = 500$ m, the SBS is located at $(80, 0)$ m, the MUE is located at $(120, 0)$ m, and the SUE is located at $(80, 40)$ m, respectively. The MBS transmits with four antennas and the power of $P_M = 46$dBm under maximal ratio transmission beamforming, each SBS has one receive antenna and transmits with one antenna and a maximal power of $P_0 = 30$dBm, and each cell has one receive antenna. The path loss follows the 3GPP channel model, namely 128.1 + 37.6 log_{10} d for the macro cell and 140.7 + 36.7 log_{10} d for the small cell, respectively, where $d$ is the distance in km [9]. A penetration loss of 20 dB is considered for the channels to the SUE. We model the interference from surrounding SBSs as noises. Define the average receive SNR of a MUE located at the macro cell edge as $\text{SNR}_{\text{edge}}$, then the noise variance $\sigma^2$ can be obtained as $\sigma^2 = P_M -(128.1+37.6 \log_{10} R_M) \cdot \text{SNR}_{\text{edge}}$.
in dBm. To evaluate the impact of imperfect self-interference cancellation for FD, we define the signal to self-interference ratio as $\text{SIR}_{\text{self}} = 1/\sigma_i^2$, which is set as $-110$ dB [5].

We simulate a LTE system with 10 MHz bandwidth ($N = 1024$), where $|\mathcal{N}|=600$ subcarriers are used for data transmission and the sampling interval is $T_s = 65$ ns [10]. The small-scale channels are generated based on WINNER II clustered delay line model [11]. Specifically, the channel from the MBS to the SBS, $h_{\text{MBS}}$, uses the typical urban macro-cell line of sight (LoS) model, the channel from the MBS to the SUE, $h_{\text{MBS}}$, uses the typical urban macro-cell Non-LoS model, and the channel from the SBS to the SUE, $h_{\text{SBS}}$, uses the typical urban micro-cell Non-LoS model. After sampling the multipath channels with the considered bandwidth, we obtain the maximal delay of $h_{\text{MBS}}$ and $h_{\text{SBS}}$ as three and nine samples, respectively. We consider the processing delay of the FD SBS as eight samples, i.e., $\tau = 0.52$ $\mu$s. Since the cyclic prefix of the LTE system is $4.7$ $\mu$s [10], i.e., 72 samples, we can obtain the maximal order of the FIR forwarding filter $L_{\text{f}}$ as 52, i.e., $L \leq 52$.

As pointed out in [5], the fICIC scheme requires the channels $h_{\text{MBS}}$, $h_{\text{SBS}}$ and $h_{\text{MBS}}$ at the SBS, where $h_{\text{MBS}}$ can be directly estimated at the SBS, $h_{\text{SBS}}$ can also be estimated at the SBS based on channel reciprocity, and $h_{\text{MBS}}$ can be first estimated at the SUE and then fed back to the SBS. In the simulations, we employ linear minimum mean-squared error estimators to estimate $h_{\text{MBS}}$, $h_{\text{SBS}}$, and $h_{\text{MBS}}$, and use analog feedback [12] to send back the estimate of $h_{\text{MBS}}$ to the SBS, where the transmit power of SUE is set as 23 dBm.

In Fig. 2 we show the equivalent ICI plus noise power (i.e., the denominator of the SINR in (10)) achieved by the relaxed frequency-domain processing in (12) and the proposed suboptimal time-domain processing, which is normalized by the maximal signal power of all subcarriers. The results are obtained under one channel realization with cell-edge SNR of 20 dB, $L = 16$, and $\kappa = 50$. We can see that the proposed method well approximates the relaxed frequency-domain processing on a large part of data subcarriers but at the penalty of wasting transmit power for forwarding noises on the null subcarriers.

In Fig. 3 the average data rate per subcarrier achieved by the proposed wideband fICIC and the HD mode is shown, where water-filling power allocation is used in the HD mode, which is also used as the initial power allocation of the proposed algorithm. We can see that at low SNR and small $L$ the wideband fICIC performs inferior to the HD mode because of imperfect channel information and bad approximation of the FIR forwarding filter. Yet, at medium-high SNR a large performance gain can be obtained by the proposed wideband fICIC scheme even for small $L$, e.g., $L = 8$.

V. CONCLUSIONS

In this paper we studied the cross-tier ICI mitigation in wideband HetNets assisted by FD. We developed a framework for the design of a wideband fICIC scheme and proposed an efficient suboptimal wideband fICIC algorithm. Simulation results showed evident performance gain of the proposed wideband fICIC scheme over the HD mode.

REFERENCES