Performance Gain of Full Duplex over Half Duplex under Bidirectional Traffic Asymmetry

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***Abstract*—Recent work has demonstrated the advantage of full-duplex (FD) network over half-duplex (HD) network in bidirectional sum rate under the assumption of full-buffer and symmetric uplink-downlink traffics. In this paper, we consider asymmetric bidirectional traffics, and study the performance gain of FD network over both the traditional time-division duplex (TDD) network with static time slot splitting for uplink and downlink and the dynamic TDD with adaptive time slot splitting according to bidirectional traffics requirements. We use the number of users supported by the networks as performance metric, which is defined as the minimum of the number of users supported in uplink and downlink given random data rate requirements of users. To maximize the number of supported users, bidirectional time slot splitting is optimized for dynamic TDD network, and bidirectional power control at both BSs and users is optimized in FD network. Numerical results show the evident gain of FD network over traditional TDD for different levels of traffic asymmetry, and the gain over dynamic TDD decreases with the increase of traffic asymmetry.**

I. INTRODUCTION

Network densification with small cells in the fifth-generation cellular systems (5G) significantly increases the variation in traffic loads between different cells [1]. Moreover, time-varying traffic asymmetry for uplink and downlink within each cell is expected due to the proliferation of diverse applications for smart wireless devices, users' mobility behaviour and application usage behaviour, etc. Therefore, technologies to deal with the spatio-temporally evolving bidirectional traffic asymmetry are needed in 5G [2].

Frequency division duplex (FDD) systems are generally recognized as difficult to handle bidirectional traffic asymmetry because of the equally paired frequency usage in downlink and uplink. Asymmetrical FDD carrier aggregation by using more frequency bands in one direction can solve the problem to a certain extent, which however requires more transceivers at users, leading to increased cost and power consumption [2]. By contrast, time division duplex (TDD) systems have the capability to handle asymmetric bidirectional traffics. In traditional macro cell deployment scenarios, an asymmetric time slot configuration can be employed for uplink and downlink, which however needs to be used in all cells across the entire network in order to avoid the detrimental opposite-directional interference, e.g., the interference generated by a downlink transmitting base station (BS) to an uplink receiving BS. The synchronous time slot configuration is highly inefficient in small cell networks because of the spatio-temporal traffic asymmetry, which motivates the investigation of cell-specific dynamic TDD technology [3, 4]. The mitigation of opposite-directional interference is a key challenge for dynamic TDD, and various interference control methods have been proposed in the literature, such as power control [5], access management [6], coordinated beamforming [7], and cell clustering [1, 8].

Full duplex (FD) communication can be regarded as an enhanced dynamic TDD in the sense that both uplink and downlink in each cell operate simultaneously so that time slot splitting is no longer necessary, which therefore can naturally support the requirements of asymmetric bidirectional traffics. FD communication was long believed impossible in wireless system design due to the severe self-interference within the same transceiver. However, the plausibility of FD technique was approved by recent tremendous progress in self-interference cancellation, and nearly doubled link performance over the half duplex (HD) system was demonstrated for short-range point-to-point communications [9, 10]. The performance of FD network has also been analyzed based on stochastic geometry in [11–14], where evident gain in bidirectional sum rate over the traditional HD network was shown when self-interference in FD nodes is well controlled. However, the existing analysis on the advantages of FD network is based on the assumption either explicitly or implicitly that both BSs and users have full buffer traffic and uplink-downlink traffics are symmetric, e.g., in [12].

In this paper, we investigate the performance gain of FD network over HD network including both traditional TDD and dynamic TDD, where bidirectional traffic asymmetry is considered. To maximize the number of users supported by the networks, which is defined as the minimum of the number of users supported in uplink and downlink, bidirectional time slot splitting is optimized for dynamic TDD network, and bidirectional power control at both BSs and users is optimized in FD network for time division multiple access (TDMA) and frequency division multiple access (FDMA), respectively. Numerical results show that under asymmetric bidirectional traffics, both FD network and dynamic TDD network exhibit large performance gain over the traditional static TDD network with equal time slot splitting, and the gain of FD network over

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dynamic TDD network decreases with the increase of traffic asymmetry.

II. System Model

Consider a small cell network consisting of multiple geographically separated cell clusters, which can be formed based on the existing cell clustering methods designed for dynamic TDD, e.g., [8]. Assume that the interference between cell clusters is negligible as in [1], so that cluster-specific uplink-downlink resource configuration can be employed based on the bidirectional traffic requirements in each cluster, and different configurations can be used between clusters. Within each cluster, following the stochastic geometry model in [11], the location of BSs follows a homogeneous Poisson point process (PPP) Ψ with density λ in the Euclidean plane, and the uplink users served in the same time-frequency resource are distributed according to an independent homogeneous PPP Ω with the same density λ. In the paper, both FD and HD BSs are respectively considered, but the users are in HD mode due to the difficulty of implementing FD at user side. We consider that each BS serves multiple users in TDMA or FDMA manner. Then, on the same time-frequency resource a HD BS can serve one uplink or downlink user, while a FD BS can serve an uplink user and a downlink user simultaneously, where every user is associated with its closest BS.

In FD network, the downlink transmission experiences the interference from both other-cell BSs and all co-scheduled uplink users. Based on the results in [11], the average downlink data rate can be expressed as

$$\bar{R}_{df}(P_b, P_u, W) = 2\pi W_0 \lambda \cdot \int_{r>0} r e^{-\lambda\pi r^2} \int_{t>0} e^{-\frac{r^{\alpha_{bu}}(e^t-1)N_0 W_0}{P_b}} F_{1f}(r,t) F_{2f}(r,t) dt dr, \tag{1}$$

where $F_{1f}(r,t) = e^{-\pi\lambda r^2 (e^t-1)^{\frac{2}{\alpha_{bu}}} \int_{(e^t-1)^{-2/\alpha_{bu}}}^{\infty} \frac{1}{1+x^{\alpha_{bu}/2}} dx}$, $F_{2f}(r,t) = e^{-\pi\lambda r^{\frac{2\alpha_{bu}}{\alpha_{uu}}} \left(\frac{P_u W_0 (e^t-1)}{P_b W}\right)^{2/\alpha_{uu}} \int_{y>0} \frac{1}{1+y^{\alpha_{uu}/2}} dy}$, α_{bu} and α_{uu} denote the pathloss exponents between BSs and users and between users, respectively, W_0 and W denote the whole bandwidth and the bandwidth allocated to each uplink user, respectively, P_b and P_u are the transmit powers of BSs and users, respectively, N_0 is the noise power spectrum density, and Rayleigh fading is considered for small-scale channels. Herein, flat transmit power spectrum is considered for both BSs and users, which is $\frac{P_b}{W_0}$ and $\frac{P_u}{W}$, respectively.

The uplink transmission in FD network suffers from the inter-cell interference generated by both other-cell uplink users and other-cell BSs, and also the self-interference from itself. The average uplink data rate can be expressed as [11]

$$\bar{R}_{uf}(P_b, P_u, W) = 2\pi W_0 \lambda \cdot \int_{r>0} r e^{-\lambda\pi r^2} \int_{t>0} e^{\frac{-r^{\alpha_{bu}}(e^t-1)(N_0 W_0 + \beta P_b) W}{P_u W_0}} F_{1f}(r,t) F'_{2f}(r,t) dt dr, \tag{2}$$

where $F'_{2f}(r,t) = e^{-\pi\lambda r^{\frac{2\alpha_{bu}}{\alpha_{bb}}} \left(\frac{P_b W (e^t-1)}{P_u W_0}\right)^{2/\alpha_{bb}} \int_{y>0} \frac{1}{(1+y^{\alpha_{bb}/2})} dy}$, α_{bb} denotes the pathloss exponent between BSs, and $\beta < 1$ reflects the level of self-interference cancellation.

From the average data rates in FD network, we can easily obtain the average data rates in HD network. Specifically, define T_d and T_u as the fractions of time slots allocated to downlink and uplink in TDD networks with $T_d + T_u = 1$. Then, the average downlink rate in HD network can be denoted by $T_d \bar{R}_{dh}(P_b)$, where $\bar{R}_{dh}(P_b)$ can be obtained from (1) by setting the transmit power of users as zeros, i.e., $P_u = 0$, which leads to $F_{2f}(r,t) = 1$ and $\bar{R}_{dh}(P_b)$ is independent of W. Similarly, the average uplink rate in HD network can be denoted by $T_u \bar{R}_{uh}(P_u)$, where $\bar{R}_{uh}(P_u)$ can be obtained from (2) by setting the transmit power of BSs as zeros, i.e., $P_b = 0$, which leads to $F'_{2f}(r,t) = 1$ and disappeared self-interference.

In the paper we focus on the interference-limited scenario, where both BSs and users have minimal transmit powers, denoted by $P_{b,min}$ and $P_{u,min}$, which ensure that the noise is negligible compared to the inter-cell interference. By setting $N_0 = 0$ in (1) and (2), we can find that the bidirectional average data rates in FD network only depend on the ratio of transmit power of BSs and users, defined as $\kappa = \frac{P_b}{P_u}$. Specifically, the bidirectional average data rates in FD and HD TDD networks in interference-limited scenario can be obtained as

$$\hat{R}_{uf}(\kappa, W) = 2\pi W_0 \lambda \cdot \int_{r>0} r e^{-\lambda\pi r^2} \int_{t>0} e^{\frac{-r^{\alpha_{bu}}(e^t-1)\beta\kappa W}{W_0}} F_{1f}(r,t) F'_{2f}(r,t,\kappa,W) dt dr, \tag{3a}$$

$$\hat{R}_{df}(\kappa, W) = 2\pi W_0 \lambda \cdot \int_{r>0} r e^{-\lambda\pi r^2} \int_{t>0} F_{1f}(r,t) F_{2f}(r,t,\kappa,W) dt dr, \tag{3b}$$

$$\hat{R}_{uh}(T_u) = T_u \hat{R}_{h0}, \ \hat{R}_{dh}(T_d) = T_d \hat{R}_{h0}, \tag{3c}$$

where $\hat{R}_{h0} = 2\pi W_0 \lambda \int_{r>0} r e^{-\lambda\pi r^2} \int_{t>0} F_{1f}(r,t) dt dr$.

We can observe from (3) that the bidirectional average data rates in FD network depend on both power ratio, κ, and bandwidth allocated to each uplink user, W, which means that the performance of FD network will be different under TDMA and FDMA. By contrast, in HD network the bidirectional average rates only depend on the time slot splitting, meaning that multiple accessing strategies have no impact on the performance.

III. Resource Configuration under Bidirectional Traffic Asymmetry

A. Performance Metric

In the paper we use the number of users supported by a network, K, as the performance metric, which is defined as the minimum of the number of users supported in uplink, K_u, and downlink, K_d, i.e., $K = \min(K_u, K_d)$. The values of K_u and K_d depend on the bidirectional traffic requirements. To model the asymmetric bidirectional traffics, we assume that the data rate requirements of all users in either direction are independent and identically distributed (i.i.d.), but no specific distributions are assumed. Let R_{ui} and R_{dj} denote the data rate requirements of an arbitrary uplink user i and downlink user j, which have the means, m_u and m_d, and variances, σ_u^2 and

σ_d^2, respectively. With the random rate requirements, we define K_u and K_d as the number of users that can be supported in uplink and downlink with the probability no less than a given value ϵ, respectively, where the value of ϵ is generally large, e.g., $\epsilon = 95\%$.

B. HD Network

In this subsection we optimize the resource configuration for HD dynamic TDD network. The optimization problem, aimed at maximizing the number of supported users under asymmetric bidirectional traffics, can be formulated as

$$\max_{T_u+T_d=1} \; K = \min(K_u, K_d) \tag{4a}$$
$$s.t. \; \mathbb{P}\{\textstyle\sum_{i=1}^{K_u} R_{ui} \leq T_u \hat{R}_{h0}\} \geq \epsilon \tag{4b}$$
$$\mathbb{P}\{\textstyle\sum_{j=1}^{K_d} R_{dj} \leq T_d \hat{R}_{h0}\} \geq \epsilon, \tag{4c}$$

where $\mathbb{P}\{\cdot\}$ denotes the probability.

To solve problem (4), we observe that $\mathbb{P}\{\cdot\}$ is a decreasing function for both K_u and K_d, therefore for any given T_u and T_d the optimal solution can be obtained when constraint (4b) and (4c) hold with equality.[1] Moreover, we can find from (4b) and (4c) that K_u is an increasing function of T_u, while K_d is a decreasing function of T_u. Therefore, in order to maximize the minimum of K_u and K_d, i.e., K in (4a), the optimal T_u should be selected to make $K_u = K_d = K$. As a result, problem (4) can be equivalently transformed as

$$\max_{T_u+T_d=1} \; K \tag{5a}$$
$$s.t. \; \mathbb{P}\{\textstyle\sum_{i=1}^{K} R_{ui} \leq T_u \hat{R}_{h0}\} = \epsilon \tag{5b}$$
$$\mathbb{P}\{\textstyle\sum_{j=1}^{K} R_{dj} \leq T_d \hat{R}_{h0}\} = \epsilon. \tag{5c}$$

To solve problem (5), we first find explicit expressions for constraint (5b) and (5c). Considering that the number of uplink and downlink users supported in current and future networks is generally large, we can employ the central limit theorem to simplify the constraints. Take constraint (5b) as an example, which can be transformed as

$$\mathbb{P}\{\textstyle\sum_{i=1}^{K} R_{ui} \leq T_u \hat{R}_{h0}\} = \Phi\Big(\frac{T_u \hat{R}_{h0} - K m_u}{\sigma_u \sqrt{K}}\Big) = \epsilon, \tag{6}$$

where the first equality follows from central limit theorem, and $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard Gaussian distribution with zero mean and unit variance. Letting $\Phi^{-1}(\cdot)$ denote the inverse function of $\Phi(\cdot)$, we can rewrite problem (4) as

$$\max_{T_u+T_d=1} \; K \tag{7a}$$
$$s.t. \; K m_u + \Phi^{-1}(\epsilon)\sigma_u\sqrt{K} = T_u \hat{R}_{h0} \tag{7b}$$
$$K m_d + \Phi^{-1}(\epsilon)\sigma_d\sqrt{K} = T_d \hat{R}_{h0}. \tag{7c}$$

From (7), the optimal K^* can be solved as

$$K^* = \frac{\Big(-\Phi^{-1}(\epsilon)\sigma_{ud} + \sqrt{\Phi^{-2}(\epsilon)\sigma_{ud}^2 + 4 m_{ud}\hat{R}_{h0}}\Big)^2}{4 m_{ud}^2}, \tag{8}$$

[1]Throughout the paper we consider that the numbers of users, K_u and K_d, are continuous variables, and the integer constraints on K_u and K_d can be easily included by using floor operation to the obtained continuous values.

where $\sigma_{ud} \triangleq \sigma_u + \sigma_d$ and $m_{ud} \triangleq m_u + m_d$.

From (7b) and (7c), we can obtain that the optimal time slot splitting, T_u^* and T_d^*, satisfy

$$\frac{T_u^*}{T_d^*} = \frac{m_u \cdot \big(\sqrt{K^*} + \Phi^{-1}(\epsilon)\frac{\sigma_u}{m_u}\big)}{m_d \cdot \big(\sqrt{K^*} + \Phi^{-1}(\epsilon)\frac{\sigma_d}{m_d}\big)}. \tag{9}$$

Under a special case with $\frac{\sigma_u}{m_u} = \frac{\sigma_d}{m_d}$, i.e., the standard deviation of data rate requirement is proportional to the average data rate requirement for both directions, we can obtain from (9) that

$$\frac{T_u^*}{T_d^*} = \frac{m_u}{m_d}. \tag{10}$$

It is shown that in this case the time slots configured to uplink and downlink are proportional to the bidirectional average data rate requirements. An example for this special case is when the data rate requirements of users follow exponential distribution with mean m_u and m_d for uplink and downlink, respectively, where the corresponding standard deviations are $\sigma_u = m_u$ and $\sigma_d = m_d$.

C. FD Network

In this subsection the resource configuration is optimized for FD network, which has two differences from the optimization of HD network. First, in FD network both uplink and downlink use all time-frequency resources, and the power ratio of BSs and users, i.e., κ, needs to be optimized instead of time slot splitting. Second, as mentioned before, the bidirectional performance in FD network depends on the bandwidth allocated to each uplink user, W, which is different in TDMA and FDMA networks. Therefore, the optimizations for FD TDMA and FDMA networks are considered, respectively, in the following.

1) FD Network with TDMA: In TDMA network each uplink user can use full bandwidth, i.e., $W = W_0$. Then, the uplink and downlink average data rates in FD network given in (3a) and (3b) only depend on the power ratio κ, which are denoted by $\hat{R}_{uf}(\kappa)$ and $\hat{R}_{df}(\kappa)$, respectively, for notational simplicity.

To obtain the optimal K in FD TDMA network, we can solve problem (4) by replacing $T_u\hat{R}_{h0}$ in (4b) and $T_d\hat{R}_{h0}$ in (4c) with $\hat{R}_{uf}(\kappa)$ and $\hat{R}_{df}(\kappa)$, respectively. It is easy to find that the optimal solution can be obtained when the constraints in the resulting problem hold with equality. Then, the optimization problem, aimed at maximizing the number of supported users, can be formulated as

$$\max_{\kappa} \; K = \min(K_u, K_d) \tag{11a}$$
$$s.t. \; \mathbb{P}\{\textstyle\sum_{i=1}^{K_u} R_{ui} \leq \hat{R}_{uf}(\kappa)\} = \epsilon \tag{11b}$$
$$\mathbb{P}\{\textstyle\sum_{j=1}^{K_d} R_{dj} \leq \hat{R}_{df}(\kappa)\} = \epsilon \tag{11c}$$
$$\kappa_{min} \leq \kappa \leq \kappa_{max}, \tag{11d}$$

where $\kappa_{min} = \frac{P_{b,min}}{P_{u,max}}$ and $\kappa_{max} = \frac{P_{b,max}}{P_{u,min}}$ denote the lower and upper bounds of power ratio κ, and $P_{b,max}$ and $P_{u,max}$ are the maximal transmit powers of BSs and users, respectively.

To solve problem (11), noting that $\hat{R}_{uf}(\kappa)$ is a decreasing function of κ and $\hat{R}_{df}(\kappa)$ is an increasing function of κ, we can obtain from (11b) and (11c) that K_u decreases with κ while K_d increases with κ. Based on this result, the maximal K can be found as follows.

First, omitting constraint (11d), we can find that the solutions to the relaxed problem, denoted by $\tilde{K}$, $\tilde{K}_u$, $\tilde{K}_d$ and $\tilde{\kappa}$, are obtained when $\tilde{K}_u = \tilde{K}_d = \tilde{K}$. By further applying the central limit theorem as in HD network, the following equations are satisfied

$$\tilde{K}m_u + \Phi^{-1}(\epsilon)\sigma_u\sqrt{\tilde{K}} = \hat{R}_{uf}(\tilde{\kappa}) \tag{12a}$$

$$\tilde{K}m_d + \Phi^{-1}(\epsilon)\sigma_d\sqrt{\tilde{K}} = \hat{R}_{df}(\tilde{\kappa}), \tag{12b}$$

from which the optimal $\tilde{\kappa}^*$ and $\tilde{K}^*$ can be easily obtained by a bisection method.

Then, recalling that K_d increases with κ and K_u decreases with κ, if $\tilde{\kappa}^* > \kappa_{max}$, we have $K_u > K_d$ for all $\kappa \in [\kappa_{min}, \kappa_{max}]$, so that $K^* = K_d$, which can be obtained from (12b) by setting $\kappa = \kappa_{max}$ and $\tilde{K} = K^*$. On the other hand, if $\tilde{\kappa}^* < \kappa_{min}$, we have $K_u < K_d$ for all $\kappa \in [\kappa_{min}, \kappa_{max}]$, so that $K^* = K_u$, which can be obtained from (12a) by setting $\kappa = \kappa_{min}$ and $\tilde{K} = K^*$. Otherwise, if $\tilde{\kappa}^* \in [\kappa_{min}, \kappa_{max}]$, we have $K^* = \tilde{K}^*$ and $\kappa^* = \tilde{\kappa}^*$.

2) FD Network with FDMA: When FDMA is considered, the bandwidth allocated to each uplink user is $W = \frac{W_0}{K_u}$, which is a function of K_u. We can observe from (3a) and (3b) that now the uplink and downlink average data rates of FD network are determined by $\frac{\kappa}{K_u}$, which are denoted by $\hat{R}_{uf}(\frac{\kappa}{K_u})$ and $\hat{R}_{df}(\frac{\kappa}{K_u})$ for simplicity in FDMA network.

Similar to (4), the number of supported users can be maximized as follows.

$$\max_{\kappa}\ K = \min(K_u, K_d) \tag{13a}$$

$$s.t.\ \mathbb{P}\{\textstyle\sum_{i=1}^{K_u} R_{ui} \le \hat{R}_{uf}(\frac{\kappa}{K_u})\} \ge \epsilon \tag{13b}$$

$$\mathbb{P}\{\textstyle\sum_{j=1}^{K_d} R_{dj} \le \hat{R}_{df}(\frac{\kappa}{K_u})\} \ge \epsilon \tag{13c}$$

$$\kappa_{min} \le \kappa \le \kappa_{max}. \tag{13d}$$

In the problems for HD dynamic TDD and FD TDMA networks, K_u and K_d are constrained separately, e.g., in (4b) and (4c), so that K is maximized when K_u and K_d reach their maximal values, respectively. In FD FDMA network, K_d is constrained only in (13c), and thus the optimal K can be obtained when K_d reaches its maximal value. For K_u, however, since it is constrained in both (13b) and (13c), it is not necessarily optimal to set K_u as its maximal value. Based on the analysis, we can rewrite constraint (13b) and (13c) as

$$K_u \le K_u^{max}(\kappa) \tag{14a}$$

$$K_d = K_d^{max}(\frac{\kappa}{K_u}), \tag{14b}$$

where $K_u^{max}(\kappa)$ and $K_d^{max}(\frac{\kappa}{K_u})$ denote the maximal values of K_u and K_d under constraint (13b) and (13c), respectively. $K_u^{max}(\kappa)$ and $K_d^{max}(\frac{\kappa}{K_u})$ have the following properties.

Proposition 1: *For any given κ, $K_u^{max}(\kappa)$ and $K_d^{max}(\frac{\kappa}{K_u})$ are obtained when constraint (13b) and (13c) hold with equality, respectively. Moreover, $K_u^{max}(\kappa)$ is a decreasing function of κ, and $K_d^{max}(\frac{\kappa}{K_u})$ is an increasing function of $\frac{\kappa}{K_u}$.* ■

Proof: See Appendix A.

Based on Proposition 1 and the central limit theorem as used in (6), the functions $K_u^{max}(\kappa)$ and $K_d^{max}(\frac{\kappa}{K_u})$ can be found from the following equations

$$K_u^{max}m_u + \Phi^{-1}(\epsilon)\sigma_u\sqrt{K_u^{max}} = \hat{R}_{uf}(\frac{\kappa}{K_u^{max}}) \tag{15a}$$

$$K_d^{max}m_d + \Phi^{-1}(\epsilon)\sigma_d\sqrt{K_d^{max}} = \hat{R}_{df}(\frac{\kappa}{K_u}). \tag{15b}$$

Since $K_d^{max}(\frac{\kappa}{K_u})$ increases with $\frac{\kappa}{K_u}$ according to Proposition 1 and further considering (14a) and (14b), we can rewrite problem (13) as

$$\max_{\kappa}\ K = \min(K_u, K_d) \tag{16a}$$

$$s.t.\ K_u \le K_u^{max}(\kappa) \tag{16b}$$

$$K_d = K_d^{max}(\frac{\kappa}{K_u}) \ge K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)}) \tag{16c}$$

$$\kappa_{min} \le \kappa \le \kappa_{max}. \tag{16d}$$

In order to maximize $K = \min(K_u, K_d)$ in problem (16), we first investigate if the feasible regions of K_u and K_d overlap. If the regions overlap, then K is maximized when $K_u = K_d$; otherwise, $K_u \neq K_d$. To this end, we need to compare the upper bound of K_u, i.e., the term $K_u^{max}(\kappa)$ in (16b), and the lower bound of K_d, i.e., the term $K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)})$ in (16c). Define $\tilde{\kappa}$ as the solution to the equation $K_u^{max}(\kappa) = K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)}) \triangleq \tilde{K}$. By substituting $\tilde{K}$ and $\tilde{\kappa}$ into (15), we have

$$\tilde{K}m_u + \Phi^{-1}(\epsilon)\sigma_u\sqrt{\tilde{K}} = \hat{R}_{uf}(\frac{\tilde{\kappa}}{\tilde{K}}) \tag{17a}$$

$$\tilde{K}m_d + \Phi^{-1}(\epsilon)\sigma_d\sqrt{\tilde{K}} = \hat{R}_{df}(\frac{\tilde{\kappa}}{\tilde{K}}). \tag{17b}$$

Since $\hat{R}_{uf}(\frac{\tilde{\kappa}}{\tilde{K}})$ and $\hat{R}_{df}(\frac{\tilde{\kappa}}{\tilde{K}})$ are respectively decreasing and increasing functions of $\frac{\tilde{\kappa}}{\tilde{K}}$, the value of $\tilde{\kappa}$ can be readily computed with a bisection method.

According to Proposition 1, we know that $K_u^{max}(\kappa)$ decreases with κ and $K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)})$ increases with κ. Therefore, if there exists $\kappa \le \tilde{\kappa}$ within its feasible region $[\kappa_{min}, \kappa_{max}]$, then $K_u^{max}(\kappa) \ge K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)})$ holds and the feasible regions of K_u and K_d overlap; otherwise, the regions have no intersection. Based on the result, we next solve problem (16) in two cases.

First, if $\kappa_{min} \le \tilde{\kappa}$, then there exists $\kappa \in [\kappa_{min}, \min(\tilde{\kappa}, \kappa_{max})]$ satisfying $\kappa \le \tilde{\kappa}$. In this case, the feasible regions of K_u and K_d overlap, and the objective function of problem (16) is maximized when $K = K_u = K_d = K_d^{max}(\frac{\kappa}{K_u})$, where the final equality comes from (16c). By substituting $K_u = K_d^{max}(\frac{\kappa}{K_u})$ into (15b), we obtain

$$K_d^{max}m_d + \Phi^{-1}(\epsilon)\sigma_d\sqrt{K_d^{max}} = \hat{R}_{df}(\frac{\kappa}{K_d^{max}}). \tag{18}$$

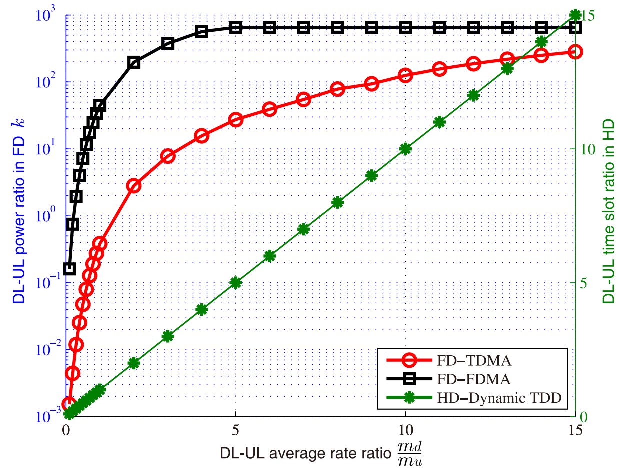

Fig. 1. Bidirectional resource configuration in FD and HD dynamic TDD networks v.s. $\frac{m_d}{m_u}$.

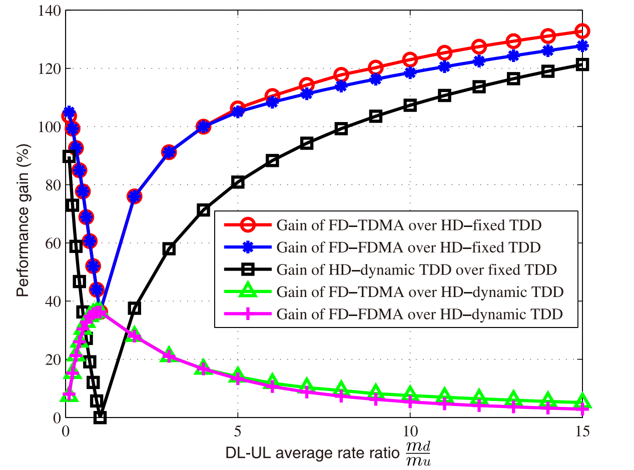

Fig. 2. Performance gain of FD network over HD static TDD and dynamic TDD networks v.s. $\frac{m_d}{m_u}$.

Since $\hat{R}_{df}(\cdot)$ is an increasing function, we know that K_d^{max} must increase with the growth of κ. This means that K_d^{max} is maximized when κ reaches its maximum, i.e., the optimal $\kappa^* = \min(\tilde{\kappa}, \kappa_{max})$. Given κ^*, the maximal K_d^{max*} can be obtained with a bisection method from (18). Finally, we have the optimal $K_u^* = K_d^* = K^* = K_d^{max*}$.

Second, if $\kappa_{min} > \tilde{\kappa}$, we have $K_u^{max}(\kappa) < K_d^{max}(\frac{\kappa}{K_u^{max}(\kappa)})$ for all $\kappa \in [\kappa_{min}, \kappa_{max}]$, i.e., the maximal possible value of K_u is smaller than the minimal possible value of K_d. Therefore, it is clear that the optimal $K^* = K_u^* = K_u^{max*}(\kappa^*)$, where $K_u^{max*}(\kappa^*)$ can be solved from (15a) by choosing the optimal $\kappa^* = \kappa_{min}$ because $K_u^{max}(\kappa)$ decreases with κ according to Proposition 1.

IV. Numerical Results

In this section, we evaluate the performance of FD and HD networks based on the proposed resource configuration methods, and investigate the performance gain of FD network over traditional static TDD and dynamic TDD networks. The following system parameter setups are considered. The system bandwidth is 10 MHz, the noise power spectrum density is –174 dBm/Hz, and the noise figure is 9 dB, from which we can obtain the total noise power is –95 dBm [15]. The density of BSs is set as $\lambda = 2\times 10^{-4} m^{-2}$, which corresponds to an average cell radius of 40 m. The pathloss for the channels between BSs, between users, and between BSs and users are calculated based on the Hata model [16], where the heights of BSs and users are set as 10 m and 1.5 m, respectively. The maximal transmit powers of BSs and users are $P_{b,max} = 23$ dBm and $P_{u,max} = 23$ dBm, respectively [15]. In order to ensure that the networks operate in interference-limited scenario, the minimal transmit power of BSs, $P_{b,min}$ is selected to satisfy the condition $\frac{\hat{R}_{h0} - \bar{R}_{dh}(P_{b,min})}{\bar{R}_{dh}(P_{b,min})} = 10^{-4}$ for HD static TDD network with $T_d = T_u$, where $\hat{R}_{h0}$ and $\bar{R}_{dh}$ are the average downlink rates in the cases without and with noise as defined in Section II, respectively. This condition means that the impact of ignoring noise on the average rate should be very small. We can obtain that $P_{b,min} = -5.2$ dBm, and the minimal transmit power of users can be obtained in the same way as $P_{u,min} = -5.2$ dBm. Then, the range of κ can be obtained as $\kappa_{min} = 0.0015$ and $\kappa_{max} = 660.7$. The parameter reflecting the residual self-interference in FD network is set as $\beta = -110$ dB. The data rate requirements of users are modeled as exponential distribution with means m_u and m_d for uplink and downlink, respectively, where m_u is set as 100 kbps and different values of m_d will be considered. The probability value used for defining the number of supported users is set as $\epsilon = 95\%$. In static TDD network, the time slots are equally allocated to uplink and downlink.

Figure 1 shows the optimal bidirectional power control for FD TDMA and FDMA networks and the optimal time slot splitting in HD dynamic TDD network, as a function of the ratio of average downlink rate requirement to average uplink rate requirement, $\frac{m_d}{m_u}$. It is shown that in FD network downlink-to-uplink transmit power ratio, κ, increases with $\frac{m_d}{m_u}$ as expected. For a given $\frac{m_d}{m_u}$, a higher κ is required by FDMA network compared to TDMA network. This means that given κ, i.e., given the transmit powers of BSs and users, TDMA network can achieve high downlink rate and low uplink rate, while FDMA network can achieve low downlink rate but high uplink rate. This is because compared to TDMA network, in FDMA network each user transmits within a narrow band in uplink, which increases the power spectrum of desired signal but also increases the interference to downlink users, where the BS needs to spread the power over full bandwidth. The result suggests that TDMA is suitable for the scenarios where downlink traffic dominates, while FDMA is suitable for the scenarios where uplink traffic dominates. Moreover, we can observe that in FDMA network κ keeps constant when $\frac{m_d}{m_u} \geq 5$, which reaches the maximal κ_{max}. Yet, for $\frac{m_d}{m_u} < 5$ in FDMA network and for any $\frac{m_d}{m_u}$ in TDMA network, we can observe from the two curves that κ increases proportionally with $\sqrt{\frac{m_d}{m_k}}$. Such a scaling result can be proven theoretically, which however is not provided in the paper due to lack of space. For

HD dynamic TDD network, we can see that the ratio of time slots configured for downlink and uplink increases linearly with $\frac{m_d}{m_u}$, which coincides with our analysis in Section III-B.

Figure 2 depicts the performance relationship between FD network and HD static TDD as well as dynamic TDD networks. It is shown that compared to the static TDD network, FD network with TDMA and FDMA can improve the network performance evidently. When symmetric bidirectional traffic requirements are considered, in which case the static TDD network operates optimally, the gain near 40% can be achieved by FD network. When the bidirectional traffic requirements are asymmetric, FD network can even achieve the gain up to 130% over the static TDD network. Moveover, in FD network TDMA can achieve higher gain over FDMA when downlink traffic dominates, which agrees with the analysis in Fig. 1. The gain of FD network over dynamic TDD network decreases with the growth of traffic asymmetry as expected. When typical values of $\frac{m_d}{m_u}$ ranging from four to six are considered [17], FD network can provide a gain of $10\% \sim 20\%$ compared to dynamic TDD network.

V. Conclusions

In this paper we investigated the performance gain of FD network over HD network including both traditional static TDD network and dynamic TDD network. We maximized the number of supported users in FD TDMA and FDMA networks as well as HD dynamic TDD network by optimizing bidirectional power control and time slot splitting, respectively, where asymmetric uplink-downlink traffic requirements were taken into account. Numerical results showed that different from HD network where multiple accessing strategy has no impact on the maximal number of supported users, in FD network TDMA is suitable for the scenarios where downlink traffic dominates, while FDMA is suitable for the scenarios where uplink traffic dominates. Under asymmetric bidirectional traffics, both FD network and dynamic TDD network exhibit evident performance gains over the traditional static TDD network. Compared to dynamic TDD network, the gain of FD network changes with the traffic asymmetry, and the gain up to 20% can be achieved for the typical downlink traffic dominant scenarios.

Appendix A
Proof of Proposition 1

For downlink, it is obvious from (13c) that K_d is maximized when (13c) holds with equality and increases with $\frac{\kappa}{K_u}$ because $\hat{R}_{df}$ increases with $\frac{\kappa}{K_u}$.

For uplink, we prove the properties of K_u by contradiction. Let K_u^{max} denote the maximal value of K_u constrained by (13b). Assume that (13b) holds with strict inequality when $K_u = K_u^{max}$, i.e., $\mathbb{P}\{\sum_{i=1}^{K_u^{max}} R_{ui} \le \hat{R}_{uf}(\frac{\kappa}{K_u^{max}})\} > \epsilon$. Then, we can always find a $\hat{K}_u > K_u^{max}$ so that $\mathbb{P}\{\sum_{i=1}^{\hat{K}_u} R_{ui} \le \hat{R}_{uf}(\frac{\kappa}{K_u^{max}})\} = \epsilon$. Since $\hat{R}_{uf}(\frac{\kappa}{K_u})$ is a decreasing function of W as shown in (3a) and hence is an increasing function with K_u, we have $\mathbb{P}\{\sum_{i=1}^{\hat{K}_u} R_{ui} \le \hat{R}_{uf}(\frac{\kappa}{\hat{K}_u})\} > \epsilon$. This means that there exists a $\hat{K}_u$ that is larger than K_u^{max} and satisfies constraint (13b). This is contradictory with the fact that K_u^{max} is defined as the maximal value of K_u. Therefore, the considered assumption is false, and K_u is maximized when (13b) holds with equality.

We proceed to prove that K_u^{max} is an increasing function of κ. Define $\bar{K}_u^{max}$ and $\hat{K}_u^{max}$ as the maximal number of K_u when κ equals to $\bar{\kappa}$ and $\hat{\kappa}$, respectively. Without loss of generality, we set $\hat{\kappa} > \bar{\kappa}$. Then, we have $\epsilon = \mathbb{P}\{\sum_{k=1}^{\hat{K}_u^{max}} R_{uk} \le \hat{R}_{uf}(\frac{\hat{\kappa}}{\hat{K}_u^{max}})\} < \mathbb{P}\{\sum_{k=1}^{\hat{K}_u^{max}} R_{uk} \le \hat{R}_{uf}(\frac{\bar{\kappa}}{\hat{K}_u^{max}})\}$, where the inequality comes from the fact that $\hat{R}_{uf}(\frac{\kappa}{K_u})$ decreases with κ. The result shows that when $\kappa = \bar{\kappa}$, $\hat{K}_u^{max}$ cannot make constraint (13b) hold with equality. It means that $\hat{K}_u^{max}$ is only a feasible point, so that $\hat{K}_u^{max} < \bar{K}_u^{max}$ must hold. Therefore, K_u^{max} decreases with κ.

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