Energy Efficient Optimization for Full-duplex Assisted Closed-loop MISO Downlink Transmission

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***Abstract*—This paper studies the energy efficient optimization for full duplex (FD) assisted closed-loop downlink transmission, where a FD base station (BS) serves multiple half duplex (HD) users in a time division multiple access manner. We optimize the durations of uplink training and downlink transmission, aimed at maximizing the energy efficiency (EE) of the system. We first show that the EE-optimal transmission scheme must use one of the two strategies: transmitting in HD mode or transmitting downlink data over all time slots so that the BS operates in FD mode during the whole uplink training phase. We then derive an approximate average net data rate of the system considering the bidirectional interference between uplink and downlink users in FD mode. Based on the result, a closed-form expression of the EE is obtained. We prove that the EE in FD mode is quasi-concave with respect to the duration of uplink training, with which the optimal durations of uplink training and downlink transmission are obtained. Simulation results show the evident gain of the proposed scheme over existing FD and HD schemes.**

I. INTRODUCTION

Full-duplex (FD) communication, enabling simultaneous transmission and reception over the same time-frequency channel, has been approved feasible at reasonable cost recently, thanks to the development of advanced self-interference cancellation techniques [1]. Its potential in doubling the throughput of current half-duplex (HD) systems was experimentally demonstrated for point-to-point communications [2, 3]. This triggers a spurt of research in redesigning the communication systems in order to gain the benefits brought by FD [4, 5].

Closed-loop beamforming is a widely used technique in downlink to exploit the multiple antennas at the base station (BS) [6]. To obtain the channel state information (CSI) at the BS, uplink training is usually used in time division duplex (TDD) systems, with which the downlink CSI can be estimated based on channel reciprocity. In HD systems, the duration of uplink training impacts the accuracy of CSI at the BS as well as the available resources for downlink transmission, the optimization with respect to which thus receives extensive studies [7]. The situation dramatically changes when FD technique is applied to the closed-loop beamforming system. Under the assumption of perfect self-interference cancellation, a novel transmission strategy, namely continuously adaptive beamforming, is proposed in [5], where the FD BS is able to send downlink data concurrently with uplink training signals collection. In the special case where the *bidirectional interference*, i.e., the interference from an uplink HD user to a downlink HD user, is negligible, [5] shows that both uplink training and downlink transmission should occupy all time-frequency resources in order to maximize the downlink spectral efficiency (SE).

However, the SE-oriented optimization for the durations of uplink training and downlink transmission is not necessarily optimal from the perspective of energy efficiency (EE), which has become one of the major performance metrics for the fifth generation (5G) cellular systems. Such a non-optimality has been confirmed for HD systems. For instance, [6] shows that the EE-oriented optimization requires longer uplink training than the SE-oriented optimization for delay-tolerant services. For FD systems, it is predictable that the existing SE-optimal strategies, e.g., transmitting in FD mode over all time-frequency resources under the above special case in [5], may not lead to a high EE, because the BS operating in FD mode will consume more circuit and signal processing powers than HD mode. To the best of our knowledge, the EE-oriented optimization for FD beamforming systems has not been addressed in the literature.

In this paper, we study the energy efficient optimization for a FD assisted closed-loop multi-input single-output (MISO) downlink transmission, where multiple users are served in a time division multiple access (TDMA) manner. We first show that the EE-optimal transmission scheme has only two possible strategies, which simplifies the expression of EE. Then, we derive an approximated average net data rate of the system considering the bidirectional interference between uplink and downlink users in FD mode. Based on the result, we prove that the EE in FD mode is a quasi-concave function of the duration of uplink training, and then obtain the optimal durations of uplink training and downlink transmission. Simulations demonstrate the performance gain of the proposed EE-optimal FD beamforming scheme over the existing SE-optimal FD and HD schemes and the EE-optimal HD scheme.

II. SYSTEM MODEL

A. FD Assisted TDD-MISO System

Consider a TDD-MISO closed-loop beamforming system consisting of one FD BS and K HD users, where the BS is equipped with M antennas and each user has a single antenna. To facilitate downlink beamforming, orthogonal uplink training signals are sent from users in time domain. Assume that each user sends τ training signals, then the total overhead of uplink training of K users is $K\tau$ time slots. Based on the

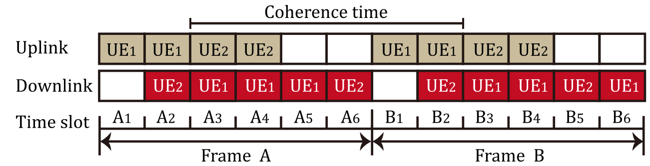

Fig. 1. Illustration of the FD closed-loop beamforming system, where the coherence time includes $T=6$ time slots, $K=2$ users are served, each user transmits uplink training in $\tau=2$ time slots, and the downlink transmission phase includes $d=5$ time slots. In Frame B, the BS operates in FD mode in time slots $B_2 \sim B_4$ and in HD mode in time slots B_1, B_5 and B_6.

received uplink training signals, the BS estimates the downlink CSI by exploiting the channel reciprocity.

We consider a frame-based transmission as shown in Fig. 1, where each frame includes T time slots and T denotes the coherence time of the channel. Each frame begins with the uplink training phase with $K\tau$ time slots. The downlink transmission phase consists of d time slots with $d \leq T$. Note that the uplink training phase and the downlink transmission phase can be overlapped since the BS has the FD capability. Nevertheless, we can find that the overlapped FD transmission will not happen if $d \leq T – K\tau$, because in this case transmitting in non-overlapped HD mode can always achieve a higher EE, considering both the bidirectional interference and larger power consumption in FD mode. Therefore, without loss of generality, we let the downlink transmission phase occupies the last d time slots of each frame. As a result, we know that within each frame there are $\min\{K\tau, T-d\}$ time slots operating in HD uplink training mode, $\min\{T-K\tau, d\}$ times slots operating in HD downlink transmission mode, and $\max\{d+K\tau-T, 0\}$ time slots operating in FD mode, during which uplink training and downlink transmission are performed simultaneously.

We assume block fading channels, i.e., the channel of a user remains constant within the coherence time, beyond which the channel changes to another independent realization. Therefore, the CSI of a user estimated by the BS will keep effective over T time slots. *This allows in each frame that the downlink transmission can take place before the uplink training phase finishes.* For example, in Fig. 1 even though the uplink training of Frame B is not finished in time slot B_2, the BS can still perform downlink transmission to UE_2 based on the CSI estimated at time slots $A_3 \sim A_4$. In different time slots, the BS may have the CSI of different users. For instance, the BS has only the CSI of the UE_2 in time slot B_2 (the uplink training of UE_1 is not fully finished), but has the CSI of both UE_1 and UE_2 in time slot B_5. We assume random scheduling for downlink transmission, with which in each time slot the BS randomly select a user from the pool of users whose CSI is available at the BS.

B. Signal Model

In the uplink training phase, UE_k sends the uplink training signals $\mathbf{s}_{\text{tr}} \in \mathbb{C}^{\tau \times 1}$, where $\mathbf{s}_{\text{tr}}^H \mathbf{s}_{\text{tr}} = P_{\text{U}}\tau$ with P_{U} denoting the transmit power of UE_k. The received signal at the BS can be expressed as

$$\mathbf{y}_{\text{U}k} = \mathbf{S}_{\text{tr}} \mathbf{h}_k + \mathbf{n}_{\text{U}k}, \tag{1}$$

where $\mathbf{S}_{\text{tr}} = \mathbf{I}_M \otimes \mathbf{s}_{\text{tr}}$ with $\mathbf{I}_M$ and $\otimes$ denoting $M \times M$ identity matrix and kronecker product, respectively, $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_{\text{h}}^2 \mathbf{I}_M)$ is the $M \times 1$ channel vector of UE_k, and $\mathbf{n}_{\text{U}k} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\text{U}}^2 \mathbf{I}_{M\tau})$ is the additive white Gaussian noise (AWGN) at the BS including both thermal noises and residual self-interference of the FD BS by assuming the self-interference is reduced to noise floor as in [5].

With the minimum mean square error (MMSE) channel estimator, we can express the relationship between the channel vector $\mathbf{h}_k$ and its estimate $\hat{\mathbf{h}}_k$ as [6]

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{2}$$

where $\hat{\mathbf{h}}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_{\hat{\text{h}}}^2 \mathbf{I}_M)$ with $\sigma_{\hat{\text{h}}}^2 = \sigma_{\text{h}}^2 \gamma_{\text{U}} \tau / (1+\gamma_{\text{U}}\tau)$, $\mathbf{e}_k \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{\text{e}}^2 \mathbf{I}_M\right)$ is the estimation error vector with $\sigma_{\text{e}}^2 = \sigma_{\text{h}}^2 / (1+\gamma_{\text{U}}\tau)$, and $\hat{\mathbf{h}}$ and $\mathbf{e}$ are uncorrelated. Herein, $\gamma_{\text{U}} \triangleq P_{\text{U}} \sigma_{\text{h}}^2 / \sigma_{\text{U}}^2$ is the uplink average signal-to-noise ratio (SNR).

During the downlink transmission phase, the maximal ratio transmission (MRT) precoder is employed. Assume that UE_k is scheduled in the i-th time slot, then the precoder is

$$\mathbf{w}_k = \sqrt{P_{\text{D}}} \hat{\mathbf{h}}_k / \|\hat{\mathbf{h}}_k\|, \tag{3}$$

where P_{D} is the transmit power of the BS.

In the i-th time slot, the BS may operate in HD or FD mode. If the BS is in HD mode in the i-th time slot, the received signal of UE_k is expressed as

$$y_{\text{D}k} = \mathbf{h}_k^H \mathbf{w}_k s_{\text{D}k} + n_{\text{D}k}, \tag{4}$$

where $s_{\text{D}k}$ is the transmit signal of UE_k with $\mathbb{E}\{|s_{\text{D}k}|^2\} = 1$ and $n_{\text{D}k} \sim \mathcal{CN}(0, \sigma_{\text{D}}^2)$ is the AWGN at UE_k.

From (2), (3) and (4), the data rate of UE_k in HD mode can be obtained as

$$R_{\text{HD},k} = \log\left(1 + \frac{P_{\text{D}}}{\sigma_{\text{D}}^2} \left| \|\hat{\mathbf{h}}_k\| + \frac{\mathbf{e}_k^H \hat{\mathbf{h}}_k}{\|\hat{\mathbf{h}}_k\|} \right|^2 \right), \tag{5}$$

where $\log(\cdot)$ is the natural logarithm throughout the paper.

If the BS operates in FD mode in the i-th time slot where the downlink user UE_k and uplink user UE_j transmit simultaneously, then the received signal of UE_k can be expressed as

$$y_{\text{D}k} = \mathbf{h}_k^H \mathbf{w}_k s_{\text{D}k} + h_{kj} s_{\text{U}} + n_{\text{D}k}, \tag{6}$$

where $h_{kj} \sim \mathcal{CN}(0, \sigma_{\text{I}}^2)$ is the bidirectional interfering channel from UE_j to UE_k, and s_{U} is the uplink training signal sent by UE_j in the i-th time slot with $|s_{\text{U}}|^2 = P_{\text{U}}$, which is an element of the training signal vector $\mathbf{s}_{\text{tr}}$.

From (2), (3) and (6), the data rate of UE_k in FD mode can be obtained as

$$R_{\text{FD},k} = \log\left(1 + \frac{P_{\text{D}}}{\sigma_{\text{D}}^2 + P_{\text{U}} |h_{kj}|^2} \left| \|\hat{\mathbf{h}}_k\| + \frac{\mathbf{e}_k^H \hat{\mathbf{h}}_k}{\|\hat{\mathbf{h}}_k\|} \right|^2 \right), \tag{7}$$

where the bidirectional interference is treated as noise.

With (5) and (7), the average net data rate in a frame can be obtained as

$$\begin{aligned}\bar{R} = &\frac{1}{T}\big(\min\{T-K\tau, d\}\mathbb{E}\{R_{\text{HD},k}\} \\ &+ \max\{d+K\tau-T, 0\}\mathbb{E}\{R_{\text{FD},k}\}\big),\end{aligned} \tag{8}$$

$$EE=\frac{\min\{T-K\tau,d\}\mathbb{E}\{R_{\mathrm{HD},k}\}+\max\{d+K\tau-T,0\}\mathbb{E}\{R_{\mathrm{FD},k}\}}{\min\{K\tau,T-d\}P_{\mathrm{RX}}+\min\{T-K\tau,d\}P_{\mathrm{TX}}+\max\{d+K\tau-T,0\}P_{\mathrm{FD}}}. \tag{13}$$

where $\min\{T-K\tau,d\}$ and $\max\{d+K\tau-T,0\}$ are the durations of downlink transmission in HD and FD modes, respectively, $\mathbb{E}\{R_{\mathrm{HD},k}\}$ is the average rate in a HD time slot, which is identical for all HD time slots under the random user scheduling as we considered, and $\mathbb{E}\{R_{\mathrm{FD},k}\}$ is the average rate in a FD time slot.

C. Power Consumption Model

Considering that the power consumption at user sides is far less than that at the BS, we only take into account the power consumed by the BS in the paper.

During the HD uplink training phase, the BS operates in pure receiving mode. The power consumption can be modeled as [8]

$$P_{\mathrm{RX}}=\frac{M\left(P_{\mathrm{RF}}^{\mathrm{RX}}+P_{\mathrm{BB}}^{\mathrm{RX}}\right)}{(1-\sigma_{\mathrm{dc}})(1-\sigma_{\mathrm{MS}})(1-\sigma_{\mathrm{cool}})}\triangleq MP_{\mathrm{C}}^{\mathrm{RX}}, \tag{9}$$

where $P_{\mathrm{RF}}^{\mathrm{RX}}$ and $P_{\mathrm{BB}}^{\mathrm{RX}}$ are the powers consumed by radio frequency (RF) links and baseband (BB) signal processing such as synchronization and channel estimation, σ_{dc}, σ_{MS} and σ_{cool} are the loss factors used to reflect the powers incurred by DC-DC power supply, mains supply, and active cooling, respectively, and $P_{\mathrm{C}}^{\mathrm{RX}}\triangleq\left(P_{\mathrm{RF}}^{\mathrm{RX}}+P_{\mathrm{BB}}^{\mathrm{RX}}\right)/(1-\sigma_{\mathrm{dc}})(1-\sigma_{\mathrm{MS}})(1-\sigma_{\mathrm{cool}})$ is the circuit power per antenna when the BS operates in pure receiving mode.

During the HD downlink transmission phase, the BS consumes both transmit and circuit power. The power consumption can be modeled as [8]

$$P_{\mathrm{TX}}=\frac{\frac{P_{\mathrm{D}}}{\eta}+M\left(P_{\mathrm{RF}}^{\mathrm{TX}}+P_{\mathrm{BB}}^{\mathrm{TX}}\right)}{(1-\sigma_{\mathrm{dc}})(1-\sigma_{\mathrm{MS}})(1-\sigma_{\mathrm{cool}})}\triangleq\mu P_{\mathrm{D}}+MP_{\mathrm{C}}^{\mathrm{TX}}, \tag{10}$$

where $P_{\mathrm{RF}}^{\mathrm{TX}}$, $P_{\mathrm{BB}}^{\mathrm{TX}}$ are the power consumed by RF links and BB signal processing such as precoding, modulation, and coding, η is the efficiency of the power amplifier (PA), $\mu\triangleq 1/(\eta(1-\sigma_{\mathrm{dc}})(1-\sigma_{\mathrm{MS}})(1-\sigma_{\mathrm{cool}}))$ is a parameter that reflects the loss of transmit power, and $P_{\mathrm{C}}^{\mathrm{TX}}\triangleq\left(P_{\mathrm{RF}}^{\mathrm{TX}}+P_{\mathrm{BB}}^{\mathrm{TX}}\right)/(1-\sigma_{\mathrm{dc}})(1-\sigma_{\mathrm{MS}})(1-\sigma_{\mathrm{cool}})$ is the circuit power per antenna when the BS operates in pure transmitting mode.

When the BS operates in FD mode, it consumes not only the powers in pure transmitting and receiving, but also the additional circuit and signal processing power for self-interference cancellation. Without loss of generality, we model the power consumption in FD mode as

$$P_{\mathrm{FD}}=\mu P_{\mathrm{D}}+\beta M\left(P_{\mathrm{C}}^{\mathrm{RX}}+P_{\mathrm{C}}^{\mathrm{TX}}\right), \tag{11}$$

where the parameter $\beta>1$ is introduced to reflect the increased power consumption for self-interference cancellation, which depends on the employed analog cancellation circuit structure and analog-digital cancellation algorithms.

With (9), (10) and (11), we can obtain the power consumption in a frame as

$$\bar{P}=\frac{1}{T}\big(\min\{K\tau,T-d\}P_{\mathrm{RX}}+\min\{T-K\tau,d\}P_{\mathrm{TX}} +\max\{d+K\tau-T,0\}P_{\mathrm{FD}}\big). \tag{12}$$

III. Energy Efficient Optimization

In this section, we maximize the EE of the FD closed-loop beamforming system by optimizing the durations of uplink training and downlink transmission phases, based on which the operation modes of the BS in each time slot can be obtained.

The EE is defined as the ratio of the average net data rate, $\bar{R}$, to the power consumption, $\bar{P}$, which can be obtained from (8) and (12) as (13), shown at the top of the page. The expression of th EE is very complicated. To maximize the EE, in the following we first strive to obtain a closed-form expression for the EE by analyzing the optimal value of d and deriving the average net data rate $\bar{R}$.

A. Optimal Downlink Transmission Duration

The downlink transmission duration affects the expression of the EE as shown in (13). To find the optimal value of d, we consider the following two cases.

1) $0\le d\le T-K\tau$: In this case, the expression of EE can be simplified as

$$EE_1=\frac{d\mathbb{E}\{R_{\mathrm{HD},k}\}}{K\tau P_{\mathrm{RX}}+dP_{\mathrm{TX}}}. \tag{14}$$

It is clear that EE_1 is an increasing function of d, meaning that the optimal value of d is $T-K\tau$ in this case.

2) $T-K\tau\le d\le T$: In this case, the expression of EE is simplified as

$$EE_2=\frac{(T-K\tau)\mathbb{E}\{R_{\mathrm{HD},k}\}+(d+K\tau-T)\mathbb{E}\{R_{\mathrm{FD},k}\}}{(T-d)P_{\mathrm{RX}}+(T-K\tau)P_{\mathrm{TX}}+(d+K\tau-T)P_{\mathrm{FD}}}. \tag{15}$$

The first-order derivative of EE_2 with respect to d can be obtained as

$$\nabla_d EE_2=\frac{A(\tau)D(\tau)-B(\tau)C}{(Cd+D(\tau))^2}, \tag{16}$$

where $A(\tau)=\mathbb{E}\{R_{\mathrm{FD},k}\}$, $B(\tau)=(T-K\tau)(\mathbb{E}\{R_{\mathrm{HD},k}\}-\mathbb{E}\{R_{\mathrm{FD},k}\})$, $C=P_{\mathrm{FD}}-P_{\mathrm{RX}}$, and $D(\tau)=TP_{\mathrm{RX}}+(T-K\tau)(P_{\mathrm{TX}}-P_{\mathrm{FD}})$.

From (16), we know that EE_2 is a monotonic function of d, increasingly or decreasingly depending on whether $A(\tau)D(\tau)-B(\tau)C$ is positive or negative when τ equals to its optimal value. Therefore, the optimal value of d has only two possible solutions, i.e.,

$$d=T-K\tau \quad\text{or}\quad d=T. \tag{17}$$

One can find that this result covers the above case 1, which gives rise to the following proposition.

Proposition 1: *The EE-optimal FD closed-loop beamforming system has only two possible strategies: performing uplink training and downlink transmission in HD mode if $A(\tau)D(\tau)-B(\tau)C<0$, or transmitting downlink data over the whole frame so that the BS operates in FD mode during the whole uplink training phase if $A(\tau)D(\tau)-B(\tau)C\geq 0$. Any other strategies, e.g., uplink training being partially overlapped with downlink transmission, will not happen. The value of τ depends on the parameters such as the SNR, power consumption, user number, etc.*

With (17), in the sequel we respectively consider the two possible values of d, and rewrite the EE as

$$EE=\begin{cases}\frac{(T-K\tau)\mathbb{E}\{R_{\mathrm{HD},k}\}}{K\tau P_{\mathrm{RX}}+(T-K\tau)P_{\mathrm{TX}}}\triangleq\widetilde{EE},\ d=T-K\tau\\ \frac{(T-K\tau)\mathbb{E}\{R_{\mathrm{HD},k}\}+K\tau\mathbb{E}\{R_{\mathrm{FD},k}\}}{(T-K\tau)P_{\mathrm{TX}}+K\tau P_{\mathrm{FD}}}\triangleq\widehat{EE},\ d=T.\end{cases}\quad(18)$$

B. Average Data Rates in HD and FD Modes

Before optimizing the uplink training duration of each user τ, we need to first find closed-form expressions of the average downlink data rates in HD and FD modes, i.e., $\mathbb{E}\{R_{\mathrm{HD},k}\}$ and $\mathbb{E}\{R_{\mathrm{FD},k}\}$ in (18). We focus on the derivation of $\mathbb{E}\{R_{\mathrm{FD},k}\}$, with which $\mathbb{E}\{R_{\mathrm{HD},k}\}$ can be readily obtained by setting the bidirectional interference as zero.

We begin with the approximation of the term $\left|\|\hat{\mathbf{h}}_k\|+\frac{\mathbf{e}_k^H\hat{\mathbf{h}}_k}{\|\hat{\mathbf{h}}_k\|}\right|^2$ in (7) following the results in [6]. First, $\|\hat{\mathbf{h}}_k\|$ is the square root of the sum of squares of M independent complex Gaussian random variables following $\mathcal{CN}(0,\sigma_{\hat{\mathrm{h}}}^2)$, while $\mathbf{e}_k^H\hat{\mathbf{h}}_k/\|\hat{\mathbf{h}}_k\|$ is a complex Gaussian variable following $\mathcal{CN}(0,\sigma_{\mathrm{e}}^2)$. Since $\sigma_{\hat{\mathrm{h}}}^2$ and σ_{e}^2 are respectively increasing and decreasing functions of the uplink SNR γ_{U}, we know that when M is large and γ_{U} is high, the term $\mathbf{e}_k^H\hat{\mathbf{h}}_k/\|\hat{\mathbf{h}}_k\|$ is much smaller than $\|\hat{\mathbf{h}}_k\|$ in a high probability. This leads to the approximation $\left|\|\hat{\mathbf{h}}_k\|+\mathbf{e}_k^H\hat{\mathbf{h}}_k/\|\hat{\mathbf{h}}_k\|\right|^2\approx\|\hat{\mathbf{h}}_k\|^2$. As analyzed in [6], this approximate is accurate even when M and γ_{U} are not very large.

Then, from (7) we have

$$\mathbb{E}\{R_{\mathrm{FD},k}\}\approx\mathbb{E}\left\{\log\left(1+\frac{P_{\mathrm{D}}\|\hat{\mathbf{h}}_k\|^2}{\sigma_{\mathrm{D}}^2+P_{\mathrm{U}}|h_{kj}|^2}\right)\right\}\triangleq\Delta_1-\Delta_2,\quad(19)$$

where $\Delta_1=\mathbb{E}\left\{\log\left(1+\frac{P_{\mathrm{U}}|h_{kj}|^2+P_{\mathrm{D}}\|\hat{\mathbf{h}}_k\|^2}{\sigma_{\mathrm{D}}^2}\right)\right\}$ and $\Delta_2=\mathbb{E}\left\{\log\left(1+\frac{P_{\mathrm{U}}|h_{kj}|^2}{\sigma_{\mathrm{D}}^2}\right)\right\}$. Since $\Delta_2=\Delta_1|_{P_{\mathrm{D}}=0}$, we put emphasis on the derivation of Δ_1.

To obtain Δ_1, we next investigate the statistics of the term $1+\frac{P_{\mathrm{U}}|h_{kj}|^2+P_{\mathrm{D}}\|\hat{\mathbf{h}}_k\|^2}{\sigma_{\mathrm{D}}^2}$, which can be recognized as the sum of Gamma distributed random variables. Exact characterization of the statistics will lead to a very complicated result, which is less useful for the subsequent optimization over τ. To obtain a closed-form expression, we employ the moment matching method to approximate this term as a Gamma distributed variable. Numerous studies in the literature, e.g., [9] and references therein, have validated that such an approximation is rather reasonable to characterize the statistics of the sum of Gamma distributed random variables. After some manipulations (details are omitted due to the lack of space), we can show that $1+\frac{P_{\mathrm{U}}|h_{kj}|^2+P_{\mathrm{D}}\|\hat{\mathbf{h}}_k\|^2}{\sigma_{\mathrm{D}}^2}$ can be approximated by a Gamma distribution $\Gamma(k_s,\theta_s)$ with

$$k_s=\frac{(k_r\theta_r+1)^2}{k_r\theta_r^2}\text{ and }\theta_s=\frac{k_r\theta_r^2}{k_r\theta_r+1},\quad(20)$$

where $k_r=\frac{(M\theta_1+\theta_2)^2}{M\theta_1^2+\theta_2^2}$, $\theta_r=\frac{M\theta_1^2+\theta_2^2}{\sigma_{\mathrm{D}}^2(M\theta_1+\theta_2)}$, $\theta_1=\frac{\sigma_{\mathrm{D}}^2\gamma_{\mathrm{U}}\gamma_{\mathrm{D}}\tau}{1+\gamma_{\mathrm{U}}\tau}$, $\theta_2=P_{\mathrm{U}}\sigma_{\mathrm{I}}^2$, and $\gamma_{\mathrm{D}}\triangleq\frac{P_{\mathrm{D}}\sigma_{\mathrm{h}}^2}{\sigma_{\mathrm{D}}^2}$ is the downlink average SNR.

For a Gamma distributed variable $Y\sim\Gamma(k_s,\theta_s)$, the equality $\mathbb{E}[\log Y]=\psi(k_s)+\log\theta_s$ holds, where $\psi(k_s)$ is the Digamma function [9]. Using this property, we can obtain $\mathbb{E}\{R_{\mathrm{FD},k}\}$ from (19) and (20) as

$$\mathbb{E}\{R_{\mathrm{FD},k}\}\approx\psi(k_s)+\log\theta_s-(\psi(k_s)+\log\theta_s)|_{\gamma_{\mathrm{D}}=0}.\quad(21)$$

When the value of M is large, k_r is large and hence k_s is large. Then, $\psi(k_s)$ can be approximated as $\psi(k_s)=\log k_s+\mathcal{O}(1/k_s)\approx\log k_s$ [9], with which $\mathbb{E}\{R_{\mathrm{FD},k}\}$ in (21) can be further derived as

$$\begin{aligned}\mathbb{E}\{R_{\mathrm{FD},k}\}&\approx\log(k_s\theta_s)-(\log(k_s\theta_s))|_{\gamma_{\mathrm{D}}=0}\quad(22)\\&=\log\left(1+\frac{M\gamma_{\mathrm{U}}\gamma_{\mathrm{D}}\sigma_{\mathrm{D}}^2\tau}{(\sigma_{\mathrm{D}}^2+P_{\mathrm{U}}\sigma_{\mathrm{I}}^2)(1+\gamma_{\mathrm{U}}\tau)}\right)\triangleq f_1(\tau).\end{aligned}$$

By setting $\sigma_{\mathrm{I}}^2=0$ in (22), we can obtain the average data rate in HD mode as

$$\mathbb{E}\{R_{\mathrm{HD},k}\}\approx\log\left(1+\frac{M\gamma_{\mathrm{U}}\gamma_{\mathrm{D}}\tau}{1+\gamma_{\mathrm{U}}\tau}\right)\triangleq f_2(\tau).\quad(23)$$

C. Maximization of EE

Based on (18), we next maximize the EE via considering two cases.

1) $d=T-K\tau$: In this case the EE maximization problem with respect to τ can be formulated as

$$\max_{\tau\in\mathbb{N}^+}\ \widetilde{EE}'\quad(24a)$$
$$s.t.\ 1\leq K\tau\leq T,\quad(24b)$$

where $\widetilde{EE}'$ denotes the approximation of $\widetilde{EE}$ by substituting $f_2(\tau)$ given by (23) into (18), and $\mathbb{N}^+$ denotes the set of positive integers.

Since the system operates in HD mode when $d=T-K\tau$, problem (24) is in fact the EE maximization problem of HD closed-loop beamforming systems. In [6], it is shown that the objective function $\widetilde{EE}'$ is a concave function of τ. Thus, by first omitting the integer constraint, the optimal continuous τ, denoted by $\tilde{\tau}$, can be readily found with standard convex optimization method. Given $\tilde{\tau}$, we can compare the values of $\widetilde{EE}'$ corresponding to $\lfloor\tilde{\tau}\rfloor$ and $\lceil\tilde{\tau}\rceil$, and then choose the one leading to the larger $\widetilde{EE}'$ as the solution of τ, where $\lfloor\tilde{\tau}\rfloor$ and $\lceil\tilde{\tau}\rceil$ denote the floor and ceiling of $\tilde{\tau}$, respectively.

2) $d=T$: Now the EE maximization problem can be formulated based on (18) as

$$\max_{\tau\in\mathbb{N}^+}\ \widehat{EE}'\triangleq\frac{(T-K\tau)f_2(\tau)+K\tau f_1(\tau)}{(T-K\tau)P_{\mathrm{TX}}+K\tau P_{\mathrm{FD}}}\quad(25a)$$
$$s.t.\ 1\leq K\tau\leq T,\quad(25b)$$

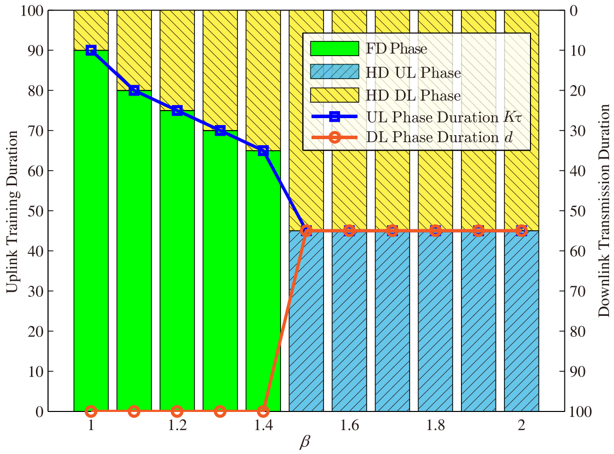

Fig. 2. Impact of the power consumption in FD mode of the BS.

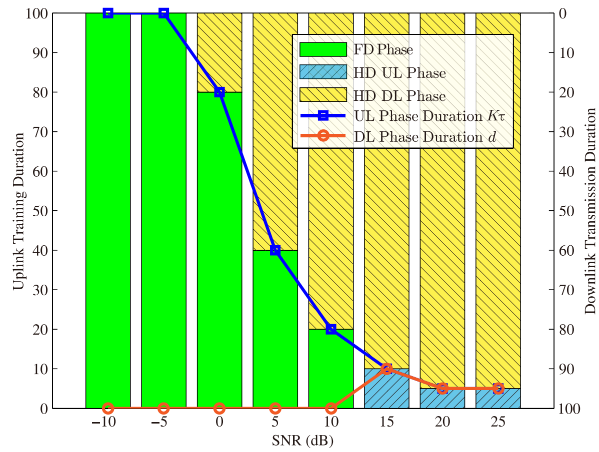

Fig. 3. Impact of the average downlink SNR with $\beta = 1.1$.

where $\widehat{EE}'$ denotes the approximation of $\widehat{EE}$ by substituting $f_1(\tau)$ and $f_2(\tau)$ given by (22) and (23) into (18).

Proposition 2: *$\widehat{EE}'$ is a quasi-convex function of τ.*

Proof: See the Appendix. ■

Then, by first ignoring the integer constraint, the resultant quasi-convex problem can be solved efficiently. The detailed algorithm is omitted for the sake of space saving. The obtained continuous τ is then quantized to a positive integer as in the above case with $d = T - K\tau$.

Finally, let τ_1, $d_1 = T - K\tau_1$, and $\widetilde{EE}'_1$ denote the solutions and the value of the objective function in the first case, and τ_2, $d_2 = T$, and $\widehat{EE}'_2$ denote the solutions and the value of the objective function in the second case. If $\widetilde{EE}'_1 > \widehat{EE}'_2$, then τ_1 and d_1 are chosen as the optimal solutions. Otherwise, τ_2 and d_2 are the optimal solutions.

IV. Simulation Results

In this section, we evaluate the performance of the proposed EE-optimal FD closed-loop beamforming scheme. Unless otherwise specified, in the simulations we consider that the BS equipped with $M = 8$ antennas serves $k = 5$ single-antenna users, the transmit powers of the BS and each user are 46 and 23 dBm, respectively, and the coherence time has $T = 100$ time slots. The power consumption parameters are configured as in [8] for a macro BS, where the per-antenna circuit power of the BS in transmitting and receiving modes is set to 47.1 W, and the parameter reflecting transmit power loss, i.e., μ, is set as 4.24. The downlink average SNR γ_{D} is set to 0 dB. Considering that the uplink average SNR is usually lower than the downlink average SNR, we set $\gamma_{\mathrm{U}} = \gamma_{\mathrm{D}} - 10$ dB. To reflect the impact of bidirectional interference between uplink and downlink users, we define the interference to noise ratio as $\mathrm{INR} = P_{\mathrm{U}}\sigma_{\mathrm{I}}^2/\sigma_{\mathrm{D}}^2$, which is set as -10 dB, where σ_{I}^2 is the variance of the channel between users as defined in (6).

We first analyze the impact of the power consumption in FD mode (i.e., β in (11)) on the optimal transmission strategy, specifically uplink training duration, downlink transmission duration, and the duration operating in FD mode. In Fig. 2, we use bar-plots to denote the ranges of uplink training phase, downlink transmission phase, and the FD phase, and use the curves to denote the uplink and downlink durations, i.e., $K\tau$ and d. Note that we use two y-axis to distinguish the uplink and downlink durations, which have the inverse scales. For example, when $\beta = 1.1$, the BS operates in FD mode in the first 80 time slots, and in the remaining 20 time slots the BS operates in HD mode and only transmits data to users in the downlink. The corresponding durations for uplink training and downlink transmission are 80 and 100 time slots, respectively.

We can obverse that the system switches between two strategies for different values of β, including transmitting downlink data in the whole frame with $d = T$ or in HD mode with $d = T - K\tau$. The duration of FD phase decreases with the growth of β, which coincides with the intuition. The shrinking of FD phase leads to the decrease of the uplink training duration with the growth of β, considering that the downlink transmission always occupies the whole frame as long as the FD phase exists.

Fig. 3 demonstrates the optimal transmission strategies under different downlink average SNRs. We can see that the duration of uplink training decreases with the increase of SNR as expected. Therefore, the duration of the FD phase decreases accordingly. When the SNR is high so that the duration of uplink training is short, the benefits of occupying the short uplink training phase to perform FD transmission is very limited, while the increase of total power consumption due to FD transmission is significant. Thus, in this scenario the system operates in HD mode to improve the EE. When the SNR increases from 10 dB to 15 dB, we can see a sudden drop of d from 100 to 90, which is due to the change of the transmission strategies from the partial FD mode into the pure HD mode.

In Fig. 4, we compare the performance of the proposed EE-oriented scheme for the FD closed-loop beamforming system (with legend "FD EE-oriented") with three relevant existing schemes, namely the SE-oriented scheme for FD systems that maximizes the average net data rate of the system (with legend "FD SE-oriented"), the EE-oriented scheme for HD systems (with legend "HD EE-oriented"), and the SE-oriented scheme for HD systems (with legend "HD SE-oriented"). For each scheme, we plot the EEs obtained from both simulations and

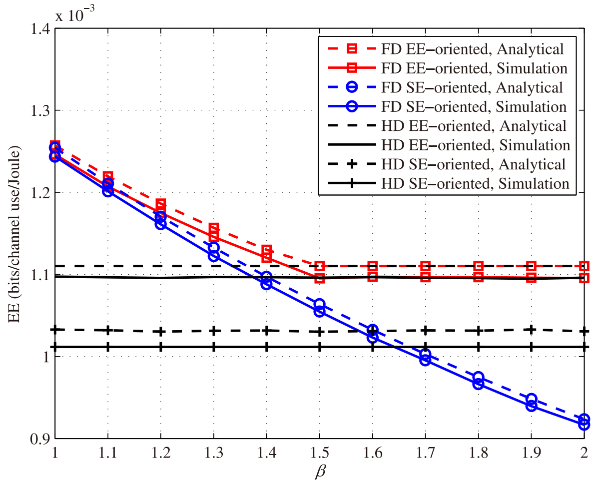

Fig. 4. Performance comparison of EE-oriented and SE-oriented schemes in FD and HD systems.

our analytical results. Specifically, in the simulation results the average data rates $\mathbb{E}\{R_{\mathrm{HD},k}\}$ and $\mathbb{E}\{R_{\mathrm{FD},k}\}$ are obtained by averaging the instantaneous data rates given by (5) and (7) over 10^4 channel realizations, while in the analytical results they are computed based on (23) and (22), respectively.

To maximize the average net data rate, the "FD SE-oriented" scheme will transmit downlink data over the whole frame under the assumption of perfect self-interference cancellation. When β is small, the proposed "FD EE-oriented" scheme also transmits downlink data over all time slots, as analyzed in Fig. 2. In this situation, the gain of the "FD EE-oriented" scheme over the "FD SE-oriented" scheme comes from the energy efficient optimization of uplink training duration. With the increase of β, the EE of the "FD SE-oriented" scheme decreases almost linearly because its FD phase has a fixed duration independent from β. As a result, when β is large, the "FD SE-oriented" scheme is inferior to both the "HD EE-oriented" and "HD SE-oriented" schemes. In contrast, the proposed "FD SE-oriented" scheme is aware of the power consumption in FD mode, and is able to adaptively adjust the durations of uplink training and downlink transmission to achieve the highest EE. We can also see that the gap between the analytical and simulation results is small, implying that the approximations used to derive the closed-form expression of EE are reasonable.

V. Conclusions

In this paper we investigated the energy efficient optimization of the durations for uplink training and downlink transmission in a FD-assisted closed-loop beamforming system. We first showed that the EE-optimal system has only two possible strategies: performing HD transmission or transmitting downlink data over the whole frame so that the BS operates in FD mode during the whole uplink training phase. Then, we derived an approximate average net data rate of the system considering the bidirectional interference between uplink and downlink users. We proved that the approximate EE in FD mode is a quasi-concave function of the duration of uplink training, and obtained the optimal durations for uplink training and downlink transmissions. Simulation results demonstrated the evident performance gain of the proposed EE-oriented FD scheme over existing SE-oriented FD and HD schemes and the EE-oriented HD scheme.

Appendix
Proof of Proposition 2

Since the term in the denominator of $\widehat{EE}'$ given in (25a) is a linear function of τ, in order to prove the quasi-convexity of $\widehat{EE}'$, it suffices to prove that the term in the numerator of $\widehat{EE}'$, denoted by $g(\tau)$, is a concave function of τ.

The second-order derivative of $g(\tau) \triangleq (T-K\tau)f_2(\tau)+K\tau f_1(\tau)$ can be obtained as

$$\begin{aligned}\nabla^2_\tau g(\tau) =& 2K\big(\nabla_\tau f_1(\tau)-\nabla_\tau f_2(\tau)\big)\\ &+K\tau\nabla^2_\tau f_1(\tau)+(T-K\tau)\nabla^2_\tau f_2(\tau). \quad \text{(A.1)}\end{aligned}$$

The first and second order derivatives of $f_2(\tau)$ can be derived as

$$\nabla_\tau f_2(\tau)=\frac{M\gamma_{\mathrm{D}}\gamma_{\mathrm{U}}}{(1+\gamma_{\mathrm{U}}\tau+M\gamma_{\mathrm{D}}\gamma_{\mathrm{U}}\tau)(1+\gamma_{\mathrm{U}}\tau)} \triangleq \phi_1(\gamma_{\mathrm{D}}) \quad \text{(A.2)}$$

$$\begin{aligned}\nabla^2_\tau f_2(\tau)=&-\frac{2M\gamma_{\mathrm{D}}\gamma_{\mathrm{U}}(\gamma_{\mathrm{U}}+\gamma_{\mathrm{U}}^2\tau+M\gamma_{\mathrm{D}}\gamma_{\mathrm{U}}^2\tau)+M^2\gamma_{\mathrm{D}}^2\gamma_{\mathrm{U}}^2}{(1+\gamma_{\mathrm{U}}\tau+M\gamma_{\mathrm{D}}\gamma_{\mathrm{U}}\tau)^2(1+\gamma_{\mathrm{U}}\tau)^2}\\ &\triangleq \phi_2(\gamma_{\mathrm{D}}). \quad \text{(A.3)}\end{aligned}$$

We note from (22) and (23) that $f_2(t)$ can be converted into $f_1(t)$ by replacing γ_{D} in $f_2(t)$ with $\bar{\gamma}_{\mathrm{D}} \triangleq \frac{\sigma_{\mathrm{D}}^2\gamma_{\mathrm{D}}}{\sigma_{\mathrm{D}}^2+P_{\mathrm{U}}\sigma_{\mathrm{I}}^2}$. Thus, we can obtain from (A.2) and (A.3) that $\nabla_\tau f_1(\tau)=\phi_1(\bar{\gamma}_{\mathrm{D}})$ and $\nabla^2_\tau f_1(\tau)=\phi_2(\bar{\gamma}_{\mathrm{D}})$. It is easy to see that $\phi_1(\cdot)$ is an increasing function, $\bar{\gamma}_{\mathrm{D}}<\gamma_{\mathrm{D}}$, and $\phi_2(\cdot)<0$. Then, we know $\nabla_\tau f_1(\tau)<\nabla_\tau f_2(\tau)$, and further from (A.1) we have $\nabla^2_\tau g(\tau)<0$. It follows that $g(\tau)$ is a concave function of τ.

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