Wideband Hybrid Precoder for Massive MIMO Systems

Lingxiao Kong, Shengqian Han, and Chenyang Yang
School of Electronics and Information Engineering, Beihang University, Beijing 100191, China
Email: {konglingxiao, sqhan, cyyang}@buaa.edu.cn

***Abstract*—This paper studies wideband hybrid precoder for downlink space-division multiple-access and orthogonal frequency-division multiple-access (SDMA-OFDMA) massive multi-input multi-output (MIMO) systems. We first derive an iterative algorithm to alternatingly optimize the phase-shifter based wideband analog precoder and low-dimensional digital precoders, then an efficient low-complexity non-iterative hybrid precoder proposes. Simulation results show that in wideband systems the performance of hybrid precoder is affected by the employed frequency-domain scheduling method and the number of available radio frequency (RF) chains, which can perform as well as narrowband hybrid precoder when greedy scheduling is employed and the number of RF chains is large.**

I. Introduction

Massive multi-input multi-output (MIMO) is a promising technique for five-generation (5G) cellular systems, which can support very high throughput by significantly increasing the number of antennas at base stations (BSs) [1]. However, the application of massive MIMO in practical systems is limited by the excessive hardware cost, transceiver complexity, and energy consumption when equipping every antenna element with a radio frequency (RF) chain. Hybrid architecture consisting of a phase-shifter based analog precoder in RF domain and low-dimensional digital precoders in baseband is an effective approach to reduce the number of required RF chains, and hence has attracted wide attention [2].

Existing studies on hybrid precoder optimization mainly focus on narrowband (i.e., single-subcarrier) massive MIMO systems, e.g., [3–5] for single-user MIMO case and [6–9] for multi-user MIMO case. The results demonstrated that the hybrid precoder with few number of RF chains performs close to the full digital precoder for massive MIMO systems operating in both millimeter wave frequency [3–7] and ultra-high frequency (UHF) band (e.g., Long-term Evolution (LTE)) [8,9]. When applied to wideband orthogonal frequency division multiplexing (OFDM) systems, these narrowband hybrid precoders, in particular those with analog precoder designed based on spatial correlation information, can work well if every user can occupy all subcarriers, i.e., frequency-domain scheduling is not employed as considered in [10].

Frequency-domain scheduling was believed as unnecessary for full digital massive MIMO systems because the sufficiently large number of antennas can harden the channels and provide plenty spatial degrees of freedom for multiplexing users [11]. However, when considering the above mentioned implementation constraints and array size limitation, the number of antennas and RF chains at the BS cannot be very large, for instance, recently 3GPP LTE Release 13 has specified that each BS can have at most 64 antennas and at least 8 RF chains [12]. When considering these practical restrictions, the necessity of frequency-domain scheduling in the near future massive MIMO systems with hybrid structure is actually unclear. With frequency-domain scheduling, different users are served over different subcarriers, making the existing narrowband hybrid precoders no longer applicable and the joint optimization of wideband analog precoder and digital precoders very challenging.

In this paper, we study the design of wideband hybrid precoder for downlink space-division multiple-access and orthogonal frequency-division multiple-access (SDMA-OFDMA) massive MIMO systems, based on which the necessity of frequency-domain scheduling and the effectiveness of hybrid structure in wideband systems are analyzed. Aimed at maximizing the sum rate of all served users, we first derive an alternating optimization algorithm to optimize the wideband hybrid precoder, then an efficient low-complexity non-iterative hybrid precoder is proposed, in both of which the phase-only constraint are considered for analog precoder. Simulation results show that both the necessity of frequency scheduling and the effectiveness of wideband hybrid precoder depend on the employed scheduling method and the number of available RF chains.

II. System Model

Consider the downlink transmission of a single-cell SDMA-OFDMA massive MIMO system, where the BS equipped with M antennas and L RF chains serves K single-antenna users over N resource blocks (RBs). Let $\mathcal{K} = \{1, \dots, K\}$ and $\mathcal{K}_n$ denote the set of candidate users and the set of the scheduled users to be served on the n-th RB, respectively, where $\mathcal{K}_n \subseteq \mathcal{K}$ for $n = 1, \dots, N$. By assuming frequency flat fading within each RB, the received signal of the k-th user served on the n-th RB (denoted by UE_{nk}) can be expressed as

$$y_{nk} = \mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nk} x_{nk} + \sum\nolimits_{\substack{j \in \mathcal{K}_n \\ j \neq k}} \mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nj} x_{nj} + z_{nk}, \quad (1)$$

where $\mathbf{h}_{nk} \in \mathbb{C}^{M \times 1}$ is the downlink channel of UE_{nk}, $\mathbf{V} \in \mathbb{C}^{M \times L}$ is the wideband analog precoder, $\mathbf{w}_{nk} \in \mathbb{C}^{L \times 1}$ is the digital beamforming for UE_{nk}, x_{nk} is the data symbol for UE_{nk} with $\mathbb{E}\{|x_{nk}|^2\} = 1$, and z_{nk} is the additive white Gaussian noise (AWGN) with zero mean and variance σ_{nk}^2.

We assume that perfect channels are available at the BS, which can be obtained in time-division duplex (TDD) systems via uplink training. Then, the hybrid precoder optimization problem, aimed at maximizing the sum rate of all served users, can be formulated as follows

$$\max_{\mathbf{V},\{\mathbf{w}_{nk}\}} \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n}\log(1+\mathrm{SINR}_{nk}) \tag{2a}$$

$$s.t. \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n}|\mathbf{V}\mathbf{w}_{nk}|^2\leq P_{max} \tag{2b}$$

$$|[\mathbf{V}]_{ml}|=1,\ m=1,\dots,M,l=1,\dots,L, \tag{2c}$$

where $\mathrm{SINR}_{nk}=\frac{|\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nk}|^2}{\sum_{j\in\mathcal{K},j\neq k}|\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nj}|^2+\sigma_{nk}^2}$ represents the signal-to-interference-plus-noise ratio (SINR) of UE_{nk}, (2b) is the sum power constraint with P_{max} denoting the maximal transmit power of the BS, and (2c) restricts that the analog precoder $\mathbf{V}$ can only adjust phases as considered in most of previous works on hybrid precoder design [3–9]. Herein, $[\cdot]_{ml}$ denotes the element at the m-th row and l-th column of a matrix.

III. Hybrid Precoder Optimization

Problem (2) is non-convex because both the objective function and the phase-only constraints (2c) are non-convex, making it difficult to find its globally optimal solution. To develop efficient solutions to problem (2), in the sequel we first optimize the analog precoder and digital precoder by omitting the phase-only constraints (2c), and then perform element-wise normalization to the obtained analog precoder to ensure (2c), based on which the digital precoder is updated accordingly.

We propose two algorithms to optimize the hybrid precoder. We start with the alternating optimization between the analog and digital precoders, which will serve as a performance baseline of the subsequently proposed low-complexity non-iterative algorithm.

A. Alternating Optimization Algorithm

By omitting the phase-only constraint (2c) and based on the equivalence between sum rate maximization problem and weighted sum mean square error (MSE) minimization problem established in [13], we can transform problem (2) as

$$\min_{\mathbf{V},\{\mathbf{w}_{nk},t_{nk}\}} \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n} t_{nk}\epsilon_{nk}-\log(t_{nk}) \tag{3a}$$

$$s.t. \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n}|\mathbf{V}\mathbf{w}_{nk}|^2\leq P_{max}, \tag{3b}$$

where ϵ_{nk} and t_{nk} are the MSE and weight for the data of UE_{nk}, respectively. Denoting g_{nk} as the receive filter of UE_{nk}, ϵ_{nk} can be expressed as

$$\epsilon_{nk}=\sum_{j\in\mathcal{K}_n}|g_{nk}\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nj}|^2-2\Re\{g_{nk}\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nk}\}+|g_{nk}|^2\sigma_{nk}^2+1,$$

where $\Re\{\cdot\}$ denotes the real part of a complex number.

Problem (3) is not jointly convex for $\mathbf{V}$, $\mathbf{w}_{nk}$, t_{nk} and g_{nk}, but is convex for each individual variable when given the others. Therefore, we use a standard alternating optimization method to solve problem (3). Due to the lack of space, we omit the detailed derivations and summarize the algorithm as follows.

1) Initialize $\mathbf{V}$ and $\{\mathbf{w}_{nk}\}$ that satisfy the constraint (3b).
2) Calculate $g_{nk}=\frac{\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nk}}{\sum_{j\in\mathcal{K}_n}|\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nj}|^2+\sigma_{nk}^2}$.
3) Calculate ϵ_{nk} and then $t_{nk}=1/\epsilon_{nk}$.
4) Calculate the digital precoder as

$$\mathbf{w}_{nk}=t_{nk}g_{nk}^H\big(\sum_{j\in\mathcal{K}_n}t_{nj}|g_{nj}|^2\mathbf{V}^H\mathbf{h}_{nj}\mathbf{h}_{nj}^H\mathbf{V}+\lambda_w\mathbf{V}^H\mathbf{V}\big)^{-1}\mathbf{V}^H\mathbf{h}_{nk}, \tag{4}$$

where λ_w is the lagrangian multiplier, which can be found easily according to the Karush-Kuhn-Tucker (KKT) complementarity condition.
5) Calculate the analog precoder as

$$\mathbf{v}=\Big(\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n}\big(t_{nk}|g_{nk}|^2\sum_{j\in\mathcal{K}_n}(\mathbf{w}_{nj}^H\mathbf{w}_{nj})^T\otimes(\mathbf{h}_{nk}^H\mathbf{h}_{nk})+\lambda_v(\mathbf{w}_{nk}\mathbf{w}_{nk}^H)^T\otimes\mathbf{I}\big)\Big)^{-1}\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_n}t_{nk}g_{nk}^H(\mathbf{w}_{nk}^H)^T\otimes\mathbf{h}_{nk}, \tag{5}$$

where $\mathbf{v}$ is the vectorization of $\mathbf{V}$, λ_v is the lagrangian multiplier and can be found from the KKT complementarity condition, and $\otimes$ denotes kronecker product.
6) Repeat steps $2\sim5$ until convergence.

The alternating optimization algorithm can converge to at least a local optimal solution because the objective function is lower bounded and monotonically decreases with the growth of iterations.

The obtained analog precoder $\mathbf{V}$ is then normalized element-wisely to ensure the phase-only constraint (2c), i.e., update $[\mathbf{V}]_{ml}\leftarrow\frac{[\mathbf{V}]_{ml}}{|[\mathbf{V}]_{ml}|}$. Given the updated phase-shifter based $\mathbf{V}$, we repeat the steps $2\sim4$ of the above algorithm until convergence to update the digital precoder $\mathbf{w}_{nk}$.

The alternating optimization algorithm is of very high complexity because of the iterative optimization between the analog and digital precoders as well as the high-dimensional $(ML\times ML)$ matrix inverse and multiplications involved in (5) when calculating the analog precoder. Next, we propose a low-complexity algorithm without requiring the iteration between the analog and digital precoders.

B. Low-complexity Algorithm

In order to avoid the iteration between analog and digital precoders, in the following method we employ the sum capacity of the equivalent downlink channels $\mathbf{h}_{nk}^H\mathbf{V}$ of all users as the objective function, which is achieved by using dirty paper coding as digital precoders and thus is an upper bound of the sum rate achieved by the considered linear digital precoders. Based on the uplink-downlink duality theory [14] and after some manipulations, we can obtain the optimization problem only with respect to the analog precoder as

$$\max_{\mathbf{V}}\max_{\{\mathbf{D}_n\}}\sum_{n=1}^{N}\log\Big(\frac{\det(\mathbf{V}^H\mathbf{H}_n\mathbf{\Sigma}_n^{\frac{1}{2}}\mathbf{D}_n\mathbf{\Sigma}_n^{\frac{1}{2}}\mathbf{H}_n^H\mathbf{V}+\mathbf{V}^H\mathbf{V})}{\det(\mathbf{V}^H\mathbf{V})}\Big) \tag{6a}$$

$$s.t. \sum_{n=1}^{N}\mathrm{Tr}(\mathbf{D}_n)\leq P_{max}, \tag{6b}$$

where $\mathbf{D}_n\in\mathbb{R}^{|\mathcal{K}_n|\times|\mathcal{K}_n|}$ is a diagonal matrix whose i-th diagonal element stands for the transmit power of $\mathrm{UE}_{n\mathcal{K}_n(i)}$ in the dual uplink, $\mathbf{H}_n\in\mathbb{C}^{M\times|\mathcal{K}_n|}$ is the channel matrix of the users in $\mathcal{K}_n$, $\mathbf{\Sigma}_n\triangleq\mathrm{diag}\{[1/\sigma_{n\mathcal{K}_n(1)}^2,\dots,1/\sigma_{n\mathcal{K}_n(|\mathcal{K}_n|)}^2]\}$,

$\mathcal{K}_n(i)$ denotes the i-th element of $\mathcal{K}_n$, $|\mathcal{K}_n|$ denotes the cardinality of $\mathcal{K}_n$, and $\text{diag}\{\mathbf{x}\}$ denotes a diagonal matrix whose main diagonal is given by the elements of vector $\mathbf{x}$.

By defining $\bar{\mathbf{H}}_n = \mathbf{H}_n \mathbf{\Sigma}_n^{\frac{1}{2}}$ and exploiting the properties of matrix determinant, we can rewrite problem (6) as

$$\max_{\mathbf{V}} \max_{\{\mathbf{D}_n\}} \sum_{n=1}^{N} \log\det(\mathbf{D}_n \bar{\mathbf{H}}_n^H \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H \bar{\mathbf{H}}_n + \mathbf{I}) \quad (7a)$$

$$s.t. \ \sum_{n=1}^{N} \text{Tr}(\mathbf{D}_n) \leq P_{max}. \quad (7b)$$

In the objective function (7a), the term $\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H$ is a projection matrix, which can be expressed as $\mathbf{U}\mathbf{U}^H$ with $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. This leads to the following proposition.

Proposition 1: *An analog precoding matrix with orthogonal columns is optimal for sum capacity maximization.*

Proposition 1 provides a theoretical basis for the existing works, e.g., [6] and [7], in which the analog precoder has orthogonal columns selected from a discrete Fourier transform (DFT) matrix.

Without loss of generality, we select $\mathbf{V}$ such that $\mathbf{V}^H\mathbf{V} = \mathbf{I}$ and rewrite problem (7) as

$$\max_{\mathbf{V}} \max_{\{\mathbf{D}_n\}} \sum_{n=1}^{N} \log\det(\mathbf{D}_n \bar{\mathbf{H}}_n^H \mathbf{V}\mathbf{V}^H \bar{\mathbf{H}}_n + \mathbf{I}) \quad (8a)$$

$$s.t. \ \sum_{n=1}^{N} \text{Tr}(\mathbf{D}_n) \leq P_{max} \quad (8b)$$

$$\mathbf{V}^H\mathbf{V} = \mathbf{I}. \quad (8c)$$

In problem (8) $\mathbf{V}$ is still coupled with $\mathbf{D}_n$. In order to reduce the complexity, we first find a suboptimal solution to $\mathbf{D}_n$. By assuming that the system is full digital, i.e., $L = M$, and the channels of all users are orthogonal, we can obtain that the optimal $\mathbf{D}_n$ maximizing the sum capacity has the water-filling structure $[\mathbf{D}_n]_{ii} = \left(\mu - \frac{\sigma^2_{n\mathcal{K}_n(i)}}{\|\mathbf{h}_{n\mathcal{K}_n(i)}\|^2}\right)^+$, where μ is the water-level variable that is chosen to ensure $\sum_{n=1}^{N} \text{Tr}(\mathbf{D}_n) = P_{max}$, and the operator $(x)^+ \triangleq \max(x, 0)$.

When given $\mathbf{D}_n$, problem (8) is still not convex for $\mathbf{V}$ due to the non-convexity of both objective function and constraint (8c). To solve the problem, we resort to the semi-definite relaxation (SDR) method [15, 16]. Specifically, by defining $\mathbf{V}\mathbf{V}^H \triangleq \mathbf{Q}$, we can convert problem (8) into

$$\max_{\mathbf{Q}} \ \sum_{n=1}^{N} \log\det(\mathbf{D}_n \bar{\mathbf{H}}_n^H \mathbf{Q} \bar{\mathbf{H}}_n + \mathbf{I}) \quad (9a)$$

$$s.t. \ \text{Tr}(\mathbf{Q}) = L \quad (9b)$$

$$\mathbf{Q} \succeq 0 \quad (9c)$$

$$\text{rank}(\mathbf{Q}) = L, \quad (9d)$$

where (9b) comes from (8c) by computing the trace of the matrices at both sides of the equality, (9c) denotes that $\mathbf{Q}$ is positive semi-definite, and (9d) comes from the definition of $\mathbf{Q}$. Note that different from the widely used vector-lifting SDR method [15] that first converts $\mathbf{V}$ into a $ML{\times}1$ vector and then introduces a rank-one constrained positive semi-definite matrix $\mathbf{Q}$ with the size of $ML \times ML$, we employ the matrix-lifting SDR method [16] that directly defines a lower-dimensional $(M \times M)$ matrix $\mathbf{Q}$ with rank L. This significantly reduce the computational complexity for the optimization of $\mathbf{Q}$.

By removing the rank constraint (9d), problem (9) is relaxed to a semi-definite programming problem, which can be efficiently solved by using the algorithm proposed in [17]. Given the SDR solution $\mathbf{Q}$, we next employ the Gaussian randomization method to obtain the analog precoder $\mathbf{V}$. The procedure is as follows.

1) Generate multiple, say S, independent and identically distributed (i.i.d.) complex Gaussian random matrices following the distribution $\mathcal{CN}(\mathbf{0}, \mathbf{Q})$, each with the size of $M \times L$.
2) Orthogonalize the columns of each generated matrix, e.g., by Gram-Schmidt method, and denote the resulting orthogonal matrices as $\hat{\mathbf{V}}_1, \ldots, \hat{\mathbf{V}}_S$.
3) Given $\hat{\mathbf{V}}_s$ for $s = 1, \ldots, S$, compute the values of the objective function (9a), denoted by $C_1, \ldots, C_S$.
4) Find $i = \arg\max\{C_1, \ldots, C_S\}$, and obtain $\mathbf{V} = \hat{\mathbf{V}}_i$.

Then, to ensure the phase-only constraint (2c), we take element-wise normalization over $\mathbf{V}$ as specified before. Given the phase-shifter based analog precoder, we can obtain the digital precoders by repeating the steps $2 \sim 4$ of the previously proposed alternating optimization algorithm.

C. Effectiveness of Hybrid Precoder

With reduced number of RF chains, in general hybrid precoder pays the penalty of performance loss compared to the full digital precoder even without the phase-only constraint for analog precoder. However, the performance loss can be avoided under the following condition.

Proposition 2: *If the number of RF chains satisfies $L \geq \min(M, \sum_{n=1}^{N} |\mathcal{K}_n|)$, then hybrid precoder without the phase-only constraint for analog precoder can achieve the same performance as full digital precoder.*

The proof is omitted due to the lack of space. We can see that the condition on the number of RF chains is natural for narrowband systems because the number of spatially multiplexed users should be no more than the number of RF chains. Yet, this condition will not hold for wideband systems when N is large. Therefore, a larger performance gap between the hybrid precoder and full digital precoder in wideband systems can be expected compared to narrowband systems.

IV. Simulation Results

In this section, we evaluate the performance of the proposed wideband hybrid precoders. Unless otherwise specified, the following parameters will be used throughout the simulations. The BS equipped $M = 64$ antennas serves in total $K = 16$ single-antenna users, where $|\mathcal{K}_n| = 8$ users are scheduled on each RB and different numbers of RF chains will be considered. The number of RBs for data transmission is set as $N = 32$, corresponding to the first 32 RBs of the total 50 RBs in LTE systems with the bandwidth of 10 MHz. The small-scale channels are generated based on the 3GPP 3D-MIMO channel model in the urban macro-cell non-line of sight (UMa-NLoS) scenario with 8×8 planar antenna array, where the users are randomly dropped in the 3D space with elevation angle ranging $[-45^\circ, 45^\circ]$ and azimuth angle ranging

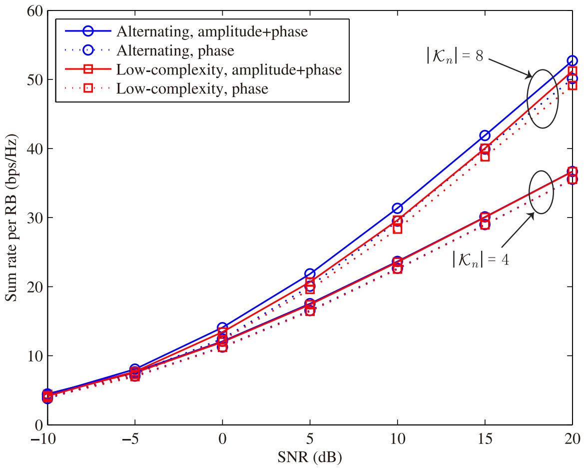

Fig. 1. Performance comparison of the proposed algorithms.

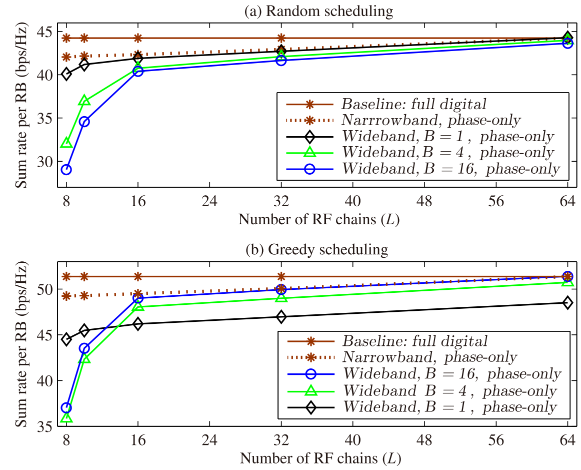

Fig. 2. Impact of the number of RF chains and frequency-domain scheduling.

$[-60°, 60°]$ [18]. The signal-to-noise ratio (SNR) of all users is 10 dB. All the results are averaged over 100 random drops.

To analyze the impact of frequency-domain scheduling, we divide all RBs into B groups and schedule the same users for the RBs in each group. If $B = 1$, all RBs schedule the same users and the system operates in SDMA-OFDM manner; otherwise, the system is in SDMA-OFDMA manner. We consider two frequency-domain scheduling methods, *random scheduling* that selects users randomly for each group and *greedy scheduling* that selects users providing the maximal sum rate for each group. Note that when greedy scheduling is applied, the achieved multiuser diversity gain increases with B, and the largest multiuser diversity gain is obtained when $B = N$, where every RB can schedule its own favourite users.

In Fig. 1, we compare the performance of the proposed algorithms, where $L = 8$ RF chains, $B = 16$ groups and random scheduling are considered. It is shown that the proposed low-complexity algorithm performs very close to the alternating optimization algorithm no matter when $|\mathcal{K}_n| = 4$ or 8 users are scheduled in each RB. We can see that the phase-only constraint on analog precoder leads to slight performance loss for both algorithms.

In Fig. 2, we analyze the necessity of frequency-domain scheduling and the effectiveness of hybrid precoder in wideband systems, where $|\mathcal{K}_n| = 8$ users are scheduled on each RB, the proposed low-complexity algorithm is applied, and per-RB sum rate is plotted for a fair comparison between narrowband and wideband systems. In Fig. 2(a) and (b) the "Baseline" curve denotes the per-RB performance of full digital precoder, which is identical for both narrowband and wideband systems.

When random scheduling and phase-only analog precoder are considered in Fig. 2(a), we can observe a larger performance gap between hybrid precoder and full digital precoder in wideband systems compared to narrowband systems even when $B = 1$, i.e., SDMA-OFDM systems, because the analog precoder is wideband which is designed based on the frequency-selective channels of all RBs rather than the specific channel of a single RB in narrowband systems. Moreover, the performance gap is enlarged when more users are scheduled (i.e., larger B) because random scheduling provides no multiuser diversity gain and serving more users only degrades the effectiveness of analog precoder on providing large array gain.

Nevertheless, the results change when greedy scheduling is applied due to the presence of multiuser diversity gain. Now scheduling more users (i.e., increasing B) has two-fold impacts, which increases the multiuser diversity gain but reduces the array gain obtained by analog precoder as mentioned before. However, the array gain loss for analog precoder can be well compensated by digital precoders when the number of RF chains L is large. Therefore, when both B and L are large, the achieved multiuser diversity gain will be greater than the array gain loss, and then the performance can be improved. It is shown in Fig.2(b) that when $L \geq 16$, scheduling more users in frequency domain can improve the performance, and the wideband hybrid precoder can perform as well as the narrowband hybrid precoder when $B = 16$.

V. Conclusions

In this paper, we studied the design of wideband hybrid precoder and investigated the necessity of frequency scheduling and the effectiveness of hybrid precoder in SDMA-OFDMA massive MIMO systems. We first derived an alternating optimization algorithm, which requires the iteration between analog precoder and digital precoders, then an efficient non-iterative algorithm was proposed to reduce the complexity. Simulation results show that both the necessity of frequency scheduling and the effectiveness of wideband hybrid precoder depend on the employed scheduling method and the number of available RF chains. When random scheduling is used, frequency scheduling degrades the performance and wideband hybrid precoder has a larger performance gap from full digital precoder compared to narrowband hybrid precoder. Nevertheless, when greedy scheduling toward maximizing the sum rate is used and the number of RF chains is large, frequency scheduling improves the performance and wideband hybrid precoder can perform as well as narrowband hybrid precoder.

References

[1] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, 2010.

[2] S. Han, C.-L. I, Z. Xu, and C. Rowell, "Large-scale antenna systems with hybrid analog and digital beamforming for millimeter wave 5G," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 186–194, 2015.

[3] Y.-Y. Lee, C.-H. Wang, and Y.-H. Huang, "A hybrid RF/baseband precoding processor based on parallel-index-selection matrix-inversion-bypass simultaneous orthogonal matching pursuit for millimeter wave MIMO systems," *IEEE Trans. Signal Processing*, vol. 63, no. 2, pp. 305–317, 2015.

[4] C.-E. Chen, "An iterative hybrid transceiver design algorithm for millimeter wave MIMO systems," *IEEE Wireless Communications Letters*, Early access.

[5] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. Heath, "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, 2014.

[6] A. Sayeed and J. Brady, "Beamspace MIMO for high-dimensional multiuser communication at millimeter-wave frequencies," in *Proc. IEEE GLOBECOM*, 2013.

[7] A. Liu and V. Lau, "Phase only RF precoding for massive MIMO systems with limited RF chains," *IEEE Trans. Signal Processing*, vol. 62, no. 17, pp. 4505–4515, 2014.

[8] T. Bogale and L. B. Le, "Beamforming for multiuser massive MIMO systems: Digital versus hybrid analog-digital," in *Proc. IEEE GLOBECOM*, 2014.

[9] L. Liang, W. Xu, and X. Dong, "Low-complexity hybrid precoding in massive multiuser MIMO systems," *IEEE Wireless Communications Letters*, vol. 3, no. 6, pp. 653–656, 2014.

[10] C. Kim, T. Kim, and J.-Y. Seol, "Multi-beam transmission diversity with hybrid beamforming for MIMO-OFDM systems," in *Proc. IEEE Globecom Workshops*, 2013.

[11] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, 2014.

[12] 3GPP TR 36.897, "Study on elevation beamforming/full-dimension (FD) MIMO for LTE," Tech. Rep., 2015.

[13] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Processing*, vol. 59, no. 9, pp. 4331–4340, 2011.

[14] W. Yu, "Uplink-downlink duality via minimax duality," *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 361–374, 2006.

[15] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Mag.*, vol. 27, no. 3, pp. 20–34, 2010.

[16] Y. Ding and H. Wolkowicz, "A low-dimensional semidefinite relaxation for the quadratic assignment problem," *Mathematics of Operations Research*, vol. 34, no. 4, pp. 1008–1022, 2009.

[17] E. Jorswieck, A. Sezgin, B. Ottersten, and A. Paulraj, "Feedback reduction in uplink MIMO OFDM systems by chunk optimization," *EURASIP Journal on Advances in Signal Processing*, no. 78, 2008.

[18] 3GPP TR 36.873, "Study on 3D channel model for LTE," Tech. Rep., 2015.