DEGREES OF FREEDOM OF GENERAL SYMMETRIC MIMO INTERFERENCE BROADCAST CHANNELS

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ABSTRACT

In this paper, we study the maximal achievable degrees of freedom (DoF) for G-cell, K-user $M \times N$ symmetric multi-input-multi-output (MIMO) interference broadcast channel (MIMO-IBC) with constant coefficients by linear interference alignment (IA). We start by analyzing the sufficient and necessary conditions of the IA feasibility, which are associated with generalized Fibonacci sequences. Except for the well-known proper condition, we find another condition necessary to ensure a kind of *irreducible interference* to be eliminated, denoted as *irreducible condition*. We proceed to find the minimal number of antennas at each base station and each user to support a required number of interference-free data streams, from which we obtain the maximal achievable DoF. We find that the feasibility conditions in terms of the required antenna resources can be divided into two regions according to the ratio of $\rho = M/N$. If ρ falls into the region that characterized by the proper condition, the achievable DoF per user with spatial extension is $(M+N)/(GK+1)$. If ρ lies in the region that characterized by the irreducible condition, the DoF is a piecewise linear function of M or N alternately.

***Index Terms*—** Interference alignment (IA), interference broadcast channel (IBC), degrees of freedom (DoF), IA feasibility conditions, irreducible interference

1. INTRODUCTION

The degrees of freedom (DoF) is a metric of great importance that reflects the potential of communication systems [1]. To derive the maximum DoF for multi-input-multi-output (MIMO) interference broadcast channel (MIMO-IBC) and MIMO interference channel (MIMO-IC) achieved by linear interference alignment (IA), many recent research devote to investigate the IA feasibility conditions.

For the MIMO-IC with constant coefficients (i.e., without time/frequency extension), a *proper condition* was first proposed in [2] by relating the linear IA feasibility to the problem of determining the solvability of a system of multivariate polynomial equations. So far, for a G-cell MIMO-IC where each base station (BS) is equipped with M antennas and each user has N antennas, the linear IA feasibility is well understood for some special cases, i.e., $M = N$ [3], either M or N is divisible by the number of data streams per user d [4], and $G = 3$ [1,5]. For the MIMO-IC with time/frequency varying channels, (i.e., with time/frequency extension), the feasibility conditions are available for $G = 3$ [1], but remain unknown for some cases of $G \geq 4$ [6]. For a G-cell K-user $M \times N$ MIMO-IBC with constant coefficients, the linear IA feasibility was analyzed for the cases where M or N is divisible by d [7]. Yet the feasibility analysis of linear IA for general symmetry MIMO-IC and MIMO-IBC is still an open problem.

In this paper, we strive to find the maximal DoF for general symmetric MIMO-IBC with constant coefficients achieved by linear IA. To this end, we first find the sufficient and necessary conditions of IA feasibility. Then, we investigate the minimal spatial resources required to transmit a given number of data streams without inter-cell interference (ICI) and multi-user interference, from which we obtain the maximal achievable DoF per user. Our study shows that except for the proper condition, the feasibility conditions also include an irreducible condition, which is comprised of a series of inequalities to ensure a sort of *irreducible ICI* to be eliminated. The irreducible ICI was defined in [7] as the ICIs whose dimensions cannot be reduced by designing the receive or transmit matrices. Based on the different impacts of the proper condition and the irreducible condition, the DoF can be divided into two regions according to the ratio of M/N. If M/N falls into the region that characterized by the proper condition, the BSs and users can share their spatial resources to remove the ICI, and the achievable DoF per user with spatial extension is $(M+N)/(GK+1)$. Otherwise, owing to the irreducible ICI, the BSs and users cannot eliminate ICI by sharing spatial resources, and the DoF is piecewise linear dependent on either M or N.

2. MAIN RESULTS

In this section, we first present the necessary and sufficient conditions of linear IA feasibility for symmetric MIMO-IBC, then derive and analyze the maximal achievable DoF. Due to the lack of space, we will not provide the proof, which can be found in [8] and checked by a test proposed in [9].

2.1. Necessary and Sufficient Conditions of IA feasibility

Theorem 1. *For a symmetric MIMO-IBC with generic channel matrices, the linear IA is feasible iff (if and only if) the following conditions are satisfied,*

$$M + N \geq (GK+1)d \tag{1a}$$

$$\max\{pM, qN\} \geq (pK+q)d, \ \forall (p,q) \in \mathcal{A} \cup \mathcal{B} \tag{1b}$$

where $\mathcal{A} \triangleq \left\{\left(p_n^A, q_n^A\right)\right\}$ *and* $\mathcal{B} \triangleq \left\{\left(p_n^B, q_n^B\right)\right\}$ *are interleaving generalized Fibonacci sequence-pairs satisfying,*

$$\begin{cases} p_{2n}^A = (G-1)p_{2n-1}^A - p_{2n-2}^A, \ p_{2n+1}^A = (G-1)Kp_{2n}^A - p_{2n-1}^A \\ q_{2n}^A = (G-1)q_{2n-1}^A - q_{2n-2}^A, \ q_{2n+1}^A = (G-1)Kq_{2n}^A - q_{2n-1}^A \end{cases}$$

$$\begin{cases} p_{2n}^B = (G-1)Kp_{2n-1}^B - p_{2n-2}^B, \ p_{2n+1}^B = (G-1)p_{2n}^B - p_{2n-1}^B \\ q_{2n}^B = (G-1)Kq_{2n-1}^B - q_{2n-2}^B, \ q_{2n+1}^B = (G-1)q_{2n}^B - q_{2n-1}^B \end{cases}$$

$$p_n^A, q_n^A, p_n^B, q_n^B \geq 0, \forall n \in \mathbb{Z}^+, \text{the set of positive integers,} \tag{2}$$

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and $(p_0^A, q_0^A) = (0,1)$, $(p_1^A, q_1^A) = (1, (G-1)K)$, $(p_0^B, q_0^B) = (1,0)$, $(p_1^B, q_1^B) = (G-1, 1)$.

(1a) is the *proper condition* for symmetric MIMO-IBC [2]. It ensures that there are enough antennas in the network to eliminate all the ICI.

(1b) contains a series of inequalities. Substituting $(p_0^A, q_0^A) = (0,1)$ and $(p_0^B, q_0^B) = (1,0)$ into (1b), we obtain $N \geq d$ and $M \geq Kd$, which ensure that there are enough antennas to convey desired signals. Substituting (p_1^A, q_1^A) and (p_1^B, q_1^B) into (1b), we have $\max\{M, (G-1)KN\} \geq GKd$ and $\max\{(G-1)M, N\} \geq ((G-1)K+1)d$, which was provided in [7] to ensure one class of *irreducible ICIs* to be eliminated. Substituting other values of $p_n^A, q_n^A, p_n^B, q_n^B$ in (2) into (1b), the resulting inequalities ensure other class of *irreducible ICIs* to be eliminated. Therefore, (1b) is called as *irreducible condition*.

2.2. Minimal Antenna Resource and Maximal Achievable DoF

The sequences in (2) are finite with length no more than six when $G = 2,\ K < 4$, and are always infinite otherwise. When the sequences are finite, we can derive the achievable DoF by enumerating all inequalities in (1b) and considering (1a). When the sequences are infinite, however, we need to find other way to derive the DoF. In the sequel, we consider the cases where $G = 2,\ K \geq 4$ or $G \geq 3$.[1] To understand the impact of (1b) on the achievable DoF, we list some values of $\{p_n^\alpha\}$ and $\{q_n^\alpha\}$, $\alpha = A, B$ in (2) in Table 1.

Remark 1. *In three-cell MIMO-IC, i.e.,* $G = 3,\ K = 1$*, we have* $p_n^A = q_n^B = n$ *and* $q_n^A = p_n^B = n+1$*, i.e., the generalized Fibonacci sequence reduces to the arithmetic sequence. All the sequences are monotonic and have explicit expressions, which allows us to analyze all the inequalities of* (1b) *in a unified way. In this case, it is easy to obtain the maximal achievable DoF. The result is the same as that found in [1, 5]. This means that the result of [1, 5] is a special case of ours.*

In two-cell MIMO-IBC, i.e., $G = 2$, $K > 1$, from Table 1 we have $\{q_n^A\} = \{1, K, K-1, K^2-2K, \cdots\}$, which is not monotonic any more. In fact, our study shows that except for the special case of three-cell MIMO-IC, $\{p_n^\alpha\}$ and $\{q_n^\alpha\}$ are infinite sequences without explicit expressions. As a result, it is non-trivial to find the minimum antenna resources or maximal achievable DoF from (1b). To circumvent this difficulty, we reexpress (1b) into a different form.

From (1b), we know that if $pM \geq qN$ (i.e., $M/N \geq q/p$), we have $pM \geq (pK+q)d$ (i.e., $M \geq (pK+q)d/p$). Otherwise, we have $N \geq (pK+q)d/q, \forall (p,q) \in \mathcal{A} \cup \mathcal{B}$. Therefore, (1b) can be rewritten as

$$\begin{cases} M \geq M_n^\alpha, & \forall \frac{M}{N} \geq C_n^\alpha \\ N \geq N_n^\alpha, & \text{otherwise.} \end{cases} \quad \forall n \in \mathbb{Z}^*, \alpha = A, B \tag{3}$$

where $\mathbb{Z}^* = \mathbb{Z}^+ \cup \{0\}$, the set of nonnegative integers,

$$C_n^\alpha \triangleq q_n^\alpha / p_n^\alpha \tag{4a}$$

$$M_n^\alpha \triangleq (Kp_n^\alpha + q_n^\alpha)d/p_n^\alpha = (K + C_n^\alpha)d \tag{4b}$$

$$N_n^\alpha \triangleq (Kp_n^\alpha + q_n^\alpha)d/q_n^\alpha = (1 + K/C_n^\alpha)\, d \tag{4c}$$

In Table 1, we also list some values of $\{C_n^\alpha\}$, $\alpha = A, B$, which show that $C_n^\alpha,\ \forall n \in \mathbb{Z}^+$ can be expressed as a form of *generalized continued fraction* [10] as,

$$\begin{aligned} C_n^A &= (G-1)K - \cfrac{1}{G-1-\cfrac{1}{(G-1)K-\cfrac{1}{\cdots}}} \\ C_n^B &= \cfrac{1}{G-1-\cfrac{1}{(G-1)K-\cfrac{1}{G-1-\cfrac{1}{\cdots}}}} \end{aligned} \tag{5}$$

According to the properties of continue fraction [10, Theorem 2.1], $C_n^\alpha,\ \forall n \in \mathbb{Z}^+$ can also be expressed as

$$C_n^A = (G-1)K - \sum_{j=1}^{n-1} \frac{1}{p_j^A p_{j+1}^A},\ C_n^B = \sum_{j=1}^{n} \frac{1}{p_j^A p_{j+1}^A} \tag{6}$$

and satisfy

$$C_{n+1}^A + \frac{K}{C_n^A} = C_n^B + \frac{K}{C_{n+1}^B} = (G-1)K,\ \forall n \in \mathbb{Z}^* \tag{7}$$

From (6) and (7), it is not difficult to derive

$$C_n^B < C_{n+1}^B < C_{n+1}^A < C_n^A,\ \forall n \in \mathbb{Z}^* \tag{8}$$

and

$$\begin{aligned} C_\infty^A &\triangleq \lim_{n\to\infty} C_n^A = \tfrac{(G-1)K + \sqrt{(G-1)^2K^2 - 4K}}{2} \\ C_\infty^B &\triangleq \lim_{n\to\infty} C_n^B = \tfrac{(G-1)K - \sqrt{(G-1)^2K^2 - 4K}}{2} \end{aligned} \tag{9}$$

By substituting (6) and (8) into (4b) and (4c), we obtain the explicit expressions of $M_n^\alpha,\ N_n^\alpha$, which are also monotonic. Since $\{C_n^\alpha\}$, $\{M_n^\alpha\}$, $\{N_n^\alpha\}$ are monotonic and have explicit expressions, we can find the required minimal antenna resources to ensure the IA feasible and the maximal achievable DoF from (3) and (1a).

Corollary 1. *For a symmetric MIMO-IBC with constant coefficients, to support* d *interference-free data streams for each user by linear IA, the minimal numbers of antennas should satisfy*

$$\begin{cases} N \geq N_n^A, & \forall D_n^A \leq \frac{M}{N} \leq C_n^A \\ M \geq M_{n+1}^A, & \forall C_{n+1}^A \leq \frac{M}{N} \leq D_n^A \end{cases},\ \forall n \in \mathbb{Z}^* \tag{10a}$$

$$M + N \geq (GK+1)d,\ \forall C_\infty^B \leq \frac{M}{N} \leq C_\infty^A \tag{10b}$$

$$\begin{cases} N \geq N_{n+1}^B, & \forall D_n^B \leq \frac{M}{N} \leq C_{n+1}^B \\ M \geq M_n^B, & \forall C_n^B \leq \frac{M}{N} \leq D_n^B \end{cases},\ \forall n \in \mathbb{Z}^* \tag{10c}$$

where $D_n^A \triangleq M_{n+1}^A / N_n^A$ *and* $D_n^B \triangleq M_n^B / N_{n+1}^B$, $\forall n \in \mathbb{Z}^*$.

Proof. See [8]. □

According to Corollary 1, we show the feasible and infeasible regions in terms of the required numbers of antennas for a G-cell MIMO-IBC in Fig. 1.

As shown in Fig. 1, the feasible region can be divided into two regions according to the ratio of M/N, i.e.,

$$\begin{aligned} &\text{Region I}: && C_\infty^B < \tfrac{M}{N} < C_\infty^A \\ &\text{Region II}: && 0 \leq \tfrac{M}{N} \leq C_\infty^B \text{ or } C_\infty^A \leq \tfrac{M}{N} \end{aligned} \tag{11}$$

The IA feasibility is only determined by (1a) (i.e., the proper condition) in Region I, and only by (1b) (i.e., the irreducible condition) in Region II.

The feasible region in Region II is divided by a piecewise linear function of (M, N), which has two classes of corner points, one class includes (M_{n+1}^A, N_n^A) and (M_n^B, N_{n+1}^B), $\forall n \in \mathbb{Z}^*$, and other class includes (M_n^A, N_n^A) and (M_n^B, N_n^B), $\forall n \in \mathbb{Z}^+$.

Substituting (7) into (4b) and (4c), we have

$$M_{n+1}^A + N_n^A = M_n^B + N_{n+1}^B = (GK+1)d,\ \forall n \in \mathbb{Z}^* \tag{12}$$

which indicates that the former class of corner points is on the curve of $M + N = (GK+1)d$.

[1] The derivations also apply for the case of $G = 2,\ K < 4$, the only difference is that the sequences will be finite here.

Table 1. The Values of the Sequences

n	0	1	2	3	4	$\cdots$
p_n^A	0	1	$G-1$	$(G-1)^2K-1$	$(G-1)^3K-2(G-1)$	$\cdots$
q_n^A	1	$(G-1)K$	$(G-1)^2K-1$	$(G-1)^3K^2-2(G-1)K$	$(G-1)^4K^2-3(G-1)^2K+1$	$\cdots$
C_n^A	∞	$(G-1)K$	$(G-1)K-\frac{1}{G-1}$	$(G-1)K-\frac{1}{G-1-\frac{1}{(G-1)K}}$	$(G-1)K-\frac{1}{G-1-\frac{1}{(G-1)K-\frac{1}{G-1}}}$	$\cdots$
p_n^B	1	$G-1$	$(G-1)^2K-1$	$(G-1)^3K-2(G-1)$	$(G-1)^4K^2-3(G-1)^2K+1$	$\cdots$
q_n^B	0	1	$(G-1)K$	$(G-1)^2K-1$	$(G-1)^3K^2-2(G-1)K$	$\cdots$
C_n^B	0	$\frac{1}{G-1}$	$\frac{1}{G-1-\frac{1}{(G-1)K}}$	$\frac{1}{G-1-\frac{1}{(G-1)K-\frac{1}{G-1}}}$	$\frac{1}{G-1-\frac{1}{(G-1)K-\frac{1}{G-1-\frac{1}{(G-1)K}}}}$	$\cdots$

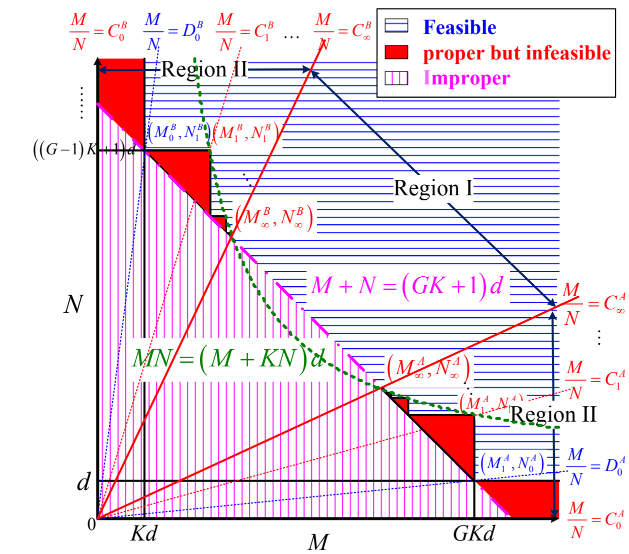

Fig. 1. Feasible and infeasible regions of linear IA for symmetric MIMO-IBC to support d data streams per user.

From (4b) and (4c), we can obtain

$$\frac{M_n^\alpha N_n^\alpha}{M_n^\alpha + KN_n^\alpha} = \frac{\frac{(Kp_n^\alpha+q_n^\alpha)^2 d^2}{p_n^\alpha q_n^\alpha}}{\frac{(Kp_n^\alpha+q_n^\alpha)d}{p_n^\alpha} + \frac{K(Kp_n^\alpha+q_n^\alpha)d}{q_n^\alpha}} = d,\ \forall n \in \mathbb{Z}^+ \tag{13}$$

which shows that the latter class of corner points is on the curve of $MN = (M+KN)d$.

Therefore, the boundary of feasible region in Region II is a piecewise linear function between $M+N = (GK+1)d$ and $MN = (M+KN)d$, and $(M_\infty^\alpha, N_\infty^\alpha)$ is the interaction point of these two curves.

Remark 2. *For a three-cell MIMO-IC, the IA feasibility cannot be derived from the proper condition, as shown in [5]. This is because for the two cases of $G=3$, $K=1$ or $G=2$, $K\leq 4$, Region I is empty. For other cases, the proper condition determines the IA feasibility if the antenna configuration falls into Region I.*

Corollary 2. *For a symmetric MIMO-IBC with constant coefficients, given the numbers of antennas at each BS and user M and N, the maximal achievable DoF per user by linear IA is bounded as*

$$d(M,N) \leq d^*(M,N) \tag{14}$$

$$d^*(M,N) = \begin{cases} \frac{q_n^A}{Kp_n^A+q_n^A}N, & \forall\, D_n^A \leq \frac{M}{N} \leq C_n^A \\ \frac{p_{n+1}^A}{Kp_{n+1}^A+q_{n+1}^A}M, & \forall\, C_{n+1}^A \leq \frac{M}{N} \leq D_n^A \end{cases},\ \forall n\in\mathbb{Z}^* \\ \frac{M+N}{GK+1}, \quad \forall C_\infty^B \leq \frac{M}{N} \leq C_\infty^A \\ \begin{cases} \frac{q_{n+1}^B}{Kp_{n+1}^B+q_{n+1}^B}N, & \forall\, D_n^B \leq \frac{M}{N} \leq C_{n+1}^B \\ \frac{p_n^B}{Kp_n^B+q_n^B}M, & \forall\, C_n^B \leq \frac{M}{N} \leq D_n^B \end{cases},\ \forall n\in\mathbb{Z}^* \tag{15}$$

Corollary 2 can be obtained from Corollary 1 immediately.

3. DISCUSSION ON THE MAIN RESULTS

3.1. Spatial Extension versus Time/Frequency Extension

The DoF per user $\lfloor d^*(M,N)\rfloor$ is achievable by linear IA without any symbol extension, where exists a DoF loss owing to rounding. It is worth to note that when considering spatial extension [1], the achievable DoF will be $d^*(M,N)$ for arbitrary M,N. In [1], the achievable DoF with spatial extension is defined as $\bar{d}(M,N) \triangleq \max_{m\in\mathbb{Z}^+}\{d(mM,mN)/m\}$, where $d(\cdot)$ is the achievable DoF without spatial extension and m is a finite integer and denoted as the spatial extension factor. It means that $m\bar{d}(M,N)$ DoF per user is achievable for the MIMO-IBC when each BS and user are equipped with mM and mN antennas. Consequently, with the finite spatial extension, the achievable DoF per user is not necessary to be an integer, which can avoid the loss of DoF caused by rounding.

According to Corollary 2, we illustrate the maximal achievable DoF with spatial extension as the solid curve in Fig. 2.

When the antenna configuration lies in Region I, from (15) the achievable DoF with spatial extension can be obtained as $\bar{d}(M,N) = (M+N)/(GK+1)$. When the antenna configuration lies in Region II, $\bar{d}(M,N)$ becomes a piecewise linear function between $MN/(M+KN)$ and $(M+N)/(GK+1)$, since the boundary of feasible region for given number of data steam per user is a piecewise linear segment between $M+N=(GK+1)d$ and $MN=(M+KN)d$, as shown in Fig. 1.

In [6], the authors investigated a *decomposition DoF bound*, which is defined as the achieved DoF in time/frequency varying channels by first decomposing the antennas at both transmitter and receiver sides and then using the asymptotic alignment (i.e., infinite time/frequency extension). The *decomposition DoF bound* for MIMO-IC was obtained as $\tilde{d}(M,N) = MN/(M+N)$. This result can be immediately extended into MIMO-IBC as follows

$$\tilde{d}(M,N) = MN/(M+KN) \tag{16}$$

which is shown as the dotted curve in Fig. 2.

As shown in Fig. 1, $(M_\infty^\alpha, N_\infty^\alpha)$ is the interaction point between

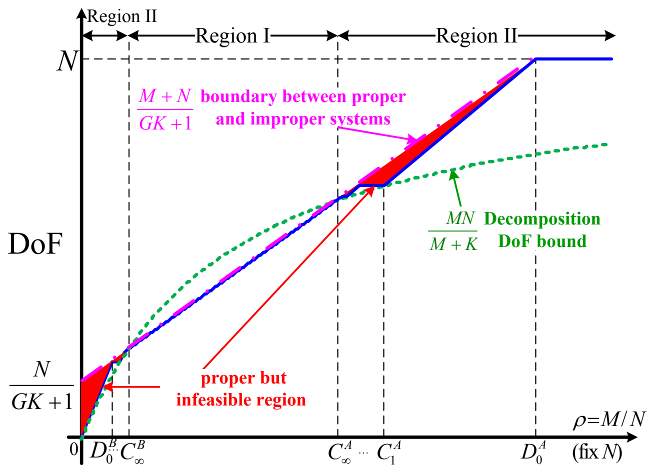

Fig. 2. Maximal achievable DoF with spatial extension.

$M+N=(GK+1)d$ and $MN=(M+KN)d$. Consequently, if $M_\infty^B/N_\infty^B = C_\infty^B < M/N < M_\infty^A/N_\infty^A = C_\infty^A$ (i.e., in Region I), $MN/(M+KN) > (M+N)/(GK+1)$. Otherwise, $MN/(M+KN) \leq (M+N)/(GK+1)$. Therefore, we have

$$\begin{array}{ll} \text{in Region I,} & \bar{d}(M,N) < \tilde{d}(M,N) \\ \text{in Region II,} & \bar{d}(M,N) \geq \tilde{d}(M,N) \end{array} \tag{17}$$

In Region I, the analysis in [6] indicates that the *decomposition DoF bound* is the information theoretic maximal DoF for MIMO-IC with time/frequcency varying channels. This conclusion also holds for MIMO-IBC. From (17), we know that the achievable DoF with spatial extension is lower than the information theoretic maximal DoF.

In Region II, the achievable DoF with spatial extension is higher than or equal to the *decomposition DoF bound*. Moreover, from the analysis in [1,5], we know that the DoF derived from the irreducible condition is the information theoretic maximal DoF for MIMO-IC with both constant coefficient and time/frequency varying channels. Again, this conclusion also holds for MIMO-IBC. This indicates that, the information theoretic maximal DoF can be achieved by linear IA with finite spatial extension[2]. In other words, in this case the infinite time/frequency extension is no longer necessary.

Remark 3. *When $G \geq 4$, $K=1$, the analysis in [6] showed the information theoretic maximal DoF for MIMO-IC with $M/N \leq (G-1)/(G^2-2G)$ as*

$$\bar{d}(M,N) = \begin{cases} M, & \forall\, 0 \leq \frac{M}{N} \leq \frac{1}{G} \\ \frac{N}{G}, & \forall\, \frac{1}{G} \leq \frac{M}{N} \leq \frac{1}{G-1} \\ \frac{(G-1)M}{G}, & \forall\, \frac{1}{G-1} \leq \frac{M}{N} \leq \frac{G}{G^2-G-1} \\ \frac{(G-1)N}{G^2-G-1}, & \forall\, \frac{G}{G^2-G-1} \leq \frac{M}{N} \leq \frac{G-1}{G^2-2G} \end{cases} \tag{18}$$

The same result can be also obtained by substituting the values of p_n^B and q_n^B for $K=1$, $n \leq 3$ into (15)*, where the linear IA can achieve the information theoretic optimal DoF based on the above analysis. Moreover, by substituting the values of p_n^B and q_n^B for $K=1$, $n \geq 3$ into* (15)*, we can obtain the information theoretic maximal DoF for the cases that are unsolved in [6].*

[2]In this study, the linear IA considers the case either with finite spatial symbol extension (without infinite time/frequency extension) or with no symbol extension at all.

3.2. ICI eliminated proportion

To support K users each with d interference-free data streams, each BS can provide $M-Kd$ antennas to eliminate ICI and each user can provide $N-d$ antennas [7]. Define

$$r_M \triangleq \frac{M-Kd}{(G-1)Kd},\ r_N \triangleq \frac{N-d}{(G-1)Kd} \tag{19}$$

as the proportions of the ICIs that can be eliminated either by all the BSs or by all the users.

When $\rho \triangleq M/N$ falls into Region I, the information theoretic maximal DoF is the *decomposition DoF bound* found in (16). From this bound, we can derive the corresponding minimal numbers of transmit and receive antennas as $M \geq (K+\rho)d$, $N \geq (1+K/\rho)d$. When ρ lies in Region II, we have shown the minimal numbers of transmit and receive antennas in (10a) and (10c). By substituting these requirements into (19), we can observe how much ICIs must be eliminated either at the BS side or at the user side, which are respectively,

$$\begin{cases} r_M \geq \frac{C_{n+1}^A}{(G-1)K},\ r_N \geq \frac{1}{(G-1)C_n^A}, & \forall C_{n+1}^A \leq \rho \leq C_n^A \\ r_M \geq \frac{\rho}{(G-1)K},\ r_N \geq \frac{1}{(G-1)\rho}, & \forall C_\infty^B \leq \rho \leq C_\infty^A \\ r_M \geq \frac{C_n^B}{(G-1)K},\ r_N \geq \frac{1}{(G-1)C_{n+1}^B}, & \forall C_n^B \leq \rho \leq C_{n+1}^B \end{cases} \tag{20}$$

In Fig. 3, we show the results of r_M and r_N in (20) versus ρ. We can see that when $\rho \leq 1/(G-1)$, all the ICIs can be canceled at the users. When $\rho \geq (G-1)K$, all the ICIs can be avoided at the BSs, which is actually the case of coordinated beamforming [11]. In other cases of ρ, neither the BSs nor the users are able to remove all the ICIs at single side. As a result, when the values of ρ is in this region, all the ICIs will be partially avoided with the antennas at the BSs and be partially canceled with the antennas at the users.

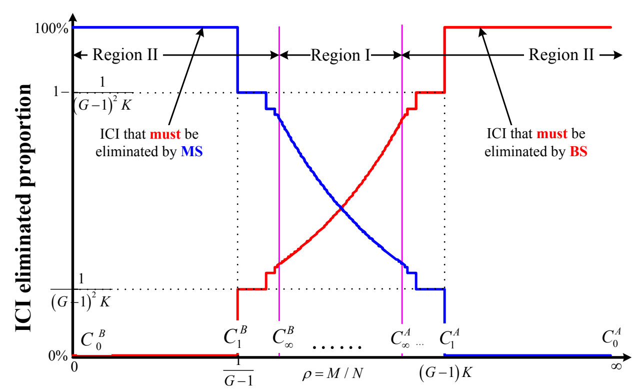

Fig. 3. ICI elimination proportion.

4. CONCLUSION

In this paper, we derived the maximal achievable DoF per user with linear IA for general symmetric MIMO-IBC with constant coefficients by analyzing the sufficient and necessary conditions of IA feasibility. We found that the minimal antenna resources required to support the desired number of data streams can be divided into two regions of M/N. In one region the DoF is limited by the proper condition and lower than the *decomposition DoF bound*. In the other region the DoF is restricted by the irreducible condition and can achieve the information theoretic maximal DoF.

A. REFERENCES

[1] C. Wang, T. Gou, S. A. Jafar, *Subspace Alignment Chains and the Degrees of Freedom of the Three-User MIMO Interference Channel*, arXiv:1109.4350v1 [cs.IT], Sep. 2011.

[2] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, “On feasibility of interference alignment in MIMO interference networks,” *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.

[3] G. Bresler, D. Cartwright, and D. Tse, *Settling the feasibility of interference alignment for the MIMO interference channel: the symmetric square case*, arXiv:1104.0888v1 [cs.IT], Apr. 2011.

[4] M. Razaviyayn, L. Gennady, and Z. Luo, “On the degrees of freedom achievable through interference alignment in a MIMO interference channel,” *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771–4782, Sep. 2012.

[5] G. Bresler, D. Cartwright, and D. Tse, “Geometry of the 3-user MIMO interference channel,” in *Proc. Allerton*, Sep. 2011.

[6] C. Wang, H. Sun, and S. A. Jafar, “Genie chains and the degrees of freedom of the K-user MIMO interference channel,” in *Proc. IEEE ISIT*, Jul. 2012.

[7] T. Liu and C. Yang, “On the Feasibility of Linear Interference Alignment for MIMO Interference Broadcast Channels,” arXiv:1207.1517 [cs.IT], Jul. 2012.

[8] ——, “IA Tree and the Degrees of Freedom of symmetric MIMO Interference Broadcast Channels,” *Journal paper in preparation*.

[9] O. Gonzalez, I. Santamaria, and C. Beltran, “A general test to check the feasibility of linear interference alignment,” in *Proc. IEEE ISIT*, Jul. 2012.

[10] W. B. Jones and W. J. Thron, *Continued Fractions: Analytic Theory and Applications* Addison-Wesley, Reading, Mass, 1980.

[11] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone, and W. Yu, “Multi-cell MIMO cooperative networks: A new look at interference,” *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.