On the Degrees of Freedom of Asymmetric MIMO Interference Broadcast Channels

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Abstract—In this paper, we study the degrees of freedom (DoF) of the asymmetric multi-input-multi-output interference broadcast channel (MIMO-IBC). By introducing a notion of connection pattern chain, we generalize the genie chain proposed in [10] to derive and prove the necessary condition of IA feasibility for the asymmetric MIMO-IBC, which is denoted as irreducible condition. It is necessary for both linear interference alignment (IA) and asymptotic IA feasibility in MIMO-IBC with arbitrary configurations. In a special class of the asymmetric two-cell MIMO-IBC, the irreducible condition is proved to be the sufficient and necessary condition for asymptotic IA feasibility, while the combination of proper condition and irreducible condition is proved to be the sufficient and necessary condition for linear IA feasibility. From these conditions, we derive the information theoretic maximal DoF and the maximal DoF achieved by linear IA. The derived maximal DoF per user for the asymmetric two-cell MIMO-IBC are also the upper-bounds of DoF per user for the asymmetric $G$-cell MIMO-IBC.

I. INTRODUCTION

The degrees of freedom (DoF) can reflect the potential of interference networks, which are the first-order approximation of sum capacity in high signal-to-noise ratio regime [1, 2]. Recently, significant research efforts have been devoted to find the information theoretic DoF for the multi-input-multi-output (MIMO) interference channel (MIMO-IC) [1–8] and MIMO interference broadcast channel (MIMO-IBC) [9, 10].

For the symmetric $G$-cell MIMO-IC where each base station (BS) and each mobile station (MS) have $M$ antennas, the study in [3] showed that the information theoretic maximal DoF per user are $M/2$, which can be achieved by asymptotic interference alignment (IA) (i.e., with infinite time/frequency extension). It implies that the sum DoF can linearly increase as $G$, and the interference networks are not interference-limited [3]. Encouraged by such a promising result, many recent works strive to analyze the DoF for the MIMO-IC and MIMO-IBC with various settings and devise interference management techniques to achieve the maximal DoF.

So far, existing studies focus on the symmetric system where each BS and each MS have $M$ and $N$ antennas, respectively. For the three-cell symmetric MIMO-IC, the information theoretic maximal DoF were obtained in [1], which can be achieved by linear IA (i.e., without any symbol extension or only with finite spatial extension) [4]. For the $G$-cell symmetric systems, the information theoretic maximal DoF were only obtained for some special configurations in the MIMO-IC [5, 6] and for many configurations in the MIMO-IBC [10]. For both MIMO-IC and MIMO-IBC, the analysis in [10] indicates that the information theoretic maximal DoF can be divided into two regions according to the ratio of $M/N$.

In the first region, the sum DoF of the system can linearly increase with the number of cells, which can be achieved by asymptotic IA. In the second region, the value of DoF is a piecewise linear function of $M$ and $N$ alternately, which can be achieved by linear IA. Considering that the asymmetric MIMO-IBC is more complex than the symmetric MIMO-IBC and expecting that the results for asymmetric cases may be extended from symmetric ones, there are only a few results on asymmetric systems. So far, a proper condition was proposed in [7] by relating the linear IA feasibility to the problem of determining the solvability of a system represented by multivariate polynomial equations. It is proved to be necessary for linear IA feasibility in the asymmetric MIMO-IC [8] and the asymmetric MIMO-IBC [9]. The following questions are still open for the asymmetric MIMO-IC and MIMO-IBC: what is the information theoretic maximal DoF when linear IA can achieve the information theoretic maximal DoF?

To understand the potential of practical networks, including both homogeneous and heterogeneous networks, we need to investigate the information theoretic maximal DoF for asymmetric systems. Moreover, in symmetric systems, all BSs or users have the same spatial resources. Consequently, it is hard to know which resources of BSs or users participate in managing interference, how the BSs or users remove the interference jointly, and what is the impact on the DoF if some nodes increase or reduce some resources (e.g., turning on or turning off antennas for energy saving)?

In this paper, we investigate the DoF of the asymmetric MIMO-IBC. By finding the difference of deriving the necessary conditions between symmetric and asymmetric systems and introducing a notion of connection pattern chain, we generalize the genie chain proposed in [10] into asymmetric MIMO-IBC and derive a necessary condition for linear IA and asymptotic IA feasibility, denoted as irreducible condition. From the irreducible condition and the combination of the proper condition and irreducible condition, we obtain the information theoretic DoF outer-bound and the DoF outer-bound achieved by linear IA in the $G$-cell asymmetric MIMO-
I. INTRODUCTION

In this paper, we study the DoF bounds of the special class of the two-cell MIMO-IBC can be achieved by asymptotic IA and linear IA, respectively. In addition, we also prove that these DoF bounds of the special class of the two-cell MIMO-IBC can be achieved by asymptotic IA and linear IA, respectively.

II. NECESSARY CONDITIONS FOR THE ASYMMETRIC G-CELL MIMO-IBC

Consider a G-cell MIMO system, where BS_i, equipped with M_i antennas transmits to K_i users each with N_{i_l}, \ldots, N_{i_{K_i}} antennas in the i-th cell, i = 1, \ldots, G. BS_i respectively transmits d_{i_1}, \ldots, d_{i_{K_i}} data streams to its K_i users, then the total number of data streams in the i-th cell is d_i = \sum_{k=1}^{K_i} d_{i_k}. Assume that there are no data sharing among the BSs and every BS has perfect CSIs of all links. This is a scenario of the asymmetric MIMO-IBC, and the configuration is denoted as \( \prod_{i=1}^{G} (M_i \times \prod_{k=1}^{K_i} (N_{i_k} \times d_{i_k})) \). When \( M_i = M, \ K_i = K, \ N_{i_k} = N \) and \( d_{i_k} = d, \ \forall i, k \), the system becomes the symmetric MIMO-IBC denoted as \( (M \times (N, d^K)^G) \).

Because both the IA with and without symbol extension will be addressed, we define two terminologies to be used throughout the paper.

Definition 1: Linear IA is the IA without any symbol extension or only with finite spatial extension [7].

Definition 2: Asymptotic IA is the IA with infinite time or frequency extension [1].

In this section, we study two necessary conditions of IA feasibility.

A. Proper Condition

When the channels are generic (i.e., drawn from a continuous probability distribution), the proper condition is one necessary condition for linear IA feasibility, which has been obtained for the asymmetric MIMO-IBC in [9]. To find the difference of deriving the necessary conditions between symmetric and asymmetric systems, we first review the proper condition in brief, which is

\[
\sum_{j \notin (l, k) \in I} (M_j - d_j) d_j + \sum_{l \notin j, (l, k) \in I} (N_{l_k} - d_{l_k}) d_{l_k} \geq \sum_{j \notin (l, k) \in I} d_j d_j, \ \forall I \subseteq J
\]

(1)

where

\[
J \triangleq \{(i, j) | 1 \leq i \neq j \leq G, 1 \leq k \leq K_i\}
\]

(2)

denotes the set of all MS-BS pairs that are mutually interfering each other and I is an arbitrary subset of J.

From (2), it is not hard to obtain that the set J has nonempty subsets, where \( |J| \) is the cardinality of J. As a result, (1) includes \( L_{\mathcal{J}} = 2^{L_{\mathcal{J}}} - 1 = 2^{G-1} \sum_{i=1}^{G} K_i - 1 \)

(3)

denotes \( \mathcal{J} \) is an arbitrary subset of J. As a result, (1) includes \( L_{\mathcal{J}} \) inequalities. By contrast, the proper condition for the symmetric MIMO-IBC includes only one inequality, which is [9]

\[
M + N \geq (GK + 1)d
\]

and obtained by only considering \( I = J \) in (1). Then, we introduce another terminology.

Definition 3: Connection pattern is a graph that represents which BSs and users that are mutually interfering each other in an arbitrarily connected MIMO-IBC.

Each subset \( \mathcal{I} \) corresponds to a connection pattern. Take a two-cell MIMO-IBC where \( K_1 = 2, K_2 = 1 \) as an example, denoted as Ex. 1. Since \( L_{\mathcal{I}} = 2^{K_1} + K_2 - 1 = 7 \), there are seven connection patterns in Ex. 1 as shown in Fig. 1. According to (2), we know that the subset \( \mathcal{I} = \mathcal{J} \) corresponds to a fully connected pattern, i.e., Pattern I, and the subset \( \mathcal{I} \subseteq \mathcal{J} \) corresponds to a partially connected pattern, e.g., each one in Patterns II~VII.

Comparing (1) and (4), we find that for the symmetric MIMO-IBC, it is enough to only consider the fully connected pattern. However, for the asymmetric MIMO-IBC, since all BSs or users have different number of antennas and suffer from different interference, it is necessary to consider all possible connection patterns rather than only consider the fully connected pattern.

B. Irreducible Condition

In [10], another necessary condition for both linear IA and asymptotic IA feasibility has been found for the symmetric MIMO-IBC, which leads to an information theoretic DoF upper-bound. It was called irreducible condition in [9] since it can ensure to eliminate a kind of irreducible inter-cell interference (ICI).

A usual way to derive the information theoretic DoF upper-bound is introducing a genie [1]. It is not easy to find a wise genie to provide the tightest possible upper-bound. The analysis in [10] indicates that when dividing the interference subspace at each BS or each user into two linearly independent subspaces, i.e., resolvable subspace and irresolvable subspace, we can construct a wise genie from the irresolvable subspace. For some antenna configurations, there may exist multiple irresolvable subspaces that constitute an irresolvable subspace chain, called subspace chain. Correspondingly, the genies in the subspace chain constitute a genie chain. To derive the DoF upper-bound (equivalently the irreducible condition) for the asymmetric MIMO-IBC, we need to first investigate the subspace chain and genie chain. Although the principle of constructing the genie chain for the asymmetric MIMO-IBC is similar with that for the symmetric MIMO-IBC, the results in [10] cannot be extended in a straightforward manner, which is shown in the forthcoming analysis.

![Fig. 1. Connection patterns in Ex. 1.](image-url)
1) Subspace Chain: The subspace chain depends on how BSs and users eliminate the interference cooperatively. When BSs first eliminate one part of ICIs and then return the remaining part to users, there exists a subspace chain that starts from the BS side, denoted as Chain A. Meanwhile, when users first eliminate one part of ICIs and then return the remaining to BSs, there also exists another subspace chain that starts from the user side, denoted as Chain B. To derive the whole necessary condition, we need to consider Chains A and B simultaneously. Since the analysis of Chain B is similar with Chain A, we only consider Chain A for conciseness.

For easy understanding, we illustrate the subspace chain for the asymmetric MIMO-IBC of Ex. 1 in Fig. 2.

As shown in the figure, the connection pattern in each chain is Pattern IV in Fig. 1, i.e., BS2 and MS11, MS12 are mutually interfering each other. For BS2, there are two interfering users and the ICIs generated by the two users occupy \( (N_1 + N_2) \)-dimensional interference subspace at BS2. Since BS2 has \( M_2 \) antennas, only \( \min\{\{(N_1 + N_2), M_2\}\} \)-dimensional subspace of the interference subspace is resolvable at BS2 and the remaining \( (N_1 + N_2 - M_2) \)-dimensional subspace is irresolvable. If \( N_1 + N_2 - M_2 \leq 0 \), there does not exist an irresolvable subspace so that the subspace chain stops. When \( N_1 + N_2 - M_2 > 0 \), the irresolvable subspace at BS2 is nonempty and the subspace chain continues.

For MS11 and MS12, there is only one interfering BS and the irresolvable ICIs occupy \( (N_1 + N_2 - M_2) \)-dimensional subspace. Since MS11 and MS12 have \( N_1 \) and \( N_2 \) antennas to resolve \( N_1 \) and \( N_2 \)-dimensional subspace, the remaining \( (N_1 + N_2 - M_2) \)-dimensional subspaces are irresolvable at MS11 and MS12, respectively.

We use \( S_{\lambda,j}^{(n)} = n = 1, 3, \ldots \) or \( S_{\lambda,i,k}^{(n)} = n = 2, 4, \ldots \) to denote BSj or MSk's irresolvable subspace in the nth Chain. For the subspace chain in Fig. 2, the dimension of the irresolvable subspace at BS2 in the first chain is

\[
S_{A,2}^{(1)} = (N_1 + N_2 - M_2)^{+} \tag{5}
\]

and the dimensions of the irresolvable subspaces at MS11 and MS12 in the second chain are

\[
S_{A,11}^{(2)} = (S_{A,2}^{(1)} - N_1)^{+} = (N_1 - M_2)^{+} \tag{6a}
\]

\[
S_{A,12}^{(2)} = (S_{A,2}^{(1)} - N_2)^{+} = (N_1 - M_2)^{+} \tag{6b}
\]

From existing studies in [1, 2], we know that there are two common ways to derive the DoF upper-bound: one is disconnecting some interfering links, and the other is allowing the cooperation either among the BSs or among the users. When disconnecting the interfering link between BS2 and MS11, the considered connection pattern of the subspace chain in Fig. 2 becomes Pattern III. As a result, the dimension of the irresolvable subspace at BS2 in the first chain becomes \( S_{A,2}^{(1)} = (N_1 - M_2)^{+} \). It indicates that the dimension of irresolvable subspace also depends on the connection pattern.

To describe the all connection patterns in the subspace chain, we introduce the fourth terminology.

Definition 4: Connection pattern chain is a chain constituted by the different connection patterns of a subspace chain, denoted as \( I^{(1)} \leftrightarrow I^{(2)} \leftrightarrow \ldots \), where \( I^{(n)} \) is the nth pattern in the chain. If a connection pattern chain satisfies \( I^{(n)} = I, \forall n \), it is equally connected pattern chain (ECPC), otherwise it is unequally connected pattern chain (UCPC). If an ECPC satisfies \( I^{(n)} = J, \forall n \), it is a full ECPC, otherwise it is a partial ECPC.

As shown in Fig. 2, the connection pattern chain is Pattern IV ↔ Pattern IV ↔ \ldots , which is an ECPC.

When disconnecting the interfering links or allowing the cooperation, some interfering links are removed from the connection pattern. Since the interference links that are removed in the current chain do not generate the interference in the next chain, the connection pattern for an arbitrary asymmetric MIMO-IBC in the next step is always a subset of that in the current step, i.e.,

\[
I^{(n)} \supseteq I^{(n+1)} \quad \forall n \geq 1 \tag{7}
\]

From the above analysis, we know that to derive the necessary condition for asymmetric systems, all possible connection pattern chains satisfying (7) need to be taken into account to construct the genie chain. In [10], only the fully ECPU is investigated in deriving the genie chain so that the obtained genie chain is also a special case of the genie chain considered for asymmetric systems. In other word, by introducing a notion of connection pattern chain, we generalize the genie chain in [10] into asymmetric MIMO-IBC.

Following the similar way in [10] and considering the difference mentioned above, for an arbitrary connection pattern chain satisfying (7), we can obtain the dimension of irresolvable subspace for the asymmetric MIMO-IBC. To express the dimension of irresolvable subspace in a unified way, we define \( S_{\lambda,i,k}^{(n)} \equiv N_{ik} \) and \( S_{\lambda,j}^{(n)} \equiv M_{j} \). Then, the dimension of irresolvable subspace in the nth chain can be expressed as

\[
S_{\lambda,j}^{(n)} = \left( \sum_{i \in C_{\lambda,j}^{(n)}} S_{\lambda,i,k}^{(n-2)} \right) - S_{\lambda,j}^{(n-2)} \quad \forall n = 2m - 1
\]

\[
S_{\lambda,i,k}^{(n)} = \left( \sum_{j \in C_{\lambda,i,k}^{(n)}} S_{\lambda,j}^{(n-2)} \right) - S_{\lambda,i,k}^{(n-2)} \quad \forall n = 2m
\]

\( \forall 0 < n \leq n_{\text{max}} \), where \( S_{\lambda,j}^{(n-2)} = \sum_{i \in C_{\lambda,j}^{(n-2)}} S_{\lambda,i,k}^{(n-2)} \) denote the dimension of irresolvable subspace at the cooperative BSs or cooperative users, when allowing the BSs in set \( C_{\lambda}^{(n)} \) or the users in set \( C_{\lambda}^{(n)} \) in cooperation, \( I_{\lambda}^{(n)} = \{i_k (i_k, j) \in I_{\lambda}^{(n)} \} \) is the set of MSs' index who are connected with BSj in the connection pattern \( I_{\lambda}^{(n)} \) the set of BSs'
index who are connected with MS$_{ik}$ in $T^{(n)}$, $n_{\text{max}}$ is the maximal length of subspace chain and satisfies $|S_{A,j}^{(n_{\text{max}})}| = 0$ or $|S_{A,j}^{(n_{\text{max}})}| = 0, \forall (i, j) \in T^{(n_{\text{max}})}$.

2) Genie Chain: From the dimension of the irresolvable subspace, we can determine the corresponding dimension of the genie and then obtain the irreducible condition.

We use $G_{A,j}^{(n)}$, $n = 1, 3, \cdots$ or $G_{A,ik}^{(n)}$, $n = 2, 4, \cdots$ to denote the introduced genie at BS$_1$ or MS$_{ik}$ in the $n$th chain. If the first chain, if the irresolvable subspace at BS$_2$, $S_{A,2}$ in (5) is nonempty, we can introduce a genie in $S_{A,2}$ (denoted as $G_{A,2}$) to help BS$_2$ to resolve all ICIs in the first chain. Then, we have

$$d_1 + d_2 \leq (M_2 - d_2) + |G_{A,2}^{(1)}| \quad (8)$$

If $S_{A,1}$ and $S_{A,2}$ in (6a) and (6b) are nonempty, we can introduce two genies in $S_{A,1}^{(2)}$ and $S_{A,2}^{(2)}$ (denoted as $G_{A,1}$ and $G_{A,2}$) to help MS$_1$ and MS$_2$ to resolve the remaining ICIs in the second chain. Then, we have

$$|G_{A,1}^{(2)}| \leq d_1 + |G_{A,1}^{(1)}|, \quad |G_{A,2}^{(2)}| \leq d_2 + |G_{A,2}^{(1)}| \quad (9)$$

Following the same way, we can obtain the dimension of the genie for the general asymmetric MIMO-IBC. To describe the genie’s dimension in a unified way, we define $|G_{A,ik}^{(0)}| \triangleq d_{ik}$ and $|G_{A,ik}^{(-1)}| \triangleq M_j - d_j$. Then, the dimension of the $n$th genie satisfies

$$\left\{ \begin{array}{l}
\sum_{i_k \in I^{(n)}} |G_{A,ik}^{(n-1)}| \leq |G_{A,j}^{(n-2)}| + |G_{A,j}^{(n)}|, \quad \forall n = 2m - 1 \\
\sum_{j \in I^{(n)}} |G_{A,ik}^{(n-1)}| \leq |G_{A,j}^{(n-2)}| + |G_{A,ik}^{(n)}|, \quad \forall n = 2m 
\end{array} \right. \quad (10)$$

$0 \leq n \leq n_{\text{max}}$, where $|G_{A,j}^{(n-2)}| = \sum_{i \in I^{(n-2)}} |G_{A,i}^{(n-2)}| or $|G_{A,ik}^{(n-2)}| = \sum_{i \in I^{(n-2)}} |G_{A,ik}^{(n-2)}|$ is the dimension of the genie at the cooperative BSs or cooperative users.

Moreover, since the genie in each chain lie in its corresponding irresolvable subspace, the dimension of the genie does not exceed that of the irresolvable subspace, i.e.,

$$\left\{ \begin{array}{l}
|G_{A,j}^{(n)}| \leq |S_{A,j}^{(n)}|, \quad \forall n = 2m - 1 \\
|G_{A,ik}^{(n)}| \leq |S_{A,ik}^{(n)}|, \quad \forall n = 2m 
\end{array} \right. \quad (11)$$

Combining the inequalities in (10) and (11) and the counterparts of Chain B, we obtain the whole irreducible condition for the asymmetric MIMO-IBC.

Remark 1: Since $|S_{A,j}^{(n)}| = N_j$ and $|G_{A,j}^{(0)}| = d_j$, $|S_{B,j}^{(0)}| = N_j$ and $|G_{B,j}^{(0)}| = d_j$, from (11) we have $d_{ik} \leq N_{ik}$ and $d_j \leq M_j$. As a result, the irreducible condition contains the condition that ensuring each BS or MS with enough antennas to convey the desired signals.

C. Comparison of Two Necessary Conditions

The irreducible condition is one necessary condition for both linear IA and asymmetric IA feasibility, while the proper condition is necessary for linear IA feasibility but no necessary for the asymmetric IA feasibility. Therefore, the information theoretical outer-bound of DoF region can be derived from the irreducible condition, while the outer-bound of DoF region achieved by linear IA needs to be derived from both the irreducible condition and the proper condition.

Remark 2: If the configuration of a MIMO-IBC satisfies the proper condition but not the reducible condition, the system is proper but infeasible for linear IA.

Since it is very difficult to obtain a general result for arbitrary configurations of asymmetric MIMO-IBC, in the following we use an example to show how to obtain the information theoretical outer-bound from the irreducible condition.

The outer-bound achieved by linear IA can be obtained in a similar way.

Consider Ex.1 again, if the configuration satisfies $|S_{A,1}^{(1)}| > 0$, $|S_{A,2}^{(0)}| = 0$ and $|S_{A,2}^{(2)}| = 0$, i.e., $N_1 + N_2 > M_2 \geq \max\{N_{11}, N_{12}\}$, from (5) we can introduce a genie to help BS$_2$, but cannot introduce any genie to help MS$_1$ and MS$_2$. Substituting into (10) and (11), the reducible condition is

$$\left\{ \begin{array}{l}
d_1 \leq N_1, d_2 \leq N_2, d_3 \leq N_2 \\
d_1 + d_2 + d_3 \leq M_2 + |G_{A,2}^{(1)}| \\
|G_{A,1}^{(2)}| \leq d_1, \quad |G_{A,1}^{(2)}| \leq d_2 \\
|G_{A,2}^{(2)}| \leq d_1 + d_2, \quad |G_{A,2}^{(2)}| \leq N_1 + N_2 - M_2 
\end{array} \right. \quad (12)$$

By solving (12), the outer-bound of DoF region is obtained as

$$\left\{ \begin{array}{l}
d_1 \leq N_1, d_2 \leq N_2, d_3 \leq N_2 \\
d_1 + d_2 \leq M_2, d_1 + d_2 \leq M_2 \\
|G_{A,1}^{(2)}| \leq d_1, \quad |G_{A,1}^{(2)}| \leq d_2 \\
|G_{A,2}^{(2)}| \leq d_1 + d_2 \leq N_1 + N_2 
\end{array} \right. \quad (13)$$

In the chain shown in Fig. 2, we only consider the connection pattern chain where $T^{(n)}$ is Pattern IV, $\forall n$. If we consider other connection pattern chain, we can obtain the other outer-bound of DoF region in a similar way.

III. DOF PER USER FOR THE TWO-CELL ASYMMETRIC MIMO-IBC

Because the proper condition and the irreducible condition for the general asymmetric MIMO-IBC are too complicated for analysis, we consider a class of special configurations to obtain the closed-form expression of the DoF. We consider that all users have the same number of data streams, i.e., $d_{ik} = d$, $\forall k, i$, then the outer-bound of DoF region can be characterized by the upper-bound of DoF per user. Besides, we consider that all users in one cell have the same number of receive antennas, i.e., $N_{ik} = N_i$, $\forall k$, then the DoF of these users can be analyzed in a unified way.

For such a special configuration, we can derive the closed-form DoF upper-bound per user for the two-cell asymmetric MIMO-IBC and prove that it is the upper-bound of DoF per user for G-cell asymmetric MIMO-IBC. Note that such a two-cell asymmetric MIMO-IBC represents a typical heterogeneous network where a macro-cell and a micro-cell interfere each other, so that its DoF results can provide useful insights into interference management in heterogeneous networks. In the following, we investigate the two-cell MIMO-IBC, denoted as $\Pi_{i=1}^2 (M_i \times (N_i, d^K_i))$. 


A. Information Theoretic Maximal DoF

To understand the potential of the two-cell asymmetric MIMO-IBC, we first investigate the information theoretic maximal DoF per user.

**Theorem 1 (Information Theoretic Maximal DoF):** For the two-cell MIMO-IBC $\prod_{i=1}^2 (M_i \times (N_i, d_i)^K)$, the information theoretic maximal DoF per user are

$$d_{\text{Info}}(M_j, N_j, K_j, K_i)$$

where

$$d_{\text{Info}}(M_j, N_j, K_j, K_i) = \begin{cases} d_{\text{Decom}}(M_j, N_j, K_j, K_i), & \text{Region I} \\ d_{\text{Quan}}(M_j, N_j, K_j, K_i), & \text{Region II} \end{cases}$$

The information theoretic DoF upper-bound and then prove that it is achievable. For the considered two-cell MIMO-IBC, the DoF bounds in (15) and (16) reduce to the decomposition DoF bound for some configurations.

For the conclusion that the decomposition DoF bound is the information theoretic upper-bound in Region I, there is no rigorous proof in existing studies. From the analysis in last section, we know that the subspace chain depends on the connection pattern chain. This means that for different connection pattern chains, we can obtain different DoF upper-bounds from the irreducible condition. Moreover, we show that when considering the full ECPC, the DoF upper-bound derived from the irreducible condition is only applicable for the antenna configurations in Region II. However, if considering the partial ECPCs or UCPUs, the derived DoF upper-bound is applicable for the antenna configurations in Region I. With more derivations, we find that for some antenna configurations in Region I, the derived DoF upper-bound is equal to the decomposition DoF bound. By considering all possible partial ECPCs or UCPUs, we can prove that the decomposition DoF bound is the information theoretic upper-bound in Region I.

To help understand the proof, we illustrate the result by an example in Fig. 3. From the boundary of feasible region, the information theoretic DoF upper-bound is shown. From the boundaries of infeasible regions, the DoF upper-bounds derived from the irreducible condition are shown. When considering the full ECPC, the DoF upper-bound is obtained only for the antenna configurations in Region II. However, when considering a given partial ECPC, the DoF upper-bound is obtained for antenna configurations in Region I, which is equal to the decomposition DoF bound for some configurations.

In the following, we prove the DoF upper-bound is achievable. Following similar derivations in [10], we can show that for the antenna configurations when both $M_1/N_2$ and $M_2/N_1$ fall in Region II, the DoF upper-bound can be achieved by linear IA and the closed-form solution of linear IA exists. Obviously, the DoF can also be achieved by asymmetric IA but the infinite extension is not necessary. For other configurations, the DoF upper-bound can only be achieved by asymptotic IA but not by linear IA.

**Remark 3:** For the considered two-cell MIMO-IBC, the information theoretic DoF upper-bound for arbitrary configurations can be obtained from the irreducible condition and can always be achieved by asymptotic IA. Therefore, the irreducible condition is the sufficient and necessary condition for asymptotic IA feasibility.

In [10], the irreducible condition is obtained as the sufficient and necessary condition of asymptotic IA feasibility for all antenna configurations in Region II and some special antenna configurations in Region I. In this study, we prove that the irreducible condition obtained from the generalized genie chain is the sufficient and necessary condition of asymptotic IA feasibility for arbitrary antenna configurations.
B. Maximal Achievable DoF of Linear IA

Considering that asymptotic IA requires infinite time/frequency extension, which is not feasible for practical systems, this motivates us to find the maximal DoF per user achieved by linear IA.

Theorem 2 (Maximal DoF achieved by Linear IA): For the two-cell MIMO-IBC $\prod_{i=1}^{G} (M_i \times (N_i, d) K_i)$, the maximal DoF per user achieved by linear IA are

$$\min_{i\neq j} \left\{ d_{\text{linear}}(M_j, N_j, K_j, K_i) \right\}$$  \hspace{1cm} (17)

where

$$d_{\text{linear}}(M_j, N_j, K_j, K_i) = \begin{cases} d_{\text{prop}}(M_j, N_j, K_j, K_i), & \text{Region I} \\ d_{\text{quan}}(M_j, N_j, K_j, K_i), & \text{Region II} \end{cases}$$

$$d_{\text{prop}}(M_j, N_j, K_j, K_i) = \frac{K_i M_j + K_i N_i}{K_j + K_i}$$  \hspace{1cm} (18)

Proof Skeleton: We first prove that (17) is the DoF upper-bound and then prove that it is achievable. After some tedious but regular manipulations, we can show that in the considered two-cell MIMO-IBC, the proper condition in (1) becomes

$$(M_j - K_j) d_K_j + (N_i - d) K_i d_j \geq K_i K_j d^2 \forall i \neq j$$  \hspace{1cm} (19)

From (19), we can obtain $d \leq d_{\text{prop}}(M_j, N_j, K_j, K_i)$ in (18) directly.

Since (18) is one DoF upper-bound obtained from the proper condition, it is called proper DoF bound. Moreover, $d_{\text{prop}}(M_j, N_j, K_j, K_i)$ is another DoF upper-bound obtained from the irreducible condition, which is proved in Theorem 1. Consequently, (17) is the upper-bound of the DoF per user achieved by linear IA.

In the proof for Theorem 1, we have known that for the antenna configurations when both $M_1/N_2$ and $M_2/N_1$ fall in Region II, the DoF can be achieved by a closed-form linear IA. For other configurations, we can prove that there exist at least one feasible solution for linear IA following a similar proof as in [9].

Remark 4: In the considered two-cell MIMO-IBC, the maximal DoF per user achieved by linear IA are obtained from the proper condition and the irreducible condition simultaneously. Therefore, the combination of the proper condition and the irreducible condition is the sufficient and necessary condition for linear IA feasibility.

Remark 5: When the proper DoF bound is achievable, (18) implies that to achieve the desired number of data streams, if one BS increases or reduces the transmit antennas, the MSs in another cell can reduce or should increase the receive antennas. Specifically, let $L = K_j/K_i$ where $K_j > K_i$. If the number of transmit antennas at BS $j$ is $M_j \pm \Delta$, the number of receive antenna at MS in cell $i$ should be $N_i \mp \Delta$. As a result, when a cell that supports more users (or equivalently that needs to transmit more data streams) has redundant antennas to help eliminate ICI, the overall antenna resource in the network can be reduced. This suggests that a heterogeneous network with different downlink traffics and antenna resources among multiple cells is more spectrally efficient than a homogeneous network, if we allow the more powerful cell (such as a macro-cell whose BS is equipped with more antennas) to help remove the ICI for the resource-limited cell (such as a micro-cell).

Remark 6: Since a two-cell MIMO-IBC can be treated as a partially connected case of a G-cell MIMO-IBC, the necessary condition for the two-cell MIMO-IBC must be the necessary condition for the G-cell MIMO-IBC. Consequently, the DoF per user upper-bounds in Theorems 1 and 2 are the DoF per user upper-bounds for G-cell MIMO-IBC $\prod_{i=1}^{G} (M_i \times (N_i, d) K_i)$ with asymptotic and linear IA, respectively.

IV. CONCLUSION

In this paper, we analyzed the DoF of the asymmetric MIMO-IBC. By generalizing the genie chain considered in [10], we found that irreducible condition is necessary for both linear IA and asymptotic IA feasibility. From the irreducible condition, we derived the information theoretic DoF upper-bound for an arbitrary G-cell MIMO-IBC and the information theoretic maximal DoF per user for a special class of the two-cell MIMO-IBC with the antenna configurations in both Regions I and II. By contrast, the reducible condition derived in [10] only leads to the information theoretic maximal DoF for the symmetric MIMO-IBC with all antenna configurations in Region II and some special configurations in Region I. By combining the proper condition and irreducible condition, we obtained the DoF outer-bound for arbitrary MIMO-IBC achieved by linear IA and the maximal achievable DoF per user for the special two-cell MIMO-IBC. By comparing the information theoretic maximal DoF and the maximal DoF achieved by linear IA, we showed when the linear IA can achieve the information theoretic maximal DoF.

REFERENCES


