Energy-Efficient Hybrid One- and Two-Way Relay Transmission

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Abstract—Two-way relaying is more spectral efficient than one-way relaying and is expected to consume less transmit power than one-way relaying to achieve the same data rate. However, when circuit power consumption is also taken into account, two-way relaying may not be more energy efficient, particularly when the bits to be transmitted from two source nodes have different numbers. To always offer high energy efficiency (EE), we propose a hybrid relay strategy, which conveys bidirectional messages with both one-way and two-way relaying. To maximize its EE, we jointly optimize the number of bits and transmission time allocated to the one- and two-way relaying stages to minimize the overall energy consumption, including the transmit power and circuit power. To reveal the behavior of the optimized hybrid relay strategy, we analyze the optimal bit allocation for the hybrid relaying under different circuit power consumption and bidirectional data amounts. Analytical and simulation results show that, in a high-traffic region where the transmit power dominates the energy consumption, the hybrid relaying degenerates into two-way relaying. By contrast, in a low-traffic region where the circuit power consumption is dominant, the proposed hybrid relaying offers significant performance gain in the sense of EE over the pure one- and two-way relaying.

Index Terms—Energy efficiency (EE), hybrid relay, one-way relay, two-way relay.

I. INTRODUCTION

S INCE the widespread demand for wireless services is sharply increasing their contribution to carbon emission and their operating costs, energy efficiency (EE) has become an important design goal for wireless systems [1]–[3].

A widely used performance metric for EE is the number of bits transmitted per unit of energy. When only transmit power is taken into account in the energy consumption, the EE monotonically decreases with the increase of the spectral efficiency (SE), at least for the point-to-point transmission in the additive white Gaussian noise channel [4]. This implies

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Digital Object Identifier 10.1109/TVT.2013.2259273

that, in such a scenario, a high SE design will lead to low EE. However, when the power consumed by various signaling and circuits to support the transmission in practical systems is also considered, the SE–EE relationship is fundamentally changed, which depends on the transmit strategy and the system setting. As a result, the optimization problem that minimizes the overall transmit power may not necessarily lead to high EE [2]. To pursue high EE meanwhile guarantee the required SE, it is paramount to investigate the relationship of SE–EE and to design the transmit strategy that maximizes the EE under the SE requirement by accounting for the overall power consumption.

Relaying plays an important role in wireless systems such as cellular and sensor networks because it is able to extend the coverage, improve the reliability, and enhance the capacity [5]. One-way relay transmission (OWRT) can reduce the one-hop communication distance or provide spatial diversity, but it suffers from a 1/2 SE loss when a half-duplex relay is applied [6]. Fortunately, two-way relay transmission (TWRT) can recover the SE loss if properly designed [7]–[9], which is, thus, more spectral efficient than the OWRT.

The EE-oriented design of one- and two-way relaying has been studied in different scenarios. In [10]-[12], the EE and the EE-oriented design of decode-and-forward (DF) OWRT systems were studied, considering both the transmit power and the circuit power consumption, respectively, with single-antenna and multiantenna nodes. In [13], after accounting for the energy cost of acquiring channel information, relay selection for an OWRT system with multiple DF relays was optimized to maximize the EE. In [14], a single-carrier frequency-division multiple-access amplify-and-forward (AF) OWRT system was designed toward EE. In [15], a three-node AF TWRT system was optimized to minimize the overall transmit power that ensures the required signal to noise ratio (SNR), where each node is equipped with multiple antennas. In [16], relay selection and power allocation were optimized for a TWRT system that minimizes the total transmit power subject to the requirement of the end-to-end rate. In [14]–[16], the circuit power consumption was not taken into account. In addition to these EE-oriented designs for the pure OWRT and TWRT, the EE of different relay strategies has been compared. In [17], TWRT is shown to be more energy efficient than OWRT via simulations, where only the transmit power was considered. In [18], the EE of the TWRT was compared with those of the OWRT and direct transmission (DT) after the relay position and transmit power at each node are optimized. It shows that the TWRT consumes less energy than the OWRT and DT, where only the transmit power was considered in the energy consumption model. In [19], the EE of OWRT, TWRT, and DT was compared, considering not

Manuscript received July 6, 2012; accepted April 7, 2013. Date of publication April 22, 2013; date of current version October 12, 2013. This work was supported in part by National Natural Science Foundation of China under Grant 61120106002 and in part by the National Basic Research Program of China (973 Program) under Grant 2012CB316003. This work was presented in part at the IEEE International Conference on Acoustics, Speech, and Signal Processing, Kyoto, Japan, March 2012. The review of this paper was coordinated by Prof. S. Muhaidat.

only the transmit power but also the circuit power consumption. It shows that the spectrally efficient TWRT is not always more energy efficient. When the numbers of bits to be transmitted in the two directions are symmetric, TWRT offers higher EE than OWRT; otherwise, it is inferior to the OWRT in terms of the EE.

To exploit the merits of both the OWRT and the TWRT, we design a hybrid relay transmission (HRT) strategy that contains both one-way relay stage and two-way relay stage in this paper. We consider an AF relay system, where two source nodes intend to exchange messages with each other with the assistance of a relay node (RN). Despite that there are other relay protocols such as DF and compress-and-forward, which can provide higher rate regions than AF, in two-way relaying, AF offers better outage performance than DF when the channel gains from two source nodes to the RN are symmetric [20], and AF is also widely applied in practical systems [5]. Moreover, different relay protocols have quite different system models. For example, the data rate in AF systems can be expressed as a function of the SNR at the destination. In DF system, the endto-end data rate depends on the lower one of the achievable data rates in two hops. This renders different optimization approaches for various relay protocols. In this paper, we study the AF relaying, whereas other relay protocols will be considered in future works.

The basic principle of EE-oriented optimization is to maximize the EE while ensuring the required quality of service (QoS). Different kinds of traffic have different QoS requirements. We consider a delay-constrained system, where the messages at the two source nodes are periodically generated and must be transmitted within a hard deadline T. Then, the QoS requirement can be represented by the overall number of bits to be transmitted in two directions within a give value of T. Such a system model is widely employed for applications with strict delay constraints on data delivery, e.g., voice over IP and wireless sensor networks [21]–[23].

We consider both the transmit power and the circuit power consumed by transmit-and-receive processing in each node. Because the transmit energy consumption decreases as the transmission time increases [4] but the circuit energy consumption increases with time, we can optimize the transmission duration to minimize the overall energy consumption. The system may complete the required transmission in a shorter duration than T and then switch to an idle status until the next block [23]. During the idle status, part of the circuits in the transceiver can be switched off to reduce the energy consumption and improve the EE.

To maximize the EE of the HRT, we jointly optimize the number of bits and the transmission time allocated to the oneand two-way relaying stages. To show the necessity of the joint optimization, we also consider an intuitive hybrid relaying scheme without optimizing the bit allocation. By analyzing the optimal bit-allocation results of the HRT, we show that the intuitive hybrid relaying scheme is suboptimal in the lowtraffic region but inferior to the optimized HRT in the hightraffic region evidently. Analysis and simulation results show that the EE of the optimal HRT depends on the circuit power consumption and the bidirectional data amounts. In the high-

TABLE I Comparison With Existing Works

Paper	Main Focus	Energy Consumption Model
[10]–[12]	EE oriented design for pure OWRT	transmit power + circuit power
[13]		transmit power + sig- naling overhead for ac- quiring channels
[14]		transmit power
[15], [16]	EE oriented design for pure TWRT	transmit power
[17], [18]	EE comparison between pure OWRT and pure TWRT, which shows that TWRT is <i>always</i> better	transmit power
[19]	EE comparison between pure OWRT and pure TWRT, which shows that TWRT is <i>not</i> al- ways better	transmit power + circuit power
This work	Hybrid relaying design, which achieves higher EE than both pure OWRT and TWRT	transmit power + circuit power

traffic region where the required SE is high and the circuit power consumption is negligible, the optimal HRT degenerates into the pure TWRT. In the low-traffic region where the circuit power consumption dominates, the optimal hybrid HRT contains both one- and two-way relaying stages. Table I provides a brief comparison between this and the prior works.

The main contribution of this paper lies in optimizing the HRT toward EE, where existing works in the context of EE only study pure OWRT or TWRT. In the proposed HRT strategy, each packet from the source node will be divided into two parts, which are respectively transmitted with one- and two-way relaying. By allocating the number of bits and duration of transmission on the one- and two-way relaying stages adaptively, the HRT can achieve higher EE than both the pure OWRT and TWRT.

The rest of this paper is organized as follows. System model including the energy consumption of the pure OWRT and TWRT are described in Section II. Then, the HRT strategy is introduced in Section III, where its EE is optimized and the optimal bit-allocation strategy is analyzed. Simulation results are given in Section IV, and Section V concludes this paper.

II. SYSTEM MODEL

Consider a three-node system, where two source nodes \mathbb{A} and \mathbb{B} intend to exchange information with the assistance of a relay node \mathbb{R} , as shown in Fig. 1. Each of the three nodes is equipped with a single antenna. Consider a delay-constrained application scenario, where B_{ab} and B_{ba} bits need to be transmitted with a hard deadline T in the $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ directions during each block, respectively. The information bits in each block are transmitted via a packet or a frame, depending on specific applications. In the sequel, we use "packet size" to denote the number of bits of each block, i.e., B_{ab} and B_{ba} . In all the following optimization and analysis, we only consider a single



Fig. 1. Three-node relay system, where nodes \mathbb{A} and \mathbb{B} exchange messages of B_{ab} and B_{ba} bits with the assistance of relay node \mathbb{R} . The channels from \mathbb{A} and \mathbb{B} to \mathbb{R} are denoted as h_{ar} and h_{br} , respectively.

block. The packet sizes in different blocks may vary, i.e., the data rates of each block may differ.

Assume that the signal is severely attenuated between the two source nodes; thus, the direct link is not considered for transmission. The noise power N_0 is assumed to be identical at each node. We consider a block-fading channel, where the channel coefficients from nodes \mathbb{A} and \mathbb{B} to \mathbb{R} , i.e., h_{ar} and h_{br} , respectively, remain constant within each block duration T but may vary from one block to another. The AF gain at the RN is chosen with the aid of instantaneous channel knowledge. In practice, channel information is never perfect, which also affects the EE. However, considering imperfect channel information will lead to rather involved derivations. To obtain an explicit expression of the EE for optimization, we assume perfect channel information as in [13] and [14].

To reduce the energy consumption, the system may not use the entire duration T for transmission in each block. After B_{ab} and B_{ba} bits have been transmitted, the system can switch to an idle mode until the next block. In other words, each node may operate in three modes: transmission, reception, and idle. The power consumption in these modes is, respectively, denoted as $P_x^t/\epsilon + P^{ct}$, P^{cr} , and P^{ci} , where P_x^t is the transmit power of node $x, x \in \{\mathbb{A}, \mathbb{B}, \mathbb{R}\}$, and $\epsilon \in (0, 1]$ denotes the power amplifier efficiency, which is the ratio of the output power to input power of a power amplifier. P^{ct} , P^{cr} , and P^{ci} are the circuit power consumption in the transmission, reception, and idle modes, respectively. The circuit power consumption at each node is assumed identical for notational simplicity.

The circuit power in transmit and receive modes P^{ct} and P^{cr} consists of the power consumed by baseband processing and that by RF circuits. The power consumption of the RF circuit is usually assumed independent of data rate [5], [23], whereas there are different assumptions for the power consumption of the baseband processing circuit. In systems with low-complexity baseband processing, the baseband power consumption can be neglected compared with the RF power consumption [5], [23]. Otherwise, it is not negligible and increases with data rate [24]. In this paper, we consider the first case, where P^{ct} and P^{cr} only consist of RF power consumption, which are modeled as constants. Modeling P^{ct} and P^{cr} as functions of the data rate leads to a different optimization problem, which will be considered in future works.

In practical systems, an idle node does not transmit or receive, but its transceiver is not shut down. Some of the hardware components still operate, such that the idle node can be "waked up" quickly whenever is necessary. The power consumption in the idle mode P^{ci} is modeled as a constant, where $0 < P^{\text{ci}} \leq P^{\text{ct}}$, and $0 < P^{\text{ci}} \leq P^{\text{cr}}$.



Fig. 2. Operation mode of each node and the corresponding duration in each block of the pure OWRT and pure TWRT. (a) Pure OWRT. (b) Pure TWRT.

Before introducing the hybrid one- and two-way relaying strategy, we first present the energy consumption models of the pure OWRT and TWRT briefly for the readers's convenience.¹

A. Energy Consumption of OWRT

During each block, the pure OWRT system transmits with duration T_{ab} and T_{ba} in the $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ directions, respectively; then, all the three nodes switch to the idle mode with duration $T - T_{ab} - T_{ba}$, as shown in Fig. 2(a).

The transmission in each direction is completed in two phases, respectively, for the source-to-RN and RN-todestination transmissions. Since we consider the AF relay protocol, where the RN simply forwards its received signal, the two phases employ identical duration. In the $\mathbb{A} \to \mathbb{B}$ direction, during the first half of duration T_{ab} , node \mathbb{A} transmits to the RN, where node \mathbb{R} is in reception mode, and node \mathbb{B} is idle. During the second half of the duration, node \mathbb{R} transmits to node \mathbb{B} , where node \mathbb{B} is in reception mode, and node \mathbb{A} is idle. Then, the energy consumed by the transmission in the $\mathbb{A} \to \mathbb{B}$ direction is given by

$$\begin{split} &\frac{T_{ab}}{2} \left(P_a^t / \epsilon + P^{\text{ct}} + P^{\text{cr}} + P^{\text{ci}} \right) + \frac{T_{ab}}{2} \left(P_{r1}^t / \epsilon + P^{\text{ct}} + P^{\text{cr}} + P^{\text{ci}} \right) \\ &\triangleq T_{ab} \left(\frac{P_a^t + P_{r1}^t}{2\epsilon} + P_O^c \right) \end{split}$$

where $P_{r_1}^t$ is the RN transmit power in the $\mathbb{A} \to \mathbb{B}$ link, and

$$P_O^c \stackrel{\Delta}{=} P^{\rm ct} + P^{\rm cr} + P^{\rm ci} \tag{1}$$

is the overall circuit power consumption during the transmission. Similarly, we can obtain the energy consumption in $\mathbb{B} \to \mathbb{A}$ transmission.

Then, the overall energy consumption of the pure OWRT to exchange the information in two directions during each block

¹The energy consumption models of the pure OWRT and TWRT presented here are simplified from those in [19] by considering that the circuit power at different nodes is identical.

is given by

$$E_{O} = T_{ab} \left(\frac{P_{a}^{t} + P_{r1}^{t}}{2\epsilon} + P_{O}^{c} \right) + T_{ba} \left(\frac{P_{b}^{t} + P_{r2}^{t}}{2\epsilon} + P_{O}^{c} \right) + (T - T_{ab} - T_{ba})(3P^{ci})$$
(2)

where P_{r2}^t is the RN transmit power in $\mathbb{B} \to \mathbb{A}$ direction, and the last term is the energy consumed by the three nodes when they are operated in idle modes.

To find the minimal transmit power to exchange the B_{ab} and B_{ba} bits within duration T_{ab} and T_{ba} , we use Shannon capacity formula and the SNR for OWRT derived from [25] to express $P_a^t + P_{r1}^t$ and $P_b^t + P_{r2}^t$ as functions of T_{ab} and T_{ba} . In this way, we can analyze the maximal EE with the given SE. In practice, there is a maximum transmit power constraint for each node. If the required transmit power exceeds the maximum value, the system cannot successfully support the required transmission (i.e., the achievable data rate is less than the required data rate); thus, an outage occurs. When analyzing the EE, it is rather involved to consider the maximum transmit power constraint, but the conclusions are almost the same no matter if it is considered, as shown in [19]. To obtain useful insight, we do not consider the maximum power constraint in the following analytical analysis.

Following the same procedure as in [19], we first optimize the transmit power at each node, such that the total transmit power in the $\mathbb{A} \to \mathbb{B}$ direction $P_a^t + P_{r1}^t$ is minimized, which is a function of B_{ab} and T_{ab} , and the total transmit power in the $\mathbb{B} \to \mathbb{A}$ direction $P_b^t + P_{r2}^t$ is minimized, which is a function of B_{ba} and T_{ba} . Then, by applying approximation $2^R - 1 \approx 2^R$ to simplify the expression of transmit power, where R denotes data rate, the overall energy consumption of the pure OWRT during each block in (2) is finally obtained as

$$E_{O} \approx T_{ab} \left[\frac{N_{0}}{2\epsilon |h_{\text{eff}}|^{2}} \left(2^{\frac{2B_{ab}}{T_{ab}W}} - 1 \right) + P_{O}^{c} \right] + T_{ba} \left[\frac{N_{0}}{2\epsilon |h_{\text{eff}}|^{2}} \left(2^{\frac{2B_{ba}}{T_{ba}W}} - 1 \right) + P_{O}^{c} \right] + (T - T_{ab} - T_{ba})(3P^{\text{ci}}) \stackrel{\Delta}{=} E_{\text{owr}}(B_{ab}, T_{ab}) + E_{\text{owr}}(B_{ba}, T_{ba}) + (T - T_{ab} - T_{ba})(3P^{\text{ci}})$$
(3)

where $|h_{\text{eff}}| \stackrel{\Delta}{=} 1/((1/|h_{\text{ar}}|) + (1/|h_{\text{br}}|))$ can be viewed as an equivalent channel gain between the two source nodes due to the usage of the RN, and

$$E_{\rm owr}(b,t) \stackrel{\Delta}{=} t \left[\frac{N_0}{2\epsilon |h_{\rm eff}|^2} \left(2^{\frac{2b}{tW}} - 1 \right) + P_O^c \right] \tag{4}$$

is defined as the energy consumption to transmit b bits of information with transmission duration t using one-way relaying. Approximation $2^R - 1 \approx 2^R$ only affects the value of transmit power, which is accurate in the high-data-rate region. In the low-data-rate region, the approximation does not affect the total power consumption because the circuit power becomes dominant.

B. Energy Consumption of TWRT

During each block, the pure TWRT system completes the bidirectional transmission with duration T_{twr} ; then, all the three nodes switch into the idle mode with duration $T - T_{twr}$, as shown in Fig. 2(b).

The bidirectional transmission is completed in two phases with identical duration, owing to the use of the AF relay. During the first half of duration T_{twr} , both nodes A and B transmit to RN R, and during the second half of the duration, RN R superimposes and broadcasts its received signals from both directions to nodes A and B. After receiving the superimposed signal, each of the source nodes A and B removes its own transmitted signal via self-interference cancelation [7] and obtains its desired signal sent from the other source node. The overall energy consumption of the pure TWRT per block is given by

$$E_T = \frac{T_{\text{twr}}}{2} \left(P_a^t / \epsilon + P_b^t / \epsilon + 2P^{\text{ct}} + P^{\text{cr}} \right)$$
$$+ \frac{T_{\text{twr}}}{2} \left(P_r^t / \epsilon + P^{\text{ct}} + 2P^{\text{cr}} \right)$$
$$+ (T - T_{\text{twr}})(3P^{\text{ci}})$$
$$= T_{\text{twr}} \left(\frac{P_a^t + P_b^t + P_r^t}{2\epsilon} + P_T^c \right) + (T - T_{\text{twr}})(3P^{\text{ci}}) \quad (5)$$

where

$$P_T^c \stackrel{\Delta}{=} (2P^{ct} + P^{cr} + P^{ct} + 2P^{cr})/2 = 1.5(P^{ct} + P^{cr})$$
 (6)

is the overall circuit power consumption in the transmission and reception modes.

Similar to the pure OWRT case, by optimizing the transmit power at each node, the overall transmit power $P_a^t + P_b^t + P_r^t$ can be minimized, which is a function of bidirectional packet sizes and the transmission time T_{twr} [19]. Then, (5) can be rewritten as

$$E_{T} \approx T_{\text{twr}} \left[\frac{N_{0}}{2\epsilon |h_{\text{eff}}|^{2}} \left(2^{\frac{2B_{ab}}{T_{\text{twr}}W}} + 2^{\frac{2B_{ba}}{T_{\text{twr}}W}} - 2 \right) + P_{T}^{c} \right] + (T - T_{\text{twr}})(3P^{\text{ci}})$$
$$\stackrel{\Delta}{=} E_{\text{twr}}(B_{ab}, B_{ba}, T_{\text{twr}}) + (T - T_{\text{twr}})(3P^{\text{ci}})$$
(7)

where

$$E_{\rm twr}(b_1, b_2, t) \stackrel{\Delta}{=} t \left[\frac{N_0}{2\epsilon |h_{\rm eff}|^2} \left(2^{\frac{2b_1}{tW}} + 2^{\frac{2b_2}{tW}} - 2 \right) + P_T^c \right]$$
(8)

is defined as the energy consumption to exchange b_1 and b_2 bits with transmission duration t using two-way relaying, and the same approximation deriving E_O has been applied.

Define EE as the number of bits transmitted per unit of energy, i.e.,

$$\eta_{\rm EE} = \frac{B_{ab} + B_{ba}}{E} \tag{9}$$



Fig. 3. Bit allocation and transmission duration of the hybrid OWRT-and-TWRT strategy. (a) A general procedure of the HRT, which consists of the $\mathbb{A} \to \mathbb{B}$ one-way relaying stage, the $\mathbb{B} \to \mathbb{A}$ one-way relaying stage, and the two-way relaying stage. (b) If $B_{ba} \leq B_{ab}$, from Proposition 1, the optimal bit allocation at node \mathbb{B} is $B_{ba}^{\text{owr-opt}} = 0$, i.e., all the bits from node \mathbb{B} should be transmitted via two-way relaying, and the $\mathbb{B} \to \mathbb{A}$ one-way relaying stage in (a) is omitted.

where E is the energy consumption per block. For a fair comparison, we compare the EE of different transmit strategies with the same packet sizes; then, the system consuming less energy E achieves higher EE.

The minimum energy consumption of the pure OWRT and pure TWRT has been compared in [19] by optimizing the transmission time. The results show that, in the high-traffic region where the packet sizes B_{ab} and B_{ba} are large, TWRT always consumes less energy than OWRT. In the low-traffic region, however, TWRT is *not always* more energy efficient. TWRT consumes less energy than OWRT when the packet sizes in the two directions B_{ab} and B_{ba} are identical; otherwise, OWRT may be more energy efficient. In the following, we propose a hybrid OWRT-and-TWRT strategy to achieve the maximal EE in all cases.

III. ENERGY-EFFICIENT HYBRID RELAY TRANSMISSION STRATEGY

With the HRT, each of the packets at nodes A and B is divided into two parts, which are transmitted using one-way and two-way relaying, respectively. The bidirectional transmission is completed in three stages. The system first uses one-way relaying to transmit the B_{ab}^{owr} bits from node A to node B with duration T_{owr-ab} and then uses one-way relaying to transmit the B_{ba}^{owr} bits from node B to node A with duration T_{owr-ba} . Then, two-way relaying is employed to transmit the remaining $B_{ab} - B_{ab}^{owr}$ and $B_{ba} - B_{ba}^{owr}$ bits bidirectionally with duration T_{twr} . Finally, the system switches into idle status during the rest of duration $T - T_{owr-ab} - T_{owr-ba} - T_{twr}$, as shown in Fig. 3(a). For notation simplicity, we use the symbols T_{ab} and T_{ba} to denote the transmission duration of one-way relaying stages T_{owr-ab} and T_{owr-ba} in the rest of this paper. Using (4) and (8), the overall energy consumption of HRT is given by

$$E_{H} = E_{\text{owr}} \left(B_{ab}^{\text{owr}}, T_{ab} \right) + E_{\text{owr}} \left(B_{ba}^{\text{owr}}, T_{ba} \right) + E_{\text{twr}} \left(B_{ab} - B_{ab}^{\text{owr}}, B_{ba} - B_{ba}^{\text{owr}}, T_{\text{twr}} \right) + \left(T - T_{\text{twr}} - T_{ab} - T_{ba} \right) (3P^{\text{ci}})$$
(10)

where the first two terms are the energy consumed in the oneway relaying stage, the third term is the energy consumed in the two-way relaying stage, and the last term is the energy consumed in the idle duration.

When $B_{ab}^{owr} = B_{ba}^{owr} = 0$, the HRT degenerates into the pure TWRT. When $B_{ab}^{owr} = B_{ab}$ and $B_{ba}^{owr} = B_{ba}$, the HRT reduces to the pure OWRT. By jointly optimizing the number of bits for one-way relaying stage and the transmission time T_{ab} , T_{ba} , and T_{twr} , we can minimize the energy consumption of the HRT, such that it will be more energy efficient than both the pure OWRT and pure TWRT. The optimization problem can be formulated as follows:

$$\begin{array}{ll}
\min_{B_{ab}^{\text{owr}}, B_{ba}^{\text{owr}}, T_{ab}, T_{ba}, T_{\text{twr}}} & E_{\text{owr}} \left(B_{ab}^{\text{owr}}, T_{ab} \right) + E_{\text{owr}} \left(B_{ba}^{\text{owr}}, T_{ba} \right) \\
& + E_{\text{twr}} \left(B_{ab} - B_{ab}^{\text{owr}}, B_{ba} - B_{ba}^{\text{owr}}, T_{\text{twr}} \right) \\
& + \left(T - T_{\text{twr}} - T_{ab} - T_{ba} \right) (3P^{\text{ci}}) \\
& \text{s.t.} & 0 \leq B_{ab}^{\text{owr}} \leq B_{ab}; & 0 \leq B_{ba}^{\text{owr}} \leq B_{ba} \\
& T_{ab} \geq 0; & T_{ba} \geq 0; & T_{\text{twr}} \geq 0 \\
& T_{ab} + T_{ba} + T_{\text{twr}} \leq T. & (11)
\end{array}$$

Considering that the exponential function is convex, $f(b) \stackrel{\Delta}{=} (N_0/2\epsilon |h_{\text{eff}}|^2)(2^{(2b/W)} - 1) + P_O^c$ is a convex function of b. From [26, Sec. 3.2.6], if f(x) is a convex function and $t \ge 0$, then g(x,t) = tf(x/t) is a convex function with respect to x and t. Therefore, $E_{\text{owr}}(b,t) = t[(N_0/2\epsilon |h_{\text{eff}}|^2)(2^{(2b/Wt)} - 1) + P_O^c]$ in (4) is convex with respect to b and t, and $E_{\text{twr}}(b_1, b_2, t)$ in (8) is convex with respect to b_1 , b_2 , and t. Then, the first three terms of E_H in (10) are all convex function, the sum of the four terms is still convex. Moreover, all the constraints in the problem are linear. Therefore, problem (11) is convex, which can be solved using efficient optimization techniques [26].

Although problem (11) is convex, it is difficult to obtain a close-form solution. Nevertheless, we can analyze the optimal values of the bit allocation to show the behavior of the optimal relay transmission strategy in different scenarios.

In the remaining part of this paper, without loss of generality, we always assume that $B_{ab} \ge B_{ba}$, whereas the analysis for the case with $B_{ab} \le B_{ba}$ is similar.

A. Optimal Bit Allocation at Node \mathbb{B}

We first study the optimal number of bits transmitted by node \mathbb{B} in the one-way relaying stage, i.e., $B_{ba}^{\text{owr-opt}}$. We start with the following two lemmas.

Lemma 1: For $b_1 \ge 0$, $b_2 \ge 0$, $t_1 > 0$ and $t_2 > 0$, we have

$$E_{\text{owr}}(b_1, t_1) + E_{\text{owr}}(b_2, t_2) \ge E_{\text{owr}}(b_1 + b_2, t_1 + t_2)$$
 (12)

where the equality holds only when $b_1/t_1 = b_2/t_2$.

Proof: See Appendix A. *Lemma 2:* For $b_1 \ge b_2 \ge 0$ and $t_1 < t_2 < 0$, we have

$$E_{\rm owr}(b_1, t_2) + E_{\rm owr}(b_2, t_1) \ge E_{\rm owr}(b_1, t_1) + E_{\rm owr}(b_2, t_2).$$
(13)

Lemma 1 indicates that, if a one-way relaying system first transmits b_1 bits in duration t_1 with data rate b_1/t_1 and then transmits b_2 bits in duration t_2 with data rate b_2/t_2 , it will consume more energy than a system that transmits $b_1 + b_2$ bits in duration $t_1 + t_2$ with data rate $(b_1 + b_2)/(t_1 + t_2)$.

Lemma 2 indicates that, if a one-way relaying system transmits a larger packet of b_1 bits with a shorter duration t_2 and then transmits a smaller packet of b_2 bits with longer duration t_1 , its energy consumption will be higher than a system that transmits the larger packet with longer duration and transmits the smaller packet with shorter duration.

With these two lemmas, we can prove the following proposition.

Proposition 1: When $B_{ab} \ge B_{ba}$, the optimal number of bits and transmission time allocated for the one-way relaying stage from node \mathbb{B} to node \mathbb{A} that minimize the energy consumption of the HRT are $B_{ba}^{\text{owr-opt}} = 0$ and $T_{ba}^{\text{opt}} = 0$.

Proof: See Appendix C.

Proof: See Appendix B.

Proposition 1 suggests that, when $B_{ab} \ge B_{ba}$, the smaller packet at node \mathbb{B} should be only transmitted via two-way relaying, i.e., there is no one-way relaying stage from node \mathbb{B} to node \mathbb{A} . This indicates that the HRT strategy can be simplified, where only the B_{ab} bits at node \mathbb{A} should be divided into two parts. The first part consisting of B_{ab}^{owr} bits is transmitted using one-way relaying from node \mathbb{A} to node \mathbb{B} with duration T_{ab} , and the remaining $B_{ab} - B_{ab}^{owr}$ bits are transmitted from node \mathbb{A} to node \mathbb{B} , together with the B_{ba} bits from \mathbb{B} to \mathbb{A} using twoway relaying with duration T_{twr} . Then, the system switches into the idle status with duration $T - T_{ab} - T_{twr}$. The procedure is shown in Fig. 3(b).

After substituting $B_{ba}^{\text{owr-opt}} = 0$ and $T_{ba}^{\text{opt}} = 0$, the energy consumption minimization problem for the HRT in (11) can be reformulated as

$$\min_{\substack{B_{ab}^{\text{owr}}, T_{ab}, T_{\text{twr}}}} E_{\text{owr}} (B_{ab}^{\text{owr}}, T_{ab}) + E_{\text{twr}} (B_{ab} - B_{ab}^{\text{owr}}, B_{ba}, T_{\text{twr}}) \\
+ (T - T_{\text{twr}} - T_{ab})(3P^{\text{ci}}) \\
\text{s.t.} \quad 0 \le B_{ab}^{\text{owr}} \le B_{ab}; \quad T_{ab} \ge 0; \quad T_{\text{twr}} \ge 0 \\
\quad T_{ab} + T_{\text{twr}} \le T$$
(14)

where the optimization variables have been reduced from five to three, which leads to a low-complexity optimization.

In the following, we proceed to analyze the optimal bit allocation at node \mathbb{A} from this optimization problem.

B. Optimal Bit Allocation at Node \mathbb{A}

It is difficult to derive the optimal bit allocation for the oneway relaying stage B_{ab}^{owr} at node A. To gain some useful insight, we consider two special cases in the following.

Case 1—High-Traffic Region: When both the values of B_{ab} and B_{ba} are large, the required data rates are high; hence, the transmit power will dominate the energy consumption.

Proposition 2: When $B_{ab} \ge B_{ba}$ and the transmit power consumption is dominant, the optimal number of bits and transmission time allocated for the one-way relaying stage from node \mathbb{A} to node \mathbb{B} that minimize the energy consumption of the HRT are $B_{ab}^{\text{owr-opt}} = 0$ and $T_{ab}^{\text{opt}} = 0$.

Proof: See Appendix D.

Proposition 2 suggests that, in the high-traffic region where the circuit power is negligible, the optimal bit-allocation strategy at node \mathbb{A} is to transmit all the B_{ab} bits using two-way relaying.

Case 2—Low-Traffic Region: When both the values of B_{ab} and B_{ba} are small, the circuit power consumption becomes dominant. Since the circuit energy consumption increases with time, the optimal transmission duration that minimizes the overall energy consumption is shorter than the block length T.

We start by introducing two lemmas.

Lemma 3: Consider $0 \le t \le T$, $b \ge 0$, and $E = E_{owr}(b,t) + (3P^{ci})(T-t)$. If the optimal value of t that minimizes E is less than T, i.e., $t^{opt} < T$, then t^{opt} will be a function of b, whereas $b/t^{opt} \triangleq R^{opt}$ will not be a function of b. The minimum value of the overall energy consumption achieved by using t^{opt} can be expressed as

$$E^{\min} = b\left(\frac{N_0 \ln 2}{W\epsilon |h_{\text{eff}}|^2} 2^{\frac{2R^{\text{opt}}}{W}}\right) \stackrel{\Delta}{=} be_{\text{owr}}^{\min}$$
(15)

where $e_{\text{owr}}^{\min} \stackrel{\Delta}{=} (N_0 \ln 2/W \epsilon |h_{\text{eff}}|^2) 2^{(2R^{\text{opt}}/W)}$. Since R^{opt} is not a function of *b*, neither does e_{owr}^{\min} .

Proof: See Appendix E.

Lemma 3 indicates that in a one-way relaying system which transmits b bits with duration t in one direction and remains in idle status with duration T - t, if $t^{\text{opt}} < T$, the corresponding optimal data rate $R^{\text{opt}} = b/t^{\text{opt}}$ does not depend on the bit number b, and the minimal energy consumed per bit is e_{owr}^{\min} .

Lemma 4: Consider $0 \le t \le T$, $b \ge 0$, $0 \le \beta \le 1$, and $E = E_{twr}[\beta b, (1 - \beta)b, t] + (3P^{ci})(T - t)$. If the optimal value of t that minimizes E is less than T, i.e., $t^{opt} < T$, then t^{opt} depends on b, but $b/t^{opt} \triangleq R^{opt}$ is not a function of b. The minimal value of overall energy consumption achieved by using t^{opt} can be expressed as

$$E^{\min} = b \left\{ \frac{N_0 \ln 2}{W \epsilon |h_{\text{eff}}|^2} \left[2^{\frac{2\beta R^{\text{opt}}}{W}} \beta + 2^{\frac{2(1-\beta)R^{\text{opt}}}{W}} (1-\beta) \right] \right\}$$
$$\stackrel{\Delta}{=} b e^{\min}_{\text{twr}}(\beta) \tag{16}$$

where $e_{\text{twr}}^{\min}(\beta) \stackrel{\Delta}{=} (N_0 \ln 2/W \epsilon |h_{\text{eff}}|^2) [2^{(2\beta R^{\text{opt}}/W)} \beta + 2^{(2(1-\beta)R^{\text{opt}}/W)} (1-\beta)]$, which is a quasi-convex function of β and achieves its minimal value when $\beta = 0.5$. Since R^{opt} is not a function of b, neither is $e_{\text{twr}}^{\min}(\beta)$.

Proof: See Appendix F.

Lemma 4 indicates that, in a two-way relaying system that transmits βb and $(1 - \beta)b$ bits in the two directions, respectively, with duration t and remains in idle status with duration T - t, if $t^{\text{opt}} < T$, its optimal bidirectional sum rate $R^{\text{opt}} = b/t^{\text{opt}}$ does not depend on the total number of bits b. The minimal energy consumed per bit $e_{\text{twr}}^{\min}(\beta)$ depends on the

ratio β . When the traffic amounts in the two directions are symmetric, i.e., $\beta = 0.5$, $e_{twr}^{min}(\beta)$ achieves its minimum.

Now, we provide the following proposition on the optimal bit allocation at node \mathbb{A} in the HRT.

Proposition 3: When $B_{ab} \ge B_{ba}$ and the circuit power consumption is dominant such that the optimal transmission duration is shorter than the block length T, the optimal bit allocation for the one-way relaying stage at node \mathbb{A} that minimizes the energy consumption of the HRT is $B_{ab}^{\text{owr-opt}} \approx B_{ab} - B_{ba}$.

Proof: See Appendix G. Proposition 3 means that, in the low-traffic region where the

rioposition's means that, in the low-traine region where the circuit power consumption is dominant and when the optimal transmission duration is shorter than the block length T, the optimal bit-allocation value at node \mathbb{A} is $B_{ab}^{\text{owr-opt}} \approx B_{ab} - B_{ba}$. In other words, approximately $B_{ab} - B_{ba}$ bits should be transmitted via one-way relaying from node \mathbb{A} to \mathbb{B} , whereas the remaining B_{ba} bits from node \mathbb{A} should be transmitted via two-way relaying. When the packet sizes $B_{ab} = B_{ba}$, approximately $B_{ab} - B_{ba} = 0$ bit is allocated to the one-way relaying stage. In this case, the optimal HRT degenerates into the pure TWRT.

C. Comparison Between the Optimal and Intuitive HRT Strategies

When the packet sizes of two source nodes are identical, the pure TWRT is always more energy efficient than the pure OWRT, as shown in [19]. Intuitively, this suggests a simple HRT strategy, which first uses two-way relaying to transmit all the B_{ba} bits from node \mathbb{B} and B_{ba} bits from node \mathbb{A} and then uses one-way relaying to transmit the remaining $B_{ab} - B_{ba}$ bits at node \mathbb{A} .

In the low-traffic region where the circuit power consumption is dominant, from Propositions 1 and 3, we know that the optimal HRT strategy should uses two-way relaying to transmit all the B_{ba} bits of the smaller packet from node \mathbb{B} to node \mathbb{A} and transmit approximately B_{ba} bits from node \mathbb{A} to node \mathbb{B} , and then uses one-way relaying to transmit the remaining $B_{ab} - B_{ba}$ bits from node \mathbb{A} to node \mathbb{B} . This is in accordance with the intuitive HRT strategy. Considering the approximation used in Proposition 3, the intuitive HRT strategy is suboptimal.

In the high-traffic region where the transmit power is dominant, from Propositions 1 and 2, we know that the optimal HRT strategy should use two-way relaying to transmit all the bits from nodes \mathbb{A} and \mathbb{B} to each other. In this scenario, the optimal HRT degenerates into the pure TWRT.

When the packet sizes B_{ab} and B_{ba} increase, there will be an EE gap between the optimized and intuitive HRT solutions, which will be large in the high-traffic region. In the following, we derive the maximal EE gap.

Considering that, in the high-traffic region with the optimal HRT, no bit is allocated for one-way relaying and the circuit power consumption can be neglected, the energy consumption minimization problem in (11) can be reformulated as

$$\min_{T_{\text{twr}}} \quad \frac{N_0 T_{\text{twr}}}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2B_{ab}}{T_{\text{twr}}W}} + 2^{\frac{2B_{ba}}{T_{\text{twr}}W}} - 2 \right)$$
s.t. $0 \le T_{\text{twr}} \le T.$ (17)

It is easy to obtain that $T_{twr}^{opt} = T$, and the corresponding minimal energy consumption is

$$E_H^{\min} = \frac{N_0 T}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2B_{ab}}{TW}} + 2^{\frac{2B_{ba}}{TW}} - 2 \right).$$
(18)

With the intuitive HRT, the bit allocation is $B_{ab}^{owr} = B_{ab} - B_{ba}$ and $B_{ba}^{owr} = 0$. Since there is no one-way relaying stage from node \mathbb{B} to node \mathbb{A} , we have $T_{ba} = 0$. Further considering that the circuit power is neglected in the high-traffic region, the energy consumption minimization problem in (11) becomes

$$\min_{T_{ab}, T_{twr}} \frac{N_0 T_{ab}}{2\epsilon |h_{eff}|^2} \left(2^{\frac{2(B_{ab} - B_{ba})}{T_{ab}W}} - 1 \right) \\
+ \frac{N_0 T_{twr}}{2\epsilon |h_{eff}|^2} \left(2^{\frac{2B_{ba}}{T_{twr}W}} + 2^{\frac{2B_{ba}}{T_{twr}W}} - 2 \right) \\$$
s.t. $T_{ab} \ge 0; \quad T_{twr} \ge 0; \quad T_{ab} + T_{twr} \le T.$ (19)

This optimization problem is convex, but the closed-form solutions for T_{ab}^{opt} and $T_{\text{twr}}^{\text{opt}}$ are hard to derive. To obtain an explicit expression of the minimal energy consumption for the intuitive HRT, we try to find its upper bound. It is easy to show that the objective function in (19) decreases as T_{ab} and T_{twr} increase. Therefore, the one-way relaying stage duration T_{ab} for link $\mathbb{A} \to \mathbb{B}$ and the two-way relaying stage duration T_{twr} should occupy the whole block duration T, i.e., $T_{ab}^{\text{opt}} + T_{\text{twr}}^{\text{opt}} = T$. We intuitively set the ratio of one-way relaying stage duration over the block length as the ratio of the bits allocated for one-way relaying stage over the total bit number at node \mathbb{A} , i.e., $T_{ab} = (B_{ab} - B_{ba}/B_{ab})T$; then, $T_{\text{twr}} = (B_{ba}/B_{ab})T$. Substituting these nonoptimal transmission duration times into the objective function in (19), we obtain an upper bound of the minimal energy consumption, which is

$$E_{H}^{\text{int-min-UB}} = \frac{N_0 T}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2B_{ab}}{TW}} - 1\right) \frac{B_{ab} + B_{ba}}{B_{ab}}.$$
 (20)

Using (18) and (20), we can obtain an upper bound for the performance gain on the EE of the optimal solution over the intuitive solution as follows:

$$\frac{\eta_{\rm EE}^{\rm opt} - \eta_{\rm EE}^{\rm int}}{\eta_{\rm EE}^{\rm opt}} = \frac{(B_{ab} + B_{ba})/E_H^{\rm min} - (B_{ab} + B_{ba})/E_H^{\rm int-min}}{(B_{ab} + B_{ba})/E_H^{\rm min}}$$
$$= 1 - \frac{E_H^{\rm min}}{E_H^{\rm int-min}}$$
$$\leq 1 - \frac{E_H^{\rm min}}{E_H^{\rm int-min-UB}} = 1 - \frac{1 + \frac{2\frac{2B_{ba}}{TW} - 1}{2\frac{TW}{-1}}}{1 + \frac{B_{ab}}{B_{ab}}}.$$
 (21)

Since we assume that $B_{ba} \leq B_{ab}$, in the high-traffic region where $B_{ab}, B_{ba} \rightarrow \infty$, it is easy to show that

$$\lim_{B_{ab} \to \infty, B_{ba} \to \infty, B_{ba} \le B_{ab}} \frac{2^{\frac{2B_{ba}}{TW}} - 1}{2^{\frac{2B_{ab}}{TW}} - 1} = 0.$$
(22)

Upon substituting (22), we obtain a concise upper bound as follows:

$$\frac{\eta_{\rm EE}^{\rm opt} - \eta_{\rm EE}^{\rm int}}{\eta_{\rm EE}^{\rm opt}} \le \frac{B_{ba}}{B_{ab} + B_{ba}}.$$
(23)

For example, if $B_{ab}/B_{ba} = 4$, i.e., the size of the packet transmitted in the $\mathbb{A} \to \mathbb{B}$ link is four times of that in the $\mathbb{B} \to \mathbb{A}$ link, the optimal HRT solution will achieve at most 1/(4+1) = 20% EE gain over the intuitive HRT solution.

IV. SIMULATION RESULTS

In this section, we evaluate the EE of the proposed HRT and validate previous analysis via simulations.

We consider that the three nodes are located on a straight line. The distance between nodes A and B is 100 m. The RN is at the midpoint of two source nodes, which is the optimal relay position [18]. In this case, the required transmit power is minimized for a given SE; thus, we can observe the maximal EE of the relay systems. The path loss is modeled as 30 + $10 \log_{10}(distance^{\alpha})$ dB, where α is the attenuation factor. The small-scale fading channels are independent and identically distributed Rayleigh block fading, which remain constant during each block but independent from one to another.

The increase in the block duration T and bandwidth W is equivalent to the reduction of the SE (i.e., the number of bits transmitted in unit time with unit bandwidth). We change the values of B_{ab} and B_{ba} for a given setting of T = 5 ms and W =10 MHz in the simulations. Define $\beta \triangleq B_{ab}/(B_{ab} + B_{ba})$. When $\beta = 0.5$, the packet sizes in the two directions are equal.

From [5] and [23], the circuit power consumption of each node in practical systems usually ranges from dozens to hundreds of milliwatts; therefore, in the simulations, we set their values in this range. Unless otherwise specified, we set the circuit power consumption in the transmission and reception modes as $P^{\rm ct} = P^{\rm cr} = 50$ mW, and we set that in the idle mode as $P^{\rm ci} = 10$ mW. The power amplifier efficiency is set as $\epsilon = 0.35$.

A. Validation of the Analytical Analysis

We first validate our former analysis by providing the simulated optimal transmission time and bit allocation, which are obtained by solving optimization problem (11). Considering that with equal bidirectional packet sizes the TWRT is always more energy efficient than OWRT [19] and the HRT degenerates to the pure TWRT, we focus on the case with unequal bidirectional packet sizes, where $\beta = 0.8$. The performance with other values of β will be considered in the following.

The simulation results are shown in Fig. 4. The x-axis is the total number of bits transmitted in the two directions normalized by the block duration and bandwidth, i.e., $(B_{ab} + B_{ba})/(TW)$, which can be viewed as the average bidirectional



Fig. 4. Optimal ratio of the bits allocated for one-way relaying and the optimal transmission duration, where $\beta = 0.8$, $P^{\text{ct}} = P^{\text{cr}} = 50 \text{ mW}$, $P^{\text{ci}} = 10 \text{ mW}$, and $\alpha = 4$.

SE per block.² The upper subfigure shows the optimal ratio of the bits allocated to the one-way relaying stage in the HRT, i.e., $B_{ab}^{\rm owr-opt}/B_{ab}$ and $B_{ba}^{\rm owr-opt}/B_{ba}$, and the lower subfigure shows the optimal transmission duration, i.e., $T_{ab}^{\rm opt} + T_{ba}^{\rm opt} + T_{twr}^{\rm opt}$.

We can see that the value of $B_{ba}^{\text{owr-opt}}/B_{ba}$ is zero, whereas the value of $B_{ab}^{\text{owr-opt}}/B_{ab}$ depends on the packet sizes. In the low-traffic region where the transmission duration is shorter than the block length of 5 ms, $B_{ab}^{\text{owr-opt}} \approx 0.7B_{ab}$. As the traffic amounts increase, $B_{ab}^{\text{owr-opt}}$ decreases. In the high-traffic region, it decreases to zero. In this case, no bit is transmitted in a one-way relaying stage, and the HRT degenerates into the pure TWRT.

Since we set $\beta = 0.8$, $B_{ab} = 4B_{ba}$, in the intuitive HRT scheme, the system uses two-way relaying to transmit B_{ba} bits in each direction, and then uses one-way relaying to transmit the remaining $B_{ab} - B_{ba} = 3B_{ba}$ bits in the $\mathbb{A} \to \mathbb{B}$ direction. Then, B_{ab}^{owr}/B_{ab} in such a scheme is $3B_{ba}/4B_{ba} = 0.75$. Comparing the results of the intuitive HRT with the optimized HRT, we see that the bit allocation in the intuitive HRT solution is approximately optimal in the low-traffic region. All these results agree well with our analysis results in Propositions 1–3.

B. Performance Comparison

In the sequel, we compare the EE of the proposed HRT with those of pure OWRT and TWRT, and analyze the impact of the path-loss factor, circuit power consumption, and the packet size difference in the two directions on the EE. In practice, the pathloss attenuation factor usually ranges from 2 to 4, depending on whether or not the line-of-sight paths exist between the RN and the source node [27]. Unless otherwise specified, we

²The average bidirectional SE per block takes into account the entire duration of a block, which includes not only the transmission time but also the idle duration as well.



Fig. 5. EE comparison among the pure OWRT, pure TWRT, and HRT, where $\beta = 0.8$, $P^{\rm ct} = P^{\rm cr} = 50$ mW, $P^{\rm ci} = 10$ mW, and $\alpha = 2.5$. (a) EE of the pure OWRT, pure TWRT, and HRT versus the average SE, where the solid and dash curves denote the cases with and without the maximum transmit power constraint, respectively. (b) Outage probabilities of different transmission strategies when considering the maximum transmit power constraint.

set $\alpha = 2.5$ and the ratio of the number of bits transmitted by node \mathbb{A} over the total bit number $\beta = 0.8$.

1) EE Versus SE: In Fig. 5(a), we compare the EE of the considered relay schemes. To show the impact of the maximum transmit power constraint on our analysis, we provide the SE–EE relationship with and without the maximum transmit power constraint, where the maximum transmit power is 30 dBm for each node for the case with the constraint.

In the case with the maximum transmit power constraint, the system may fail to support the required SE. As the SE increases, the required transmit power at each node goes up. When the required transmit power exceeds the maximum value, an outage occurs. The outage probabilities are shown in Fig. 5(b). In practice, the system should keep the outage probability lower than a threshold, e.g., 10%. It indicates that, when considering the maximum transmit power constraint, there is a limit for the maximum SE. For example, the outage probability of the pure OWRT is higher than 10% when the SE is higher than 6.6 b/s/Hz, which is the maximum SE able to be supported by



Fig. 6. Normalized EE of the pure OWRT, pure TWRT, and HRT versus the average SE, where $\beta = 0.8$, $P^{\rm ct} = P^{\rm cr} = 50$ mW, $P^{\rm ci} = 10$ mW, and $\alpha = 2.5$.

the pure OWRT. Therefore, the corresponding EE curve of the pure OWRT in Fig. 5(a) stops at 6.6 b/s/Hz when considering the maximum transmit power constraint.

In the case without the maximum transmit power constraint, the system can always complete the required transmission successfully, i.e., no outage occurs. By comparing the cases with and without the maximum transmit power constraint, we see that the EE curves are almost identical, except that the curves with maximum transmit power constraint stop at certain SE due to the outage. It indicates that the maximum transmit power constraint does not affect our analysis on the EE. In the following, we always consider the case without the transmit power constraint, such that we can observe the EE in both highand low-SE regions.

It is shown in Fig. 5(a) that the TWRT is not always more energy efficient than the OWRT. In the low-traffic region, the TWRT is inferior to the OWRT. By jointly optimizing the transmission time and bit allocation, the proposed HRT always achieves the highest EE. In the high-traffic region, the EE of the HRT and the TWRT overlap, both are higher than the OWRT as expected. The optimized HRT achieves higher EE than the intuitive HRT, but their performance gap seems minor because the EE of both HRT schemes is too low to be distinguishable in this region.

In Fig. 6, we present the normalized EE of different transmit strategies, considering the large dynamic range of the values of the EE. For each given SE, we normalize the EE of all strategies with the EE of the optimized HRT. It is shown that the performance of the intuitive HRT is almost the same as that of the optimized HRT in the low-traffic region as we have analyzed, but the performance gap grows with the SE. The relative performance gain of the optimized HRT over the intuitive HRT is about 18% in the high-traffic region, which is close to our derived upper bound of 20% in Section III-C.

2) Impact of the Path-Loss Factor and Circuit Power Consumption: In the sequel, we compare the normalized EE of different transmit strategies versus the path-loss attenuation factor and the circuit power consumption for a given average SE per block, which is set as $(B_{ab} + B_{ba})/(TW) = 5$ bps/Hz.



Fig. 7. Normalized EE versus the path-loss attenuation factor, where $P^{\rm ct} = P^{\rm cr} = 50 \,$ mW, $P^{\rm ci} = 10 \,$ mW, $(B_{ab} + B_{ba})/(TW) = 5 \,$ bps/Hz, and $\beta = 0.8$.



Fig. 8. Normalized EE versus the circuit power consumption, where $(B_{ab} + B_{ba})/(TW) = 5$ bps/Hz, $\alpha = 2.5$, $\beta = 0.8$, and $P^{ci} = 0.2P^{ct}$.

In Fig. 7, the EE versus the path-loss attenuation factor α is shown, which varies from 2 to 4 [27]. In Fig. 8, we demonstrate the impact of the circuit power consumption, where $P^{\text{ct}} = P^{\text{cr}}$, which ranges from 0 to 1000 mW [5], [23]. Because no practical model is available for the circuit power consumption in idle mode, we set $P^{\text{ci}} = 0.2P^{\text{ct}}$.

It is shown that the optimized HRT always provides the highest EE, and the performance gains over the pure TWRT and OWRT depend on the values of the attenuation factor and circuit power consumption. Considering that as the attenuation factor increases the system needs more transmit power to achieve the same SE, the transmit power will contribute more to the overall energy consumption when the attenuation factor is larger. Then, from the figures we can conclude that, when the transmit power is dominant and the circuit power consumption can be neglected, the spectrally efficient TWRT is also energy efficient. In this case, the EE of the pure TWRT and the EE of the optimized HRT overlap with each other, and both are much higher than that of the OWRT. As the circuit power consumption becomes dominant, the performance gap between the optimized HRT and the pure TWRT increases,



Fig. 9. Normalized EE of the pure OWRT, pure TWRT, and HRT versus the average SE for various bidirectional packet size ratios, where $P^{\text{ct}} = P^{\text{cr}} = 50 \text{ mW}$, $P^{\text{ci}} = 10 \text{ mW}$, and $\alpha = 2.5$.

whereas the gap between the pure OWRT and the optimized HRT shrinks. We can also see that the intuitive HRT is only near optimal in the case of a low attenuation factor or high circuit power consumption, where the circuit power consumption is dominant; otherwise, it is far from optimal.

3) Impact of the Packet Size Difference in Two Directions: In Fig. 9, we compare the normalized EE of the optimized HRT and those of the pure OWRT and TWRT, given different values of β . Considering that the normalized EE of the optimized HRT always equals 1, it is not shown for conciseness. We can see from the results that the EE of the pure TWRT is always higher than that of the OWRT with symmetric bidirectional packet sizes where $\beta = 0.5$. In this case, the optimized HRT degenerates into the pure TWRT; therefore, the normalized EE of the pure TWRT always equals 1. With asymmetric bidirectional packet sizes where $\beta > 0.5$, in the high-traffic region, the normalized EE of the pure TWRT still equals 1, which is the same as that of the optimized HRT. In the lowtraffic region, however, there is a considerable performance gap between the optimized HRT and the pure TWRT. As β increases, i.e., the bidirectional packet sizes B_{ab} and B_{ba} become more different, the performance gap between the optimized HRT and the pure TWRT increases, but the gap between the optimized HRT and the pure OWRT shrinks. This is because in the low-traffic region as β increases, B_{ba} approaches zero. Therefore, the bidirectional transmission is more and more similar to a unidirectional transmission, and the pure OWRT can offer better performance. By adjusting the bit allocation to the one- and two-way relaying parts, the proposed HRT can always achieve higher EE than both the pure OWRT and the pure TWRT.

V. CONCLUSION

In this paper, we have proposed a hybrid OWRT-and-TWRT strategy. By jointly optimizing the number of bits and transmission time allocated to the one- and two-way relaying stages that minimize the overall energy consumption of the system, the HRT strategy can provide the maximal EE under various scenarios. The HRT strategy bridges the gap between the pure OWRT and TWRT. Both analytical and simulation results showed that the performance of the optimal HRT strategy depends on the circuit power consumption and the bidirectional data amounts. When the circuit power is negligible or the data amounts in the two directions are large where high SE is required, the optimized HRT strategy degenerates into the pure TWRT. By contrast, in the low-traffic region where the circuit power consumption becomes dominant, the optimized HRT strategy provides substantial EE gain over the pure OWRT or TWRT.

By comparing the optimized HRT strategy with an intuitive HRT scheme, we analyzed the necessity of optimizing the bit allocation. It was shown that the intuitive solution only with the optimized transmission time is energy efficient in the lowtraffic region but is far from optimal when the bidirectional data amounts increase.

APPENDIX A Proof of Lemma 1

Since the exponential function $f(x) = (N_0/2\epsilon |h_{\text{eff}}|^2)(2^{(2x/W)} - 1) + P_O^c$ is convex, we have

$$\theta f(x_1) + (1 - \theta) f(x_2) \ge f(\theta x_1 + (1 - \theta) x_2)$$
 (24)

where the equality holds when $x_1 = x_2$.

Define $\theta = t_1/(t_1 + t_2)$, $x_1 = b_1/t_1$, and $x_2 = b_2/t_2$. By substituting them into (24) and using the definition of $E_{owr}(b, t)$ in (4), we can obtain (12).

Since the equality in (24) holds only when $x_1 = x_2$, the equality in (12) holds only when $b_1/t_1 = b_2/t_2$.

APPENDIX B Proof of Lemma 2

By substituting the expression of $E_{owr}(b, t)$ in (4), we have

$$\begin{split} & [E_{\rm owr}(b_1, t_1) + E_{\rm owr}(b_2, t_2)] - [E_{\rm owr}(b_1, t_2) + E_{\rm owr}(b_2, t_1)] \\ &= \frac{N_0}{2\epsilon |h_{\rm eff}|^2} \left[t_1 \left(2^{\frac{2b_1}{t_1W}} - 2^{\frac{2b_2}{t_1W}} \right) - t_2 \left(2^{\frac{2b_1}{t_2W}} - 2^{\frac{2b_2}{t_2W}} \right) \right] \\ & \stackrel{\Delta}{=} \frac{N_0}{2\epsilon |h_{\rm eff}|^2} \left[f(t_1) - f(t_2) \right] \end{split}$$
(25)

where $f(t) \stackrel{\Delta}{=} t(2^{(2b_1/tW)} - 2^{(2b_2/tW)}).$

The derivative of f(t) can be obtained as

$$f'(t) = 2^{\frac{2b_1}{tW}} \left(1 - \ln 2 \frac{2b_1}{tW} \right) - 2^{\frac{2b_2}{tW}} \left(1 - \ln 2 \frac{2b_2}{tW} \right)$$
$$\stackrel{\Delta}{=} g(b_1) - g(b_2) \tag{26}$$

where $g(b) \stackrel{\Delta}{=} 2^{(2b/tW)} (1 - \ln 2(2b/tW)).$

The derivative of g(b) is $g'(b) = -(\ln 2)^{2}2^{(2b/tW)}(4b/t^2W^2) \leq 0$ for b > 0. Then, g(b) is a decreasing function of b for b > 0. Considering that $b_1 \geq b_2$, we have $f'(t) = g(b_1) - g(b_2) \leq 0$, which means that f(t) is a decreasing function of t. Considering that $t_1 \geq t_2$, the expression in (25) is less than 0, i.e., Lemma 2 is true.

APPENDIX C PROOF OF PROPOSITION 1

Since it is difficult to directly find the optimal values of the bit allocation and transmission time, we prove this proposition using the following idea.

For an arbitrary system parameter set of $\mathbb{P}^1 \stackrel{\Delta}{=} (B_{ab}^{\mathrm{owr1}}, B_{ba}^{\mathrm{owr1}}, T_{ab}^1, T_{ba}^1, T_{twr}^1)$, with which the energy consumption of the HRT is E_H^1 , if we can find a set of $\mathbb{P}^2 \stackrel{\Delta}{=} (B_{ab}^{\mathrm{owr2}}, B_{ba}^{\mathrm{owr2}}, T_{ab}^2, T_{ba}^2, T_{twr}^2)$ satisfying $B_{ba}^{\mathrm{owr2}} = 0$ and $T_{ba}^2 = 0$ that yields lower energy consumption of E_H^2 , i.e., $E_H^2 \leq E_H^1$, Proposition 1 will hold true. Therefore, to prove Proposition 1, we only need to find such a specific set of \mathbb{P}^2 for an arbitrary set of \mathbb{P}^1 .

Considering an arbitrary system parameter set \mathbb{P}^1 , the corresponding energy consumption can be obtained from (10) as follows:

$$E_{H}^{1} = E_{owr} \left(B_{ab}^{owr1}, T_{ab}^{1} \right) + E_{owr} \left(B_{ba}^{owr1}, T_{ba}^{1} \right) + E_{twr} \left(B_{ab} - B_{ab}^{owr1}, B_{ba} - B_{ba}^{owr1}, T_{twr}^{1} \right) + \left(T - T_{ab}^{1} - T_{ba}^{1} - T_{twr}^{1} \right) (3P^{ci})$$
(27)

where the transmission time should not exceed the deadline T, i.e., $T_{ab}^1 + T_{ba}^1 + T_{twr}^1 \le T$.

From the definition of $E_{owr}(b,t)$ in (4) and that of $E_{twr}(b_1, b_2, t)$ in (8), we have

$$E_{\rm twr}(b_1, b_2, t) - tP_T^c = E_{\rm owr}(b_1, t) + E_{\rm owr}(b_2, t) - 2tP_O^c.$$
(28)

Then, after substituting (28), we can rewrite (27) as

$$E_{H}^{1} = E_{owr} \left(B_{ab}^{owr1}, T_{ab}^{1} \right) + E_{owr} \left(B_{ba}^{owr1}, T_{ba}^{1} \right) + E_{owr} \left(B_{ab} - B_{ab}^{owr1}, T_{twr}^{1} \right) + E_{owr} \left(B_{ba} - B_{ba}^{owr1}, T_{twr}^{1} \right) + T_{twr}^{1} \left(P_{T}^{c} - 2P_{O}^{c} \right) + \left(T - T_{ab}^{1} - T_{ba}^{1} - T_{twr}^{1} \right) \left(3P^{ci} \right) \geq E_{owr} \left(B_{ab}, T_{ab}^{1} + T_{twr}^{1} \right) + E_{owr} \left(B_{ba}, T_{ba}^{1} + T_{twr}^{1} \right) + \left(P_{T}^{c} - 2P_{O}^{c} \right) T_{twr}^{1} + \left(T - T_{ab}^{1} - T_{ba}^{1} - T_{twr}^{1} \right) \left(3P^{ci} \right)$$
(29)

where the inequality is obtained by applying (12) in Lemma 1. Define

$$T^{*1} \stackrel{\Delta}{=} \max \left\{ T^{1}_{twr} + T^{1}_{ab}, T^{1}_{twr} + T^{1}_{ba} \right\}$$
$$T^{*2} \stackrel{\Delta}{=} \min \left\{ T^{1}_{twr} + T^{1}_{ab}, T^{1}_{twr} + T^{1}_{ba} \right\}$$
$$\rho \stackrel{\Delta}{=} T^{*2} / T^{*1}.$$
(30)

Since $B_{ab} \ge B_{ba}$, we have

$$E_{owr} \left(B_{ab}, T_{ab}^{1} + T_{twr}^{1} \right) + E_{owr} \left(B_{ba}, T_{ba}^{1} + T_{twr}^{1} \right)$$

$$\geq E_{owr} (B_{ab}, T^{\star 1}) + E_{owr} (B_{ba}, T^{\star 2})$$

$$= E_{owr} (B_{ab}\rho, T^{\star 2}) + E_{owr} (B_{ba}, T^{\star 2})$$

$$+ E_{owr} \left(B_{ab} (1-\rho), T^{\star 1} - T^{\star 2} \right)$$

$$= E_{twr} (B_{ab}\rho, B_{ba}, T^{\star 2}) + E_{owr} \left(B_{ab} (1-\rho), T^{\star 1} - T^{\star 2} \right)$$

$$+ (2P_{O}^{c} - P_{T}^{c}) T^{\star 2}$$
(31)

where the inequality comes from Lemma 2, the first equality is from the equality condition of (12), and for the second equality, we have applied (28).

Define the following set of bit allocation and transmission time for the HRT as $B_{ab}^{owr2} = B_{ab}(1-\rho)$, $B_{ba}^{owr2} = 0$, $T_{ab}^2 = T^{\star 1} - T^{\star 2}$, $T_{ba}^2 = 0$, and $T_{twr}^2 = T^{\star 2}$. It is easy to show that $T_{ab}^2 + T_{ba}^2 + T_{twr}^2 = T^{\star 1} = \max\{T_{twr}^1 + T_{ab}^1, T_{twr}^1 + T_{ba}^1\} \le T$. The corresponding energy consumption can be obtained by substituting the set $(B_{ab}^{owr2}, B_{ba}^{owr2}, T_{ab}^2, T_{ba}^2, T_{twr}^2)$ into (10) as follows:

$$E_{H}^{2} = E_{\text{owr}}(B_{ab}(1-\rho), T^{\star 1} - T^{\star 2}) + E_{\text{twr}}(B_{ab}\rho, B_{ba}, T^{\star 2}) + (T - T^{\star 1})(3P^{\text{ci}}).$$
(32)

By substituting (31) and (32) into (29), we obtain

$$E_{H}^{1} \ge E_{H}^{2} + \left(T^{\star 2} - T_{\text{twr}}^{1}\right) \left(2P_{O}^{c} - P_{T}^{c}\right) \\ + \left(T^{\star 1} - T_{\text{twr}}^{1} - T_{ab}^{1} - T_{ba}^{1}\right) \left(3P^{\text{ci}}\right).$$
(33)

From the definitions of P_O^c and P_T^c in (1) and (6), we have

$$2P_O^c - P_T^c = 0.5P^{\rm ct} + 0.5P^{\rm cr} + 2P^{\rm ci} \ge 3P^{\rm ci}$$
(34)

where the inequality is because $P^{ct} \ge P^{ci}$ and $P^{cr} \ge P^{ci}$.

Substituting (34) into (33), and considering that $T^{\star 2} = \min\{T_{\text{twr}}^1 + T_{ab}^1, T_{\text{twr}}^1 + T_{ba}^1\} \ge T_{\text{twr}}^1$, we have

$$E_{H}^{1} \ge E_{H}^{2} + \left(T^{\star 1} + T^{\star 2} - 2T_{\text{twr}}^{1} - T_{ab}^{1} - T_{ba}^{1}\right)(3P^{\text{ci}}) = E_{H}^{2}$$
(35)

where the equality comes from the definitions of $T^{\star 1}$ and $T^{\star 2}$ in (30).

Now, we have found such a specific set of $(B_{ab}^{owr2}, B_{ba}^{owr2}, T_{ab}^2, T_{ba}^2, T$

APPENDIX D Proof of Proposition 2

Considering that $P^{ct} = P^{cr} = P^{ci} \approx 0$, (14) becomes

$$\min_{\substack{B_{ab}^{owr}, T_{ab}, T_{twr}}} \frac{N_0}{2\epsilon |h_{eff}|^2} \left[T_{ab} \left(2^{\frac{2B_{ab}^{owr}}{T_{ab}W}} - 1 \right) + T_{twr} \left(2^{\frac{2(B_{ab} - B_{ab}^{owr})}{T_{twr}W}} + 2^{\frac{2B_{ba}}{T_{twr}W}} - 2 \right) \right]$$
s.t. $0 \le B_{ab}^{owr} \le B_{ab}; \quad T_{ab} \ge 0; \quad T_{twr} \ge 0$
 $T_{ab} + T_{twr} \le T.$ (36)

It is easy to show that the objective function in (36) is a decreasing function of T_{ab} and T_{twr} . Therefore, to minimize the energy consumption of the HRT, the duration for the one-way relaying stage and the two-way relaying stage should occupy the whole block duration, i.e., $T_{ab} + T_{twr} = T$. By

substituting this expression into both the objective function and the constraints of (36), the problem is reformulated as

$$\begin{split} \min_{\substack{B_{ab}^{\text{owr}}, T_{ab}}} & \frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left[T_{ab} \left(2^{\frac{2B_{ab}^{\text{owr}}}{T_{ab}W}} - 1 \right) \\ & + (T - T_{ab}) \left(2^{\frac{2(B_{ab} - B_{ab}^{\text{owr}})}{(T - T_{ab})W}} - 1 \right) \\ & + (T - T_{ab}) \left(2^{\frac{2B_{ba}}{(T - T_{ab})W}} - 1 \right) \right] \\ \text{s.t.} & 0 \le B_{ab}^{\text{owr}} \le B_{ab}, \quad 0 \le T_{ab} \le T. \end{split}$$
(37)

We employ a similar approach as in [28] to solve such a joint optimization problem for B_{ab}^{owr} and T_{ab} . We first optimize B_{ab}^{owr} , given that the transmission time T_{ab} is fixed; then, the optimal value of B_{ab}^{owr} will be a function of T_{ab} . After substituting this function into the original problem, we will obtain an optimization problem only related to T_{ab} , which is equivalent to the original joint optimization problem.

Using the same method that we used to prove the convexity of the problem (11), we can show that the objective function in (37) is a convex function. Therefore, by taking the derivative of the objective function in (37) with respect to B_{ab}^{owr} and setting it to be zero, it is easy to obtain the optimal B_{ab}^{owr} as a function of T_{ab} , i.e.,

$$B_{ab}^{\rm owr-opt} = B_{ab} \frac{T_{ab}}{T}$$
(38)

which satisfies the constraints in (37).

By substituting (38) into (37), the original problem is *equivalent* to the following problem:

$$\min_{T_{ab}} \frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left[T \left(2^{\frac{2B_{ab}}{TW}} - 1 \right) + (T - T_{ab}) \left(2^{\frac{2B_{ba}}{(T - T_{ab})W}} - 1 \right) \right]$$
s.t. $0 \le T_{ab} \le T$. (39)

It is easy to show that the objective function is an increasing function of T_{ab} ; therefore, the optimal solution $T_{ab}^{\text{opt}} = 0$. Then $B_{ab}^{\text{owr-opt}} = B_{ab}T_{ab}^{\text{opt}}/T = 0$, and $T_{\text{twr}}^{\text{opt}} = T - T_{ab}^{\text{opt}} = T$. Now, the proof is complete.

APPENDIX E Proof of Lemma 3

When the optimal value of t that minimizes the energy consumption $E = E_{owr}(b,t) + (T-t)(3P^{ci})$ is less than T, there is no need to consider the maximum length constraint on t. The optimal value of t can be obtained by solving the following problem:

$$\min_{t} E_{owr}(b,t) + (T-t)(3P^{c_{1}})$$
s.t. $t > 0.$ (40)

It is not hard to show that the objective function is convex by calculating its second-order derivative. Therefore, the objective function should achieve its minimum value at the stationary point, where the first-order derivative of the objective function equals zero, i.e.,

$$\frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2b}{tW}} - 1 \right) + P_O^c - 3P^{\text{ci}} - \frac{N_0 \ln 2}{2\epsilon |h_{\text{eff}}|^2} 2^{\frac{2b}{tW}} \frac{2b}{tW} = 0|_{t=t^{\text{opt}}}.$$
(41)

However, it is difficult to directly find the stationary point from (41); thus, it is hard to show whether or not the stationary point satisfies the constraint in (40). In this case, the solution of t for optimization problem (40) is either the stationary point or the boundary point t = 0. From the definition of $E_{owr}(b,t)$ in (4) and the L' Hopital's rule, it is not hard to show that $E_{owr}(b,t)$ goes to infinity as t approaches zero with any nonzero number of bits b. Therefore, the optimal solution is not the boundary point t = 0. Instead, it is the stationary point of the objective function, which satisfies (41).

Denote $R \stackrel{\Delta}{=} b/t$ and substitute it into (41). Then, we have

$$\frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2R}{W}} - 1 \right) + P_O^c - 3P^{\text{ci}} - \frac{N_0 \ln 2}{2\epsilon |h_{\text{eff}}|^2} 2^{\frac{2R}{W}} \frac{2R}{W} = 0|_{R=R^{\text{opt}}}$$
(42)

which shows that the optimal value of $R^{\text{opt}} = b/t^{\text{opt}}$ is not a function of b.

Since the optimal value of t must satisfy (41), we can obtain the minimum value of E with t^{opt} as follows:

$$E^{\min} = E_{\text{owr}}(b, t^{\text{opt}}) - (3P^{\text{ci}})(T - t^{\text{opt}})$$

$$= t^{\text{opt}} \left[\frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left(2^{\frac{2b}{t^{\text{opt}}W}} - 1 \right) + P_O^c - 3P^{\text{ci}} \right]$$

$$= b \frac{N_0 \ln 2}{W\epsilon |h_{\text{eff}}|^2} 2^{\frac{2b}{Wt^{\text{opt}}}}$$

$$= b \frac{N_0 \ln 2}{W\epsilon |h_{\text{eff}}|^2} 2^{\frac{2R^{\text{opt}}}{W}} \stackrel{\Delta}{=} b e_{\text{owr}}^{\min}$$
(43)

where for the second equality we have substituted the expression of $E_{owr}(b, t)$ in (4); for the third equality we have applied (41); for the fourth equality we have used the definition $R^{opt} \stackrel{\Delta}{=} b/t^{opt}$; and e_{owr}^{min} is defined as

$$e_{\rm owr}^{\rm min} \stackrel{\Delta}{=} \frac{N_0 \ln 2}{W\epsilon |h_{\rm eff}|^2} 2^{\frac{2R^{\rm opt}}{W}}.$$
 (44)

Since R^{opt} is not a function of b, neither does e_{owr}^{\min} .

APPENDIX F Proof of Lemma 4

When the optimal value of t that minimizes the energy consumption $E = E_{twr}(\beta b, (1 - \beta)b, t) + (T - t)(3P^{ci})$ is less than T, there is no need to consider the maximum length constraint on t. The optimal value of t can be obtained by solving the following problem:

$$\min_{t} \quad E_{twr}\left(\beta b, (1-\beta)b, t\right) + (T-t)(3P^{ci})$$
s.t. $t \ge 0.$
(45)

Following the similar analysis that we used to prove Lemma 3, we can obtain that the optimal solution of t is the stationary point of the objective function, where the first-order derivative of the objective function in (45) with respect to t is zero, i.e.,

$$\frac{N_0}{2\epsilon|h_{\text{eff}}|^2} \left[2^{\frac{2\beta b}{tW}} + 2^{\frac{2(1-\beta)b}{tW}} - 2 \right] + P_T^c - 3P^{\text{ci}} - \frac{N_0 \ln 2}{2\epsilon|h_{\text{eff}}|^2} \left[2^{\frac{2\beta b}{tW}} \frac{2\beta b}{tW} + 2^{\frac{2(1-\beta)b}{tW}} \frac{2(1-\beta)b}{tW} \right] = 0|_{t=t^{\text{opt}}}.$$
(46)

Denote $R \stackrel{\Delta}{=} b/t$ and substitute it into (46). Then, we have

$$\frac{N_0}{2\epsilon |h_{\text{eff}}|^2} \left[2^{\frac{2\beta R}{W}} + 2^{\frac{2(1-\beta)R}{W}} - 2 \right] + P_T^c - 3P^{\text{ci}} - \frac{N_0 \ln 2}{2\epsilon |h_{\text{eff}}|^2} \left[2^{\frac{2\beta R}{W}} \frac{2\beta R}{W} + 2^{\frac{2(1-\beta)R}{W}} \frac{2(1-\beta)R}{W} \right] = 0|_{R=R^{\text{opt}}}$$
(47)

which shows that the optimal value of $R^{\text{opt}} = b/t^{\text{opt}}$ depends on β but is not a function of b.

By taking the derivative of the left-hand side of (47) with respect to β and after some regular manipulations, we have

$$\frac{d2^{\frac{2\beta R_{\rm twr}^{\rm opt}}{W}}}{d\beta}\beta + \frac{d2^{\frac{2(1-\beta)R_{\rm twr}^{\rm opt}}{W}}}{d\beta}(1-\beta) = 0.$$
(48)

Based on these results, we can obtain the minimum energy consumption with t^{opt} as follows:

$$E^{\min} = E_{twr} \left(\beta b, (1-\beta)b, t^{opt}\right) - (3P^{ci})(T-t^{opt})$$

$$= t^{opt} \left\{ \frac{N_0}{2\epsilon |h_{eff}|^2} \left[2^{\frac{2\beta b}{topt_W}} + 2^{\frac{2(1-\beta)b}{topt_W}} - 2 \right] + P_T^c - 3P^{ci} \right\}$$

$$= b \left\{ \frac{N_0 \ln 2}{W\epsilon |h_{eff}|^2} \left[2^{\frac{2\beta b}{Wt^{opt}}} \beta + 2^{\frac{2(1-\beta)b}{Wt^{opt}}} (1-\beta) \right] \right\}$$

$$= b \left\{ \frac{N_0 \ln 2}{W\epsilon |h_{eff}|^2} \left[2^{\frac{2\beta R^{opt}}{W}} \beta + 2^{\frac{2(1-\beta)R^{opt}}{W}} (1-\beta) \right] \right\}$$

$$\stackrel{\Delta}{=} b e_{twr}^{\min}(\beta) \qquad (49)$$

where for the second equality we have substituted the expression of $E_{twr}(b_1, b_2, t)$ in (8); for the third equality we have applied (46); for the fourth equality we have used the definition $R^{opt} = b/t^{opt}$; and $e_{twr}^{min}(\beta)$ is defined as

$$e_{\rm twr}^{\rm min}(\beta) \stackrel{\Delta}{=} \frac{N_0 \ln 2}{W\epsilon |h_{\rm eff}|^2} \left[2^{\frac{2\beta R^{\rm opt}}{W}} \beta + 2^{\frac{2(1-\beta)R^{\rm opt}}{W}} (1-\beta) \right].$$
(50)

Since R^{opt} is not a function of b, neither is $e_{\text{twr}}^{\min}(\beta)$.

In the following, we show that $e_{twr}^{min}(\beta)$ is a quasi-convex function of β , which achieves its minimum value when $\beta = 0.5$, i.e., the numbers of bits to be transmitted in the two directions of the TWRT are equal. Take the derivative of $e_{twr}^{min}(\beta)$ as follows:

$$\frac{de_{\text{twr}}^{\min}(\beta)}{d\beta} = \frac{N_0 \ln 2}{W\epsilon |h_{\text{eff}}|^2} \left[2^{\frac{2\beta R^{\text{opt}}}{W}} - 2^{\frac{2(1-\beta)R^{\text{opt}}}{W}} + \beta \frac{d2^{\frac{2\beta R^{\text{opt}}}{W}}}{d\beta} + (1-\beta) \frac{d2^{\frac{2(1-\beta)R^{\text{opt}}}{W}}}{d\beta} \right] \\
= \frac{N_0 \ln 2}{W\epsilon |h_{\text{eff}}|^2} \left[2^{\frac{2\beta R^{\text{opt}}}{W}} - 2^{\frac{2(1-\beta)R^{\text{opt}}}{W}} \right]$$
(51)

where at the second equality, we have substituted (48).

From (51), it is easy to see that when $\beta < 0.5$, $(de_{twr}^{min}(\beta)/d\beta) < 0$, i.e., $e_{twr}^{min}(\beta)$ decreases as β increases. On the other hand, when $\beta > 0.5$, $(de_{twr}^{min}(\beta)/d\beta) > 0$, i.e., $e_{twr}^{min}(\beta)$ increases with β . Therefore, $e_{twr}^{min}(\beta)$ is quasi-convex with respect to β . When $\beta = 0.5$, $(de_{twr}^{min}(\beta)/d\beta) = 0$, and $e_{twr}^{min}(\beta)$ achieves its minimum value.

APPENDIX G PROOF OF PROPOSITION 3

Since the optimal transmission duration is shorter than the block duration T, the maximal value constraint on the transmission duration in problem (14) can be omitted. Then, the problem can be reformulated as

$$\begin{array}{ll} \min_{B_{ab}^{\text{owr}},T_{ab},T_{\text{twr}}} & \left[E_{\text{owr}} \left(B_{ab}^{\text{owr}},T_{ab} \right) + (3P^{\text{ci}})(T-T_{ab}) \right] \\ & + \left[E_{\text{twr}} \left(B_{ab} - B_{ab}^{\text{owr}}, B_{ba}, T_{\text{twr}} \right) \\ & + (3P^{\text{ci}})(T-T_{\text{twr}}) \right] - 3P^{\text{ci}}T \\ \text{s.t.} & 0 \le B_{ab}^{\text{owr}} \le B_{ab}; \quad T_{ab} \ge 0; \quad T_{\text{twr}} \ge 0. \ (52)
\end{array}$$

We solve this problem using the same approach that we used to solve problem (37). We first find the optimal transmission duration in the one-way relaying stage T_{ab} and the optimal duration in the two-way relaying stage T_{twr} , given that the bit allocation B_{ab}^{owr} is fixed. Then, the optimal values of T_{ab} and T_{twr} will be functions of B_{ab}^{owr} . Finally, the original problem can be *equivalently* converted into a problem only related to B_{ab}^{owr} [28].

In the objective function of (52), the terms in the first square bracket only depend on T_{ab} , the terms in the second square bracket only depend on T_{twr} , and the last term is a constant. Considering that the constraints for T_{ab} and T_{twr} in the problem are decoupled, we can optimize T_{ab} and T_{twr} separately. Using Lemmas 3 and 4, we can, respectively, obtain

$$\begin{split} & \left[E_{\rm owr} \left(B_{ab}^{\rm owr}, T_{ab} \right) + (3P^{\rm ci})(T - T_{ab}) \right]_{\rm min} \\ &= B_{ab}^{\rm owr} e_{\rm owr}^{\rm min} \big|_{T_{ab} = T_{ab}^{\rm opt}} \tag{53a} \\ & \left[E_{\rm twr} \left(B_{ab} - B_{ab}^{\rm owr}, B_{ba}, T_{\rm twr} \right) + (3P^{\rm ci})(T - T_{\rm twr}) \right]_{\rm min} \\ &= \left(B_{ab} - B_{ab}^{\rm owr} + B_{ba} \right) e_{\rm twr}^{\rm min} (\beta) \big|_{T_{\rm twr} = T_{\rm twr}^{\rm opt}} \tag{53b} \end{split}$$

where
$$\beta \stackrel{\Delta}{=} (B_{ab} - B_{ab}^{owr} / B_{ab} - B_{ab}^{owr} + B_{ba}).$$

Upon substituting (53), problem (52) can be *equivalently* reformulated as

$$\min_{\substack{B_{ab}^{\text{owr}}\\B_{ab}}} \quad B_{ab}^{\text{owr}} e_{\text{owr}}^{\min} + (B_{ab} - B_{ab}^{\text{owr}} + B_{ba}) e_{\text{twr}}^{\min}(\beta) - 3P^{\text{ci}}T$$
s.t. $0 \le B_{ab}^{\text{owr}} \le B_{ab}$ (54)

where only the bit allocation needs to be optimized.

To solve this problem, we add a small increment δ on B_{ab}^{owr} and observe whether the objective function in problem (54) decreases. Denote $\beta' = (B_{ab} - B_{ab}^{owr} - \delta/B_{ab} - B_{ab}^{owr} - \delta + B_{ba})$; then, the increment of the objective function due to the increment of δ on B_{ab}^{owr} can be obtained as follows:

$$\Delta E_{H} = \left[\left(B_{ab}^{\text{owr}} + \delta \right) e_{\text{owr}}^{\min} + \left(B_{ab} - B_{ab}^{\text{owr}} - \delta + B_{ba} \right) e_{\text{twr}}^{\min}(\beta') \right] - \left[B_{ab}^{\text{owr}} e_{\text{owr}}^{\min} + \left(B_{ab} - B_{ab}^{\text{owr}} + B_{ba} \right) e_{\text{twr}}^{\min}(\beta) \right] = \delta \left[e_{\text{owr}}^{\min} - e_{\text{twr}}^{\min}(\beta') \right] + \left(B_{ab} - B_{ab}^{\text{owr}} + B_{ba} \right) \left[e_{\text{twr}}^{\min}(\beta') - e_{\text{twr}}^{\min}(\beta) \right].$$
(55)

When $\delta \ll (B_{ab} - B_{ab}^{owr} + B_{ba})$, we can approximately omit the first term in the equality; therefore

$$\Delta E_H \approx (B_{ab} - B_{ab}^{\text{owr}} + B_{ba}) \left[e_{\text{twr}}^{\min}(\beta') - e_{\text{twr}}^{\min}(\beta) \right].$$
(56)

Because $e_{\text{twr}}^{\min}(\beta)$ is quasi-convex with respect to β and achieves the minimum value at $\beta = 0.5$, from the expressions of β and β' , it is not hard to show that $\beta' < \beta$. Then, when $\beta > 0.5$, $\Delta E_H < 0$, and when $\beta < 0.5$, $\Delta E_H > 0$. Since $\beta \stackrel{\Delta}{=} (B_{ab} - B_{ab}^{\text{owr}} / B_{ab} - B_{ab}^{\text{owr}} + B_{ba})$, when $\beta > 0.5$, $B_{ab}^{\text{owr}} < B_{ab} - B_{ba}$, and when $\beta < 0.5$, $B_{ab}^{\text{owr}} > B_{ab} - B_{ba}$. Therefore, we have

$$\Delta E_H \begin{cases} <0, & B_{ab}^{\text{owr}} < B_{ab} - B_{ba} \\ >0, & B_{ab}^{\text{owr}} > B_{ab} - B_{ba} \end{cases}$$
(57)

which means that, when $B_{ab}^{owr} < B_{ab} - B_{ba}$, adding a δ on B_{ab}^{owr} will reduce the energy consumption, but when $B_{ab}^{owr} > B_{ab} - B_{ba}$, increasing B_{ab}^{owr} will increase the energy consumption. Then, the optimal value of B_{ab}^{owr} that minimizes the energy consumption should be $B_{ab}^{owr-opt} \approx B_{ab} - B_{ba}$. The approximation is because we omit the first term in (55).

Now, the proof is complete.

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