This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

Is Two-way Relay More Energy Efficient?

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***Abstract*— Two-way relay transmission (TWRT) is more spectral efficient than direct transmission (DT) and one-way relay transmission (OWRT). In this paper, we try to find out if TWRT is also more energy efficient in practice. To this end, we first derive and compare the energy efficiency of DT, OWRT and TWRT systems, when either the receiver processing power consumption is taken into account or not. We then compare the maximal energy efficiency achieved by TWRT and OWRT, as well as the corresponding spectral efficiency. Both analytical and simulation results show that TWRT is not always more energy efficient than DT and OWRT, depending on the channel condition, processing power consumption and the bidirectional date rate requirements. Generally speaking, in high spectral efficiency and large path loss attenuation region, the TWRT provides high energy efficiency.**

I. Introduction

Relaying has been extensively studied since it is a promising strategy for wireless systems [1]. To recover the spectral efficiency (SE) loss led by the half-duplex constraint in one-way relay transmission (OWRT), two-way relay transmission (TWRT) is proposed. Considerable efforts have been devoted to analyze and design various TWRT systems, see [2]–[5] and references therein.

Since explosive growth of wireless services is sharply increasing their contribution to the carbon footprint, energy efficiency (EE) has drawn more and more attention recently as a new goal for designing systems [6], [7]. A widely used metric for EE is the number of transmitted bits per unit energy. In practical systems, not only the power for transmitting information bits but also various signaling and transceiver circuits contribute to the system energy consumption. Therefore, the optimization problem to minimize the overall transmit power does not necessarily lead to a high EE design [7].

Relaying has been viewed as an energy saving technique because it can reduce the transmit power by breaking long range transmission into short range transmission. In [8], considering both transmit power and receiver processing power, the EE of decode-and-forward (DF) OWRT systems is studied. In [9], after taking into account the energy cost of acquiring channel information, relay selection for an OWRT system with multiple DF relays is optimized to maximize the EE. In [10], [11], the EE of relay and base station cooperation transmission is compared by considering the overall energy costs including those from manufacture or deployment.

Although TWRT systems are spectral efficient, it is not clear whether they are also energy efficient. In [12], the authors show by simulation that TWRT is more energy efficient than OWRT, where their main focus is on the SE. However, only transmit power is considered in their energy consumption model. When we take into account the energy costs other than that contributed by transmit power, will TWRT still be more energy efficient?

In this paper, we attempt to answer this question by studing a simple amplify-and-forward (AF) relay system. We consider both transmit power and receiver processing power consumption, and respectively derive the SE-EE relationship for direct transmission (DT), OWRT, and TWRT. After comparing the EE of the three strategies, we show that TWRT is *not* always more energy efficient considering non-zero receiver processing power consumption. We obtain the EE crossover points between these strategies, and analyze the impact of various system and channel settings on the results.

This work was supported by the International S&T Cooperation Program of China (ISCP) under Grant No. 2008DFA12100.

II. System Model

We consider a system consisting of two source nodes $\mathbb{A}$ and $\mathbb{B}$, and an AF relay node $\mathbb{R}$. All the nodes are equipped with single antenna. The channel between the nodes $\mathbb{A}$ and $\mathbb{B}$ is denoted as h_{ab}, the channels from nodes $\mathbb{A}$ and $\mathbb{B}$ to node $\mathbb{R}$ are respectively denoted as h_{ar} and h_{br}. We assume that the noise power N_0 is identical at each node. The nodes $\mathbb{A}$ and $\mathbb{B}$ intend to exchange their signals x_a and x_b respectively with the transmit powers P_A and P_B. We assume that $|x_a|^2 = |x_b|^2 = 1$. Three strategies are considered to complete the bidirectional transmission, i.e., DT, OWRT and TWRT.

A. Direct Transmission

When two source nodes transmit to each other without the assistance of the relay, two time slots are required to complete the bidirectional transmission. One is for $\mathbb{A} \rightarrow \mathbb{B}$ transmission, and the other is for $\mathbb{B} \rightarrow \mathbb{A}$ transmission. The received signals at the two nodes are respectively given as $y_b = \sqrt{P_A} h_{ab} x_a + n_b$ and $y_a = \sqrt{P_B} h_{ab} x_b + n_a$, where n_a and n_b are the noises at the two nodes. Then the bidirectional data rates with unit bandwidth are respectively

$$R_{ab} = \frac{1}{2} \log_2(1 + \frac{|h_{ab}|^2 P_A}{N_0}), \quad (1)$$

$$R_{ba} = \frac{1}{2} \log_2(1 + \frac{|h_{ab}|^2 P_B}{N_0}), \quad (2)$$

where the factor $1/2$ is due to the two time slots duration.

Denote the receiver processing power consumption as P_{re}, which is assumed identical at the receiver of each node. During the two time slots, the transmitter of node $\mathbb{A}$ and the receiver of node $\mathbb{B}$ are active in the 1st time slot, the transmitter of node $\mathbb{B}$ and the receiver of node $\mathbb{A}$ are active in the 2nd time slot. We denote T as the duration of one time slot. Then the average power consumed by the DT system is given by

$$\begin{aligned} P_S &= [(P_A + P_{re})T + (P_B + P_{re})T]/(2T) \\ &= (P_A + P_B + 2P_{re})/2. \end{aligned} \tag{3}$$

B. One-way Relay Transmission

In OWRT, each of the $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$ transmission is divided into two hops, thus the bidirectional transmission needs four time slots. In $\mathbb{A} \to \mathbb{B}$ transmission, the node $\mathbb{A}$ transmits to the relay, the received signal at relay is $y_r = \sqrt{P_A} h_{ar} x_a + n_r$, where n_r is the noise at relay. The relay then amplifies and forwards y_r to node $\mathbb{B}$. The received signal at node $\mathbb{B}$ is

$$y_b = \alpha_r \sqrt{P_A} h_{ar} h_{br} x_a + \alpha_r h_{br} n_r + n_b, \tag{4}$$

where $\alpha_r = \sqrt{P_{R1}/(|h_{ar}|^2 P_A + N_0)}$ is the amplifying factor at the relay, P_{R1} is the transmit power of the relay. The data rate of link $\mathbb{A} \to \mathbb{B}$ with unit bandwidth is computed as

$$R_{ab} = \frac{1}{4}\log_2(1 + \frac{|h_{ar}|^2 |h_{br}|^2 P_A P_{R1}}{|h_{ar}|^2 P_A N_0 + |h_{br}|^2 P_{R1} N_0 + N_0^2}), \tag{5}$$

where the factor $1/4$ is due to the four time slots duration. Similarly, the data rate of link $\mathbb{B} \to \mathbb{A}$ can be obtained as

$$R_{ba} = \frac{1}{4}\log_2(1 + \frac{|h_{ar}|^2 |h_{br}|^2 P_B P_{R2}}{|h_{ar}|^2 P_{R2} N_0 + |h_{br}|^2 P_B N_0 + N_0^2}), \tag{6}$$

where P_{R2} is the transmit power of the relay for $\mathbb{B} \to \mathbb{A}$ transmission.

Assume that the receiver processing power consumption at the relay is also P_{re}. The average power consumed by the OWRT system is obtained as follows by applying the similar procedure as that in the DT system,

$$P_S = (P_A + P_{R1} + P_B + P_{R2} + 4P_{re})/4. \tag{7}$$

C. Two-way Relay Transmission

In TWRT, two time slots are required for the bidirectional transmission. First, both node $\mathbb{A}$ and node $\mathbb{B}$ transmit to the relay. The received signal at the relay is given as $y_r = \sqrt{P_A} h_{ar} x_a + \sqrt{P_B} h_{br} x_b + n_r$. Then the relay amplifies and forwards y_r to both node $\mathbb{A}$ and node $\mathbb{B}$. The received signal at node $\mathbb{B}$ is

$$y_b = \alpha_r \sqrt{P_A} h_{ar} h_{br} x_a + \alpha_r \sqrt{P_B} h_{br}^2 x_b + \alpha_r h_{br} n_r + n_b, \tag{8}$$

where the second term $\alpha_r \sqrt{P_B} h_{br}^2 x_b$ is the transmitted signal of node $\mathbb{B}$, which is known by node $\mathbb{B}$ and thus can be removed via self interference cancelation (SIC) [2], $\alpha_r = \sqrt{P_R/(|h_{ar}|^2 P_A + |h_{br}|^2 P_B + N_0)}$ is the amplifying factor at the relay, P_R is the transmit power of the relay. The data rate of link $\mathbb{A} \to \mathbb{B}$ with unit bandwidth is then

$$R_{ab} = \frac{1}{2}\log_2(1 + \frac{|h_{ar}|^2 |h_{br}|^2 P_A P_R / N_0}{|h_{ar}|^2 P_A + |h_{br}|^2 P_B + |h_{br}|^2 P_R + N_0}), \tag{9}$$

where the factor $1/2$ is led by the two time slots duration. Similarly, the data rate of link $\mathbb{B} \to \mathbb{A}$ with unit bandwidth is

$$R_{ba} = \frac{1}{2}\log_2(1 + \frac{|h_{ar}|^2 |h_{br}|^2 P_B P_R / N_0}{|h_{ar}|^2 P_A + |h_{br}|^2 P_B + |h_{ar}|^2 P_R + N_0}). \tag{10}$$

In TWRT, the receiver at the relay consumes identical processing power P_{re} as that in OWRT, while each of the receivers at the nodes $\mathbb{A}$ and $\mathbb{B}$ consumes more processing power than that in DT or OWRT owing to the SIC. We use P_{SIC} to denote the extra processing power consumption for SIC. Similarly, the average power consumed by the TWRT system can be obtained as follows,

$$P_S = (P_A + P_B + P_R + P_{re} + 2(P_{re} + P_{SIC}))/2. \tag{11}$$

III. Energy Efficiency of Three Systems

In this section, we will derive the EE for the DT, OWRT and TWRT systems.

We refer the bidirectional sum rate per unit bandwidth, i.e., $R_S = R_{ab} + R_{ba}$, as the SE. We define EE of three strategies in an unified form, which is the bidirectional transmitted bit number per unit bandwidth per unit energy, i.e., $\eta = R_S/P_S$.

In practice, the data rates R_{ab} and R_{ba} may differ. Moreover, for a given R_S, there exist various rate pairs $\{R_{ab}, R_{ba}\}$ that satisfy $R_{ab} + R_{ba} = R_S$, but the EE of the systems with different rate pairs are different. We define $R_{ab}/R_S = \beta$ and $R_{ba}/R_S = 1 - \beta$ to reflect such an asymmetric rate scenario, where $0 < \beta < 1$.

A. Direct Transmission

To achieve a given rate pair of $R_{ab} = \beta R_S$ and $R_{ba} = (1-\beta)R_S$, the required transmit powers for the nodes $\mathbb{A}$ and $\mathbb{B}$ can be obtained from (1) and (2). From (3) the average power consumption is

$$P_S^D = \frac{(2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2)N_0}{2|h_{ab}|^2} + P_{re}. \tag{12}$$

Then the EE of DT is $\eta^D = R_S/P_S^D$.

B. One-way Relay Transmission

To achieve a given rate pair of R_{ab} and R_{ba}, it is shown from (5) and (6) that we need to find four transmit powers P_A, P_B, P_{R1} and P_{R2} from two equations. Apparently, the solution is not unique, which leads to multiple solutions of P_S to achieve the given bidirectional rates. This allows us to find the minimum P_S to maximize the EE, which can be achieved by solving the following problem,

$$\begin{aligned} \min_{P_A, P_{R1}, P_B, P_{R2}} \quad & (P_A + P_{R1})/4 + (P_B + P_{R2})/4 + P_{re} \\ \text{s.t.} \quad & R_{ab} = \beta R_S, \; R_{ba} = (1-\beta)R_S. \end{aligned} \tag{13}$$

As shown in (5) and (6), R_{ab} is not a function of P_B and P_{R2}, while R_{ba} is not a function of P_A and P_{R1}. Therefore, the above problem can be equivalently decoupled into two subproblems as

$$\min_{P_A, P_{R1}} \quad P_A + P_{R1} \tag{14a}$$
$$\text{s.t.} \quad R_{ab} = \beta R_S, \tag{14b}$$

$$\min_{P_B, P_{R2}} \quad P_B + P_{R2} \tag{15a}$$
$$\text{s.t.} \quad R_{ba} = (1-\beta) R_S, \tag{15b}$$

Upon substituting (5), (14b) can be rewritten as

$$P_A = \frac{C(|h_{br}|^2 P_{R1} N_0 + N_0^2)}{|h_{ar}|^2(|h_{br}|^2 P_{R1} - C N_0)} \triangleq f(P_{R1}), \tag{16}$$

where $C \triangleq 2^{4\beta R_S} - 1$. Substituting (16) into (14a), the problem (14) is reformulated as follows,

$$\min_{P_{R1}} \quad f(P_{R1}) + P_{R1}. \tag{17}$$

The optimal P_{R1} can be found by setting the derivative of $f(P_{R1}) + P_{R1}$ to zero. Substituting the optimal P_{R1} into (16), the optimal P_A is obtained. Following the same procedure, the optimal P_B and P_{R2} can be solved. Detailed derivation is omitted due to the lack of space. Finally, we can obtain the minimum average power consumption as,

$$\begin{aligned} P_S^O =& \frac{N_0}{4}\Big[\big(\frac{1}{|h_{ar}|^2} + \frac{1}{|h_{br}|^2}\big)(2^{4\beta R_S} + 2^{4(1-\beta)R_S} - 2) \\ &+ \frac{2}{|h_{ar}||h_{br}|}\Big(\sqrt{2^{4(1-\beta)R_S}(2^{4(1-\beta)R_S} - 1)} \\ &+ \sqrt{2^{4\beta R_S}(2^{4\beta R_S - 1})}\Big)\Big] + P_{re}. \end{aligned} \tag{18}$$

Then the EE of OWRT is $\eta^O = R_S / P_S^O$.

C. Two-way Relay Transmission

Analogous to OWRT, there exist multiple solutions of the transmit power for the three nodes to achieve the given rate pair R_{ab} and R_{ba} in TWRT. The minimum P_S can be solved from the following optimization problem,

$$\begin{aligned} \min_{P_A, P_B, P_R} & (P_A + P_B + P_R + P_{re} + 2(P_{re} + P_{SIC}))/2 \\ \text{s.t.} \quad & R_{ab} = \beta R_S, \; R_{ba} = (1-\beta) R_S. \end{aligned} \tag{19}$$

Using similar approach as that we used to solve the problem (14), we can find the minimum average power consumption in the TWRT system as

$$\begin{aligned} P_S^T =& N_0\Big[\frac{2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2}{2}\big(\frac{1}{|h_{ar}|^2} + \frac{1}{|h_{br}|^2}\big) + \\ & \frac{\sqrt{(2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2)(2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 1)}}{|h_{ar}||h_{br}|}\Big] \\ &+ 1.5 P_{re} + P_{SIC}, \end{aligned} \tag{20}$$

and the EE of TWRT is $\eta^T = R_S / P_S^T$.

IV. Energy Efficiency Comparison

In this section, we will compare the EE of TWRT with those of DT and OWRT. As a baseline, we first consider a case where the receiver processing power consumption is zero. Then we consider a more practical case when the processing power consumption is non-zero.

A. Zero Receiver Processing Power

In this case, a system with less average transmit power is more energy efficient.

1) TWRT vs. OWRT:

To compare the EE of TWRT and OWRT, we find the EE difference between them, i.e.,

$$\eta^T - \eta^O = \frac{R_S}{P_S^T} - \frac{R_S}{P_S^O} = \frac{R_S}{P_S^T P_S^O}(P_S^O - P_S^T). \tag{21}$$

Since both P_S^O and P_S^T are functions of R_S as shown in (18) and (20), we define

$$F(R_S) = P_S^O - P_S^T. \tag{22}$$

By substituting (18) and (20) into (22), and considering that $P_{re} = P_{SIC} = 0$, we find that $F(0) = 0$.

When the bidirectional data rates are symmetric, i.e., $\beta = 0.5$, it is not hard to prove that the derivative of $F(R_S)$ satisfies $F'(R_S) \geq 0$ for any $R_S \geq 0$. This means that $F(R_S) \geq F(0) = 0$, i.e., $P_S^O - P_S^T \geq 0$, and therefore $\eta^T - \eta^O \geq 0$.

Observation 1: If the receiver processing power consumption is zero, and the bidirectional data rates are symmetric, TWRT is always more energy efficient than OWRT.

When the bidirectional rates are asymmetric, we can not rigorously draw the same conclusion. Nevertheless, simulation in next section will show that TWRT is almost always more energy-efficient when $\beta \neq 0.5$.

2) TWRT vs. DT:

Considering that $P_{re} = P_{SIC} = 0$, the EE of DT can be derived as follows by substituting (12),

$$\eta^D = \frac{R_S}{P_S^D} = \frac{2 R_S |h_{ab}|^2}{(2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2) N_0}. \tag{23}$$

Since P_S^T in (20) is very complicated, the expression of η^T is also too complex. To gain useful insight into the problem, we consider the following approximation for high R_S,

$$2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 1 \approx 2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2. \tag{24}$$

Upon substituting (24) and (20), and considering that $P_{re} = P_{SIC} = 0$, the EE of TWRT is obtained as

$$\begin{aligned} \eta^T &= R_S / P_S^T \\ &\approx R_S \Big/ \left[N_0\big(\frac{2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2}{2}\big)\big(\frac{1}{|h_{ar}|} + \frac{1}{|h_{br}|}\big)^2\right] \\ &= \frac{2 R_S |h_{eff}|^2}{(2^{2\beta R_S} + 2^{2(1-\beta)R_S} - 2) N_0}, \end{aligned} \tag{25}$$

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where $|h_{eff}| \triangleq 1/\left(\frac{1}{|h_{ar}|} + \frac{1}{|h_{br}|}\right)$ can be viewed as an equivalent channel gain between the two source nodes after the usage of the relay.

With this definition, the EE in (25) and the EE in (23) are expressed in an unified form. It implies that to achieve a given R_S, the EE of a TWRT system is the same as the EE of a DT system with an equivalent channel gain of $|h_{eff}|$.

Observation 2: If $|h_{eff}| > |h_{ab}|$, TWRT is more energy efficient than DT. Otherwise, DT is more energy efficient.

To gain further insight into this issue, we consider an AWGN channel[1], where $|h_{ab}|^2$ is normalized as 1, the distance from relay $\mathbb{R}$ to node $\mathbb{A}$ and node $\mathbb{B}$ are respectively d and $1-d$. Then $|h_{ar}|^2 = (\frac{1}{d})^\alpha$ and $|h_{br}|^2 = (\frac{1}{1-d})^\alpha$, where α is the path loss attenuation factor. Then the equivalent channel gain becomes

$$|h_{eff}| = \frac{1}{d^{\alpha/2} + (1-d)^{\alpha/2}}. \tag{26}$$

When $d = 0.5$, the relay node is placed in the middle of two source nodes, $|h_{eff}|$ achieves its maximal value, and thus the EE of TWRT is maximized. In this case, $|h_{eff}| = 2^{\alpha/2}/2 > |h_{ab}| = 1$ only when $\alpha > 2$. It means that in this scenario, TWRT is more energy efficient than DT only when the path loss attenuation factor is larger than 2.

B. Non-zero Receiver Processing Power

1) TWRT vs. DT and TWRT:

To derive a simple expression of the EE of TWRT, we again employ the high R_S approximation in (24). We also consider the following approximations when deriving the EE of OWRT,

$$2^{4(1-\beta)R_S} \approx 2^{4(1-\beta)R_S} - 1, \tag{27a}$$
$$2^{4\beta R_S} \approx 2^{4\beta R_S} - 1. \tag{27b}$$

Upon substituting the approximations (24) and (27), and P_S^D, P_S^O and P_S^T in (12), (18) and (20), we obtain the EE of the three strategies as follows,

$$\eta^D = \frac{R_S}{P_S^D} = \frac{R_S}{\frac{N_0(2^{2\beta R_S}+2^{2(1-\beta)R_S}-2)}{2|h_{ab}|^2} + P_{re}}, \tag{28}$$

$$\eta^O = \frac{R_S}{P_S^O} \approx \frac{R_S}{\frac{N_0(2^{4\beta R_S}+2^{4(1-\beta)R_S}-2)}{4|h_{eff}|^2} + P_{re}}, \tag{29}$$

$$\eta^T = \frac{R_S}{P_S^T} \approx \frac{R_S}{\frac{N_0(2^{2\beta R_S}+2^{2(1-\beta)R_S}-2)}{2|h_{eff}|^2} + (1.5P_{re} + P_{SIC})}. \tag{30}$$

At the denominator in η^D or η^O or η^T, the first term is the transmit power, while the second term is the receiver processing power consumption.

If $R_S \to \infty$, all the transmit powers will grow to infinity in the three strategies, and the receiver processing power consumption will have little impact on the EE. Then the comparison result among η^D, η^O and η^T will be the same as that in last subsection. On the other hand, if $R_S \to 0$, all the transmit powers will reduce to zero. In this case it is easy to see from (28), (29) and (30) that both η^D and η^O are higher than η^T.

Observation 3: If R_S is high, the EE of TWRT is higher than OWRT, and also higher than DT if $|h_{eff}| > |h_{ab}|$. Otherwise, both OWRT and DT are more energy efficient than TWRT.

2) Crossover point of Energy Efficiency:

The Observation 3 indicates that there exists a EE crossover point between TWRT and DT when $|h_{eff}| > |h_{ab}|$, which can be obtained by substituting (28) and (30) into the equation $\eta^T = \eta^D$. When the bidirectional data rates are symmetric, i.e., $\beta = 0.5$, we can derive the closed-form of this crossover point as

$$R_{S,co1} = \log_2\left(1 + \frac{0.5P_{re} + P_{SIC}}{N_0(1/|h_{ab}|^2 - 1/|h_{eff}|^2)}\right). \tag{31}$$

If $R_S > R_{S,co1}$, TWRT is more energy efficient, otherwise, DT is better. Since TWRT requires more receiver processing power consumption than DT, as P_{re} and P_{SIC} become larger, the crossover point increases.

Similarly, the EE crossover point between TWRT and OWRT can be obtained by substituting (29) and (30) into the equation $\eta^T = \eta^O$. When $\beta = 0.5$, the closed-form of crossover point is derived as

$$R_{S,co2} = \log_2\left(1 + \sqrt{\frac{2|h_{eff}|^2(0.5P_{re} + P_{SIC})}{N_0}}\right). \tag{32}$$

Again, we can see that the increase in P_{re} and P_{SIC} will lead to a higher crossover point. Moreover, when $|h_{eff}|^2/N_0$ increases, both the TWRT and OWRT systems need less transmit power to achieve a given R_S, and the receiver processing power consumption will become dominant. Then OWRT will be more energy efficient than TWRT, and the crossover point increases.

3) Maximal Energy Efficiency and the Corresponding Spectral Efficiency:

Finally, we compare the maximal EE and the corresponding R_S achieving the maximal EE of different strategies.

It is hard to derive closed-form expressions of the maximal EE for DT, OWRT and TWRT systems. Fortunately, we still can compare the maximal EE between OWRT and TWRT systems by rewriting the EE of them into an unified form. To this end, we define $x_1 \triangleq 2R_S$ in (29), $x_2 \triangleq R_S$ in (30), and define a function as follows,

$$g(x) = \frac{N_0(2^{2\beta x} + 2^{2(1-\beta)x} - 2)}{2|h_{eff}|^2}. \tag{33}$$

Then the maximal EE of them can be written as

$$\max_{R_S}(\eta^O) \approx \max_{x_1}\left(\frac{x_1}{g(x_1) + 2P_{re}}\right) = \max_{x}\left(\frac{x}{g(x) + 2P_{re}}\right), \tag{34a}$$

$$\max_{R_S}(\eta^T) \approx \max_{x_2}\left(\frac{x_2}{g(x_2) + 1.5P_{re} + P_{SIC}}\right) = \max_{x}\left(\frac{x}{g(x) + 1.5P_{re} + P_{SIC}}\right). \tag{34b}$$

[1]It should be noted that AWGN channel is appropriate for modeling free space propagation where $\alpha = 2$. We consider different path loss attenuation factor here, which may be an abuse of the concept of "AWGN channel".

Apparently, the relationship between the maximal η^O and η^T depends on the values of $2P_{re}$ and $1.5P_{re} + P_{SIC}$. When $2P_{re} \geq 1.5P_{re} + P_{SIC}$, i.e., $P_{SIC} \leq 0.5P_{re}$, we can find that $\max(\eta^O) \leq \max(\eta^T)$. Otherwise, $\max(\eta^O) > \max(\eta^T)$.

Observation 4: If the P_{SIC} is lower than or equal to half of the P_{re}, the maximal EE of TWRT is higher or equal to that of OWRT. Otherwise, the maximal EE of TWRT is lower than that of OWRT.

When $2P_{re} = 1.5P_{re} + P_{SIC}$, the expressions of the maximal η^O and η^T in (34) have the same form. In this case, $\max(\eta^T) = \max(\eta^O)$. Moreover, the optimal x_1 that maximizes η^O equals the optimal x_2 that maximizes η^T. It is not hard to find that,

$$x_{1,opt} = 2R^O_{S,opt} = x_{2,opt} = R^T_{S,opt}, \tag{35}$$

i.e., the optimal rate that achieves the maximal EE in TWRT is twice of that in OWRT. It means that TWRT offers better SE-EE tradeoff than OWRT, because when operating at their respective maximal EE, TWRT achieves higher SE than OWRT.

It is hard to analyze the maximal EE of the DT system, which will be shown by simulations later.

V. Simulation Results

In this section, we compare EE of DT, OWRT and TWRT by simulations to validate our analysis.

We consider that the nodes $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{R}$ are located on a line. The distance between node $\mathbb{A}$ and node $\mathbb{B}$ is denoted as D, and those from node $\mathbb{R}$ to nodes $\mathbb{A}$ and $\mathbb{B}$ are d and $D-d$, respectively. The noise power is N_0. The path loss is set as $PL = 30 + 10\log_{10}(\text{distance}^{\alpha})$.

Since the increase in D, N_0, or α will all result in higher transmit power to achieve a given data rate, the impacts of D and N_0 on the system are similar to that of α. We then only consider α as a variant in the following, and set $D =$ 100 m, $N_0 = -94$ dBm, considering -174 dBm/Hz noise spectral density, $10M$ Hz bandwidth, and 10 dB receiver noise figure. We assume that all the small scale fading channels are i.i.d. Rayleigh fading channels. Unless otherwise specified, we always consider symmetric relay position and symmetric bidirectional data rates, i.e., $d = 50$ m and $\beta = 0.5$. All the results are averaged over 500 Monte-carlo trails of fading channels.

A. Zero Receiver Processing Power

As a baseline for comparison, we first compare the EE of three strategies considering zero receiver processing power consumption in scenarios of $\alpha = 2$ and 4, $\beta = 0.5$. We also consider the case where $\beta = 0.1$ and $\alpha = 4$ to compare the OWRT and TWRT with asymmetric bidirectional data rates.

For the two path poss attenuation factors, the EE differs distinctively. Since we are more interested in comparing the EE rather than its absolute value, we normalize the EE by the maximum EE of DT system for each α. The normalized EE versus SE of the three strategies are presented in Fig. 1.

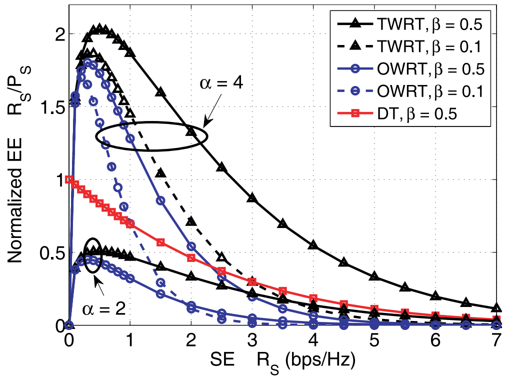

Fig. 1. EE comparison with zero receiver processing power consumption

It is shown that the EE of TWRT is almost always higher than that of OWRT in all cases. Because of the normalization, the curves of DT under different α overlap. We see that when the attenuation factor is large, i.e., $\alpha = 4$, TWRT is more energy efficient than DT, but when $\alpha = 2$, the EE of DT is higher. All these results are in accordance with our analysis.

In addition, we see that for low R_S the DT is more energy efficient than the two relay schemes. This is because in this scenario a large portion of the transmit power at the relay is consumed to forward the noise.

B. Non-zero Receiver Processing Power

The receiver processing power varies significantly in different hardware models. According to [1], [13], the receiver processing power consumption is usually from dozens mW to more than 100 mW. Here we set P_{re} as 100 mW. To the best of our knowledge, there is no model for the SIC power consumption. We respectively set P_{SIC} as 30, 50 and 70 mW. The EE versus SE of the three strategies are presented in Fig. 2.

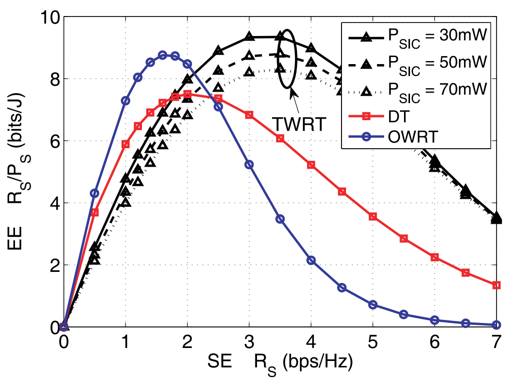

Fig. 2. EE comparison with receiver processing power consumption, $\alpha = 4$

It shows that TWRT is not always more energy efficient now. Generally, TWRT is more energy efficient than OWRT and DT in high R_S region. When R_S is low, both OWRT and DT outperform TWRT. As the P_{SIC} increases, the EE crossover points between TWRT and the other two schemes shift to larger values.

Now let us see the maximal values of EE of TWRT and OWRT. When $P_{SIC} = 50$ mW, since $P_{SIC} = 0.5P_{re}$, the maximal values of EE of OWRT and TWRT are almost identical, which are about 8.9 bits/J. The optimal R_S that achieves the maximal EE in TWRT is 3.5 bps/Hz, which is about twice of that in OWRT, i.e., 1.75 bps/Hz. All these results validate our analysis in last section.

Finally, we show energy efficient regions for the three strategies. In Fig. 3, OWRT is more energy efficient than the other two strategies in the region filled with circle markers, TWRT is more energy efficient in the region filled with triangle markers, and the EE of DT is the highest in the region with square markers. It is shown that TWRT is more energy efficient in high R_S and large α region, OWRT is better in low R_S and moderate α region, while DT outperforms both relay schemes in small α region. This result implies that the most energy efficient strategy depends on the data rate requirements and the environments. In practice, the system should adaptively switch among DT, OWRT and TWRT to achieve a specific bidirectional sum rate under different channel conditions.

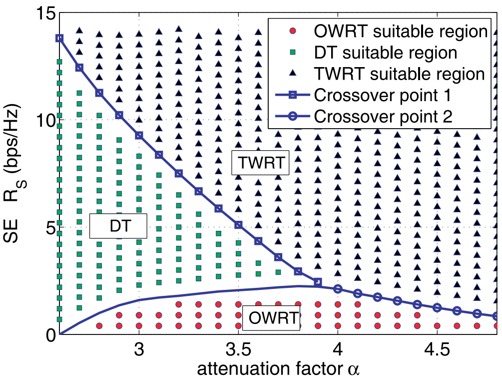

Fig. 3. Energy efficient regions for the three strategies

We also highlight the EE crossover points between TWRT and DT, and those between TWRT and OWRT in the figure, which are respectively denoted as “Crossover Point 1” and “Crossover Point 2”. Intuitively, this can be explained as follows. As α increases, the gap between the equivalent channel gain $|h_{eff}|$ when using TWRT and the direct link channel gain $|h_{ab}|$ also increases. Therefore, TWRT can save more transmit power than DT, which reduces the EE crossover point between them. On the other hand, to achieve a given rate both the TWRT and OWRT need more transmit power for a larger α, which occupies more proportion in the overall energy consumption. Then the comparison between TWRT and OWRT will be more and more similar to the case with zero receiver processing power consumption. Therefore, the EE crossover point between TWRT and OWRT decreases. All these results agree well with our former analysis.

Due to the lack of space, the simulation results under asymmetric bidirectional data rates are not shown here, which illustrate the similar phenomenon as that in Fig. 3.

VI. Conclusion

In this paper, we studied the energy efficiency of two-way relay transmission and compared it with those of direct transmission and one-way relay transmission. Analysis shows that to achieve the same spectral efficiency, two-way relay transmission consumes less transmit power than the other two strategies. However, since it consumes more receiver processing power, it is not always more energy efficient. Specifically, when the system operates at the region of high spectral efficiency, the two-way relaying provides high energy efficiency. Otherwise, either direct transmission or one-way relay transmission will be superior. The energy efficiency of the strategies depend on the path loss attenuation factor, the ratio of the bidirectional data rate and the power consumption model. We also show that the comparison result of the maximal energy efficiency between one-way relay and two-way relay transmission only depends on their receiver processing power consumption. Our analysis indicates that to minimize the energy consumption meanwhile to achieve a given bidirectional sum rate, a system should adaptively switch among the three strategies depending on the settings.

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