Power and Channel Allocation for Cooperative Relay in Cognitive Radio Networks

Guodong Zhao, *Student Member, IEEE*, Chenyang Yang, *Senior Member, IEEE*, Geoffrey Ye Li, *Fellow, IEEE*, Dongdong Li, *Member, IEEE*, and Anthony C. K. Soong, *Senior Member, IEEE*

***Abstract*—In this paper, we investigate power and channel allocation for cooperative relay in a three-node cognitive radio network. Different from conventional cooperative relay channels, cognitive radio relay channels can be divided into three categories: direct, dual-hop, and relay channels, which provide three types of parallel end-to-end transmission. In the context, those spectrum bands available at all three nodes may either perform relay diversity transmission or assist the transmission in direct or dual-hop channels. On the other hand, the relay node involves both dual-hop and relay diversity transmission. In this paper, we develop power and channel allocation approaches for cooperative relay in cognitive radio networks that can significantly improve the overall end-to-end throughput. We further develop a low complexity approach that can obtain most of the benefits from power and channel allocation with minor performance loss.**

***Index Terms*—Channel allocation, cognitive radio, cooperative relay, power control, resource allocation.**

I. Introduction

COGNITIVE radio (CR), a key technology for future wireless communications, is capable of sensing and adapting to environments [1]–[5]. It can adjust its transmission parameters, such as spectrum bands, transmission power, coding rates, and modulation levels, to opportunistically access available spectrum bands without interfering with *primary users* (PUs). In general, a CR user can access the spectrum bands that are not used by PUs. Spectrum sensing detects the availability of spectrum bands. Recent studies [6]–[8] have shown that the available spectrum bands may vary with different CR users. It happens when the transmission range of CR users is larger than or similar to that of PUs. Then different CR users may obtain different sensing results since they are at different locations and have different impacts on PU systems. For example, an available spectrum band at a CR transmitter may not be available at its intended CR receiver and vice versa [9]. Nevertheless, secondary communication can only be established through common available bands between a pair of CR users. If there are no available bands in common, then no direct link can be established.

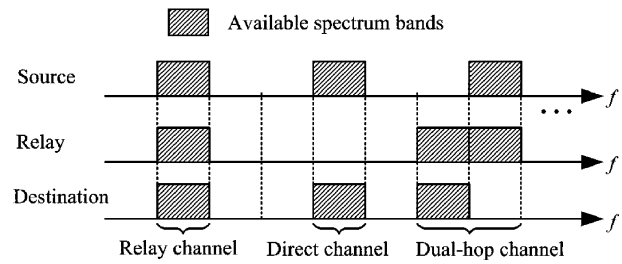

Fig. 1. CRRC in CR networks.

In order to solve the problem, cooperative relay has been introduced into CR networks [10]. With the assistance of a CR user as a relay that has rich available spectrum bands, some of non-common spectrum bands between the CR source and the CR destination can be bridged to exploit more spectrum opportunities. Recent literature [11]–[15] has discussed cooperative relay in CR from various perspectives. In [11], a cognitive space-time–frequency coding technique is proposed to maximize spectrum opportunities. In [12], relays are used for balancing the traffic requests and available spectrum resources. In [13], [14], *signal-to-interference-plus-noise ratio* (SINR) is enhanced by relays through spatial diversity. In [15], directional transmission of relays is used for exploiting spatial spectrum holes.

In this paper, we investigate power and channel allocation for cooperative relay in a three-node CR network, which consists of a source, a relay, and a destination and can operate in multiple spectrum bands. In the context, *CR relay channels* (CRRCs) can be divided into three categories as shown in Fig. 1. If a spectrum band is available at all three CR nodes, it is called a *relay channel* since it can provide end-to-end communication using cooperative relay protocols in [16]. If a spectrum band is available at both the source and the destination but not at the relay, it is called a *direct channel* for it can provide end-to-end communication directly. If one spectrum band is available at the source and the relay, and another one is available at the relay and the destination, it is called a *dual-hop channel* since end-to-end communication can be established via the relay. Each kind of the above channels has been studied in [17]–[19]. Dual-hop channels may increase throughput, extend coverage,

Manuscript received November 06, 2009; revised March 29, 2010; accepted May 21, 2010. Date of publication June 14, 2010; date of current version January 19, 2011. This work was supported in part by the National Science Foundation under Grant 0721580, in part by the Research Gift from Huawei Technologies Co., Ltd., in part by the Program of Introducing Talents of Discipline to Universities (111 Program), and in part by the Innovation Foundation of BUAA for Ph.D. Graduates. The associate editor coordinating the review of this manuscript and approving it for publication was Vincent K. N. Lau.

G. Zhao and C. Yang are with the School of Electronics and Information Engineering, Beihang University, Beijing 100191, China (e-mail: zhaoguodong@ee.buaa.edu.cn; cyyang@buaa.edu.cn).

G. Y. Li is with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: liye@ece.gatech.edu).

D. Li and A. C. K. Soong are with Huawei Technologies, Plano, TX 75075 USA (e-mail: ldd@ieee.org; asoong@huawei.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JSTSP.2010.2052784

TABLE I
LIST OF ABBREVIATIONS

BD	Band
CR	Cognitive radio
CRRC	Cognitive radio relay channel
CSI	Channel state information
DF	Decode and forward
RD	Relay-destination
SD	Source-destination
SR	Source-relay
PU	Primary user

and reduce interference [20]. Relay channels improve the performance through spatial diversity by using additional paths between source and destination [21]. However, in CRRC, if CR nodes are with rich available bands, some of the bands can be used as a relay, a direct, or a dual-hop channel, and different combinations may result in different performance. Allocating each available band to one of the three kinds of channels will bring us new degrees of freedom, which has not been discussed in CR.

Instead of addressing the transmission for each kind of channels separately, we design transmission schemes to exploit all channels jointly to optimize overall system performance. In general, a dual-hop channel has a bottleneck in throughput whereas a relay channel loses half of its throughput due to its half duplex constraint in practice. We propose to assign the spectrum band of the relay channel to assist the transmission in dual-hop or direct channels. This not only compensates for the bottleneck of the dual-hop channel but also enables the spectrum band of the relay channel to work in full-duplex mode. As a result, the overall end-to-end throughput can be significantly improved. Furthermore, we apply power allocation for CRRC so that the maximum overall end-to-end throughput can be achieved.

The rest of this paper is organized as follows. In Section II, we introduce cooperative relay channel in a three-node CR network, discuss power constraints for both the source and the relay, and derive throughput expression for CRRC. In Section III, we study power and channel allocation to maximize the overall end-to-end throughput. In Section IV, we present numerical results to illustrate the performance of the proposed power and channel allocation and develop a low complexity approach from practical considerations. Then we conclude the paper in Section V.

The abbreviations in this paper are summarized in Table I.

II. Cooperative Relay Channel in CR Networks

In this section, we will first introduce CRRC in a CR network with four typical spectrum bands, and then discuss power constraints for both the source and the relay. Finally, we will obtain the end-to-end throughput of CRRC.

A. Cooperative Relay Channel

Fig. 2 shows an example of cooperative relay in a three-node CR network. In the figure, the source intends to send data to the destination, and the relay may assist the transmission. We assume that a central controller can obtain sensing results and *channel state information* (CSI) among all three CR nodes through dedicated control channels. We further assume that every CR node is equipped with an omnidirectional antenna and can simultaneously sense four licensed spectrum bands,[1] $\mathrm{BD}i$ for $i = 1, \ldots, 4$. Each of them belongs to a PU exclusively. Specifically, PU3 in Fig. 2 may be a base station or a TV tower with large coverage and uses BD3. All three CR nodes are assumed to obtain the same sensing result on BD3. If PU3 is not transmitting, BD3 can be used as a relay channel to provide relay diversity transmission,[2] i.e., the source broadcasts its data to both the relay and the destination in a time slot while the relay forwards the data to the destination in the subsequent one. Meanwhile, the source is silent when the relay transmits signals. Three links are involved in BD3, *source-relay* (SR), *relay-destination* (RD), and *source-destination* (SD). As shown in Fig. 2, their channel powers are denoted as g_3^{sr}, g_3^{rd}, and g_3^{sd}, respectively. By contrast, the rest of the spectrum bands, BD1, BD2, and BD4, are assumed to belong to short-range primary users, PU1, PU2, and PU4, respectively. Those users can only affect the sensing results of their nearby CR nodes. Even when they are active, their spectrum bands can still be used by the CR system. As shown in Fig. 2, BD1 and BD2 can be used as a dual-hop channel, where BD1 and BD2 bridge the source and the relay, and the relay and the destination, respectively. Then the data can be sent to the relay through BD1 and be forwarded to the destination through BD2.[3] Furthermore, BD4 that is available at both the source and the destination but not at the relay can provide direct transmission. Therefore, the overall end-to-end communication is composed of the three kinds of channels occupying four spectrum bands.

Although CR systems may have multiple available spectrum bands, each can be regarded as one of the above four typical bands. Without loss of generality, we will develop our algorithms for a CR network with four typical spectrum bands to gain insights on the performance of power and channel allocation.

B. Transmit Power Constraint

Denote p_i^S and p_i^R as the transmit power of the source and the relay for the ith spectrum band, respectively. Then their power allocation vectors can be expressed as $\mathbf{p}^S = [p_1^S, p_2^S, p_3^S, p_4^S]$ and $\mathbf{p}^R = [p_1^R, p_2^R, p_3^R, p_4^R]$. In particular, $p_2^S = 0$ implies that BD2 cannot be used at the source. Similarly, $p_1^R = 0$ and $p_4^R = 0$ denote that BD1 and BD4 can not be used at the relay.

In CR, secondary communication can only be established when it does not cause intolerable interference to PUs. To protect each PU, we assume that the transmit power of the source

[1]Ideal spectrum sensing is considered here; therefore, no false alarm and missed detection events happen.

[2]Even though there are many cooperative relay protocols [19], we only consider the *decode-and-forward* (DF) protocol in [16] to facilitate the development of our idea.

[3]In the dual-hop transmission, full-duplex relays are used here, which requires the SR and RD links, e.g., BD1 and BD2 in Fig. 2, are available at the same time. Otherwise, if they are not available at the same time, the dual-hop transmission will be disabled and other modes will be used for providing the end-to-end throughput.

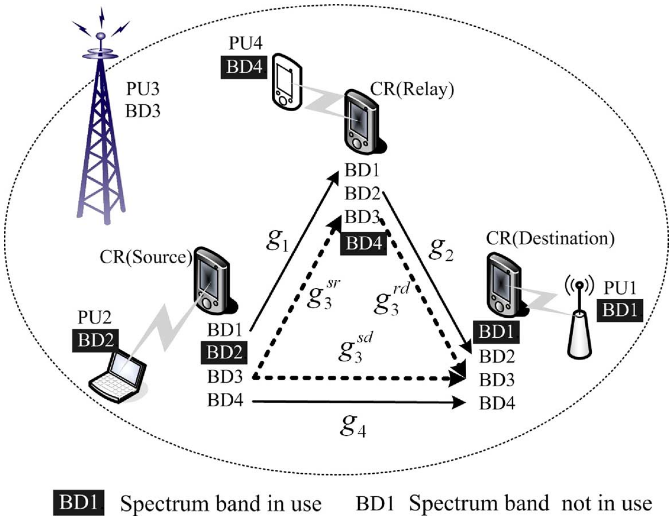

Fig. 2. System setup of cooperative relay in CR networks.

and the relay on each band has a power constraint of $P_{\max}$, which is called the *per band power constraint* and can be expressed as

$$p_i^S \leq P_{\max} \text{ and } p_i^R \leq P_{\max}, \quad i = 1, \ldots, 4. \tag{1}$$

We notice that secondary transmission can be designed under various power constraints [22]–[24], which need the CSI from CR transmitter to primary receiver that is in general very hard to obtain. In this paper, we focus on developing the method of cooperation among the three kinds of channels in CRRC. Therefore, the above per band power constraint is used for simplicity and it can be obtained by proactive spectrum sensing [25].

Furthermore, due to some implementation issues, such as *radio front-end* (RF) capability, power, and cost budgets, the total transmit power at the source and the relay will be limited, which can be expressed by the *sum power constraint*

$$\sum_{i=1}^{4} p_i^S \leq P_{\max}^S \text{ and } \sum_{i=1}^{4} p_i^R \leq P_{\max}^R \tag{2}$$

where $P_{\max}^S$ and $P_{\max}^R$ are the maximum powers that the source and the relay are able to transmit, respectively.

C. End-to-End Throughput

In CRRC, the three kinds of channels can provide three parallel end-to-end transmission. For the direct transmission in BD4, the end-to-end throughput can be expressed as

$$R_{\text{direct}} = C(p_4^S g_4) \tag{3}$$

where $C(x) = B\log(1+x)$ is the Shannon capacity with bandwidth B and g_4 is the channel power of BD4.[4] Here we assume that the bandwidth of each band, BDi for $i = 1, \ldots, 4$, is the same for simplicity.

For the dual-hop transmission in BD1 and BD2, since the two hops are serially connected at the relay, the end-to-end throughput equals to the smaller throughput of the two hops. In addition, the full duplex can be realized because the two hops, i.e., the SR and the RD links, work in different spectrum bands and the relay is able to transmit and receive at the same time. Then the end-to-end throughput can be expressed as [26]

$$R_{\text{dual}} = \min\left\{C(p_1^S g_1), C(p_2^R g_2)\right\}. \tag{4}$$

For the relay diversity transmission in BD3, all three links involved in the DF protocol are in the same spectrum band. Only half duplex can be realized in practice since the relay cannot receive and transmit signals simultaneously at the same spectrum band. According to [21], [26], and [27], the end-to-end throughput can be expressed as

$$R_{\text{relay}} = \frac{1}{2}\min\left\{C(p_3^S g_3^{sr}), C(p_3^S g_3^{sd}) + C(p_3^R g_3^{rd})\right\}. \tag{5}$$

Therefore, the overall end-to-end throughput of CRRC is contributed by the three parallel transmission as follows:

$$R_{all}(\mathbf{p}^S, \mathbf{p}^R) = R_{\text{direct}} + R_{\text{dual}} + R_{\text{relay}}. \tag{6}$$

Remark: In (6), the overall end-to-end throughput is given by the summation of the three kinds of transmission since multiple transmission may happen simultaneously in CRRC. Thus, $R_{all}(\cdot)$ or $J(\cdot)$ in the following sections represents the instantaneous throughput, where the state of PUs is constant during CR's communication.[5]

III. Power and Channel Allocation

In a three-node CR network with a cooperative relay, the relay improves performance by exploiting available spectrum bands at the three nodes. When an available band at the relay is also available at the source and the destination, such as BD3 in Fig. 2, it can introduce the extra SR and RD links to enhance the existing SD link by relay diversity transmission. On the other hand, when two different spectrum bands are available at the source and the destination, respectively, and are both available at the relay, such as BD1 and BD2 in Fig. 2, then they can be bridged by the relay through dual-hop transmission. As indicated before, the relay diversity transmission loses half of the end-to-end throughput as shown in (5) whereas the dual-hop transmission experiences a bottleneck in end-to-end throughput as shown in (4). In order to maximize the overall end-to-end throughput, all three kinds of channels should be used cooperatively.

In this section, we will investigate power and channel allocation to fully exploit CRRC by considering the heterogeneous spectrum availability at different CR nodes. In principle, our idea is to use the relay channel to compensate for the bottleneck of the dual-hop channel, which can be realized by channel allocation[6] To further improve the overall end-to-end throughput, power allocation for different kinds of channels is also considered, which is different from the conventional power allocation in parallel relay channels [21]. Since joint power and channel allocation design is very complicated, we can perform power

[4]If h_i represents a complex channel coefficient, $g_i = |h_i|^2$ is defined as *channel power*, where the subscript denotes the index of the channels.

[5]The state of PUs (on/off) may change with time, i.e., state transition. This leads to the power and channel allocation in both time and frequency, and will complicate our system. Since the models describing the state transition of PUs have not been well developed, the case with multiple PU states is beyond the scope of this paper.

[6]In addition, when the SD link of the relay channel is good enough, it can also be used to provide end-to-end transmission directly.

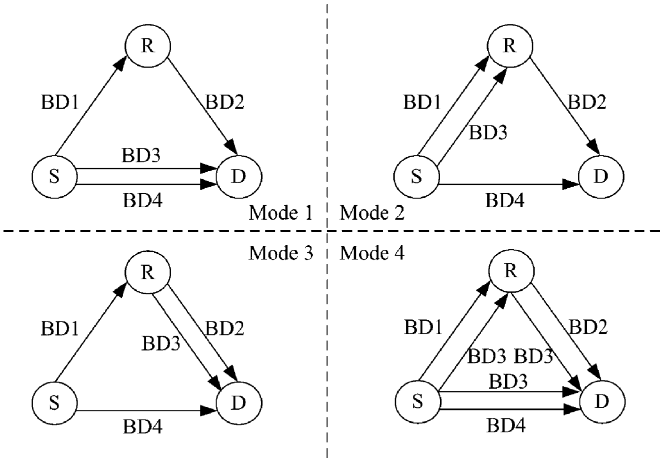

Fig. 3. Different modes of channel allocation.

allocation for all possibilities of channel allocation and then select the case with the highest throughput. In the following, we will first present all the possible channel allocations in CRRC and then develop the corresponding power allocation for each case.

A. Channel Allocation

In CRRC, some available spectrum bands can be used in different ways, which will result in different overall end-to-end throughputs. Here, BD3 that is available to all three CR nodes can be used in the four different modes in Fig. 3:

- *Mode 1:* Direct transmission from the source to the destination.
- *Mode 2:* Dual-hop transmission from the source to the relay.
- *Mode 3:* Dual-hop transmission from the relay to the destination.
- *Mode 4:* Relay diversity transmission by using all three links with cooperative relay protocols [16].

If one of the first three modes is used, i.e., Modes 1, 2, and 3, the overall end-to-end throughput consists of the throughputs of the direct and the dual-hop transmission with enhanced SD, SR, and RD links and can be expressed as in (7), shown at the bottom of the page. If the last mode is used, the overall end-to-end throughput can be obtained by (6).

The channel allocation is to select a proper mode to maximize the overall end-to-end throughput. From (6) and (7), the throughput in each mode is also determined by power allocation. To achieve the maximum overall end-to-end throughput, the throughput in each mode should be maximized by power allocation. Afterwards, we can pick the mode with the highest throughput to perform cooperative transmission in CRRC. Next, we will develop power allocation approaches for each mode of the channel allocation.

TABLE II
CHANNEL ALLOCATION SETS FOR MODES 1, 2, AND 3

Mode	Γ_{SD}	Γ_{SR}	Γ_{RD}
1	$\{4,3\}$	$\{1\}$	$\{2\}$
2	$\{4\}$	$\{1,3\}$	$\{2\}$
3	$\{4\}$	$\{1\}$	$\{2,3\}$

B. Power Allocation

From the above discussion, there may be two or three types of transmission working simultaneously in CRRC. Consequently, this will lead to different power constraints for different spectrum bands, and makes the power allocation more challenging. Since there are two types of transmission for the first three modes and three types for the last mode, we will discuss them separately in the following.

1) Modes 1, 2, and 3: In these modes, without relay diversity transmission, the direct and dual-hop transmission provide end-to-end communication simultaneously. Define Γ_{SD}, Γ_{SR}, and Γ_{RD} as three sets of the spectrum band indices allocated for the SD, SR, and RD links, respectively. Then they can be expressed as in Table II for different channel allocation modes.

To maximize the throughput from the relay to the destination, we allocate power at the relay with both sum and per band power constraints, i.e.,

$$J_{RD} = \max_{\mathbf{p}^R}\left\{\sum_{i\in\Gamma_{RD}} C(p_i^R g_i)\right\} \tag{8}$$

$$= \max_{\mathbf{p}^R}\left\{\sum_{i\in\Gamma_{RD}} B\log(1+p_i^R g_i)\right\} \tag{9}$$

$$\text{subject to} \quad \sum_{i\in\Gamma_{RD}} p_i^R \le P_{\max}^R \tag{10}$$

$$p_i^R \le P_{\max}, \quad i\in\Gamma_{RD} \tag{11}$$

$$p_i^R \ge 0, \quad i\in\Gamma_{RD} \tag{12}$$

where $g_3 = g_3^{rd}$. According to [28], the power allocation can be obtained by a water-filling solution, i.e.,

$$p_{io}^R = \left[\frac{B}{\ln 2(\lambda+\mu_i)} - \frac{1}{g_i}\right]^+ \tag{13}$$

where $1/(\lambda+\mu_i)$ represents the water-level of the ith spectrum band. Since it is a convex problem, it can be solved efficiently by using the algorithms in [29]. Then the throughput of the second hop in the dual-hop transmission can be expressed as

$$R^* = \sum_{i\in\Gamma_{RD}} C(p_{io}^R\, g_i). \tag{14}$$

$$R_{all}(\mathbf{p}^S,\mathbf{p}^R) = R_{\text{direct}} + R_{\text{dual}} = \begin{cases} \left(C(p_4^S g_4) + C(p_3^S g_3^{sd})\right) + \min\left\{C(p_1^S g_1), C(p_2^R g_2)\right\} & : \text{Mode 1} \\ C(p_4^S g_4) + \min\left\{C(p_1^S g_1) + C(p_3^S g_3^{sr}), C(p_2^R g_2)\right\} & : \text{Mode 2} \\ C(p_4^S g_4) + \min\left\{C(p_1^S g_1), C(p_2^R g_2) + C(p_3^R g_3^{rd})\right\} & : \text{Mode 3} \end{cases} \tag{7}$$

The total power of the source can be divided into two parts, one for direct transmission and the other for dual-hop transmission. The former can provide the end-to-end throughput directly. The latter can only provide the throughput of the first hop from the source to the relay, which can improve the end-to-end throughput of the dual-hop transmission as in (7) only when the first hop is no better than the second one. Therefore, the source should assign more power for the dual-hop transmission only when the throughput of the first hop is no larger than that of the second one. Therefore, the following power constraint on dual-hop transmission will apply to the source, i.e.,

$$\sum_{i\in\Gamma_{SR}} C(p_i^S g_i) \le R^* \tag{15}$$

where R^* is from (14) and it is the throughput that the second hop can provide. Consequently, to maximize the overall end-to-end throughput, the power allocation at the source can be formulated as follows:

$$J_{SD} = \max_{\mathbf{p}^S} \left\{ \sum_{i\in\Gamma_{SR}\cup\Gamma_{SD}} C(p_i^S g_i) \right\} \tag{16}$$

$$\text{subject to} \quad \sum_{i\in\Gamma_{SR}\cup\Gamma_{SD}} p_i^S \le P_{\max}^S, \tag{17}$$

$$p_i^S \le P_{\max}, \quad i \in \Gamma_{SR}\cup\Gamma_{SD} \tag{18}$$

$$p_i^S \ge 0, \quad i \in \Gamma_{SR}\cup\Gamma_{SD} \tag{19}$$

$$\sum_{i\in\Gamma_{SR}} C(p_i^S g_i) \le R^* \tag{20}$$

where $g_3 = g_3^{sr}$ and $\cup$ denotes the union of two sets.

In Modes 1 and 3, since there is only one spectrum band for the first hop of the dual-hop transmission, the constraint (20) can be converted into the following inequality:

$$p_1^S \le \frac{1}{g_1}\left(2^{R^*/B} - 1\right). \tag{21}$$

Then the power allocation at the source is a convex optimization problem and has the same water-filling solution as in (13).

In Mode 2, when there are more than one spectrum bands for the first hop of the dual-hop transmission, the constraint (20) is non-convex, which makes the optimization complicated. Next, we will convert the non-convex constraint to inequality constraints by the following steps:

- *Step 1*: Perform power allocation without considering the constraint (20) and obtain the power allocation vector $\mathbf{p}^S$.
- *Step 2*: Check whether $\mathbf{p}^S$ meets the constraint (20). If so, it is the power allocation vector that we need for the source. Otherwise, reduce the sum power constraint of the source $P_{\max}^S$ and perform power allocation until $\mathbf{p}^{S*}$ meets

$$R^* - \epsilon \le \sum_{i\in\Gamma_{SR}} C(p_i^{S*} g_i) \le R^* + \epsilon \tag{22}$$

  where ϵ is a small error that is tolerable for the system and $\mathbf{p}^{S*} = [p_1^{S*},\ 0,\ p_3^{S*},\ 0]$.
- *Step 3*: Obtain the inequality constraints by $p_1^S \le p_1^{S*}$ and $p_3^S \le p_3^{S*}$.

Then we can solve the problem efficiently by replacing the non-convex constraint in (20) by the two inequality constraints.

2) Mode 4: As indicated in (6), the overall end-to-end throughput consists of the throughputs of the three transmission. In particular, the relay diversity transmission uses all three links in the same spectrum band. When the SD link is better than the SR link, i.e., $g_3^{sd} > g_3^{sr}$, there is no need to ask the relay for help and direct transmission should be used in BD3 [27]. Then the power allocation is the same as that in Mode 1.

On the other hand, when the SD link is no better than the SR link, i.e., $g_3^{sd} \le g_3^{sr}$, all three links will take part in the relay diversity transmission. In this case, the power allocations at the source and the relay interact with each other. In order to simplify the problem, we introduce a factor $0 \le \alpha \le 1$ to divide the total transmit power of the relay into two parts, $\alpha P_{\max}^R$ and $(1-\alpha)P_{\max}^R$, for dual-hop and relay transmission, respectively. In this way, the power allocation at the source can be designed for a given power allocation at the relay that is fixed by α. Then the overall end-to-end throughput can be maximized by searching for the optimal α. For a given α and considering the per band power constraint in (1), we have $p_2^R(\alpha) = \min\{\alpha P_{\max}^R, P_{\max}\}$ and $p_3^R(\alpha) = \min\{(1-\alpha)P_{\max}^R, P_{\max}\}$. In other words, the power allocation vector at the relay depends on the factor α and can be expressed as

$$\mathbf{p}^R(\alpha) = [0, p_2^R(\alpha), p_3^R(\alpha), 0]. \tag{23}$$

Now we develop power allocation at the source for the given $\mathbf{p}^R(\alpha)$. In this case, the source needs to divide its power into three parts for direct, dual-hop, and relay diversity transmission, respectively. In the direct transmission, from (3), we have

$$R_{\text{direct}}(\alpha) = C(p_4^S g_4). \tag{24}$$

It involves only SD link and provides end-to-end throughput directly. In the dual-hop transmission, from (4), we have

$$R_{\text{dual}}(\alpha) = \min\left\{C(p_1^S g_1), C(p_2^R(\alpha) g_2)\right\}. \tag{25}$$

It involves the SR and RD links for the first and second hops, respectively. As indicated before, for a given power allocation at the relay $p_2^R(\alpha)$, the throughput of the first hop should be no more than that of the second hop, i.e.,

$$C(p_1^S g_1) \le R^* = C(p_2^R(\alpha) g_2). \tag{26}$$

From (21) and (26), we have the following inequality constraint:

$$p_1^S \le \frac{g_2}{g_1} p_2^R(\alpha). \tag{27}$$

Therefore, the end-to-end throughput of the dual-hop transmission becomes $R_{\text{dual}}(\alpha) = C(p_1^S g_1)$ with constraint (27). In the relay diversity transmission, from (5), we have

$$R_{\text{relay}}(\alpha) = \frac{1}{2}\min\left\{C(p_3^S g_3^{sr}), C(p_3^S g_3^{sd}) + C(p_3^R(\alpha) g_3^{rd})\right\}. \tag{28}$$

It involves three links, the SR, SD, and RD links. Similar to (25), the end-to-end throughput of the relay diversity transmission is also limited by the smaller term in (28). The first one represents the throughput of the SR link, which is determined by the power allocation at the source p_3^S. The second one denotes the summation of the throughputs of the SD and RD links, which is determined by the power allocations at both the source and the relay, i.e., p_3^S and p_3^R. For a given power allocation at the relay $p_3^R(\alpha)$, even though the throughput of the RD link $C(p_3^R(\alpha)g_3^{rd})$ is a constant, the end-to-end throughput will still increase as the power p_3^S grows. Generally speaking, if the power p_3^S is low enough,

$$R_{\text{relay}}(\alpha) = \frac{1}{2}C(p_3^S g_3^{sr})$$

since $C(p_3^S g_3^{sr}) \leq C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})$. On the other hand, if the power p_3^S is high enough,

$$R_{\text{relay}}(\alpha) = \frac{1}{2}\left(C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})\right)$$

since $C(p_3^S g_3^{sr}) > C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})$.

Therefore, the overall end-to-end throughput that consists of the throughputs of direct, dual-hop, and relay diversity transmission can be maximized as follows:

$$J(\alpha) = \max_{\mathbf{P}^S}\{R_{\text{direct}}(\alpha) + R_{\text{dual}}(\alpha) + R_{\text{relay}}(\alpha)\} \tag{29}$$

subject to

$$\sum_{i=\{1,3,4\}} p_i^S \leq P_{\max}^S \tag{30}$$

$$p_i^S \leq P_{\max}, \quad i = 1,3,4 \tag{31}$$

$$p_i^S \geq 0, \quad i = 1,3,4 \tag{32}$$

$$p_1^S \leq \frac{g_2}{g_1}p_2^R(\alpha). \tag{33}$$

When $C(p_3^S g_3^{sr}) \leq C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})$, the objective function becomes

$$J(\alpha) = \max_{\mathbf{P}^S}\left\{C(p_1^S g_1) + \frac{1}{2}C(p_3^S g_3^{sr}) + C(p_4^S g_4)\right\}. \tag{34}$$

When $C(p_3^S g_3^{sr}) > C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})$, the objective function becomes

$$J(\alpha) = \max_{\mathbf{P}^S}\{C(p_1^S g_1) + \frac{1}{2}(C(p_3^S g_3^{sd}) + C(p_3^R(\alpha)g_3^{rd})) + C(p_4^S g_4)\}. \tag{35}$$

Each of the above case is a convex problem and the power allocation has the water-filling solution as follows:

$$p_{1o}^S = \left[\frac{B}{\ln 2(\lambda + \mu_1)} - \frac{1}{g_1}\right]^+ \tag{36}$$

$$p_{3o}^S = \left[\frac{B}{2\ln 2(\lambda + \mu_3)} - \frac{1}{g_3}\right]^+ \tag{37}$$

$$p_{4o}^S = \left[\frac{B}{\ln 2(\lambda + \mu_4)} - \frac{1}{g_4}\right]^+ \tag{38}$$

where $g_3 = g_3^{sr}$ is for (34) and $g_3 = g_3^{sd}$ for (35). Since whether using (34) or (35) depends on the power allocation at the source for the relay diversity transmission p_3^S, we can perform the power allocation by using the objective function in (34) to obtain the power p_3^S. Then we check whether its condition is satisfied. If so, the power allocation for the source is obtained. Otherwise, it can be calculated by using the objective function in (35).

In brief, the power and channel allocation in CRRC can be summarized as follows:

- List all possible modes of the channel allocation as in Section III-A.
- Perform power allocation for each mode as in Section III-B.
- Pick the mode with the largest overall end-to-end throughput by exhaustive search.

Even though we have developed the power and channel allocation in this typical case with four spectrum bands, we will show performance in the case with an arbitrary number of spectrum bands in the next section, where a low complexity approach will be considered.

IV. Numerical Results

In this section, we first present numerical results to illustrate the performance of our power and channel allocation in the typical case with four spectrum bands. Based on the observation from the results, we will find a low complexity method and show its performance in the case with an arbitrary number of spectrum bands.

In this example, we assume that three CR nodes are with equal distance[7] and they experience independent Rayleigh fading channels. We further assume that each spectrum band has 1 MHz bandwidth, the noise power at each CR node is -126 dBW, and the path loss between two CR nodes is 126 dB. Furthermore, all curves are averaged over 200 channel realizations and the per band power constraint is set to 3 W, i.e., $P_{\max} = 3$ W, which represents the maximum allowable transmit power on each spectrum band. In the following, we will present the overall end-to-end throughput, R_{all}, for different power and channel allocation schemes. Legends "PA+CA," "PA only," "CA only," and "No PA No CA" represent the scheme with both power and channel allocation, the scheme that only works in Mode 4 with power allocation, the scheme that performs channel allocation with equal power allocation, and the scheme that only works in Mode 4 with equal power allocation, respectively.

A. Different Source Power Constraints

Fig. 4 illustrates the overall end-to-end throughputs of different schemes versus different sum power constraints at the source, where the sum power constraint at the relay is set to be 6 W. It is shown that the power and channel allocation can significantly improve the throughput. When the power constraint is low, for example, 2 W, the throughput is improved from about 1.5 Mbps to about 2.5 Mbps, i.e., around 67% improvement.

[7]For simplicity, we consider the scenario with equal distance among three CR nodes.

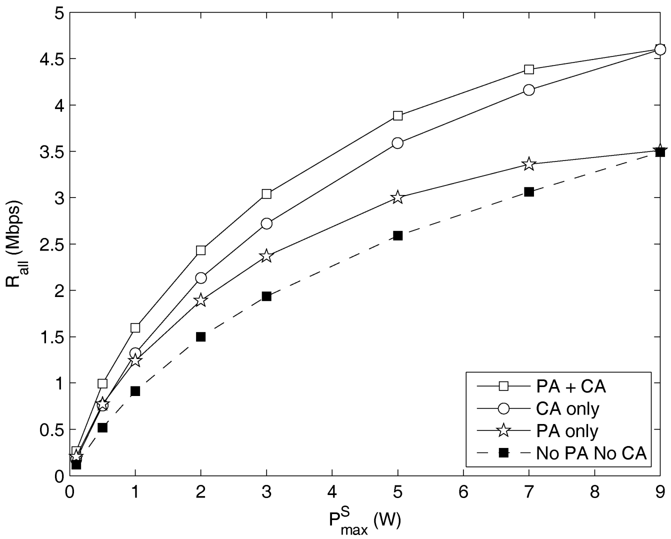

Fig. 4. End-to-end throughput versus the power constraint at the source.

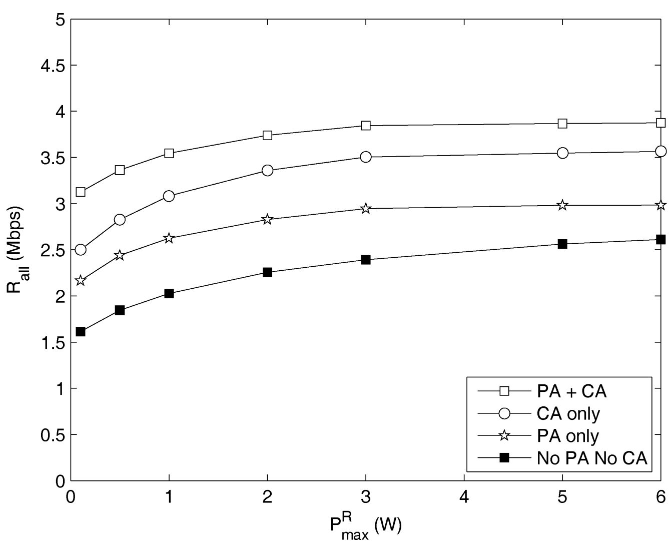

Fig. 5. End-to-end throughput versus the power constraint at the relay.

When the power constraint equals to 9 W, all throughputs become the same value because the per band power constraint limits the transmit power so as to protect PUs.

From Fig. 4, the schemes with either power or channel allocation can improve the throughput as well but in different manners. The scheme without power and channel allocation is a baseline for comparison. From the figure, as the sum power constraint grows, the throughput improvement of channel allocation increases. On the other hand, the power allocation can only improve the throughput when $P_{\max}^S < 9$ W. This is because when the sum power constraint is large enough, the per band power constraint will limit the transmit power. Then the source sends signals with maximum allowable transmit power $P_{\max}$ on each spectrum band. This is equivalent to equal power allocation, i.e., no power allocation. Therefore, channel allocation is more effective than power allocation in CRRC.

B. Different Relay Power Constraints

Fig. 5 shows the overall end-to-end throughputs of different schemes versus different sum power constraints at the relay,

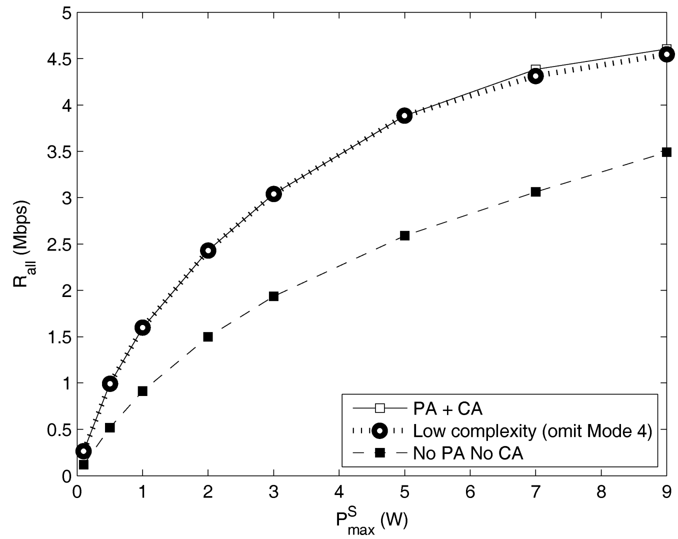

Fig. 6. Performance of the low complexity approach in the typical case with four spectrum bands.

where the sum power constraint at the source is set to be 5 W. From the figure, as the sum power constraint at the relay increases from 0 to 6 W, the throughputs of different schemes grow almost at similar scales. As before, when the sum power constraint is large enough, the throughput will be capped by per band power constraint.

C. Low Complexity Approach

In practice, if the CR system works in Mode 4, the relay has to conduct both dual-hop and relay diversity transmission, which complicates the system. Therefore, we omit Mode 4 and only consider Modes 1, 2, and 3 for the power and channel allocation. In this way, the relay only needs to conduct dual-hop transmission, which significantly simplifies the CR system.

Fig. 6 compares the overall end-to-end throughput of the approach without Mode 4 and those of the schemes with and without power and channel allocation. Here, the sum power constraint at the relay is set to be 6 W. We can find that the low complexity approach of omitting Mode 4 has similar performance to the method of considering all four modes. Furthermore, when the sum power constraint at the source is larger than 9 W, it only decreases the throughput from about 4.6 Mbps to about 4.5 Mbps compared to the scheme with power and channel allocation, i.e., about 2% performance loss. Therefore, the low complexity approach can obtain most of the benefits from power and channel allocation with minor performance loss.

D. Performance in Multiple Spectrum Bands

We consider a relay CR network with N spectrum bands available. For the channel allocation of the low-complexity approach without Mode 4, each relay channel can be used in one of the first three modes in Fig. 3. For the power allocation, the developed method in Section III-B1 can be directly applied. We also compare the performance of the power and channel allocation with all four modes.

Fig. 7 compares the overall end-to-end throughput of the low complexity approach and those schemes with and

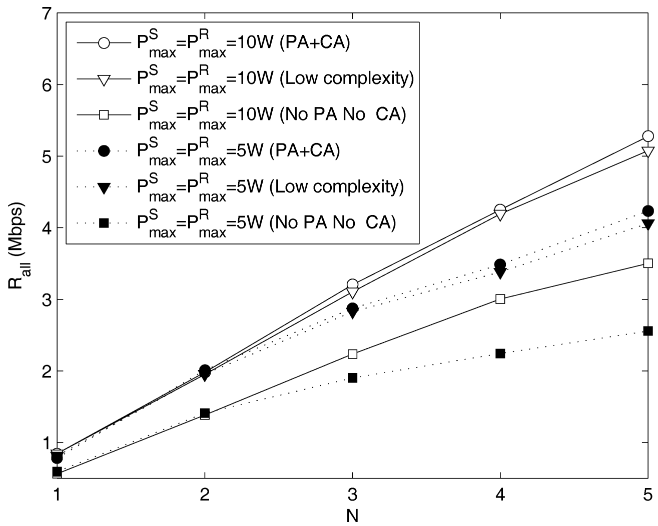

Fig. 7. Performance of the low complexity approach in the case with N spectrum bands.

without power and channel allocation in different sum power constraints. We assume that the spectrum availability of N spectrum bands is independent and each of them may be one of the four typical spectrum bands in CRRC with equal probability. Then there are $L = N/4$ relay channels on average. We further assume that the source and the relay have the same sum power constraint. We compare the performance curves of difference schemes when the sum power constraints are 10 W. It is shown from the figure that the low complexity approach outperforms the scheme without power and channel allocation, and has the similar performance to the power and channel allocation scheme with all four modes. In particular, the throughput improvement of the low complexity approach increases as the number of spectrum bands grows. This is because the more available spectrum bands there are, the more relay channels may appear, and the more benefits from power and channel allocation can be obtained. Similar results are observed when the sum power constraints are 5 W.

Now, we compare the curves with different power constraints for the same approach. From the figure, when the number of the spectrum bands is small, i.e., $N = 1, 2$, the throughputs with different sum power constraints are similar. When the number of the spectrum bands becomes large, i.e., $N \geq 3$, the throughput with 5 W power constraint is smaller than that of 10 W. This is because when there are a few spectrum bands, the per band power constraint limits the throughputs, which leads to the similar performance. On the contrary, when there are many spectrum bands, the different sum power constraints limit the throughputs, which leads to the different performance.

V. Conclusion

In this paper, we have studied power and channel allocation for cooperative relay in a three-node CR network, where a CR relay channel consists of three kinds of channels: direct, dual-hop, and relay channels. In order to maximize the overall end-to-end throughput, we propose to use some relay channels that have spectrum bands available at all three CR nodes to assist the transmission in direct or dual-hop channels. Our study shows that the power allocation can improve the throughput when the power at the source is limited. The channel allocation is more effective than the power allocation and can improve the throughput with various power constraints. Based on the observation from the numerical results, we suggest to use the relay only for dual-hop transmission since this can significantly simplify the system with only minor performance loss.

References

[1] J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, Royal Inst. of Technol. (KTH), Stockholm, Sweden, 2000.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] I. A. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, pp. 2127–2159, May 2006.

[4] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[5] S. Haykin, J. H. Reed, G. Y. Li, and M. Shafi, "Scanning the issue," *Proc. IEEE: Special Iss. Cognitive Radio*, vol. 97, no. 4/5, Apr./May 2009.

[6] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.

[7] J. Ma, Y. (G.) Li, and B. H. Hwang, "Signal processing in cognitive radio," *Proc. IEEE: Special Iss. Cognitive Radio*, vol. 97, no. 5, pp. 805–823, May 2009.

[8] S. M. Mishra, R. Tandra, and A. Sahai, "Coexistence with primary users of different scales," in *Proc. IEEE DySPAN*, Jun. 2007, pp. 158–167.

[9] W. Ren, Q. Zhao, and A. Swami, "Power control in cognitive radio networks: How to cross a multi-lane highway," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1283–1296, Sep. 2009.

[10] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Commun. Mag.*, vol. 47, no. 2, pp. 111–117, Feb. 2009.

[11] K. Letaief and W. Zhang, "Cooperative communications for cognitive radio networks," *Proc. IEEE: Special Iss. Cognitive Radio*, vol. 97, no. 5, pp. 878–893, May 2009.

[12] J. Jia, J. Zhang, and Q. Zhang, "Cooperative relay for cognitive radio networks," in *Proc. IEEE Infocom.*, Apr. 2009, pp. 2304–2312.

[13] X. Gong, W. Yuan, W. Liu, W. Cheng, and S. Wang, "A cooperative relay scheme for secondary communication in cognitive radio networks," in *Proc. IEEE Globecom.*, Nov. 2008, pp. 1–6.

[14] J. Mietzner, L. Lampe, and R. Schober, "Performance analysis for a fully decentralized transmit power allocation scheme for relay-assisted cognitive-radio systems," in *Proc. IEEE Globecom*, Nov. 2008, pp. 1–6.

[15] G. Zhao, J. Ma, G. Y. Li, T. Wu, Y. Kwon, A. Soong, and C. Yang, "Spatial spectrum holes for cognitive radio with relay-assisted directional transmission," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 5270–5279, Oct. 2009.

[16] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient Protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[17] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25, no. 5, pp. 572–584, Sep. 1979.

[18] A. Hadsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.

[19] K. J. L. Ray, A. K. Sadek, W. Su, and A. Kwasinski, *Cooperative Communication and Networking*. Cambridge, U.K.: Cambridge Univ. Press, 2009.

[20] M. Hasna and M. Alouini, "A performance study of dual-hop transmission with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.

[21] Y. Liang, V. Veeravalli, and H. Poor, "Resource allocaiton for wireless fading relay channels: Max-min solution," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3432–3453, Oct. 2007.

[22] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[23] L. Musavian and S. Aissa, "Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 148–156, Jan. 2009.

[24] X. Kang, Y.-C. Liang, A. Nallanathan, H. Krishna, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[25] G. Zhao, G. Y. Li, and C. Yang, "Proactive detection of spectrum opportunities in primary systems with power control," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4815–4823, Sep. 2009.

[26] F. H. P. Fitzek and M. D. Katz, *Cooperation in Wireless Networks: Principles and Applications*. New York: Springer, 2006.

[27] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1446–1454, Apr. 2007.

[28] X. Zhou, G. Y. Li, D. Li, D. Wang, and A. Soong, "Probability-based resource allocation in cognitive radio networks," in *Proc. IEEE Globecom*, Dec. 2009, pp. 1–6.

[29] D. Palomar and J. Fonollosa, "Practical algorithms for a family of waterfilling solutions," *IEEE Trans. Signal Process.*, vol. 53, no. 2, pp. 686–695, Feb. 2005.

Guodong Zhao (S'07) received the B.E. degree from the School of Electrical Engineering, Xidian University, Xi'an, China, in 2005. He is currently pursuing the Ph.D. degree at the School of Electronics and Information Engineering, Beihang University, Beijing, China.

From September 2007 to October 2008, he worked as a Visiting Student in the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta. His research interests include cognitive radio and MIMO communications.

Chenyang Yang (M'99–SM'08) received the M.S.E. and Ph.D. degrees in electrical engineering from Beijing University of Aeronautics and Astronautics (BUAA, now renamed as Beihang University), Beijing, China, in 1989 and 1997, respectively.

She is now a Full Professor in the School of Electronics and Information Engineering, BUAA. She has published various papers and filed many patents in the fields of signal processing and wireless communications.

Dr. Yang was nominated as an Outstanding Young Professor of Beijing in 1995 and was supported by the 1st Teaching and Research Award Program for Outstanding Young Teachers of Higher Education Institutions by Ministry of Education (PRC "TRAPOYT") during 1999–2004. Currently, she serves as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, an Associate Editor-in-Chief of the *Chinese Journal of Communications*, and an Editor of the *Chinese Journal of Signal Processing*. She is the chair of Beijing Chapter of the IEEE Communications Society. She was the symposium co-chair of CoNet'07 and ChinaCom'09. She has served as TPC members for many IEEE conferences such as ICC and GlobeCom. Her recent research interests include signal processing in MIMO, ultra-wideband systems, wireless sensor networks, and cognitive radio.

Geoffrey Ye Li (S'92–M'95–SM'97–F'06) received the B.S.E. and M.S.E. degrees from the Department of Wireless Engineering, Nanjing Institute of Technology, Nanjing, China, in 1983 and 1986, respectively, and the Ph.D. degree from the Department of Electrical Engineering, Auburn University, Auburn, Alabama, in 1994.

He was a Teaching Assistant and then a Lecturer with Southeast University, Nanjing, from 1986 to 1991, a Research and Teaching Assistant with Auburn University from 1991 to 1994, and a Post-Doctoral Research Associate with the University of Maryland at College Park, MD, from 1994 to 1996. He was with AT&T Labs—Research, Red Bank, NJ, as a Senior and then a Principal Technical Staff Member from 1996 to 2000. Since 2000, he has been with the School of Electrical and Computer Engineering, Georgia Institute of Technology, as an Associate and then a Full Professor. He is also holding the Cheung Kong Scholar title at the University of Electronic Science and Technology of China since March 2006. His general research interests include statistical signal processing and telecommunications, with emphasis on OFDM and MIMO techniques, cross-layer optimization, and signal processing issues in cognitive radios. In these areas, he has published about 200 papers in refereed journals or conferences and filed about 20 patents. He also has two books, entitled, *Blind Equalization and Identification* (coauthored with Z. Ding) (Mercel Dekker, 2000) and *OFDM for Wireless Communications* (coauthored with G. Stüber) (Springer, 2006). He once served or is currently serving as an editor, a member of editorial board, and a guest editor for about ten technical journals. He organized and chaired many international conferences, including technical program vice-chair of the IEEE 2003 International Conference on Communications. He has been awarded an IEEE Fellow for his contributions to signal processing for wireless communications in 2005, selected as a Distinguished Lecturer from 2009–2010 by the IEEE Communications Society, and won the 2010 IEEE Communications Society Stephen O. Rice Prize Paper Award in the field of communications theory.

Dongdong Li (S'03–M'05) received the B.Sc. and M.Sc. degrees in electrical engineering from Beijing University of Posts and Telecommunications, Beijing, China, in 1997 and 2000, respectively, and the Ph.D. degree in electrical engineering from the University of Texas at Arlington, in 2005.

Since 2009, he has been with Tellabs, Inc., as a Senior Network Consultant for AT&T Mobility. Most recently, he was with the R&D Department, Huawei Technologies, Plano, TX, as a Senior Researcher, where he was focusing on the research of LTE and Wimax networks and products. Before that, he was working for Cerion, Inc., Frisco, TX, USA, as a technology manager, for AT&T Cingular wireless network planning and optimizations. His current research interests include wireless network capacity planning, performance modeling, and advanced wireless technologies and solution development. He has published extensively in prestigious journals and conferences and has several patents pending.

Anthony C. K. Soong (S'88–M'91–SM'02) received the B.Sc. degree in animal physiology and physics from the University of Calgary, Calgary, AB, Canada, and the B.Sc. degree in electrical engineering, the M.Sc. degree in biomedical physics, and Ph.D. degree in electrical and computer engineering from the University of Alberta, Edmonton, AB, Canada.

He is currently a Principal Engineer for advance research and standards at Huawei Technologies Co. Ltd, Plano, TX. From 2007 to 2009, he served as the chair for 3GPP2 TSG-C NTAH (the next-generation radio access network technology development group). Currently, he is the vice chair for 3GPP2 TSG-C WG3 (the physical layer development group for CDMA 2000). Prior to joining Huawei, he was with the Systems Group for Ericsson, Inc., and Qualcomm, Inc. His research interests are in statistical signal processing, robust statistics, wireless communications, spread spectrum techniques, multicarrier signaling, multiple antenna techniques, and physiological signal processing. He has published numerous scientific papers and has over 50 patents granted or pending.

Dr. Soong received the 2005 award of merit for his contribution to 3GPP2 and cdma2000 development. He has served on the technical program committee and has chaired at numerous major conferences in the area of communications engineering. He has acted as a Guest Editor for the IEEE COMMUNICATIONS MAGAZINE and is a technical reviewer for the *EURASIP Journal on Wireless Communications and Networking*, the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, the IEEE TRANSACTIONS ON SIGNAL PROCESSING, and the IEEE COMMUNICATION LETTERS.