Spatial Spectrum Holes for Cognitive Radio with Relay-Assisted Directional Transmission

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***Abstract*—*Spectrum hole* (SH) is defined as a spectrum band that can be utilized by unlicensed users, which is a basic resource for *cognitive radio* (CR) systems. Most of existing contributions detect SHs by sensing whether a primary signal is present or absent and then try to access them so that the CR and primary users use the spectrum band either at different time slots or in different geographic regions. In this paper, we propose a novel scheme with relays or directional relays for CR users to exploit new spectrum opportunity, called spatial SH. It can provide higher spectrum efficiency by coexistence of primary and CR users at the same region, time, and spectrum band. In particular, when the spectrum opportunity of a direct link from a CR transmitter to a CR receiver does not appear, our scheme may still establish the communication through indirect links, i.e., other CR users act as relay stations to assist the communication by using other spatial domains. Furthermore, we analyze the successful communication probabilities of CR users and demonstrate that the spectrum efficiency can be considerably improved by our scheme.**

***Index Terms*—Cognitive radio, directional transmission, relay, spectrum hole.**

I. Introduction

COGNITIVE radio (CR), a very promising technology for future wireless communications, enables much higher spectrum efficiency by means of dynamic and opportunistic spectrum access. It was first proposed in late 1990s [1] and a comprehensive overview on CR has been provided in [2]. The basic concept of CR is to allow unlicensed CR users, also called secondary users, to use licensed spectrum bands as long as they cause no intolerable interference to licensed users, also called *primary users* (PUs). Therefore, secondary users must identify and use the spectrum bands that are not being used by PUs. In practice, the available spectrum bands for secondary users vary with time and location. A region of location-time-frequency available for a secondary user is called a *spectrum hole* (SH). Generally, a CR system is able to coexist with a primary system by accessing SHs.

Apparently, SH is a basic resource for CR users. A report from Federal Communication Commission [3] shows that with a very large probability, there are some licensed spectrum bands that are not being used by PUs in a certain location or time. Thus spectrum sensing [4]-[12], which seeks SHs in time or location for secondary users to utilize them, is critical to the development of CR. Three traditional signal detection techniques, matched filter, energy detector, and cyclostationary feature detector, have been introduced in [5] for local spectrum sensing at an individual CR user. In particular, the performance of energy detection over fading channel has been well studied in [6]. Since fading and shadowing in wireless channels degrade the performance of local spectrum sensing significantly [5], cooperative spectrum sensing has been proposed to improve detection capabilities by exploiting spatial diversity in multi-user wireless networks [7]-[9]. Soft combination of sensing information from different CR users in centralized CR networks has been investigated in [10]. In [11] and [12], a cooperative sensing method with amplify-and-forward relay has been developed. Based on sensing results, most of existing works enable CR users to share spectrum bands with PUs at either different time slots or different locations, i.e., a CR user can use the licensed spectrum band when PUs are not operating in a certain location. Alternatively, a secondary system can coexist with a primary system at the same location, time, and frequency band as long as the CR does not cause intolerant interference to PUs. This is the scenario we will study in this paper.

Motivated by relay, smart antenna, and spatial division multiple access technologies [13][14], we will propose a novel scheme to exploit *spatial spectrum holes* (SSHs), however at different spatial domains with the help of relay and directional transmission techniques. There have been some contributions considering the utilization of relays and multiple antennas to explore SSHs for CR communications but in a different way from us. Since relay techniques are able to create multiple links between a *CR transmitter* (CRtx) and a *CR receiver* (CRrx), the spatial diversity can be achieved by CR to enhance either *signal-to-interference-plus-noise ratio* (SINR) of CR transmissions [15][16] or the performance of cooperative spectrum sensing [17]. Our scheme in this paper exploits spectrum opportunities by means of relay transmissions. Intuitively, if a

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TABLE I
ABBREVIATION TABLE

CR	Cognitive radio
CRR	Cognitive radio relay station
CRrx	Cognitive radio receiver
CRtx	Cognitive radio transmitter
EAB	Equivalent available bandwidth
GSH	Geographic spectrum hole
NAT	Normalize achievable throughput
PU	Primary user
SCP	Successful communication probability
SH	Spectrum hole
SINR	Signal-to-interference-plus-noise-ratio
SSH	Spatial spectrum hole
TSH	Temporal spectrum hole

CR user is with a low transmit power and a short transmission range, interference to PUs will be effectively reduced. Then an indirect link from the CRtx to the CRrx can be established with the assistance of relay stations. Furthermore, when directional antennas or multiple antennas with beamforming are available at CR users, the interference to PUs can be further mitigated. Recently, there are some works [18]-[21] discussing SSHs with path loss propagations and beamforming techniques. Nevertheless, they only focus on point-to-point CR links. In this paper, we propose a relay-assisted directional CR transmission scheme to exploit SSHs in CR networks, which enables primary and CR systems to coexist at the same region, time, and frequency band. Our analysis indicates that it significantly improves the successful communication probability of CR.

The rest of this paper is organized as follows. In Section II, we propose our relay-assisted directional transmission scheme to use SSHs. Moreover, based on whether or not the directional transmission is available for CR, we divide the scheme into basic and advanced ones. In Section III, we address the basic scheme for the relay-assisted secondary transmission and analyze successful communication probabilities to demonstrate the advantages of the proposed scheme. In Section IV, we propose the advanced scheme that exploits directional relays. Finally, we conclude the paper in Section V.

The abbreviations in this paper are summarized in Table I.

II. SPATIAL SPECTRUM HOLES

Generally, whether a spectrum hole exists or not at a certain time and location can be described as the presence or absence of the primary signal according to the observation,

$$y(t) = \begin{cases} n(t), & \mathcal{H}_0, \\ hs(t) + n(t), & \mathcal{H}_1, \end{cases} \tag{1}$$

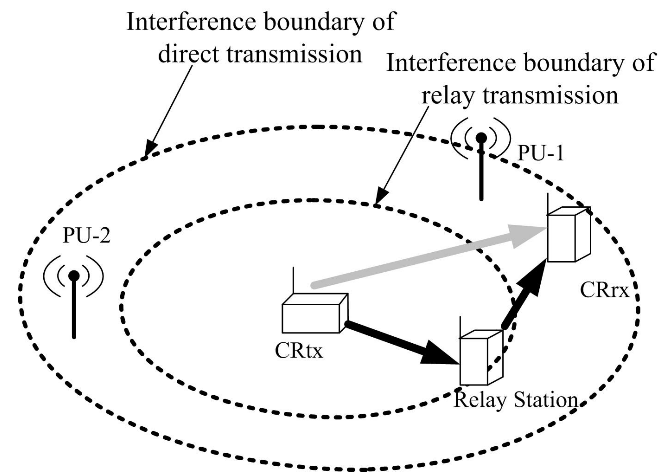

Fig. 1. Coexistence of CR and primary systems.

where $s(t)$ is the primary signal, h denotes the channel coefficient between the primary transmitter and the CR user*, $n(t)$ represents additive white Gaussian noise, and Hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the absence and the presence of the primary signal, respectively.

Conventionally, there are two scenarios involved in CR communications. The first one is that the CR user may access a spectrum band when the PU is not using it temporarily, which means that the CR and PUs can be deployed at the same spectrum band and geographic area, but at different time slots. In this case, $\mathcal{H}_1$ and $\mathcal{H}_0$ in (1) represent that the PU turns on and off, respectively, and secondary transmission is realized by utilizing the silent time slots of the PUs, which is called *temporal spectrum holes* (TSHs) in this paper. The second scenario for a CR communication is that the CR user can access the spectrum band when the CR and PUs are in different geographic regions. In this scenario, path-loss and shadowing in wireless channels separate the CR and PUs and make it possible for both to work at the same time without interfering each other. In this case, $\mathcal{H}_0$ and $\mathcal{H}_1$ in (1) represent whether or not the CR and PUs can be separated by path-loss, respectively. Such secondary communication opportunities are called *geographic spectrum holes* (GSHs) in this paper.

Generally, it is hard for CR users and PUs to work in the same geographic region, at the same time, and on the same frequency band. However, this can be realized by the utilization of some advanced techniques, such as relay and directional transmissions. Figure 1 shows an example of the coexistence of the primary and secondary systems in this scenario. As shown in the figure, if a CRtx sends data directly to a CRrx through a licensed spectrum band, it has a large interference region, which is defined as an area where primary transmission is interfered by the CR transmission [23]. However, if the CRtx lowers its transmit power by using other CR users as relay stations†, the interference region will be effectively reduced. Therefore, the CR user will still be able to

*We will only consider the path-loss in this paper since it is the dominant component compared to shadowing and small scale fading in wireless channels [22].

†The relay user can be either a special relay station or an ordinary CR user and we use "relay station" throughout this paper.

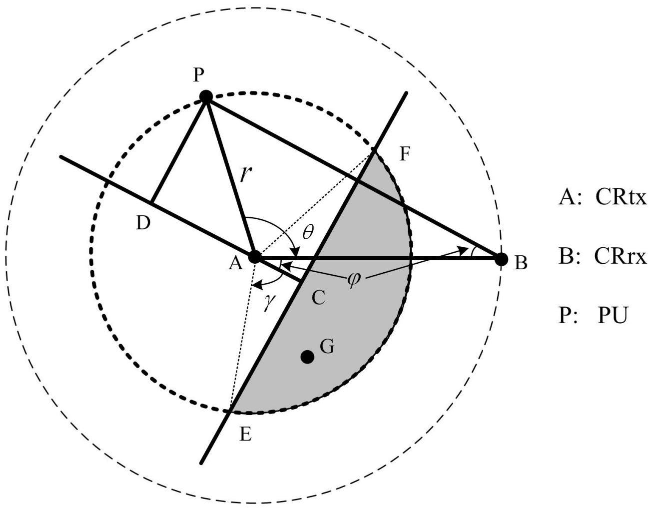

Fig. 2. Illustration of the relay-enabling region.

access the spectrum band without interfering with PUs. Moreover, if CR users are equipped with directional antennas or antenna arrays, they are able to send and forward a CR signal directionally, which further reduces the interference region and achieves more spectrum opportunities. Here, $\mathcal{H}_1$ and $\mathcal{H}_0$ in (1) represent whether or not the CR's transmission interferes with PUs. Such secondary communication opportunities existing in primary systems are called SSHs since primary and CR links are established through different spatial domains (or regions). In other words, the CR signals are forwarded through relay stations when a direct link between the CRtx and the CRrx is not available. Obviously, relay stations play a critical role to implement the proposed scheme and they need to satisfy the following conditions:

- The CRtx sends its signal to a relay station without interfering with PUs.
- The relay station forwards the signal to the CRrx without interfering with PUs.

Depending on whether directional transmissions are available or not, we will develop our scheme with relays and directional relays, respectively. For simplicity, we name the former one as a basic system and the latter one as an advanced system.

III. Basic System

In this section, we will introduce the principle of the relay-assisted CR transmission for SSH exploitation and then analyze its performance. We will assume the spectrum sensing and data transmissions are preformed ideally throughout this paper.

A. Relay-Assisted Scheme

Figure 2 demonstrates the relay-enabling region, which is defined as an area where a relay station can be located to ensure a successful CR transmission without interfering with PUs. In the figure, A, B, and P represent the positions of the CRtx, the CRrx, and the PU, respectively. To facilitate analysis, the polar coordinator is applied. Without loss of generality, let A be the origin and the angle of the position B be zero. Then the position of the PU can be represented as (r, θ), where r is the distance from A to P and θ is the angle between Line AP and Line AB. Assume G in the figure is the position of a certain relay station and $\overline{GA}$, $\overline{GB}$, and $\overline{GP}$ denote the distances from the relay station to the CRtx, the CRrx, and the PU, respectively. A CR user can act as a relay station for the CRtx if its position satisfies the following requirements:

- The relay station, G, is closer to the CRtx than the PU, P, so that the CRtx can communicate with G without interfering with the PU, i.e., $\overline{GA} < \overline{PA}$[‡]. In Fig. 2, the area inside the circle with center A and radius r satisfies this condition.
- The relay station, G, is closer to the CRrx than the PU, P, so that the relay station can forward the data without interfering with the PU, i.e., $\overline{GB} < \overline{GP}$. In Fig. 2, the right half region of Line EF satisfies this condition, where Line EF is the perpendicular bisector of Segment BP.

According to the above two requirements, any CR user in the shaded area in Fig. 2 can be used as a relay station. Then the area is called the *relay-enabling region*. Apparently, the relay-enabling region is determined by the relative positions of the CRtx, the CRrx, and the PU. The larger the relay-enabling region is, the easier it is for the CRtx to find a CR user satisfying the two requirements.

B. Performance Analysis

Here, we will analyze *successful communication probability* (SCP) of the proposed relay-assisted CR transmission scheme. Since the area of the relay-enabling region depends on the position of the PU, we will first obtain the relay-enabling region for a given position of the PU, and then derive the average SCP in a certain region.

As shown in Fig. 2, we consider that the area inside the circle is with center CRtx and radius d, where d is the distance between the CRtx and the CRrx. Then the considered region can be expressed as

$$\mathbb{S} = \{(r, \theta) | r < d\}, \tag{2}$$

for $0 \leq \theta < 2\pi$. Assume that N CR users are uniformly distributed in the considered area, then the density of CR users is $\zeta = \frac{N}{\pi d^2}$. Suppose a PU is inside the considered region and the CRtx intends to send data to the CRrx without interfering with the PU. Based on geometrical analysis in Appendix A, we can obtain the area of the relay-enabling region for a given position of the PU, (r, θ), as

$$S_{relay}(r, \theta) = r^2 \arccos\left(\frac{\overline{AC}}{r}\right) - r \sin\left[\arccos\left(\frac{\overline{AC}}{r}\right)\right] \cdot \overline{AC}, \tag{3}$$

‡In practice, due to the fading and the shadowing in wireless channels, there are some chances that the signal strength with a long distance to the CRtx may be larger than that with a short distance, which may lead to interference to the PU. Thus a guard distance, ϵ, is usually used as a margin to guarantee the protection to PUs [4], i.e., $\overline{GA} + \epsilon < \overline{PA}$. In this paper, we omit the guard distance for simplicity. Except for smaller values of communication chances, the conclusions are similar no matter whether the guard distance is considered or not.

where $\overline{AC}$ denotes the distance between A and Line EF in Fig. 2. If $S_{relay}(r,\theta) = 0$ in (3), it is impossible for CR users to access the spectrum band by two-hop relays. If $S_{relay}(r,\theta) > 0$ and at least one CR user acts as a relay station, the CRtx is able to communicate with the CRrx. Therefore, we call the region satisfying $S_{relay}(r,\theta) > 0$ accessible region and it can be expressed as

$$\mathbb{S}_{access} = \{(r,\theta)|(r,\theta) \in \mathbb{S}, S_{relay}(r,\theta) > 0\}. \quad (4)$$

From above discussions, whether or not a CR link can be assisted by relays depends on both the position of the PU and the density of CR users. In other words, a successful communication between the CRtx and the CRrx can be achieved when the PU appears in the accessible region and the density of CR users is high enough.

Define SCP as the probability that the CRtx could send data to the CRrx successfully. Apparently, for a given position of the PU, the larger the density of CR users is, the higher the SCP is. But in reality, the actual SCP depends on a certain density of CR users and we will derive the average SCP in the following.

For a given position of the PU, (r,θ), the probability that a CR user falls in the relay-enabling region can be expressed as

$$P(r,\theta) = \frac{S_{relay}(r,\theta)}{\pi d^2}. \quad (5)$$

If there is at least one CR user in the relay-enabling region, the CRtx will be able to successfully communicate with the CRrx via the relay station. Therefore, the SCP for a given position of a PU can be expressed as

$$P_{SCP}(r,\theta) = 1 - (1 - P(r,\theta))^N. \quad (6)$$

Consequently, for a PU randomly located in the considered area, the corresponding average SCP can be obtained by

$$\tilde{P}_{SCP} = \frac{1}{\pi d^2}\int_0^{2\pi}\int_0^d (1 - (1 - P_{SCP}(r,\theta))^N) dr d\theta, \quad (7)$$

which can be evaluated numerically.

IV. Advanced System

In this section, we will develop our scheme when CR users have the ability of directional transmissions. We will study the requirements for directional relays to realize the CR communication, analyze its performance, and then give corresponding numerical results.

A. Relay-Assisted Scheme with Directional Transmission

Figure 3 illustrates how SSHs can be utilized in a CR network. As shown in the figure, two PUs are communicating with each other and a number of CR users are operating in the same region and spectrum band. Each CR user is with directional transmitting and omni-directional receiving abilities. We assume that the direction of the transmission at each CR user is random. When the CRtx intends to send messages to the CRrx, it first senses the spectrum band for the coverage of its directional antenna. If the sensing result is idle, the spectrum band can be used by the CRtx; otherwise, the CRtx needs to vary its direction of the transmission randomly. When the sensing result of the CRtx is idle and the CRrx is within the coverage of the CRtx, a direct link from the CRtx to the CRrx can be established. But when the spectrum opportunity of a direct link does not appear, relay stations can be used to assist the CR's communication. For a CR user to be able to act as a relay station, it must satisfy the following two requirements:

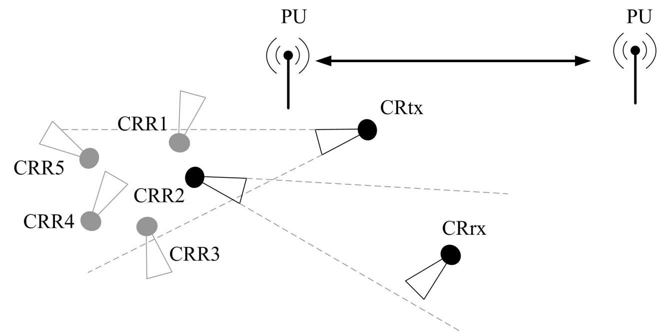

Fig. 3. Demonstration of the relay-assisted directional CR transmission.

- The coverage of its directional transmission is clear, i.e., the sensing result of the relay station is idle so as not to cause interference to the PUs.
- Its directional transmission faces the CRrx or another relay station if multi-hop relays are assumed.

As demonstrated in Fig. 3, *CR relay station 2* (CRR2) satisfies the two requirements. Then a two-hop relay link can be established among the CRtx, the CRR2, and the CRrx.

Apparently, CR users will undergo interference from the PUs and hence appropriate interference avoidance or cancellation techniques need to be employed. We assume that there are dedicated control channels to organize all nodes in the CR network. While a lot of issues need to be investigated regarding the proposed scheme, we will focus on the probability of successfully establishing CR links, which aims at revealing the potential of exploiting the SSHs by relay and directional transmission techniques.

B. Performance Analysis

In this subsection, the SCP from the CRtx to the CRrx will be analyzed first. Then the SCP within multiple consecutive time slots will be further investigated. Also, the coexistence of multiple CR links with primary links will be considered.

1) Successful Communication Probability: Here, we only focus on CR users with directional antennas even though the discussion here can be easily applied to CR users with beamforming. Denote the beamwidth of directional antennas as φ degrees and the normalized beamwidth as $\gamma = \frac{\varphi}{360}$. We assume that the area of interest is large enough so that the boundary can be ignored. In other words, both primary and CR users here are within their transmission ranges. Consider that there are L PUs and N CR relay stations randomly located in an area and the directions of them are uniformly distributed.

Direct Transmission

According to the above definitions and assumptions, the probability that a PU within the coverage of the CRtx is γ and

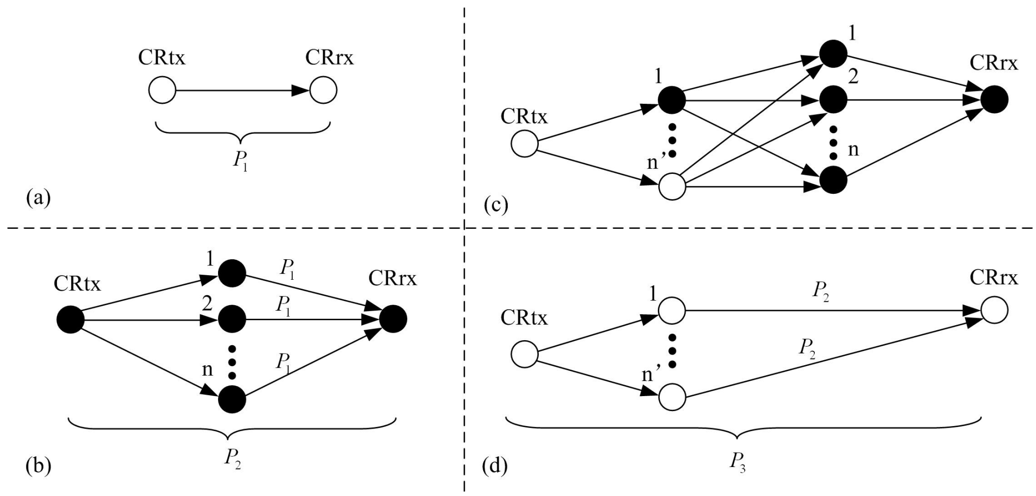

Fig. 4. Analysis of multi-hop relay transmissions.

there are $(1-\gamma)$ chances for the CRtx to access the spectrum band. For L PUs, the probability of the idle sensing result at the CRtx can be obtained by

$$P_{CRtx,idle} = (1-\gamma)^L. \quad (8)$$

At the same time, the probability that the CRrx falls in the coverage of the CRtx is γ. Therefore, the probability of the one-hop direct communication between the CRtx and the CRrx is given by

$$P_1 = \gamma(1-\gamma)^L, \quad (9)$$

which can be shown in Fig. 4-(a).

Two-Hop and Multi-Hop Transmissions

When a two-hop communication between the CRtx and the CRrx is considered, a relay station needs to be selected. Since there are N independent relay stations in the considered region and the normalized beamwidth is γ, the number of relay stations in the coverage of the CRtx, say n in Fig. 4-(b), is a random variable with binomial distribution and its probability mass function is given by

$$f(n) = \binom{N}{n}\gamma^n(1-\gamma)^{N-n},\ 0 \le n \le N, \quad (10)$$

where $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ and ! denotes factorial operation.

We define the link from one of n relay stations to the CRrx as an one-hop relay, then the probability that a relay station to be qualified as an available relay station is P_1. Therefore, for n relay stations, the probability that there exists at least one available relay station to assist CR transmission can be expressed as

$$P_{relay,n} = 1 - [1-P_1]^n\,. \quad (11)$$

According to the derivation of Appendix B and denote P_{relay} as the average probability of $P_{relay,n}$, we can obtain

$$P_{relay} = \mathbb{E}\{P_{relay,n}\} = 1-(1-\gamma+\gamma A)^N, \quad (12)$$

where $A = 1-P_1$. Therefore, when at least one relay station is in the coverage of the CRtx and its sensing result is idle, the relay station is able to forward the signal from the CRtx to the CRrx successfully. Then the SCP of the two-hop relay can be obtained by

$$\begin{aligned} P_2 &= P_{CRtx,idle}\cdot P_{relay} \\ &= (1-\gamma)^L[1-(1-\gamma+\gamma A)^N]. \end{aligned} \quad (13)$$

Substitute $A = 1-P_1$ into (13), then

$$P_2 = (1-\gamma)^L[1-(1-\gamma P_1)^N]. \quad (14)$$

Figure 4-(c) illustrates a three-hop relay scenario, where the CRtx sends data to the CRrx via two relay stations. We group the last two hops together, e.g., the solid CR users in Fig. 4-(c) are grouped and its SCP equals to P_2. Denote n' as the number of relay stations in the coverage of the CRtx, then there are n' groups with identical SCP as shown in Fig. 4-(d). With this way, the three-hop relay can be equivalent to a two-hop relay and the SCP can be expressed as

$$P_3 = (1-\gamma)^L[1-(1-\gamma P_2)^N]. \quad (15)$$

Similarly, the SCP of m-hop transmission can be derived recursively by

$$P_m = (1-\gamma)^L[1-(1-\gamma P_{m-1})^N]. \quad (16)$$

The CR communication can be established by either a direct transmission or a multi-hop transmission. If at least one case is available, a successful communication can be achieved. Then based on (16), the SCP of a maximum M-hop relay can be calculated as

$$Q_M = \Pr\left[\bigcup_{m=1}^{M} P_m\right] = 1-\prod_{m=1}^{M}(1-P_m), \quad (17)$$

where $\cup$ represents the union operator and $\Pr[a]$ denotes the probability of a.

2) SCP in Multiple Consecutive Time Slots: In the previous discussion, we investigated the instantaneous SCP within one single time slot. In the following, we will further investigate the SCP between the CRtx and the CRrx in multiple consecutive time slots. Assume that the direction of the CR transmission varies independently from one to another, which may be caused by the CR user's random movements or by different beam pattern vectors utilized at different time slots. Define ς as the time slot index, P_ς as the SCP in each time slot, then the overall SCP within consecutive V time slots is given by

$$P_V = \Pr\left[\bigcup_{\varsigma=1}^{V} P_\varsigma\right] = 1 - (1 - P_\varsigma)^V. \tag{18}$$

Apparently, a higher SCP can be achieved by letting CR users attempt to communicate in more time slots.

3) Spectrum Multiplexing among CR Users: In essence, we have studied the possibility of enabling the coexistence of CR and primary systems without interfering with PUs. With our approach, a CR link can be established simultaneously with existing primary radio links in the same geographical region. Thus the spectrum band of the primary system can be accessed by CR users, which improves the spectrum efficiency accordingly. While in this section, we will further investigate the possibility of enabling multiple CR links to coexist with primary links at the same region, time, and spectrum band. In contrast to the approach in one CR link, our multiple CR links method not only accesses the spectrum band of a primary system, but also reuses the spectrum band with the CR system itself. Thus we name it as spectrum multiplexing to distinguish the one CR link approach. Obviously, the spectrum efficiency can be further increased by means of the spatial reuse.

We divide CR links into different classes according to their priorities. The link in Class I has the highest priority and it only needs to avoid interfering with the PUs. In contrast, the link in Class II has to avoid interfering with both PUs and the CR users in Class I. Apparently, there will be fewer SSHs for Class II than for Class I. Similarly, the secondary transmission of the CR users in Class III, Class IV, and so on, can be realized. Therefore, multiple CR links can work simultaneously, which may dramatically increase the spectrum efficiency.

Assume there is a primary network and a secondary network coexisting in the same region using the same frequency band, whose bandwidth is W and it is the *equivalent available bandwidth* (EAB). Suppose there are L' PUs and N' CR users operating in the considered area. Define M as the maximum number of hops in relay-assisted directional transmission. For the link in Class I, the SCP of m_1-hop, $1 \leq m_1 \leq M$, $P_{m_1}^{(1)}(L = L', N = N')$, can be calculated from (9) or (16) where the superscript of P represents the class index. For the link in Class II, the link in Class I should be considered as an equivalent primary link to avoid interference. Therefore, the corresponding SCP of m_2-hop link ($1 \leq m_2 \leq M$) in Class II can be expressed as

$$P_{m_2}^{(2)} = \Pr\left[\bigcup_{m_1=1}^{M} P_{m_2}^{(2)}(L' = L + m_1, N = N' - (m_1 + 1))\right].$$

Similarly, the SCP of m_k-hop link in Class k is given by (19) and the EAB of the link in Class k with a maximum of M hops is given by

$$W_k = W \cdot \Pr\left[\bigcup_{m_k=1}^{M} P_{m_k}^{(k)}\right] = W Q_M^{(k)}, \tag{20}$$

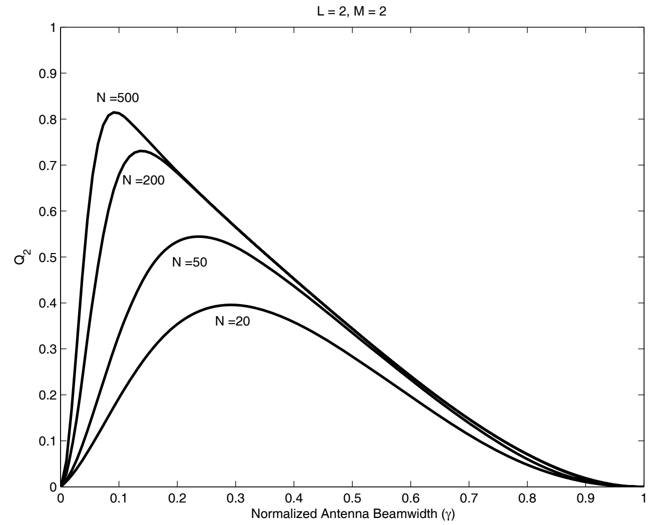

Fig. 5. SCP versus γ for different numbers of CR users, N.

where $Q_M^{(k)} = 1 - \prod_{k=1}^{K}(1 - P_{m_k}^{(k)})$. Therefore, the overall EAB for the CR network can be obtained by

$$W_{CR} = \sum_{k=1}^{K} W_k = W \sum_{k=1}^{K} Q_M^{(k)}, \tag{21}$$

where K is the number of active classes or layers in secondary links. Thus the equivalent reuse ratio of the EAB for the CR network can be expressed as

$$\eta = \frac{W_{CR}}{W} = \sum_{k=1}^{K} Q_M^{(k)}. \tag{22}$$

From (19), (20), and (22), the reuse ratio, η, is determined by the density of the primary and CR users, the CR's directional antenna beamwidth, and the maximum number of hops.

C. Numerical Results

In this subsection, we present numerical results to show the performance of the proposed relay-assisted directional CR transmission scheme. We assume that a primary network and a secondary network are deployed in the same region. We further assume that the number of CR users is much larger than that of PUs, i.e. $N \gg L$, which may happen when the CR users have a much shorter communication range than PUs do.

1) SCP of Relay-Assisted Directional Transmission: Figure 5 illustrates the CR's SCP, Q_M, versus normalized beamwidth γ for different numbers of relay stations, N. Here, assume that two PUs are communicating with each other in the area and a maximum of two hops relay can be used for the relay-assisted directional transmission scheme, i.e., $L = 2$ and $M = 2$. Figure 5 indicates that the maximum Q_2 increases with the number of CR users, which is reasonable since more CR users could exploit SSHs more sufficiently. Also, it can be observed that the larger the number of CR users is, the narrower the optimal CR directional antenna beamwidth is. This is because a large number of CR users means that the CRtx can easily find a proper relay station and the interference from the PUs

$$P_{m_k}^{(k)} = \Pr\left[\bigcup_{m_{k-1}=1}^{M} \ldots \bigcup_{m_1=1}^{M} P_{m_k}^{k}\left(L = L' + m_{k-1} + \ldots + m_1, N = N' - (m_{k-1}+1) - \ldots - (m_1+1)\right)\right], \qquad (19)$$

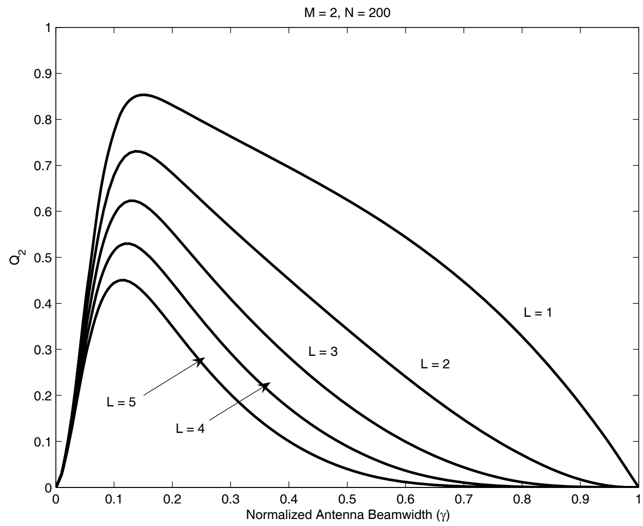

Fig. 6. SCP versus γ for different numbers of primary users, L.

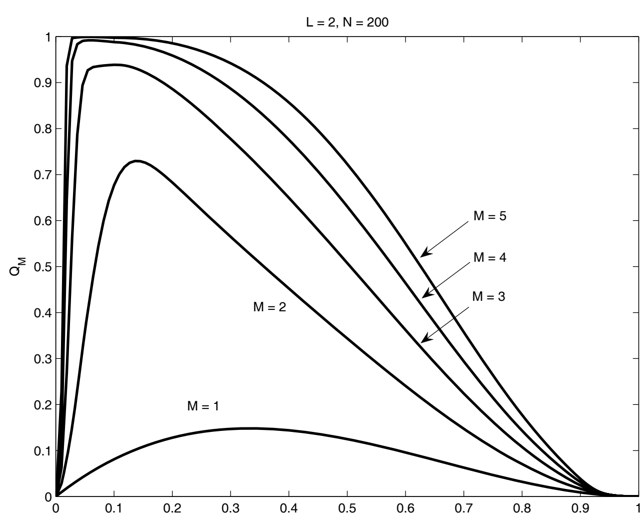

Fig. 7. SCP versus γ for different numbers of hops, M.

will be the bottleneck for SCP. As a result, a narrow antenna beamwidth is required to avoid the interference from the PUs and therefore achieves a higher SCP. On the other hand, when the number of CR users is small, lacking of relay stations will be the bottleneck and therefore a large antenna beamwidth can help the CR transmitter to find relay stations easily.

Figure 6 shows the performance of the SCP versus the normalized beamwidth γ for different numbers of PUs, L, where $M = 2$ and $N = 200$. It can be observed that both the maximum Q_2 and the corresponding optimal antenna beamwidth go down as the number of PU, L, increases. This is because, as L increases, the size of SSHs decreases and the antenna beamwidth needs to be decreased accordingly so as not to cause interference to PUs. Figure 7 shows SCP versus γ for different numbers of hops, M, where $L = 2$ and $N = 200$. It can be observed from Fig. 7 that Q_M increases dramatically as M goes up, indicating that the utilization of more hops of relays significantly improves the SCP.

2) Tradeoff between Achievable Throughput and SCP: As indicated in Fig. 7, the SCP, Q_M, increases as the number of hops goes up. That is because more relay paths from the CRtx to the CRrx can be selected by CR users. In the meantime, more sophistic protocols, however, need to be applied to coordinate those relay stations and they lead to protocol overheads and delays. Thus the achievable throughput reduces as the number of hops increases. In this subsection, we will investigate the relationship between the achievable throughput and the SCP for different numbers of hops.

In practice, the throughput loss depends on specific protocols, channel models, and so on. Usually, it is very hard to obtain an exact relationship among them. In order to facilitate analysis, we consider all factors resulting in the loss of throughput are equivalent to the energy consumption. Assume that some data at the CRtx is to be transmitted to the CRrx. We further assume that the transmissions of such data by either the CRtx or relay stations spend the same unit energy. Obviously, the more relay stations are involved in such a transmission, the more energy needs to be spent. In other words, given a certain energy, the achievable throughput decreases as the number of hops increases. Then the equivalent achievable throughput for M-hop relay can be expressed as

$$\mathcal{C} = \mathcal{B}\log\left(1 + \frac{P_s}{MP_n}\right), \qquad (23)$$

where $\mathcal{B}$ denotes the bandwidth, P_s is the transmission power of CR users, and P_n is the power of the noise. Figure 8 shows the *normalize achievable throughput* (NAT) versus the SCP for different numbers of hops. The SCP in the figure represents the maximum SCP, i.e., maximum Q_M. From Fig. 8, as the number of hops increases, the SCP increases while the NAT decreases. In particular, only NAT decreases and the SCP is almost constant when the number of hops is larger than five, i.e., $M > 5$. It means that the number of hops should be a proper value to achieve a resealable performance between NAT and SCP.

3) SCP of Spectrum Multiplexing: Figure 9 shows SCP versus γ when $L = 2$, $M = 2$, and $N = 200$. The solid curves in the figure denote the SCP performance in different classes and the dashed curve is the summation of them, which represents the overall SCP in the CR network. Figure 9 illustrates that the maximum SCP goes down as the class number k increases, which is reasonable since a larger k means a lower priority and more high priority CR users to be protected. From the dashed curve, it can be observed that the proposed spectrum multiplexing method can considerably increase the spectrum efficiency. Figure 9 also indicates that the optimal γ is different for different classes. Therefore, an appropriate antenna beamwidth should be selected according

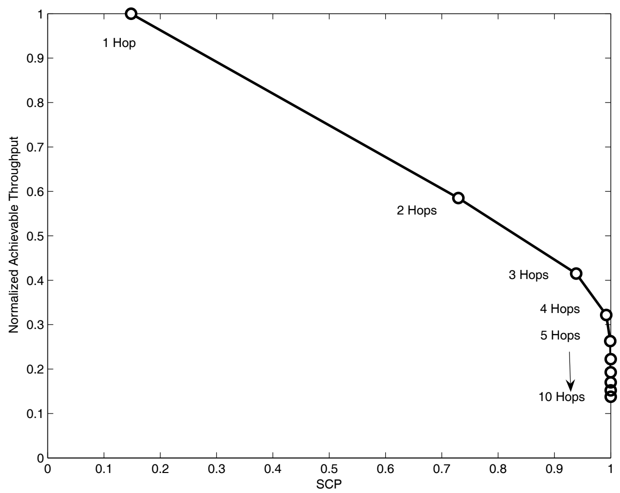

Fig. 8. Tradeoff between normalized achievable throughput and SCP.

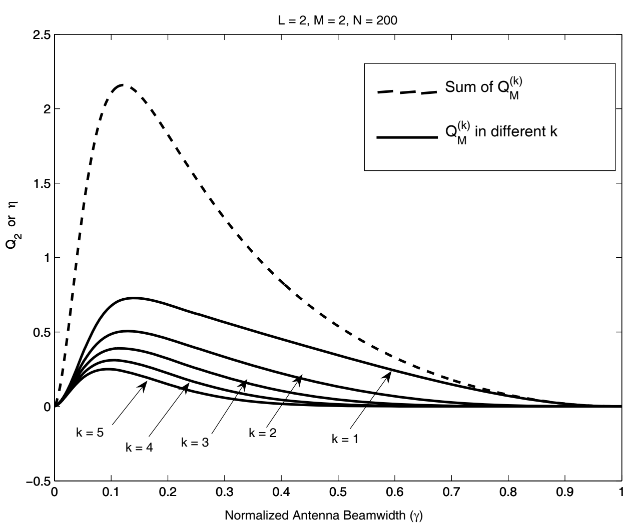

Fig. 9. Performance of the spectrum multiplexing in the advanced system.

to the CR user's density in the CR network and the quality-of-service requirement. For instance, the antenna beamwidth that maximizes η can be chosen to optimize the overall network performance.

V. Conclusion

In this paper, we exploit the spatial spectrum holes by using relay and directional transmission techniques. Compared to direct transmissions from a CR transmitter to a CR receiver, the data can be sent to its destination via relays with a higher success probability. If CR users have the ability of directional transmission, it may achieve a good successful communication probability with a proper beamwidth. Since relay and directional transmissions are able to provide significant potentials of spectrum reuse, the spectrum efficiency can be further improved by spectrum multiplexing. As a result, CR and primary networks are able to coexist at the same time, in the same spectrum band and geographic region, but through different spatial domains. As the number of relay hops increases, more complicated protocols are involved, which decreases the CR's throughput. Our analysis demonstrates the tradeoff between achievable throughput and successful communication probability for different number of hops. In practice, a proper relay strategy should be applied according to application scenarios.

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Appendix A
Derivation of $S_{relay}(r, \theta)$

In this appendix, we will get the area of the relay-enabling region for a given position of the PU, (r, θ). In other words, we need to obtain the area of the shaded region in Fig. 2. As shown in the figure, $\triangle ABP$ denotes a triangle with three points A, B, and P, and Line EF is the perpendicular bisector of Segment BP. Line AC is parallel with Line BP and Line DP is perpendicular to Line AC. Assume $\overline{AP} = r$ and $\overline{AB} = d$, then $\overline{BP}$ can be obtained by

$$\begin{aligned} \overline{BP} &= \sqrt{\overline{AP}^2 + \overline{AB}^2 - 2\overline{AP} \cdot \overline{AB}\cos\theta} \\ &= \sqrt{r^2 + d^2 - 2rd\cos\theta}. \end{aligned} \quad (24)$$

Denote $\angle ABP$ and $\angle BAC$ as the angles between Lines AB and BP, and Lines BA and BC. Since Lines BP and CD are parallel, $\angle ABP = \angle BAC = \phi$, where

$$\begin{aligned} \phi &= \arccos\left(\frac{\overline{BP}^2 + d^2 - r^2}{2\overline{BP}d}\right) \\ &= \arccos\left(\frac{d - 2r\cos\theta}{\sqrt{r^2 + d^2 - 2rd\cos\theta}}\right). \end{aligned} \quad (25)$$

Thus the lengths of Lines AD and AC can be expressed as

$$\overline{AD} = \begin{cases} r \cdot \cos(\pi - \theta - \phi), & \text{if } (\theta + \phi) > \pi/2, \\ r \cdot \cos(\theta + \phi), & \text{if } (\theta + \phi) \le \pi/2, \end{cases} \quad (26)$$

and

$$\overline{AC} = \begin{cases} \overline{BP}/2 - \overline{AD}, & \text{if } (\theta + \phi) > \pi/2, \\ \overline{BP}/2 + \overline{AD}, & \text{if } \theta + \phi) \le \pi/2, \end{cases} \quad (27)$$

respectively. Accordingly, the angle of $\angle CAE$ is γ, which can be expressed as

$$\gamma = \arccos\left(\frac{\overline{AC}}{r}\right). \quad (28)$$

The area of $\triangle AEF$ and sector $\sphericalangle AEF$ can be expressed as

$$S_{\triangle AEF} = r\sin\gamma \cdot r\cos\gamma, \quad (29)$$

and

$$S_{\sphericalangle AEF} = \pi r^2 \frac{2\gamma}{2\pi}, \quad (30)$$

respectively. Therefore, the area of the shaded region in Fig. 2 can be obtained by

$$\begin{aligned} S_{relay}(r, \theta) &= S_{\sphericalangle AEF} - S_{\triangle AEF} \\ &= r^2\arccos\left(\frac{\overline{AC}}{r}\right) - r\sin\left[\arccos\left(\frac{\overline{AC}}{r}\right)\right] \cdot \overline{AC} \end{aligned} \quad (31)$$

APPENDIX B
DERIVATION OF $\mathbb{E}\{P_{relay,n}\}$

Define $A = 1 - P_1$, then $P_{relay,n} = 1 - A^n$ and the mean of $P_{relay,n}$ can be obtained by

$$P_{relay} = \mathbb{E}\{P_{relay,n}\} = 1 - \mathbb{E}\{A^n\} = 1 - \mathbb{E}\{e^{n \ln A}\}, \quad (32)$$

where $\mathbb{E}[\cdot]$ represents expectation operation. Recall that the *moment-generating function* of the binomial random variable, n, is given by [24]

$$M_n(t) = \mathbb{E}(e^{nt}) = (1 - \gamma + \gamma e^t)^N. \quad (33)$$

According to (32) and (33), we obtain

$$\mathbb{E}\{P_{relay,n}\} = 1 - (1 - \gamma + \gamma A)^N. \quad (34)$$

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