Spatial Spectrum Holes in Cognitive Radio with Relay Transmission†

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Abstract—In this paper, we propose a relay-assisted transmission scheme in cognitive radio (CR) to exploit spatial spectrum holes, which are generated by relay techniques. The proposed scheme enables CR users to coexist with primary users at the same time in the same geographic area and spectrum band. Compared to conventional schemes, a higher spectrum efficiency is achieved by our method. We further analyze the successful communication probability and present numerical results to show advantages of our method.

I. INTRODUCTION

Cognitive radio [1] is a paradigm for wireless communications in which the system is able to change its transmission parameters to communicate without interfering with primary users (PUs). It promises a high spectrum efficiency by means of accessing the spectrum band when PUs are not using it in a certain time and location. Such spectrum opportunity is called spectrum holes (SHs) and is a kind of basic resource in CR systems.

Most of existing works are to sense SHs and try to access them efficiently by spectrum sensing and dynamic spectrum access. In spectrum sensing, a comprehensive overview has been provided in [2], three traditional signal detection techniques, matched filter, energy detector, and cyclostationary feature detector, have been introduced in [3], and energy detection has been extensively studied in [4]. Furthermore, cooperative sensing has been proposed in [5][6] to improve the sensing performance in fading and shadowing channels. In addition, there are some works detecting primary receivers (PRs) [7][8] since the ultimate purpose of spectrum sensing is to avoid interfering with PRs. In [7], PRs are detected by exploiting local oscillator leakage emitted by radio-frequency (RF) front end. In [8], when a primary system is with power control, CR users can identify PRs by sending some sounding signals and detecting the response of such power control. On the other hand, in order to access SHs more efficiently, the durations of sensing and data frames have been optimized in [9] so that the interference to PUs can be minimized and the throughput for CR users can be maximized.

While all above techniques are to detect and access SHs either in different time slots or in different locations with PUs, the possibilities that a CR system coexists with a primary system at the same time in the same geographic area and spectrum band have not been well investigated. In [10], beamforming techniques have been applied in point-to-point CR systems. In our previous work in [11], spatial SHs generated in CR networks have been studied if CR users are equipped with directional antennas. In this paper, we propose a concept of spatial SHs that are generated by relay transmissions, and then present a relay-assisted scheme for a CR system when directional antennas with beamforming are not available for CR users. Principally, instead of a direct transmission from the position of a CR transmitter, the data is sent to its receiver via other CR users. With the help of relay techniques, CR users are able to achieve more communication opportunities even when direct transmissions are not available.

The rest of the paper is organized as follows. In Section II, we describe the problem and propose the concept of spatial SHs. In Section III, we develop the principle of our relay-assisted method. Then, we analyze the successful communication probability of our proposed scheme in Section IV. In Section V, we present the numerical results to show its performance and conclude the paper in Section VI.

II. PROBLEM DESCRIPTION AND CHANNEL MODEL

Conventionally, a SH is identified by detecting the presence or absence of a primary signal at certain time and location and can be roughly divided into temporal spectrum holes and geographic spectrum holes. In former one, a CR user accesses a spectrum band when PUs are not using it temporarily, which means that both the CR and PUs can be deployed in the same spectrum band and area but at different time slots, and the secondary transmission is realized by utilizing the silent time slots of PUs. While in latter one, a CR user can access a spectrum band when the CR and PUs are in different geographic areas since path-loss and shadowing of wireless channels separate them and make it possible for both to work at the same time without interfering each other.

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A. Spatial Spectrum Hole

Usually, it is hard for CR and PUs to work simultaneously in the same geographic area and spectrum band. However, Figure 1 shows a schematic representation of a coexistence of the primary and secondary systems in this scenario when relay technique is available. Assume CR and PUs work in the same spectrum band. As shown in the figure, if a CR user sends data directly to its receiver, it has a large interference boundary and a large area inside the boundary needs to be cleared, which means the CR user can only communicate through the spectrum band if there are no PUs within the interference boundary. Otherwise, the CR user may generate harmful interference to PUs when it works. However, if the CR user lowers its transmit power and use other CR users as relay stations to forward data to its destination, it can still access the spectrum band without interfering with PUs since the smaller interference boundary can be obtained in this way. Therefore, such secondary communication opportunities at the same time and in the same geographic area with the PUs are called spatial spectrum holes (SSHs) since they are using different spatial domains which primary and CR links are in.

B. Channel Model

Wireless channels usually experience both large-scale and small-scale fadings [12]. The large-scale fading is the dominant component and it contains path-loss and shadowing. For simplicity, we ignore shadowing and model the path-loss of a wireless channel as

\[
G_{PL} = \frac{c}{d^\varsigma},
\]

where \( c \) is a constant, \( \varsigma \) is a path-loss exponent, and \( d \) is the distance between the transmitter and its receiver. Given a transmit power, \( P_{tx} \), the power at the CR receiver can be expressed as

\[
P_{rx} = G_{PL} \cdot P_{tx} = \frac{cP_{tx}}{d^\varsigma}.
\]

1Usually, the SNR requirement for signal detection is lower than that for data decoding, and a guard margin is usually needed for protecting PUs. Thus, the radius of spectrum sensing boundary in CR is larger than that of interference boundary to PU. However, we omit the margin and assume the spectrum sensing and interference boundaries are the same for facilitating analysis in this paper.

From (2), \( P_{rx} \) is determined by \( d \) because \( \varsigma \) is usually a constant in a given application scenario. Therefore, instead of path-loss, we only consider \( d \) in the following analysis.

III. Relay-Assisted Transmission

In this section, we will introduce the principle of our relay-assisted transmission in CR system, which enables CR users to communicate through SSHs. Throughout the paper, we consider two-hop relay rather than multiple hops, which may be feasible in theoretically, for the overhead, complexity, and implementation considerations.

As shown in Figure 1, a CR transmitter (CRtx) intends to send data to the CR receiver (CRrx) through a licensed spectrum band. Thus the CRtx will perform spectrum sensing to detect whether such a CR transmission in the spectrum band will cause interference to PUs or not. If the PU is close to the CRrx, such as PU-1 in the figure, it is hard for CRrx to communicate with the CRtx without causing interference to the PU. On the other hand, if the PU is far away from the CRrx, such as PU-2 in the figure, the CRtx may communicate with the CRrx with the help of relays. In this case, the CRtx lowers its transmit power and shortens the radius of the interference region, and as a result, the CRtx may still transmit without causing interference to PU-2. Furthermore, with the help of relay stations, the data will be conveyed from the CRtx to the CRrx successfully on a non-interfering basis.

Figure 2 demonstrates the principle of relay-enabling region, which is defined as an area where a relay station can be located to ensure successful CR transmission without interfering PUs. In Figure 2, \( A, B, \) and \( P \) represent the positions of the CRtx, the CRrx, and the PU, respectively. To facilitate analysis, the polar coordinator is applied. Without loss of generality, let \( A \) be the origin and the angle of the position \( B \) be zero. Then, the position of the PU can be represented as \((r, \theta)\), where \( r \) is the distance from \( A \) to \( P \) and \( \theta \) is the angle between Line \( AP \) and Line \( AB \). Assume \( G \) in the figure is the position of a certain relay station and \( GA, GB, \) and \( GP \) denote the distances from the relay station to the CRtx, the CRrx, and the PU, respectively. Thus a CR user can act as a relay station for the CRtx if its position satisfies the following requirements:

- The relay station, \( G \), is closer to the CRtx than the PU, \( P \), so that the CRtx can communicate with \( G \) without...
interfering the PU, i.e., $\overline{GA} < \overline{PA}$. In Figure 2, the area inside the circle with center $A$ and radius $r$ satisfies this condition.

- The relay station, $G$, is closer to the CRtx than the PU, $P$, so that the relay station can forward the data without interfering with the PU, i.e., $\overline{GB} < \overline{GP}$. In Figure 2, the right half region of Line $EF$ satisfies this condition, where Line $EF$ is the perpendicular bisector of Segment $BP$.

According to the above two requirements, any CR user in the shaded area in Figure 2 can be used as a relay station. In this paper, this area is called the relay-enabling region. Apparently, the relay-enabling region is determined by the relative positions of the CRtx, the CRrx, and the PU. The larger the relay-enabling region is, the easier it is for the CRtx to find a CR user satisfying the two requirements.

Principally, a relay station equivalently changes the transmit facility analysis, we assume the ideal communication, which means there is no error during the data transmission. Thus the successful communication of the CRtx could be achieved if there is at least one CR user acting as a relay station within the relay-enabling area. In the following, we will derive the relay-enabling area in a given position of a PU since the size of the relay-enabling area varies with the position of the PU. Then, we will find the SCP bound of our scheme, which is the maximum communication opportunity when the density of CR users goes to infinity. After that, we will develop the average SCP under a given density of CR users for practical consideration.

**IV. Performance Analysis**

There are many issues in the proposed scheme, such as scheduling, resource allocation, power control, and so on. However, we will focus on the analysis of successful communication probability (SCP) in the proposed scheme. To facility analysis, we assume the ideal communication, which means there is no error during the data transmission. Thus the successful communication of the CRtx could be achieved if there is at least one CR user acting as a relay station within the relay-enabling area. In the following, we will derive the relay-enabling area in a given position of a PU since the size of the relay-enabling area varies with the position of the PU. Then, we will find the SCP bound of our scheme, which is the maximum communication opportunity when the density of CR users goes to infinity. After that, we will develop the average SCP under a given density of CR users for practical consideration.

**A. Relay-enabling Area for A Given Position of A PU**

As shown in Figure 2, we denote $\triangle ABP$ as a triangle with three point $A$, $B$, and $P$. Line $EF$ is the perpendicular bisector of Segment $BP$. Then we plot Line $AC$ which is parallel to Segment $BP$ and touches Line $EF$ at Point $C$. Also, we plot another Line $DP$, which is perpendicular to Line $AC$. We further assume $\overline{AP} = r$ and $\overline{AB} = d$. So we obtain the length of Line $BP$ by cosine theorem as follows:

$$BP = \sqrt{AP^2 + AB^2 - 2AP \cdot AB \cos \theta},$$

$$= \sqrt{r^2 + d^2 - 2rd \cos \theta}. \quad (3)$$

Denote $\angle ABP$ as the angle between Line $AB$ and Line $BP$ and $\angle ABP = \phi$. Since Line $BP$ and Line $CD$ are parallel, $\angle ABP = \angle BAC$. According to cosine theorem again, we obtain

$$\phi = \arccos \left( \frac{BP^2 + d^2 - r^2}{2BPd} \right). \quad (4)$$

Substitute (3) into (4), we can get

$$\phi = \arccos \left( \frac{d - 2r \cos \theta}{\sqrt{r^2 + d^2 - 2rd \cos \theta}} \right). \quad (5)$$

Then,

$$AD = \begin{cases} r \cdot \cos(\pi - \theta - \phi), & (\theta + \phi) > \pi/2, \\ r \cdot \cos(\theta + \phi), & (\theta + \phi) \leq \pi/2, \end{cases} \quad (6)$$

and

$$AC = \begin{cases} BP/2 - AD, & (\theta + \phi) > \pi/2, \\ BP/2 + AD, & (\theta + \phi) \leq \pi/2. \end{cases} \quad (7)$$

Thus the angle of $\angle CAE$ is $\gamma$, which can be expressed as

$$\gamma = \arccos \left( \frac{AC}{r} \right). \quad (8)$$

Then, the area of $\triangle AEF$ and sector $\angle AEF$ can be expressed as

$$S_{\triangle AEF} = r \cdot \sin \gamma, \quad (9)$$

and

$$S_{\angle AEF} = \frac{\pi r^2 2\gamma}{2\theta}, \quad (10)$$

respectively. Therefore, the shaded area in Figure 2 can be obtained by

$$S_{\text{relay}}(r, \theta) = S_{\triangle AEF} - S_{\angle AEF}, \quad (11)$$

which is the function of $r$ and $\theta$.

**B. SCP Bound**

According to the relative positions of the CRtx, the CRrx, and the PU as analyzed above, the considered area can be divided into two components, the accessible and non-accessible areas. If a PU is within the accessible area, the reliable CR communication assisted by relay transmission can be guaranteed when the density of CR users is high enough. Otherwise, if the PU is within the non-accessible area, it is impossible for the CRtx to communicate with the CRRx through two-hop relay no matter how large the density of CR user is. Therefore, the proportion of the accessible area out of the considered area represents the SCP bound for our relay-enabling method. So, given the distance between the CRtx and the CRrx, denoted as $d$, it is the maximum SCP when the direct communication is not available. In other words, the SCP for relay transmissions is the communication opportunity that is distinguished from most of existing works.

Figure 3 shows an example of SCP bound. In the figure, the shaded area represents the accessible area, where $S_{\text{relay}}(r, \theta) > 0$. From our numerical calculation, the SCP bound is 0.763 when the number of PUs in the considered area is $10^6$. It means that compared to direct transmission there is 76.3% more chance that the CRtx could access the spectrum band without interfering with the PU under the assumption that the density of CR user is infinity.

**C. SCP for A Given Density of CR users**

In reality, the actual average SCP depends on the density of CR users. Suppose there are $N$ CR users uniformly distributed
in the considered area, then the density of CR users is \( \eta_{CR} = \frac{N}{\pi d^2} \). For a certain position of the PU, \( (r, \theta) \), the probability that a CR user falls in the relay-enabling region can be expressed as

\[
P(r, \theta) = \frac{S_{relay}(r, \theta)}{\pi d^2}.
\]  

(12)

If there is at least one CR user in the relay-enabling region, the CRtx will be able to successfully communicate with the CRtx via the relay station. Therefore, the SCP for a given position of PU can be expressed as

\[
P_{SCP}(r, \theta) = 1 - (1 - P(r, \theta))^N.
\]  

(13)

Consequently, for a PU randomly located in the considered area, the corresponding average SCP can be obtained by

\[
P_{SCP} \approx \dfrac{1}{2\pi d^2} \int_0^{2\pi} \int_0^d \! P_{pdf}(r, \theta) \! \left(1 - (1 - P_{SCP}(r, \theta))^N\right) \! dr \! d\theta,
\]  

(14)

where \( P_{pdf}(r, \theta) \) is the probability that a PU appear in location \( (r, \theta) \). In the following section, \( P_{SCP} \) will be evaluated numerically.

V. NUMERICAL RESULTS

In this section, we present numerical results to show the performance of the proposed relay-assisted transmission scheme. Assume that a CRtx intends to communicate with a CRtx, where the distance between them is \( d \). The area inside a circle with radius \( d \) and center CRtx is considered. There is a PU within it and the probabilities that the PU appears in each positions are the same, and \( N \) CR users are uniformly distributed in it at the same time.

Figure 4 shows that average SCP versus the number of CR users and we also plot the SCP bound for comparison. From the figure, the average SCP dramatically increases as the number of CR users goes up, which represents the density of CR users. The average SCP is over 0.5 when \( N > 20 \), which means there is over 50% chance that the CR user is able to successfully send data to its receiver if there are more than 20 CR users in the considered area.

VI. CONCLUSION

In this paper, we proposed a relay-assisted CR transmission scheme to exploit spatial spectrum holes, which may enable a CR transmitter to access the spectrum band assigned to a primary system when the direct transmission is not available. Therefore, the higher spectrum efficiency could be achieved through our method by coexistence of CR and primary users at the same time in the same geographic area and spectrum band. The successful communication probability bound has been found by simulation, and numerical results show the advantages of our proposed method.

REFERENCES


