# Asymmetric Two-Way Relay with Doubly Nested Lattice Codes

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***Abstract*—Two-way relaying can significantly improve the spectrum efficiency of bi-directional information transmission between two nodes with the help of half-duplex relays. However, in the general case of asymmetric channel gains, the achievable rates of both directions are restricted by the link with the worse channel quality. In this paper, we propose a new three-phase cooperative transmission protocol, which exploits the direct link and the time resource of a two-way relay system more efficiently. We derive the rate region outer bound of this protocol, and propose a practical transmission scheme based on doubly nested lattice codes, which can achieve this outer bound within 0.5 bit. Numerical results show the superiority of this protocol over two other classical protocols in asymmetric channels.**

***Index Terms*—Asymmetric channel, lattice code, rate region, two-way relay.**

## I. Introduction

**W**IRELESS relaying can improve the coverage or data rate of wireless systems, and is particularly useful if the direct source-to-destination link is weak [1]. However, the half-duplex nature of most practical relays almost halves the spectrum efficiency of one-way relaying (transmission from one source to one destination via the relay). It is for this reason that there is great interest in two-way relaying, where the relay simultaneously receives signals from two users who expect to exchange information with each other, and transmits a combination of these signals [2]–[5]. The introduction of network coding on the physical layer breaks the capacity restrictions of the traditional combination of multiple-access and broadcast transmissions.

The classical protocol for two-way relays is the Multiple-Access and Broadcast (MABC) protocol, the rate region outer bound of which is given in [6]–[8]. Two time slots are allocated in this protocol. The first time slot is for multiple-access, when two users transmit signals simultaneously to the relay. The second time slot is for broadcast, when the relay broadcasts a signal to both users. Several practical coding schemes implementing this protocol have been proposed [9]–[13]. In [9], a joint decoding involving both channel decoding and bit-wise network coding operations was developed. The repeat-accumulate channel code is adopted at the two end nodes and a belief propagation algorithm is redesigned at the relay. Ref. [10] and [11] considered a symbol-wise network coding strategy where the two received modulated symbols, which can having different modulations, are mapped to a transmit symbol directly at the relay. Ref. [12] and [13] studied codeword-wise network coding where nested lattice codes are applied in symmetric and asymmetric channels, respectively. With different transmit powers of the two end nodes in [13], two codes with different rates were constructed from a lattice partition chain, and the achievable rate region is within 0.5 bit from the rate region for each user. However, as the outer bound of the rate region shows, the MABC protocol suffers from the asymmetric channel gains, the data rates of both direction are restricted by the weaker of the two node-to-relay links.

In practice, asymmetric channels are frequently encountered due to relay deployment, terminal roaming, or channel fading, such that one node has a stronger link to the relay than the other. The question thus naturally arises whether there is any better protocol to explore the potential of two-way relaying in asymmetric channels? What is the rate region outer bound of this system, and how to achieve it?

A three-phase protocol named Time Division Broadcast (TDBC) was proposed in [8], which outperforms the MABC protocol in some asymmetric channel conditions. The TDBC protocol considers sequential transmissions from the two users followed by a broadcasting from the relay. The direct link between the two users is also exploited in the first two phases. In the TDBC protocol, the relay first decodes the data of each user separately, then network coding is implemented on the decoded bits. Unfortunately, TDBC loses efficiency in symmetric channel and performs worse than MABC protocol when the channel gains tend to be identical.

We propose an alternative three-phase Cooperative MABC (CoMABC) protocol in this paper, which outperforms the TDBC and MABC protocols in all circumstances. After the two phases of a MABC protocol, a third phase is introduced to help the weaker link with a transmission that uses cooperation between the relay and the stronger node[1]. In this protocol, the broadcast phase is finished just after the stronger node decodes

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[1]In a slight abuse of notation, we call the "stronger node" the node that has the link with the higher channel gain to the relay, and similarly for the "weaker node".

its codewords, while the weaker node will continue receiving until the end of the third phase. With the help of cooperative transmission, the network throughput is no longer restricted by the weaker node-to-relay link. We derive the rate region outer bound of this protocol and optimize the time slot allocations to maximize the sum rate.

A transmission scheme for the CoMABC protocol is proposed which can approach the rate region outer bound within 0.5 bit for each user. Comparing with the transmission scheme in [13], which is to achieve the rate region outer bound of MABC protocol, more elaborated encoding and decoding schemes in all the three transmission phases should be considered. We use a sequence of doubly nested lattice codes in the multiple-access phase for network coding, and we use another sequence of doubly nested lattice codes in the broadcast and cooperation phases for joint encoding. In the multiple-access phase, the two sources transmit with different rates according to their channel gains to the relay. The relay decodes two network coded codewords which respectively belong to a fine lattice and a coarse lattice, and transmits them in the second and third phases. In the second (broadcast) phase, the relay re-encodes the fine codeword to another doubly nested codeword. While the stronger node can directly decode its information, the weaker node can also decode its information with the help of information it received in the third phase.

The rest of this paper is organized as follows. The system model and the transmission procedure of the CoMABC protocol are introduced in Section II. Section III derives the rate region outer bound of the CoMABC protocol and discusses the optimization of the time slots. The transmission scheme with doubly nested lattice codes is presented in section IV and numerical comparisons with MABC and TDBC protocols are provided in section V. Finally, section VI concludes the paper.

## II. System Model and CoMABC Protocol

### A. System Model

Consider a two-way relay system with a time-division half-duplex relay node $r$ and two end nodes $a$ and $b$ who intend to exchange information with the assistance of the relay. The whole transmission process is divided into $M$ time slots or phases, the $m$-th time slot is denoted as $t_m$, and $s_i^{(m)}$ denotes the message that node $i$ wants to transmit in the $m$-th time slot. The message $s_i^{(m)}$ is mapped to a codeword $\boldsymbol{x}_i^{(m)} = \mathcal{M}_{i,j}^{(m)}(s_i^{(m)})$ and sent to node $j$ with rate $\tilde{R}_{i,j}^{(m)}$, where $\mathcal{M}_{i,j}^{(m)}$ denotes the mapping function applied to the message from node $i$ to node $j$. The codewords satisfy power constraints of each node, *i.e.*, $E(\|\boldsymbol{x}_i^{(m)}\|^2)/n = P_i$, where $n$ is the codeword length. The noise vector received at node $i$ in the $m$-th time slot is denoted as $\boldsymbol{n}_i^{(m)}$, which is Gaussian with zero mean and covariance matrix $\sigma_i^2\boldsymbol{I}$.

The channel coefficient between nodes $a$ and $r$ is denoted as $h_{a,r} = h_{r,a} = h_1$, correspondingly we have $h_{b,r} = h_{r,b} = h_2$ and $h_{a,b} = h_{b,a} = h_3$, where channel reciprocity is assumed for all links. We consider channels with real coefficients and additive white Gaussian noise (AWGN) in this paper, and the channel coefficients remain invariant for the duration of transmission. The channel state information of the three links is available at all nodes.

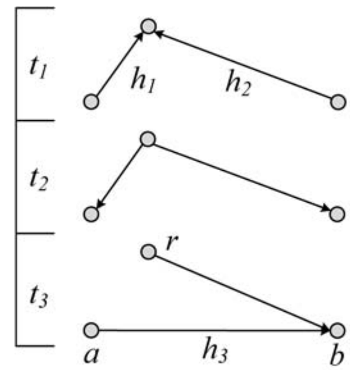


Fig. 1. The three-phase CoMABC transmission protocol in asymmetric channels.

### B. CoMABC Protocol

Without loss of generality, assume that link $a \leftrightarrow r$ is stronger than link $b \leftrightarrow r$ unless otherwise specified. Since the two links have different point-to-point channel capacities, the relay may receive a signal from node $a$ at a higher rate but can only forward it to node $b$ at a lower rate. The opposite happens to the data flow from node $b$ to node $a$. When using the MABC protocol, in the MA phase the relay may receive much more bits from node $a$ than from node $b$, but in the BC phase node $b$ may receive at a much lower rate than that of node $a$. Thus node $a$ will quickly complete its decoding and wait in an idle state for node $b$ to decode its data. The waiting period may be long, depending on the difference between the channel gains of the two links.

We introduce a third phase in the MABC protocol to improve the time efficiency and therefore to increase the throughput of the two-way relay system. This new protocol is the so called CoMABC, where after the MA and BC phases the idle node $a$ joins the $r \to b$ transmission in the third time slot. Any cooperative transmission technique can be applied, either diversity where node $a$ re-transmits part of the data it had transmitted to node $r$, or multiplexing where node $a$ transmits new information. Similarly, if link $b \leftrightarrow r$ is stronger than link $a \leftrightarrow r$, it would be $b$ helping $r$ in the third phase. The three-phase CoMABC protocol is illustrated in Fig. 1.

In time slot $t_1$, node $a$ and node $b$ simultaneously transmit codewords $\boldsymbol{x}_a^{(1)} = \mathcal{M}_a^{(1)}(s_a^{(1)})$ and $\boldsymbol{x}_b^{(1)} = \mathcal{M}_b^{(1)}(s_b^{(1)})$. The received vector signal at the relay is

$$\boldsymbol{y}_r^{(1)} = h_1\boldsymbol{x}_a^{(1)} + h_2\boldsymbol{x}_b^{(1)} + \boldsymbol{n}_r^{(1)}. \tag{1}$$

The relay can either decode messages $s_a^{(1)}$ and $s_b^{(1)}$ separately using a successive interference cancelation (SIC) algorithm, or decode a network coded codeword directly for the transmission in phase 2 and 3. We will not specify the behavior of the relay node at this moment.

In time slot $t_2$, the relay broadcasts a codeword $\boldsymbol{x}_r^{(2)}$ to nodes $a$ and $b$; then the received vectors at both nodes are

$$\boldsymbol{y}_a^{(2)} = h_1\boldsymbol{x}_r^{(2)} + \boldsymbol{n}_a^{(2)}, \tag{2}$$

$$\boldsymbol{y}_b^{(2)} = h_2\boldsymbol{x}_r^{(2)} + \boldsymbol{n}_b^{(2)}. \tag{3}$$

With the assumption of a stronger $a \leftrightarrow r$ link, node $a$ is able to decode the message $s_b^{(1)}$ with the knowledge of its

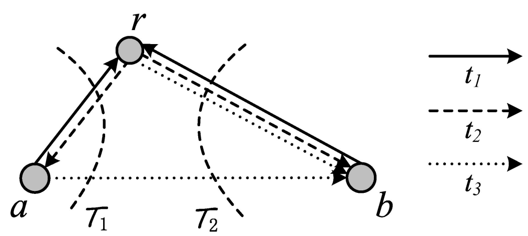


Fig. 2. The node set separations in two-way relay network, where the two cuts are illustrated.

own transmitted message $s_a^{(1)}$. Therefore, at the end of phase 2, it can revert to the transmitting state. However, at this time node $b$ is not able to decode the message $s_a^{(1)}$ yet, and requires additional signals transmitted from the relay and node $a$ in the third time slot.

In time slot $t_3$, the relay node will transmit a codeword $\boldsymbol{x}_r^{(3)}$ to node $b$ to continue forwarding its received data from node $a$, and node $a$ will also transmit a codeword $\boldsymbol{x}_a^{(3)}$ to node $b$ either for diversity or for spatial multiplexing. Thus node $b$ will receive a superimposed vector as

$$\boldsymbol{y}_b^{(3)} = h_3\boldsymbol{x}_a^{(3)} + h_2\boldsymbol{x}_r^{(3)} + \boldsymbol{n}_b^{(3)}. \quad (4)$$

After receiving $\boldsymbol{y}_b^{(3)}$, node $b$ should be able to decode all the information transmitted from node $a$.

## III. The Rate Region Outer Bound

### A. The Outer Bound of Rate Region

*Theorem 1*: The rate region of the half-duplex two-way relay channel with CoMABC protocol is outer bounded by

$$R_a \le \min\{t_1 I(X_a^{(1)};Y_r^{(1)}|X_b^{(1)}) + t_3 I(X_a^{(3)};Y_b^{(3)}|X_r^{(3)}), \\ t_2 I(X_r^{(2)};Y_b^{(2)}) + t_3 I(X_a^{(3)},X_r^{(3)};Y_b^{(3)})\}, \quad (5)$$

$$R_b \le \min\{t_1 I(X_b^{(1)};Y_r^{(1)}|X_a^{(1)}), t_2 I(X_r^{(2)};Y_a^{(2)})\}, \quad (6)$$

where $\sum_{m=1}^{3} t_m = 1$ with $0 \le t_m \le 1$, and their values are to be optimized.

*Proof:* The proof is based on the cut-set bound theorem for networks with multiple states [1], [14], which is a multiple-state expansion of the well-known cut-set bound theorem [15]. The network we considered has three states which correspond to the three transmission phases shown in Fig. 1. The two cuts of interest, $\mathcal{T}_1$ and $\mathcal{T}_2$, are shown in Fig. 2.

For the data flow from node $a$ to node $b$, the three nodes are divided into two sets $S = \{a\}$ and $S^C = \{r, b\}$ by cut $\mathcal{T}_1$, and $S = \{a, r\}$ and $S^C = \{b\}$ by cut $\mathcal{T}_2$. Thus the rate of information transfer from $a$ to $b$ is bounded by

$$R_a \le \sup_{t_m} \min_{S} \sum_{m=1}^{3} t_m I\left(X_S^{(m)}; Y_{S^C}^{(m)}|X_{S^C}^{(m)}\right), \quad (7)$$

where the supremum is over all non-negative $t_m$ subject to $\sum_{m=1}^{M} t_m = 1$, the minimum is over different cut sets. The conditional mutual information $I(X_S^{(m)}; Y_{S^C}^{(m)}|X_{S^C}^{(m)})$ is the maximum achievable rate between transmit nodes in set $S$ and receive nodes in set $S^C$, given the transmit signal from set $S^C$.

With the CoMABC protocol, in phase 1, node $a$ and node $b$ transmit to node $r$; in phase 2, node $r$ transmits to node $a$ and node $b$; in phase 3, node $r$ and node $a$ transmit to node $b$. Therefore, by cut $\mathcal{T}_1$, only in phase 1 and phase 3 there is information transfer from $S$ to $S^C$, and

$$\sum_{m=1}^{3} t_m I\left(X_S^{(m)}; Y_{S^C}^{(m)}|X_{S^C}^{(m)}\right) \\ = t_1 I(X_a^{(1)};Y_r^{(1)}|X_b^{(1)}) + t_3 I(X_a^{(3)};Y_b^{(3)}|X_r^{(3)}). \quad (8)$$

By cut $\mathcal{T}_2$, only in phase 2 and phase 3 there is information transfer from $S$ to $S^C$, and

$$\sum_{m=1}^{3} t_m I\left(X_S^{(m)}; Y_{S^C}^{(m)}|X_{S^C}^{(m)}\right) \\ = t_2 I(X_r^{(2)};Y_b^{(2)}) + t_3 I(X_a^{(3)},X_r^{(3)};Y_b^{(3)}). \quad (9)$$

Taking the minimization over these two cuts, we obtain (5).

Similarly, we can obtain the upper bound of the information transfer rate from node $b$ to node $a$ as in (6). ∎

For Gaussian channels with coefficients $h_{i,j}$, the rate region outer bound is given by

$$R_a \le \min\{t_1 C_{a,r} + t_3 C_{a,b},\ t_2 C_{r,b} + t_3 C_{\text{MA},b}\}, \quad (10)$$

$$R_b \le \min\{t_1 C_{b,r},\ t_2 C_{r,a}\}, \quad (11)$$

where $C_{i,j} = \frac{1}{2}\log(1 + \frac{P_i h_{i,j}^2}{\sigma_j^2})$ is the point-to-point channel capacity from node $i$ to node $j$ with a transmit power $P_i$, $i, j \in a, b, r$, and $C_{\text{MA},b} = \frac{1}{2}\log(1 + \frac{P_r h_2^2 + P_a h_3^2}{\sigma_b^2})$ is the sum capacity of the multiple-access channel from nodes $r$ and $a$ to node $b$.

### B. The Optimization of Time Slots

In (10) and (11), the time slot durations $t_1$, $t_2$ and $t_3$ are not specified. The rate region should be the closure of all possible pairs of rates $R_a$ and $R_b$. Since this rate region is in two dimension, it does not have a unique maximum, and therefore there is no optimal configuration of the time slots. As an alternative, a commonly used performance metric, the throughput of the network, can be maximized. Given the objective function $R_a + R_b$, an optimization problem is formulated as

$$\begin{aligned} \max_{t_1,t_2,t_3}\ & R_a + R_b \\ s.t.\ & R_a - t_1 C_{a,r} - t_3 C_{a,b} \le 0 \\ & R_a - t_2 C_{r,b} - t_3 C_{\text{MA},b} \le 0 \\ & R_b - t_1 C_{b,r} \le 0 \\ & R_b - t_2 C_{r,a} \le 0 \\ & t_1 + t_2 + t_3 = 1 \\ & 0 < t_1, t_2, t_3 < 1 \end{aligned} \quad (12)$$

This is a linear programming problem with $t_1$, $t_2$ and $t_3$ as the optimization parameters, which can be easily transformed to the standard form and solved.

The linear constraints in (12) define a feasible region of a convex polyhedron. The linear objective function is a convex function and is bounded above on the feasible region. According to the Corollary 32.3.4 in [16], the maximum is attained at one of the vertices of the polyhedron. If we want $R_a > 0$ and $R_b > 0$, the only vertex is that all the inequality constraints are realized at the upper boundary, which means that the first four constraints in (12) are satisfied with equality. In this case, the data flows through cut $\mathcal{T}_1$ and cut $\mathcal{T}_2$ are exactly equal.

## IV. Transmission Scheme with Doubly Nested Lattice Codes

In this section, we will present a transmission scheme which can achieve the rate region outer bound within 0.5 bit. The proof of the achievable rate region is also given along with the three transmission phases. Doubly nested lattice codes are used twice in the multiple-access phase and the broadcast phase, respectively. In the multiple-access phase, such codes are used for network coding which is computed on the nested lattices. In the broadcast phase, they are used for joint encoding with side information transmitted in the third phase.

### A. Preliminaries on Lattice Codes

Some definitions on lattice codes are necessary before we introduce the transmission scheme; these definitions are following [17] and [18].

*Definition 1 (Lattice):* A lattice $\Lambda$ is a discrete subgroup of the Euclidean space $\mathbb{R}^n$ under vector addition. Thus, if $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are in $\Lambda$, their sum and difference are also in $\Lambda$. The zero vector is always an element in a lattice. A lattice $\Lambda$ may be specified by its generator matrix $\boldsymbol{G} \in \mathbb{R}^{n\times n}$ as

$$\Lambda = \{\boldsymbol{\lambda} = \boldsymbol{G}\boldsymbol{w} : \ \boldsymbol{w} \in \mathbb{Z}^n\}. \tag{13}$$

That is, the lattice is generated by taking all integer linear combinations of the basis vectors.

*Definition 2 (Quantizer):* A lattice quantizer $Q_\Lambda$ is a mapping from a point $\boldsymbol{x} \in \mathbb{R}^n$ to a nearest point $\boldsymbol{\lambda} \in \Lambda$ with Euclidean distance,

$$Q_\Lambda(\boldsymbol{x}) = \arg\min_{\boldsymbol{\lambda}\in\Lambda} \|\boldsymbol{x} - \boldsymbol{\lambda}\|. \tag{14}$$

*Definition 3 (Modulus):* Let $[\boldsymbol{x}] \bmod \Lambda$ denote the quantization error of $\boldsymbol{x} \in \mathbb{R}^n$ with respect to the lattice $\Lambda$, *i.e.*,

$$[\boldsymbol{x}] \bmod \Lambda = \boldsymbol{x} - Q_\Lambda(\boldsymbol{x}). \tag{15}$$

*Definition 4 (Voronoi Region):* The fundamental Voronoi region, $\mathcal{V}$, of a lattice, is the set of all points in $\mathbb{R}^n$ that are closest to the zero vector, *i.e.*, $\mathcal{V} = \{\boldsymbol{x} : Q_\Lambda(\boldsymbol{x}) = \boldsymbol{0}\}$. The volume of $\mathcal{V}$ is denoted as $\mathrm{Vol}(\mathcal{V})$.

*Definition 5 (Moments):* The second moment of a lattice $\Lambda$ is defined as the second moment per dimension of a uniform distribution over the fundamental Voronoi region

$$\sigma^2(\Lambda) = \frac{1}{n\mathrm{Vol}(\mathcal{V})}\int_{\mathcal{V}} \|\boldsymbol{x}\|^2 d\boldsymbol{x}. \tag{16}$$

*Definition 6 (Nested Lattices):* A lattice $\Lambda_2$ is said to be nested in a lattice $\Lambda_1$ if $\Lambda_2 \subseteq \Lambda_1$. The lattice $\Lambda_1$ is often referred to as a fine lattice and $\Lambda_2$ as a coarse lattice. More generally, a sequence of lattices $\Lambda_L, \ldots, \Lambda_1$ are nested if $\Lambda_L \subseteq \cdots \subseteq \Lambda_1$.

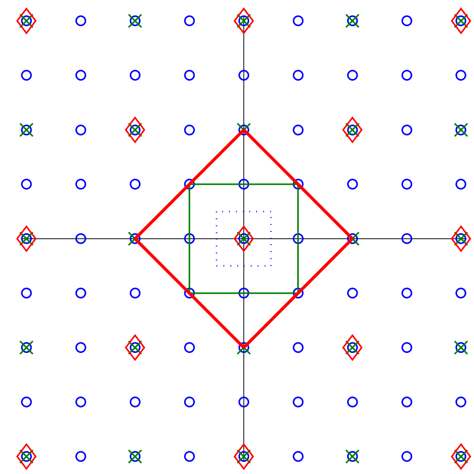

Fig. 3. The 'o' denotes fine lattice $\Lambda$ with the dashed line as its Voronoi region; the 'x' denotes the first level coarse lattice $\Lambda_b$ with the thin solid line as its Voronoi region; the '$\diamond$' denotes the second level coarse lattice $\Lambda_a$ with the bold solid line as its Voronoi region.

*Definition 7 (Nested Lattice Codes):* A nested lattice code $\mathcal{L}$ is the set of all points of a fine lattice $\Lambda_1$ that are within the fundamental Voronoi region $\mathcal{V}_2$ of a coarse lattice $\Lambda_2$, *i.e.*, $\mathcal{L} = \Lambda_1 \cap \mathcal{V}_2$. The rate of a nested lattice code is

$$R = \frac{1}{n}\log|\mathcal{L}| = \frac{1}{n}\log\frac{\mathrm{Vol}(\mathcal{V}_2)}{\mathrm{Vol}(\mathcal{V}_1)}. \tag{17}$$

In [17], Erez and Zamir showed that nested lattice codes can approach the capacity of point-to-point AWGN channels if $\Lambda_2$ is simultaneously good for covering, quantization and AWGN channel coding and $\Lambda_1$ is good for AWGN channel coding. Actually, there exists a sequence of lattices that is simultaneously covering, quantization, and coding good [18], [19].

For any $P_a \geq P_b \geq 0$, [13] shows that there exists doubly nested $n$-dimensional lattices $\Lambda_3 \subseteq \Lambda_2 \subseteq \Lambda_1$, as $n \to \infty$, $\sigma^2(\Lambda_3) = P_a$, $\sigma^2(\Lambda_2) = P_b$, and the coding rate of $\mathcal{L}' = \Lambda_1 \cap \mathcal{V}_3$ satisfies

$$R' = \frac{1}{n}\log\frac{\mathrm{Vol}(\mathcal{V}_3)}{\mathrm{Vol}(\mathcal{V}_1)} = R + \frac{1}{2}\log\left(\frac{P_a}{P_b}\right). \tag{18}$$

Fig. 3 shows a simple example of three doubly nested lattices $\Lambda_a \subset \Lambda_b \subset \Lambda$ with their corresponding Voronoi regions. Fig. 4 shows an example to construct the nested lattice codes, where $\mathcal{L}_a = \Lambda \cap \mathcal{V}_a$, $\mathcal{L}_b = \Lambda \cap \mathcal{V}_b$, $\mathcal{L}_c = \Lambda_b \cap \mathcal{V}_a$. Since the fine lattices on the boundary of the Voronoi regions $\mathcal{V}_a$ and $\mathcal{V}_b$ are shared with adjacent coarse lattices, the codeword sets should be selected based on a non-redundant pattern, *i.e.*, $\mathcal{L}_a + \Lambda_a = \Lambda$, $\mathcal{L}_b + \Lambda_b = \Lambda$, $\mathcal{L}_c + \Lambda_a = \Lambda_b$.

The examples in Figs. 3 & 4 are not good in the sense of covering, quantization and coding, they are only chosen to help understanding the mathematical expressions. The general construction methods of lattice codes are out of the scope of this paper, we refer the interested readers to [20]–[22].

### B. Multiple-Access Phase

*1) Encoding:* Given the time slot allocations, $t_1$, $t_2$ and $t_3$, we set the codeword length in each phase as $n_1 = t_1/T_s$, $n_2 = t_2/T_s$ and $n_3 = t_3/T_s$, where $T_s$ is the sampling interval.

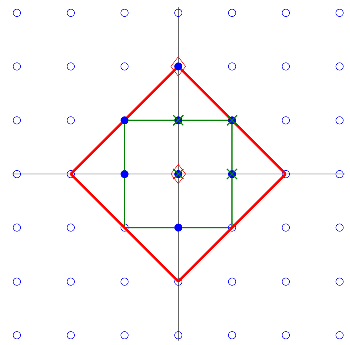

Fig. 4. The '•' denotes codeword set $\mathcal{L}_a$, the 'x' denotes codeword set $\mathcal{L}_b$, and the '⋄' denotes codeword set $\mathcal{L}_c$.

Define a sequence of three doubly nested lattices $\Lambda_a \subset \Lambda_b \subset \Lambda$ with dimension $n_1$, and the codeword sets $\mathcal{L}_a$, $\mathcal{L}_b$, and $\mathcal{L}_c$ are constructed as in the above example. The codebook $\mathcal{L}_a$ and $\mathcal{L}_b$ are used for encoding by node $a$ and node $b$, respectively. The codebook $\mathcal{L}_c$ is used for decoding by the relay.

Given two source messages $s_a^{(1)}$ and $s_b^{(1)}$, map $s_a^{(1)}$ into a codeword $\boldsymbol{\lambda}_a \in \mathcal{L}_a$ and map $s_b^{(1)}$ into a codeword $\boldsymbol{\lambda}_b \in \mathcal{L}_b$. Adding random dithers on the codewords, node $a$ and $b$ transmit

$$\boldsymbol{x}_a^{(1)} = \frac{1}{h_1}\left[(\boldsymbol{\lambda}_a + \boldsymbol{u}_a) \bmod \Lambda_a\right], \tag{19}$$

$$\boldsymbol{x}_b^{(1)} = \frac{1}{h_2}\left[(\boldsymbol{\lambda}_b + \boldsymbol{u}_b) \bmod \Lambda_b\right], \tag{20}$$

where $\boldsymbol{u}_a$ is a random dither vector uniformly distributed in $\mathcal{V}_a$, *i.e.*, $\boldsymbol{u}_a \sim \text{Unif}(\mathcal{V}_a)$; similarly, $\boldsymbol{u}_b \sim \text{Unif}(\mathcal{V}_b)$. The dither vectors are known to both source nodes and the relay.

Since the channels of $a \to r$ and $b \to r$ have different path losses, and the fine lattices should be aligned at the relay, we pre-amplify the transmit signals. According to the transmit powers, the second moments of $\Lambda_a$ and $\Lambda_b$ satisfy

$$\sigma^2(\Lambda_a) = P_a h_1^2, \quad \sigma^2(\Lambda_b) = P_b h_2^2. \tag{21}$$

*2) Decoding:* At the relay, a noisy signal $\boldsymbol{y}_r^{(1)}$ is received as in (1). The relay implements a minimum mean-square error (MMSE) estimate on $\boldsymbol{y}_r^{(1)}$ and then cancels the dither, *i.e.*,

$$\hat{\boldsymbol{y}}_r^{(1)} = \alpha_r \boldsymbol{y}_r^{(1)} - \boldsymbol{u}_a - \boldsymbol{u}_b, \tag{22}$$

where the MMSE coefficient is

$$\alpha_r = \frac{P_a h_1^2 + P_b h_2^2}{P_a h_1^2 + P_b h_2^2 + \sigma_r^2}. \tag{23}$$

By (19), (20), (1) and (22),

$$\begin{aligned}\hat{\boldsymbol{y}}_r^{(1)} &= h_1\boldsymbol{x}_a^{(1)} - \boldsymbol{u}_a + h_2\boldsymbol{x}_b^{(1)} - \boldsymbol{u}_b \\ &\quad -(1-\alpha_r)h_1\boldsymbol{x}_a^{(1)} - (1-\alpha_r)h_2\boldsymbol{x}_b^{(1)} + \alpha_r\boldsymbol{n}_r^{(1)},\end{aligned} \tag{24}$$

where the last three terms denote the effective noise in the decoding at relay. Define the sum of these three terms as $\boldsymbol{z}_r$, which has a variance as

$$\frac{1}{n_1}E\left\{\|\boldsymbol{z}_r\|^2\right\} = \frac{(P_a h_1^2 + P_b h_2^2)\sigma_r^2}{P_a h_1^2 + P_b h_2^2 + \sigma_r^2}. \tag{25}$$

Since

$$\begin{aligned}h_1\boldsymbol{x}_a^{(1)} - \boldsymbol{u}_a &= \left[\boldsymbol{\lambda}_a + \boldsymbol{u}_a - Q_{\Lambda_a}(\boldsymbol{\lambda}_a + \boldsymbol{u}_a)\right] - \boldsymbol{u}_a \\ &= \boldsymbol{\lambda}_a - Q_{\Lambda_a}(\boldsymbol{\lambda}_a + \boldsymbol{u}_a),\end{aligned} \tag{26}$$

and with similar derivation of $h_2\boldsymbol{x}_b^{(1)} - \boldsymbol{u}_b$, (24) becomes

$$\begin{aligned}\hat{\boldsymbol{y}}_r^{(1)} &= \left[\boldsymbol{\lambda}_a - Q_{\Lambda_a}(\boldsymbol{\lambda}_a + \boldsymbol{u}_a)\right] \\ &\quad + \left[\boldsymbol{\lambda}_b - Q_{\Lambda_b}(\boldsymbol{\lambda}_b + \boldsymbol{u}_b)\right] + \boldsymbol{z}_r.\end{aligned} \tag{27}$$

Decoding $\hat{\boldsymbol{y}}_r^{(1)}$ on the fine lattice $\Lambda$ within the Voronoi region $\mathcal{V}_b$ give us

$$\begin{aligned}\hat{\boldsymbol{\lambda}}_{r,1} &= Q_\Lambda\left[\hat{\boldsymbol{y}}_r^{(1)} \bmod \Lambda_b\right] \\ &= Q_\Lambda\left[(\boldsymbol{\lambda}_a + \boldsymbol{\lambda}_b + \boldsymbol{z}_r) \bmod \Lambda_b\right],\end{aligned} \tag{28}$$

where $\hat{\boldsymbol{\lambda}}_{r,1} \in \mathcal{L}_b$. The codeword $\hat{\boldsymbol{\lambda}}_{r,1}$ is a network coding result over two codewords with different data rates. With correct decoding, the information of $\boldsymbol{\lambda}_b$ is completely preserved in $\hat{\boldsymbol{\lambda}}_{r,1}$, but the information of $\boldsymbol{\lambda}_a$ is partially lost.

To preserve the information of $\boldsymbol{\lambda}_a$, we need to compute another codeword $\hat{\boldsymbol{\lambda}}_{r,2} \in \mathcal{L}_c$, that is

$$\begin{aligned}\hat{\boldsymbol{\lambda}}_{r,2} &= Q_{\Lambda_b}\left[\hat{\boldsymbol{y}}_r^{(1)} \bmod \Lambda_a\right] \\ &= Q_{\Lambda_b}\left[(\boldsymbol{\lambda}_a + \boldsymbol{\lambda}_b - \boldsymbol{\lambda}_u + \boldsymbol{z}_r) \bmod \Lambda_a\right],\end{aligned} \tag{29}$$

where $\boldsymbol{\lambda}_u = Q_{\Lambda_b}(\boldsymbol{\lambda}_b + \boldsymbol{u}_b)$. The codeword $\hat{\boldsymbol{\lambda}}_{r,2}$ is decoded on the coarse lattice $\Lambda_b$ within the Voronoi region $\mathcal{V}_a$.

To forward the message of node $b$ to node $a$, it is enough for the relay to just broadcast $\hat{\boldsymbol{\lambda}}_{r,1}$ in the second phase. The partial information of the message from node $a$, *i.e.*, $\hat{\boldsymbol{\lambda}}_{r,2}$, is only required by node $b$ in the third phase.

We know that with correct decisions,

$$\boldsymbol{\lambda}_{r,1} = (\boldsymbol{\lambda}_a + \boldsymbol{\lambda}_b) \bmod \Lambda_b, \tag{30}$$

$$\boldsymbol{\lambda}_{r,2} = Q_{\Lambda_b}\left[(\boldsymbol{\lambda}_a + \boldsymbol{\lambda}_b - \boldsymbol{\lambda}_u) \bmod \Lambda_a\right]. \tag{31}$$

Therefore, the probabilities of decoding errors

$$\Pr(\hat{\boldsymbol{\lambda}}_{r,1} \neq \boldsymbol{\lambda}_{r,1}) = \Pr(\boldsymbol{z}_r \bmod \Lambda_b \notin \mathcal{V}) \leq \Pr(\boldsymbol{z}_r \notin \mathcal{V}), \tag{32}$$

$$\Pr(\hat{\boldsymbol{\lambda}}_{r,2} \neq \boldsymbol{\lambda}_{r,2}) = \Pr(\boldsymbol{z}_r \bmod \Lambda_a \notin \mathcal{V}) \leq \Pr(\boldsymbol{z}_r \notin \mathcal{V}). \tag{33}$$

According to the Theorem 3 in [13], the error probabilities vanish as $n_1 \to \infty$ if

$$\sigma^2(\Lambda) > \frac{1}{n_1}E\left\{\|\boldsymbol{z}_r\|^2\right\}. \tag{34}$$

By (17), (18), and (21), we obtain the achievable rate region of link $a \to r$ and $b \to r$ as

$$R_{a,r}^* = \left[\frac{1}{2}\log\left(\frac{P_a h_1^2}{P_a h_1^2 + P_b h_2^2} + \frac{P_a h_1^2}{\sigma_r^2}\right)\right]^+, \tag{35}$$

$$R_{b,r}^* = \left[\frac{1}{2}\log\left(\frac{P_b h_2^2}{P_a h_1^2 + P_b h_2^2} + \frac{P_b h_2^2}{\sigma_r^2}\right)\right]^+, \tag{36}$$

where $[x]^+ \triangleq \max\{x, 0\}$.

Given the transmit power $P_a$ and $P_b$, if the transmission rates $\tilde{R}_{a,r}^{(1)} < R_{a,r}^*$ and $\tilde{R}_{b,r}^{(1)} < R_{b,r}^*$, the second moment of $\Lambda$ will be larger than the effective noise variance of $\boldsymbol{z}_r$, and we will recover $\boldsymbol{\lambda}_{r,1}$ and $\boldsymbol{\lambda}_{r,2}$ with probability 1.

Comparing (35), (36) with the AWGN channel capacities $C_{a,r}$ and $C_{b,r}$, the difference of each rate is at most 0.5 bit. In fact, for $R^*_{a,r}$, the largest gap happens when

$$\frac{P_a h_1^2}{P_a h_1^2 + P_b h_2^2} = 0, \quad \frac{P_a h_1^2}{\sigma_r^2} = 1.$$

On the contrary, when the power ratio $(P_a h_1^2)/(P_b h_2^2)$ is large, or the received SNR $P_a h_1^2/\sigma_r^2$ is large, the gap vanishes. A similar argument applies to $\tilde{R}^{(1)}_{b,r}$.

### C. Broadcast Phase

*1) Encoding:* Since the relay has a transmit power $P_r$, the downlink may have a different capacity than the uplink. Thus the codeword $\boldsymbol{\lambda}_{r,1}$ is demapped to a message $s_r^{(2)}$ and re-encoded with a new codebook.

The $r \to b$ link is weaker than the $r \to a$ link, to decode $s_r^{(2)}$, node $b$ requires help from the information transmitted in the third phase. Thus joint encoding between the last two phases should be considered.

Define another sequence of doubly nested lattices $\Lambda'_a \subset \Lambda'_b \subset \Lambda'$ with dimension $n_2$. The corresponding Voronoi regions of these three lattices are $\mathcal{V}'_a$, $\mathcal{V}'_b$ and $\mathcal{V}'$, respectively. The second moment of $\Lambda'_a$ is $\sigma^2(\Lambda'_a) = P_r$. Define the codeword sets $\mathcal{L}'_a = \Lambda' \cap \mathcal{V}'_a$, $\mathcal{L}'_b = \Lambda' \cap \mathcal{V}'_b$, and $\mathcal{L}'_c = \Lambda'_b \cap \mathcal{V}'_a$.

Mapping the message $s_r^{(2)}$ to $\boldsymbol{\lambda}'_r \in \mathcal{L}'_a$, the relay sends

$$\boldsymbol{x}_r^{(2)} = \left(\boldsymbol{\lambda}'_r + \boldsymbol{u}_r\right) \bmod \Lambda'_a, \tag{37}$$

where $\boldsymbol{u}_r \sim \text{Unif}(\mathcal{V}'_a)$ and is known to the relay and both end nodes. According to the definitions, the codeword $\boldsymbol{\lambda}'_r$ can be decomposed as $\boldsymbol{\lambda}'_{r,1} + \boldsymbol{\lambda}'_{r,2}$, where $\boldsymbol{\lambda}'_{r,1} \in \mathcal{L}'_b$ and $\boldsymbol{\lambda}'_{r,2} \in \mathcal{L}'_c$.

*2) Decoding:* At node $a$, a noisy vector $\boldsymbol{y}_a^{(2)}$ is received as in (2). From $r$ to $a$, it is a point-to-point AWGN channel, the decoding error

$$\Pr(\hat{\boldsymbol{\lambda}}'_r \neq \boldsymbol{\lambda}'_r) \to 0, \tag{38}$$

as $n_2 \to \infty$, if the data rate $\tilde{R}^{(2)}_{r,a}$ is less than the AWGN channel capacity $C_{r,a}$ [17].

After retrieving the message $s_r^{(2)}$, node $a$ can recover the codeword $\boldsymbol{\lambda}_{r,1} \in \mathcal{L}_b$. By (30), the transmitted codeword of node $b$ is computed at node $a$ as

$$\boldsymbol{\lambda}_b = (\boldsymbol{\lambda}_{r,1} - \boldsymbol{\lambda}_a) \bmod \Lambda_b, \tag{39}$$

where $\boldsymbol{\lambda}_a$ is the transmitted codeword of node $a$ in the first phase and is therefore known *a priori*. The source message $s_b^{(1)}$ is consequently obtained by node $a$. This completes the data flow under direction $b \to a$.

At node $b$, a noisy vector $\boldsymbol{y}_b^{(2)}$ is received as in (3). After MMSE estimate and dither cancelation, node $b$ computes

$$\begin{aligned}\tilde{\boldsymbol{y}}_b^{(2)} &= \left[\frac{\alpha_b}{h_2}\boldsymbol{y}_b^{(2)} - \boldsymbol{u}_r\right] \bmod \Lambda'_a \\ &= \left[\boldsymbol{\lambda}'_{r,1} + \boldsymbol{\lambda}'_{r,2} + \boldsymbol{z}_b^{(2)}\right] \bmod \Lambda'_a,\end{aligned} \tag{40}$$

where the MMSE coefficient is

$$\alpha_b = \frac{P_r h_2^2}{P_r h_2^2 + \sigma_b^2}, \tag{41}$$

and the effective noise $\boldsymbol{z}_b = -(1-\alpha_b)\boldsymbol{x}_r^{(2)} + \frac{\alpha_b}{h_2}\boldsymbol{n}_b^{(2)}$ having a variance

$$\frac{1}{n_2} E\left\{\|\boldsymbol{z}_b^{(2)}\|^2\right\} = \frac{P_r \sigma_b^2}{P_r h_2^2 + \sigma_b^2}. \tag{42}$$

If $\boldsymbol{\lambda}'_{r,1}$ is known, we can decode a codeword $\hat{\boldsymbol{\lambda}}'_{r,2} \in \mathcal{L}'_c$ as

$$\hat{\boldsymbol{\lambda}}'_{r,2} = Q_{\Lambda'_b}\left[\left(\tilde{\boldsymbol{y}}_b^{(2)} - \boldsymbol{\lambda}'_{r,1}\right) \bmod \Lambda'_a\right]. \tag{43}$$

The probability of decoding error is

$$\Pr(\hat{\boldsymbol{\lambda}}'_{r,2} \neq \boldsymbol{\lambda}'_{r,2}) = \Pr(\boldsymbol{z}_b \bmod \Lambda'_a \notin \mathcal{V}'_b). \tag{44}$$

Similar with (34), the error probability vanishes as $n_2 \to \infty$ if

$$\sigma^2(\Lambda'_b) > \frac{1}{n_2} E\left\{\|\boldsymbol{z}_b\|^2\right\}. \tag{45}$$

Thus the data rate of link $r \to b$ should satisfy

$$\tilde{R}^{(2)}_{r,b} < \frac{1}{2}\log\left(1 + \frac{P_r h_2^2}{\sigma_b^2}\right), \tag{46}$$

where the right-hand side is exactly the AWGN channel capacity $C_{r,b}$.

Actually, the codeword $\boldsymbol{\lambda}'_{r,1}$ is transmitted to node $b$ in the third phase. Thus the decoding of $\boldsymbol{\lambda}'_{r,2}$ can not be implemented before the decoding of $\boldsymbol{\lambda}'_{r,1}$.

### D. Cooperative Transmission Phase

*1) Encoding:* There are two codewords $\boldsymbol{\lambda}_{r,2} \in \mathcal{L}_c$ and $\boldsymbol{\lambda}'_{r,1} \in \mathcal{L}'_b$ that should be transmitted to node $b$ in this phase. Not only the relay has the information of these two codewords, node $a$ has them as well. Since node $a$ knows both messages $s_a^{(1)}$ and $s_b^{(1)}$ at the end of the second phase, it can calculate these two codewords by itself.

The codewords $\boldsymbol{\lambda}_{r,2}$ and $\boldsymbol{\lambda}'_{r,1}$ are demapped to a message $s_r^{(3)}$ (a data packet), each of node $a$ and node $r$ transmits part of its information bits. This is a conventional multiple-access channel, any capacity-achieving code can be used.

Actually, if the data rate of $s_r^{(3)}$ is less than the sum capacity of the multiple-access channel, node $a$ can transmit another data stream $s_a^{(3)}$ in this phase.

*2) Decoding:* Due to the nested relationship of the codes, the data rate of $\boldsymbol{\lambda}_{r,2}$ is $t_1(\tilde{R}^{(1)}_{a,r} - \tilde{R}^{(1)}_{b,r})$, and the data rate of $\boldsymbol{\lambda}'_{r,1}$ is $t_2(\tilde{R}^{(2)}_{r,a} - \tilde{R}^{(2)}_{r,b})$. Given the data rate of $s_a^{(3)}$ as $\tilde{R}^{(3)}_{a,b}$, to make the decoding error probabilities vanish as $n_3 \to \infty$, we have the following constraints

$$\tilde{R}^{(3)}_{a,b} < C_{a,b}, \tag{47}$$

$$t_1(\tilde{R}^{(1)}_{a,r} - \tilde{R}^{(1)}_{b,r}) + t_2(\tilde{R}^{(2)}_{r,a} - \tilde{R}^{(2)}_{r,b}) + t_3\tilde{R}^{(3)}_{a,b} < t_3 C_{\text{MA},b}. \tag{48}$$

Note that $t_1\tilde{R}^{(1)}_{b,r} = t_2\tilde{R}^{(2)}_{r,a}$, (48) is simplified as

$$t_1\tilde{R}^{(1)}_{a,r} - t_2\tilde{R}^{(2)}_{r,b} + t_3\tilde{R}^{(3)}_{a,b} < t_3 C_{\text{MA},b}. \tag{49}$$

After correct decoding of $\boldsymbol{\lambda}'_{r,1}$, we obtain $\boldsymbol{\lambda}'_r = \boldsymbol{\lambda}'_{r,1} + \boldsymbol{\lambda}'_{r,2}$. Mapping from $\mathcal{L}'_a$ to $\mathcal{L}_b$, $\boldsymbol{\lambda}_{r,1}$ is recovered from $\boldsymbol{\lambda}'_r$. Then with the knowledge of $\boldsymbol{\lambda}_b$ and $\boldsymbol{\lambda}_u = Q_{\Lambda_b}(\boldsymbol{\lambda}_b + \boldsymbol{u}_b)$, and by (30) and (31), node $b$ computes

$$\boldsymbol{\lambda}_a = [\boldsymbol{\lambda}_{r,1} + \boldsymbol{\lambda}_{r,2} + \boldsymbol{\lambda}_u - \boldsymbol{\lambda}_b] \bmod \Lambda_a. \tag{50}$$

The message of node $a$ is consequently obtained at node $b$, and the two-way information exchange is accomplished.

### E. Achievable Rate Region

For the data flow from $a$ to $b$, the rate constraints in the three transmission phases are summarized as follows.

$$\tilde{R}_{a,r}^{(1)} < R_{a,r}^*,$$
$$\tilde{R}_{r,b}^{(2)} < C_{r,b},$$
$$\tilde{R}_{a,b}^{(3)} < C_{a,b},$$
$$t_1\tilde{R}_{a,r}^{(1)} + t_3\tilde{R}_{a,b}^{(3)} < t_2\tilde{R}_{r,b}^{(2)} + t_3 C_{\mathrm{MA},b}.$$

Thus the sum rate of the two streams $a \to r \to b$ and $a \to b$ satisfies

$$t_1\tilde{R}_{a,r}^{(1)} + t_3\tilde{R}_{a,b}^{(3)} < t_1 R_{a,r}^* + t_3 C_{a,b}, \tag{51}$$
$$t_1\tilde{R}_{a,r}^{(1)} + t_3\tilde{R}_{a,b}^{(3)} < t_2 C_{r,b} + t_3 C_{\mathrm{MA},b}. \tag{52}$$

For the data flow from $b$ to $a$, the rate constraints are

$$\tilde{R}_{b,r}^{(1)} < R_{b,r}^*, \quad \tilde{R}_{r,a}^{(2)} < C_{r,a}.$$

The achievable rate region of the two-way relay system with CoMABC protocol is consequently obtained as

$$R_a \le \min\{t_1 R_{a,r}^* + t_3 C_{a,b},\ t_2 C_{r,b} + t_3 C_{\mathrm{MA},b}\}, \tag{53}$$
$$R_b \le \min\{t_1 R_{b,r}^*,\ t_2 C_{r,a}\}. \tag{54}$$

Comparing with the rate region outer bound (10) and (11), the gap is at most 0.5 bit for each user.

## V. Numerical Results

In this section, we compare the rate region outer bound, in a sum rate sense, of the three two-way relay transmission protocols: CoMABC, MABC and TDBC. The achievable sum rates of CoMABC and MABC protocols are also investigated. We will see the gap between the achievable sum rate and the sum rate upper bound in different SNRs and received power ratios. There is no available scheme for TDBC yet to approximately achieve its rate region outer bound. In the end, we will also analyze the time slot allocation results.

The rate region outer bounds of the MABC and TDBC protocols are given in [8]. With the same definition of the AWGN channel capacity as in (10) and (11), the rate region of the MABC protocol is outer bounded as

$$R_a \le \min\{t_1 C_{a,r},\ t_2 C_{r,b}\}, \tag{55}$$
$$R_b \le \min\{t_1 C_{b,r},\ t_2 C_{r,a}\}, \tag{56}$$

and the achievable rate region of this protocol given by [13] is

$$R_a \le \min\{t_1 R_{a,r}^*,\ t_2 C_{r,b}\}, \tag{57}$$
$$R_b \le \min\{t_1 R_{b,r}^*,\ t_2 C_{r,a}\}, \tag{58}$$

where $R_{a,r}^*$ and $R_{b,r}^*$ are the same with that in (53) and (54).

The rate region of the TDBC protocol is outer bounded as

$$R_a \le \min\{t_1 C_{\mathrm{BC},a},\ t_1 C_{a,b} + t_3 C_{r,b}\}, \tag{59}$$
$$R_b \le \min\{t_2 C_{\mathrm{BC},b},\ t_2 C_{b,a} + t_3 C_{r,a}\}, \tag{60}$$

where $C_{\mathrm{BC},a} = \log_2(1 + P_a|h_{a,r}|^2 + P_a|h_{a,b}|^2)$ is the sum capacity of the broadcast channel from node $a$ to nodes $r$ and $b$, $C_{\mathrm{BC},b}$ has a similar definition denoting the sum capacity of the broadcast channel from node $b$ to nodes $r$ and $a$.

When we calculate the sum rate upper bound of the MABC and TDBC protocols or the achievable sum rate of the CoMABC and MABC protocols, similar linear programming problems as in (12) can be formulated and optimal time slot allocations are found.

To compare the sum rates in different channel conditions, we fix the positions of node $a$ at -1 and node $b$ at 1. Let the relay $r$ move along the line between them. When $r$ is at position $d$, the distance between $a$ and $r$ is $1 + d$ and the distance between $b$ and $r$ is $1 - d$. Normalize the channel gain when the relay is at the middle point of the two end nodes, *i.e.*, $h_1 = h_2 = 1$ when $d = 0$. With positions $d \in (-1, 1)$, $h_1 = (1 + d)^{-\gamma/2}$, $h_2 = (1 - d)^{-\gamma/2}$ and $h_3 = 2^{-\gamma/2}$, where $\gamma$ is the path loss exponent. Consider that all nodes have the same transmit power $P$ and the same noise variance $\sigma^2$, and the transmit SNR is defined as $P/\sigma^2$. The received SNRs also depend on the path losses.

The sum rate upper bounds with different $d$ for CoMABC, MABC and TDBC protocols are given in Fig. 5, where the achievable sum rates of CoMABC and MABC are also shown. The comparisons are evaluated at SNR=5 dB and 15 dB, respectively, with $\gamma = 3$. The received SNRs of the two uplink signals at the relay change with $d$, as do their power ratios. As we have remarked in Section IV-B, when SNR=15 dB, the achievable sum rates of both protocols are almost overlapped with the sum rate upper bounds; when SNR=5 dB, there are apparent gaps. For CoMABC, the maximal gap happens at $d = \pm 0.3$, and for MABC, the maximal gap happens at $d = \pm 0.4$, both gaps are within 0.2 bit. Interestingly, when $d \to \pm 1$ the gaps vanish again. This is because the sum rate is dominated by one data flow at this time. For example, when $d \to -1$, $R_{a,r}^*$ is close to $C_{a,r}$ since $\frac{P_a h_1^2}{P_a h_1^2 + P_b h_2^2} \to 1$.

If we look at the upper bounds, CoMABC is always higher than the other two. In the perfectly symmetric case, CoMABC is degenerated to MABC, and the third time slot decreases to zero. In the extreme asymmetric case, *e.g.*, $h_1/h_2 \to \infty$, both CoMABC and TDBC will degenerate to the one-way cooperative transmission, where the relay behaves like an additional transmit antenna for CoMABC and an additional receive antenna for TDBC. The sum rate will reduce to $C_{\mathrm{MA},b}$. In other words, the gap between CoMABC and MABC gradually increases if the channel asymmetry grows, and the gap between CoMABC and TDBC rises to maximum in the symmetric case.

The superiority of CoMABC comes from the efficient utilization of the direct link and the time resource. To see the impact of the strength of the direct link, we compare the upper bounds of sum rates with different path loss exponents. The larger $\gamma$ is, the weaker is the direct link relative to the relay links. Fig. 6 shows the results with $\gamma$=2 and 4, where SNR=10 dB. Since the channel gain is normalized at $d = 0$, the two groups of curves meet at this position. With larger $\gamma$, the gap between CoMABC and MABC shrinks, while the gap between CoMABC and TDBC enlarges. When the direct link disappears, CoMABC will degenerate to MABC since there is no the third phase any longer. In this case, TDBC loses time efficiency since it decodes the messages from $a$ and $b$ separately in the first two phases.

Fig. 7 shows the optimal time slot allocation results when

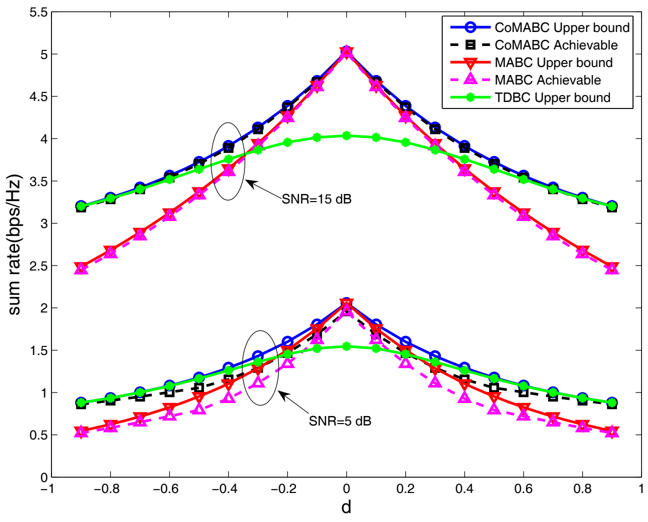


Fig. 5. Compare the sum rate upper bounds of CoMABC, MABC and TDBC protocols, and the achievable sum rates of CoMABC and MABC protocols, with different SNRs and the relay positions.

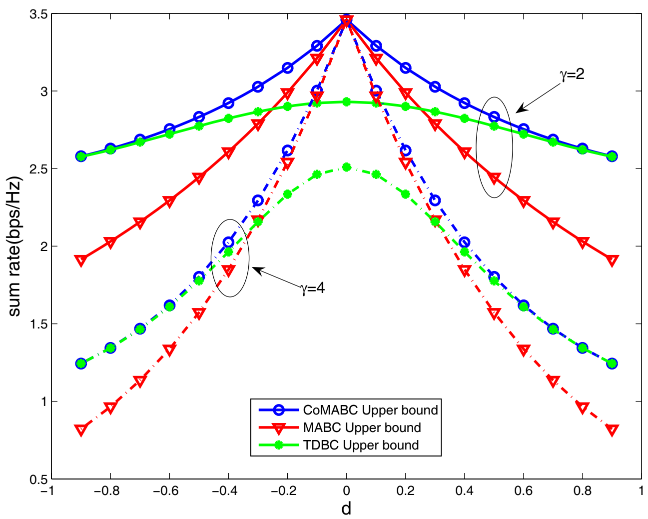


Fig. 6. Compare the sum rate upper bounds of CoMABC, MABC and TDBC protocols, with different path loss exponents and the relay positions.

the upper bounds of the sum rate are maximized, where SNR=10 dB and $\gamma = 3$ are applied. The MABC protocol always divide the two slots equally, so it is not present in this figure. For the CoMABC protocol, the first two phases are multiple-access and broadcast, and the third phase is cooperative transmission. When $r$ is close to $a$, the third phase transmission is from $r$ and $a$ to $b$, while when $r$ is close to $b$, the third phase transmission is from $r$ and $b$ to $a$. Thus the time slot allocation results, $t_1$, $t_2$ and $t_3$, are symmetric around $d = 0$. For the TDBC protocol, the first phase is $a$ to $b$ and $r$, the second phase is $b$ to $a$ and $r$, and the third phase is $r$ to $a$ and $b$. The transmission sequence is not changed along with the position of $r$. So that the time slot allocation results are asymmetric around $d = 0$. When $r$ is close to $a$, the first phase would be shorter, and the second phase would be longer, as we have seen in the figure.

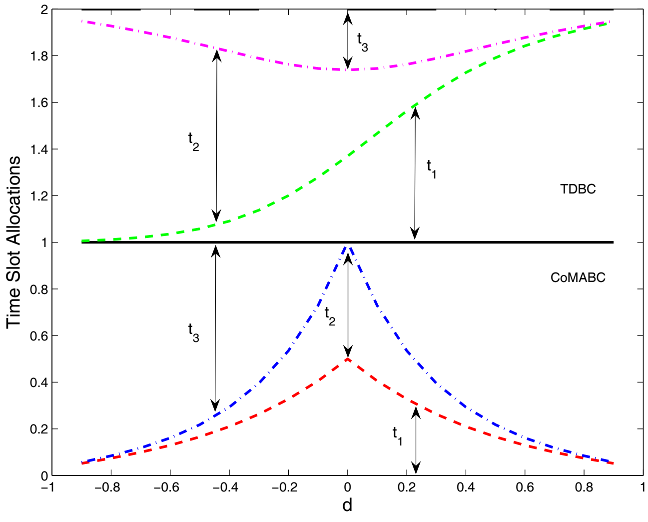


Fig. 7. Optimal time slot allocation results when the upper bounds of the sum rate are maximized.

In the symmetric case, $t_1 = t_2 = 0.5$ and $t_3 = 0$ for CoMABC, while $t_1 = t_2 = 0.37$ and $t_3 = 0.26$ for TDBC. In the asymmetric cases, for example $d$ moves to -1, CoMABC allocates more time to the third time slot while TDBC relies more on the second time slot. This distinction comes from the different transmission strategies of two protocols. For CoMABC, the relay $r$ receives a lot of data from $a$ in the first phase but can not transfer them to $b$ in the second phase, thus the third phase is more relied to complete the information exchanging. For TDBC, however, the broadcast phase is in the final and $r \to a$ link is fast, so that we must allocate more time to the second phase in order to accumulate more data from $b$ at the relay.

## VI. Conclusion

In this paper, we studied two-way relay in asymmetric channels. After analyzing the time efficiency of a conventional MABC protocol, a new three-phase CoMABC protocol was proposed, which allows the stronger link to convey more data and assists the weaker link by introducing an additional cooperative transmission phase. We then derived the rate region outer bound of this protocol and showed it to be achievable by doubly nested lattice codes within 0.5 bit. The encoding and decoding processes were described for each node in each transmission phase. The sum rate of CoMABC is higher than that of MABC and TDBC under any circumstance. In symmetric channels, the CoMABC degenerates to the MABC protocol, while in extremely asymmetric channels, it degenerates to the one-way cooperative transmission.

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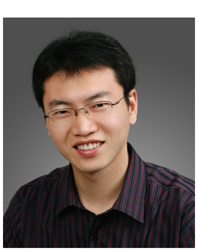

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