Spectral-Efficiency of TDD Multiuser Two-Hop MC-CDMA Systems Employing Egocentric-Altruistic Relay Optimization

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***Abstract*— In this contribution we investigate the spectral-efficiency of a two-hop cooperative network using multicarrier code-division multiple-access (MC-CDMA) transmission scheme. The two-hop network constitutes K source users transmitting signals to K destinations with the aid of N relays. Our focus is on the relay optimization, when assuming that the N relays cooperate or do not cooperate with each other. Specifically, in this contribution the egocentric-altruistic (E-A) optimization is introduced, which constitutes an E-optimization motivating to suppress the multiuser interference (MUI) of the source-relay channels and an A-optimization aiming to pre-mitigate the potential MUI of the relay-destination channels. Both the minimum mean-square error (MMSE) and zero-forcing (ZF) optimization criteria are considered. Furthermore, the spectral-efficiency performance of the two-hop MC-CDMA systems using the proposed E-A relay optimization is investigated by simulations, when assuming communications over frequency-selective fading channels.**

I. Introduction

In order to increase the link quality, reliability, and data rate of wireless networks, future wireless communication systems are required to be developed for cooperation rather than merely for coexistence [1]. In order to accomplish such cooperation, relay nodes may be deployed to assist the communications between sources and destinations, making the signal processing at sources and destinations simple in addition to the other advantages. Specifically, in a two-hop network operated in time-division duplex (TDD) principles, the main signal processing may be implemented at the relays so that the complexity of source and destination devices is as low as possible.

In TDD-based networks, owing to the reciprocal characteristics of the bidirectional communication channels, it is usually convenient for relays to attain the channel state information (CSI) required for carrying out some signal processing. Assuming that the CSI of both the source-relay and relay-destination channels is known to the relays, then, signal detection and transmission at the relays can be represented by one joint optimization problem, which is referred to as the joint detection/transmission relay optimization. This joint relay optimization problem may be solved in the principles of, such as, MMSE [2–4] or ZF [5]. However, this joint detection/transmission relay optimization problem is usually hard to solve. Therefore, in this contribution we propose and investigate a relatively simple relay optimization scheme, which is developed based on the concepts of egocentric (E)-optimization and altruistic (A)-optimization [8, 9], referred to as the *E-A optimization* or *E-A relay optimization* for convenience. As our forthcoming discourse shown, the E-A relay optimization is constituted by an E-optimization resulting in a multiuser detector (MUD) and an A-optimization leading to a multiuser transmitter (MUT). The MUD motivates to suppress the MUI from the source-relay channels and the MUT aims to pre-mitigate the potential MUI of the relay-destination channels. Furthermore, it can be shown that the solution to the joint detection/transmission relay optimization is a special case of our E-A optimization [4].

In comparison with the joint detection/transmission relay optimization, the E-A optimization has the following characteristics. Firstly, it decomposes the optimization into two optimization problems, an E-optimization for the first hop and an A-optimization for the second hop. Hence, the E-A optimization employs more freedom to choose the optimization schemes for the first and second hops of optimization. Secondly, due to the cumulated MUI associated with the two-hop transmissions, the end-to-end signal-to-interference-plus-noise ratio (SINR) in the joint detection/transmission relay optimization becomes hard to analyze. By contrast, with the E-A relay optimization, the SINR of the E-optimization and that of the A-optimization can usually be derived, and the end-to-end SINR can be conveniently approximated by the existing formulas.

Following the E-A optimization, in this contribution we investigate the spectral-efficiency of the TDD-based two-hop network in conjunction with the MC-CDMA [6, 7]. Specifically, the spectral-efficiency of the two-hop MC-CDMA systems is investigated, when assuming that the relays are in cooperation or they do not cooperate with each other. Both the MMSE- and ZF-assisted relay optimization schemes are considered. Furthermore, impact of the frequency-selectivity of fading channels on the spectral-efficiency of the two-hop MC-CDMA systems is investigated.

In this paper bold uppercase/lowercase variables denote matrices/vectors. Conjugation, transpose, Hermitian transpose and expectation are in the form of $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $\mathbb{E}\{\cdot\}$. The trace of a square matrix is denoted by $\mathrm{tr}\{\cdot\}$ and, finally, 's.t' means 'subject to'.

II. System Description

We consider a two-hop multiuser (nodes) cooperative network as shown in Fig. 1, where transmissions between K source-destination pairs are assisted by N amplify-and-forward (AF) relays operated in half-duplex protocol. The N relays $\mathcal{R}_1, \ldots, \mathcal{R}_N$ receive signals from the K sources $\mathcal{S}_1, \ldots, \mathcal{S}_K$ in the first hop and, after the MUD/MUT processing, forward the transformed signals to the K destinations $\mathcal{D}_1, \ldots, \mathcal{D}_K$ in the second hop. In order to support multiple users to access the network, multicarrier code-division multiple-access (MC-CDMA) scheme using M subcarriers [10] is assumed in our study.

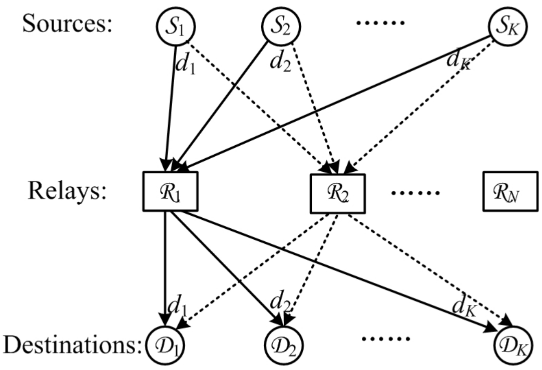

Fig. 1. Two-hop multiuser cooperative network with K source-destination pairs assisted by N relays.

Let the elements of the diagonal matrix $\boldsymbol{H}_{\mathcal{SR}}^{(n,k)} \in \mathbb{C}^{M\times M}$ denote the frequency (F)-domain channel responses over the M subcarriers between the kth source and nth relay. Correspondingly, let the elements of the diagonal matrix $\boldsymbol{H}_{\mathcal{DR}}^{(n,k)} \in \mathbb{C}^{M\times M}$ denote the F-domain channel responses over the M subcarriers between the nth relay and the kth destination. Furthermore, let

$$\boldsymbol{H}_{\mathcal{SR}} = [\boldsymbol{H}_{\mathcal{SR}1}^T, \boldsymbol{H}_{\mathcal{SR}2}^T, \ldots, \boldsymbol{H}_{\mathcal{SR}N}^T]^T$$
$$\boldsymbol{H}_{\mathcal{DR}} = [\boldsymbol{H}_{\mathcal{DR}1}^T, \boldsymbol{H}_{\mathcal{DR}2}^T, \ldots, \boldsymbol{H}_{\mathcal{DR}N}^T]^T$$

where

$$\boldsymbol{H}_{\mathcal{SR}n} = [\boldsymbol{H}_{\mathcal{SR}}^{(n,1)}, \boldsymbol{H}_{\mathcal{SR}}^{(n,2)}, \cdots, \boldsymbol{H}_{\mathcal{SR}}^{(n,K)}]$$
$$\boldsymbol{H}_{\mathcal{DR}n} = [\boldsymbol{H}_{\mathcal{DR}}^{(n,1)}, \boldsymbol{H}_{\mathcal{DR}}^{(n,2)}, \cdots, \boldsymbol{H}_{\mathcal{DR}}^{(n,K)}]$$

Then, it can be shown that the received signal vector of the N relays from the first hop can be expressed as

$$\boldsymbol{y}_{\mathcal{R}} = \boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C}\boldsymbol{d} + \boldsymbol{n}_{\mathcal{R}} \tag{1}$$

where $\boldsymbol{y}_{\mathcal{R}} = [\boldsymbol{y}_{\mathcal{R}1}^T, \boldsymbol{y}_{\mathcal{R}2}^T, \cdots, \boldsymbol{y}_{\mathcal{R}N}^T]^T$ is an MN-length observation vector and $\boldsymbol{y}_{\mathcal{R}n} = \boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}\boldsymbol{d} + \boldsymbol{n}_{\mathcal{R}n}$ is an M-length observation vector of the nth relay, $\boldsymbol{n}_{\mathcal{R}} = [\boldsymbol{n}_{\mathcal{R}1}^T, \boldsymbol{n}_{\mathcal{R}2}^T, \cdots, \boldsymbol{n}_{\mathcal{R}N}^T]^T$ and $\boldsymbol{n}_{\mathcal{R}n}$ is an M-length Gaussian noise vector of the nth relay with zero-mean and a covariance matrix $\mathbb{E}\{\boldsymbol{n}_{\mathcal{R}n}\boldsymbol{n}_{\mathcal{R}n}^H\} = \sigma_{\mathcal{R}}^2\boldsymbol{I}_M$, where σ_R^2 denotes the noise variance at the relays. $\boldsymbol{d} = [d_1, d_2, \cdots, d_K]^T$, where $d_1, d_2, \cdots, d_K$ are independent identically distributed (iid) random variables satisfying $\mathbb{E}\{d_k\} = 0$ and $\mathbb{E}\{|d_k|^2\} = P_{\mathcal{S}}$, where $P_{\mathcal{S}}$ is the transmit power per symbol at the sources. Finally, in (1)

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}_2 & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{c}_K \end{bmatrix}$$

where $\boldsymbol{c}_k$ is an M-length spreading sequence assigned to the kth user, which satisfies $\boldsymbol{c}_k^H\boldsymbol{c}_k = 1$.

During the second hop, the N relays forward a transformed signal vector, which can be expressed as

$$\boldsymbol{x}_{\mathcal{R}} = \boldsymbol{G}\boldsymbol{y}_{\mathcal{R}} \tag{2}$$

where $\boldsymbol{x}_{\mathcal{R}} = [\boldsymbol{x}_{\mathcal{R}1}^T, \boldsymbol{x}_{\mathcal{R}2}^T, \cdots, \boldsymbol{x}_{\mathcal{R}N}^T]^T$ is an MN-length transmit vector and $\boldsymbol{x}_{\mathcal{R}n}$ is an M-length transmit vector of the nth relay, $\boldsymbol{G} \in \mathbb{C}^{MN\times MN}$ is a linear processing matrix to be determined according to certain optimization criterion. The processing matrix $\boldsymbol{G}$ satisfies the power constraint of $\text{tr}\{\boldsymbol{G}\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}}\boldsymbol{y}_{\mathcal{R}}^H\}\boldsymbol{G}^H\} \leq KP_{\mathcal{R}}$, where $P_{\mathcal{R}}$ is the transmit power per symbol at the relays. In this contribution, we assume that the sources and relays share the total transmit power per symbol of P, implying that $P = P_{\mathcal{S}} + P_{\mathcal{R}}$. Furthermore, it is assumed that the total transmit power at the relays is independent of N of the number of relays.

The received signal vector at the kth destination during the second hop can be written as

$$\boldsymbol{y}_{\mathcal{D}k} = \sum_{n=1}^{N}(\boldsymbol{H}_{\mathcal{DR}}^{(n,k)})^T\boldsymbol{x}_{\mathcal{R}n} + \boldsymbol{n}_{\mathcal{D}k},\ k = 1, 2, \ldots, K \tag{3}$$

where $\boldsymbol{n}_{\mathcal{D}k}$ is an M-length Gaussian noise vector at the kth destination with zero-mean and a covariance matrix $\mathbb{E}\{\boldsymbol{n}_{\mathcal{D}k}\boldsymbol{n}_{\mathcal{D}k}^H\} = \sigma_{\mathcal{D}}^2\boldsymbol{I}_M$, in which $\sigma_{\mathcal{D}}^2$ denotes the noise variance at the destinations. Thus, the MK-length received signal vector $\boldsymbol{y}_{\mathcal{D}} = [\boldsymbol{y}_{\mathcal{D}1}^T, \boldsymbol{y}_{\mathcal{D}2}^T, \cdots, \boldsymbol{y}_{\mathcal{D}K}^T]^T$ at the K destinations becomes

$$\boldsymbol{y}_{\mathcal{D}} = \boldsymbol{H}_{\mathcal{DR}}^T\boldsymbol{x}_{\mathcal{R}} + \boldsymbol{n}_{\mathcal{D}} \tag{4}$$

where $\boldsymbol{n}_{\mathcal{D}} = [\boldsymbol{n}_{\mathcal{D}1}^T, \boldsymbol{n}_{\mathcal{D}2}^T, \cdots, \boldsymbol{n}_{\mathcal{D}K}^T]^T$ is the MK-length noise vector.

In this contribution, we assume that at the destinations the low-complexity matched filters (MFs) are employed for estimation of the desired symbols. Finally, after the correlation of $\boldsymbol{y}_{\mathcal{D}k}$ with $\boldsymbol{c}_k$ of the kth user's spreading sequence, the decision variable of d_k can be expressed as

$$\hat{d}_k = \boldsymbol{c}_k^T\boldsymbol{y}_{\mathcal{D}k},\ k = 1, 2, \ldots, K \tag{5}$$

Then, the decision vector $\hat{\boldsymbol{d}} = [\hat{d}_1, \hat{d}_2, \cdots, \hat{d}_K]^T$ can be formed as

$$\hat{\boldsymbol{d}} = \boldsymbol{C}^T\boldsymbol{y}_{\mathcal{D}} = \tilde{\boldsymbol{H}}\boldsymbol{d} + \tilde{\boldsymbol{n}} \tag{6}$$

where $\tilde{\boldsymbol{H}} = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T\boldsymbol{G}\boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C}$ is referred to as the *equivalent channel matrix*, which is $(K \times K)$ dimensional, and $\tilde{\boldsymbol{n}} = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T\boldsymbol{G}\boldsymbol{n}_{\mathcal{R}} + \boldsymbol{C}^T\boldsymbol{n}_{\mathcal{D}}$ is a K-length *equivalent noise vector* with zero-mean and a covariance matrix $\boldsymbol{R}_{\tilde{\boldsymbol{n}}} = \sigma_{\mathcal{R}}^2\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T\boldsymbol{G}\boldsymbol{G}^H\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^* + \sigma_{\mathcal{D}}^2\boldsymbol{I}_K$.

III. Egocentric-Altruistic Relay Optimization

In this section we consider the E-A relay optimization, when two operation scenarios are considered. The first scenario assumes that all the N relays share their channel knowledge and hence are cooperative. By contrast, the second scenario assumes that the N relays cannot share their channel knowledge. Therefore, their optimization is distributed.

A. Cooperative Relays

When all the N relays as shown in Fig. 1 are connected via a high speed backbone and can thus share their received signals as well as channel knowledge, we say that they are cooperative relays. In this case all the relays can access $\boldsymbol{y}_{\mathcal{R}}$, which is referred to as the *global observations*, as well as exploit the *global channel knowledge* of $\boldsymbol{H}_{\mathcal{SR}}$ and $\boldsymbol{H}_{\mathcal{DR}}$. Consequently, the processing matrix after the E-A relay optimization can be written as

$$\boldsymbol{G} = \sqrt{\alpha}\boldsymbol{P}_{\mathcal{RD}}\boldsymbol{W}_{\mathcal{SR}}^H = \sqrt{\alpha}\sum\nolimits_{k=1}^{K}\boldsymbol{p}_{\mathcal{RD}k}\boldsymbol{w}_{\mathcal{SR}k}^H \tag{7}$$

where $\alpha = KP_{\mathcal{R}}/\text{tr}\{\boldsymbol{P}_{\mathcal{RD}}\boldsymbol{W}_{\mathcal{SR}}^H\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}}\boldsymbol{y}_{\mathcal{R}}^H\}\boldsymbol{W}_{\mathcal{SR}}\boldsymbol{P}_{\mathcal{RD}}^H\}$ is the amplification factor for achieving the power constraint on relays' transmitted signals, $\boldsymbol{W}_{\mathcal{SR}} = [\boldsymbol{w}_{\mathcal{SR}1}, \cdots, \boldsymbol{w}_{\mathcal{SR}K}]$, $\boldsymbol{P}_{\mathcal{RD}} = [\boldsymbol{p}_{\mathcal{RD}1}, \cdots, \boldsymbol{p}_{\mathcal{RD}K}]$, where $\boldsymbol{w}_{\mathcal{SR}k}$ and $\boldsymbol{p}_{\mathcal{RD}k}$ are the MN-length post-processing and pre-processing vectors in terms of d_k.

According to the principles of E-A optimization [8], we can readily obtain the post-processing matrix for MUD, which is

$$\boldsymbol{W}_{\mathcal{SR}} = \left(\boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C}\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}}^H + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_{MN}\right)^{-1}\boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C} \tag{8}$$

and the pre-processing matrix for MUT, which is

$$\boldsymbol{P}_{\mathcal{RD}} = \left(\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^*\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_{MN}\right)^{-1}\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^* \tag{9}$$

In (8) and (9) $\lambda_{\mathcal{S}} = \sigma_{\mathcal{R}}^2/P_{\mathcal{S}}$, $\lambda_{\mathcal{D}} = \sigma_{\mathcal{D}}^2/P_{\mathcal{R}}$ and $\rho > 0$ is a noise-suppression factor [8]. According to (8) and (9), explicitly, $\boldsymbol{W}_{\mathcal{SR}}$ and $\boldsymbol{P}_{\mathcal{RD}}$ achieve respectively the ZF-MUD and ZF-MUT when $\rho \to 0$, while MMSE-MUD and MMSE-MUT if $\rho = 1$.

Note that, when joint relay optimization [2–4] is considered, the optimization problem in MMSE sense can be stated as

$$\min_{a,\boldsymbol{G}}\mathbb{E}\{\|a\hat{\boldsymbol{d}} - \boldsymbol{d}\|^2\}\ \ s.t\ \ \text{tr}\{\boldsymbol{G}\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}}\boldsymbol{y}_{\mathcal{R}}^H\}\boldsymbol{G}^H\} \leq KP_{\mathcal{R}} \tag{10}$$

where a is an amplification factor. The solution to the above optimization problem can be expressed as [4]

$$\begin{aligned}\boldsymbol{G} =& \tilde{\alpha}\left(\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^*\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T + \lambda_{\mathcal{D}}\boldsymbol{I}_{MN}\right)^{-1}\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^* \\ &\times \boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}}^H\left(\boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C}\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}}^H + \lambda_{\mathcal{S}}\boldsymbol{I}_{MN}\right)^{-1}\end{aligned} \tag{11}$$

where $\tilde{\alpha}$ achieves power constraint. Explicitly, the relays first carry out the MMSE-MUD and then transmit the signals over relay-destination channels in the MMSE-MUT principles.

When comparing (7) with (11), we can readily find that the proposed E-A optimization associated with $\rho = 1$ results in the same solution as the joint relay optimization in MMSE sense. This observation explains that the joint relay optimization can be divided into two independent optimization problems, one for detection and one for transmission, without making trade-off of the optimality. However, since the E-optimization and A-optimization can be carried out independently, the E-A relay optimization is capable of providing us higher flexibility to choose relay processing methods than the joint relay optimization. For example, the E-optimization and A-optimization may be operated under different optimization criteria instead of one in the joint relay optimization.

B. Distributed Relays

When the relays are distributed and cannot cooperate with each other, the nth relay can only access $\boldsymbol{y}_{\mathcal{R}n}$, which is referred to as the nth relay's *local observation*, and make use of its *local channel knowledge* of $\boldsymbol{H}_{\mathcal{SR}n}$ and $\boldsymbol{H}_{\mathcal{DR}n}$. In this case, each relay can only carry out local detection and transmission processing. Consequently, the processing matrix of the N relays can be represented as $\boldsymbol{G} = \text{diag}\{\boldsymbol{G}_1, \boldsymbol{G}_2, \cdots, \boldsymbol{G}_N\}$, where $\boldsymbol{G}_n \in \mathbb{C}^{M\times M}$ is the processing matrix of the nth relay, which is independent of the other relays' processing. According to the principles of E-A optimization, the processing matrix $\boldsymbol{G}_n$ can be written as

$$\boldsymbol{G}_n = \sqrt{\alpha_n}\boldsymbol{P}_{\mathcal{RD}n}\boldsymbol{W}_{\mathcal{SR}n}^H,\ n = 1, 2, \ldots, N \tag{12}$$

where α_n is for achieving the power constraint on the n relay, while $\boldsymbol{W}_{\mathcal{SR}n}$ and $\boldsymbol{P}_{\mathcal{RD}n}$ are the post-processing and pre-processing matrices of the nth relay, which can be obtained based on the E-optimization and A-optimization, respectively.

In this paper we assume that all the relays have an equal transmit power. Then, it can be shown that $\alpha_n = KP_{\mathcal{R}}/\text{tr}\{N\boldsymbol{P}_{\mathcal{RD}n}\boldsymbol{W}_{\mathcal{SR}n}^H\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}n}\boldsymbol{y}_{\mathcal{R}n}^H\}\boldsymbol{W}_{\mathcal{SR}n}\boldsymbol{P}_{\mathcal{RD}n}^H\}$. Furthermore, according to the principles of E-A optimization [8], we can readily obtain $\boldsymbol{W}_{\mathcal{SR}n}$ and $\boldsymbol{P}_{\mathcal{RD}n}$, which are

$$\begin{aligned}\boldsymbol{W}_{\mathcal{SR}n} &= \left(\boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}\boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}n}^H + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}\\ \boldsymbol{P}_{\mathcal{RD}n} &= \left(\boldsymbol{H}_{\mathcal{DR}n}^*\boldsymbol{C}^*\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_{\mathcal{DR}n}^*\boldsymbol{C}^*\end{aligned} \tag{13}$$

In this case, the decision vector of (6) can be expressed as

$$\begin{aligned}\hat{\boldsymbol{d}} &= \sum_{n=1}^{N}\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T\boldsymbol{G}_n\boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}\boldsymbol{d} + \sum_{n=1}^{N}\boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T\boldsymbol{G}_n\boldsymbol{n}_{\mathcal{R}n} + \boldsymbol{C}^T\boldsymbol{n}_{\mathcal{D}}\\ &= \sum_{n=1}^{N}\hat{\boldsymbol{d}}_n\end{aligned} \tag{14}$$

where $\hat{\boldsymbol{d}}_n = \tilde{\boldsymbol{H}}_n\boldsymbol{d} + \tilde{\boldsymbol{n}}_n$ is the decision vector provided by the nth relay, in which $\tilde{\boldsymbol{H}}_n = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T\boldsymbol{G}_n\boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}$ and $\tilde{\boldsymbol{n}}_n = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T\boldsymbol{G}_n\boldsymbol{n}_{\mathcal{R}n} + \boldsymbol{C}^T\boldsymbol{n}_{\mathcal{D}}/N$ are the equivalent channel matrix and equivalent noise vector over the nth relay, respectively.

Note that, it can be shown that, when $\rho = 1$, $\boldsymbol{G}_n$ of (12) associated with $\boldsymbol{W}_{\mathcal{SR}n}$ and $\boldsymbol{P}_{\mathcal{RD}n}$ given in (13) is in fact the solution of the following joint MMSE optimization problem

$$\min_{a_n, \boldsymbol{G}_n}\mathbb{E}\left\{\|a_n\hat{\boldsymbol{d}}_n - \boldsymbol{d}\|^2\right\}\quad s.t\ \text{tr}\{\boldsymbol{G}_n\mathbb{E}\{\boldsymbol{y}_{\mathcal{R}n}\boldsymbol{y}_{\mathcal{R}n}^H\}\boldsymbol{G}_n^H\} \leq KP_{\mathcal{R}}/N \tag{15}$$

Therefore, again, the joint relay optimization problem can be divided into two independent relay optimization problems, one E-optimization problem for the source-relay multiple-access channel and one A-optimization problem for the relay-destination broadcast channel.

IV. Performance analysis

The spectral-efficiency of the cooperative MC-CDMA systems supporting two-hop relay communications as shown in Fig. 1 can be expressed as [10]

$$\eta = \frac{1}{2M}\sum_{k=1}^{K}\log_2(1 + \gamma_k) \tag{16}$$

where the factor $1/2$ reflects the fact that a transmission consists of two hops and γ_k is the end-to-end SINR of the kth user. Explicitly, we need first to analyze the SINR of γ_k, in order to evaluate the spectral-efficiency of (16).

A. Exact Analysis of SINR γ_k

We can find from (6) that, given the equivalent channel matrix $\tilde{\boldsymbol{H}}$ and the equivalent noise covariance matrix $\boldsymbol{R}_{\tilde{\boldsymbol{n}}}$, the end-to-end SINR of d_k can be denoted as

$$\gamma_k = \frac{\left|\boldsymbol{e}_k^T\tilde{\boldsymbol{H}}\boldsymbol{e}_k\right|^2}{\boldsymbol{e}_k^T\left(\tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^H + \boldsymbol{R}_{\tilde{\boldsymbol{n}}}\right)\boldsymbol{e}_k - \left|\boldsymbol{e}_k^T\tilde{\boldsymbol{H}}\boldsymbol{e}_k\right|^2} \tag{17}$$

where $\boldsymbol{e}_k$ is a basic vector with its kth entry being one while all the other entries being zeros.

Furthermore, for the two-hop MC-CDMA systems using cooperative relays, we can obtain from (7) that

$$\tilde{\boldsymbol{H}} = \sqrt{\alpha}\boldsymbol{\Phi}_{\mathcal{D}}\left(\boldsymbol{\Phi}_{\mathcal{D}} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\left(\boldsymbol{\Phi}_{\mathcal{S}} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Phi}_{\mathcal{S}}$$

and

$$\begin{aligned}\boldsymbol{R}_{\tilde{\boldsymbol{n}}} &= \sigma_{\mathcal{R}}^2\alpha\boldsymbol{\Phi}_{\mathcal{D}}\left(\boldsymbol{\Phi}_{\mathcal{D}} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\left(\boldsymbol{\Phi}_{\mathcal{S}} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\\ &\quad\times\boldsymbol{\Phi}_{\mathcal{S}}\left(\boldsymbol{\Phi}_{\mathcal{S}} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\left(\boldsymbol{\Phi}_{\mathcal{D}} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Phi}_{\mathcal{D}} + \sigma_{\mathcal{D}}^2\boldsymbol{I}_K\end{aligned}$$

where $\boldsymbol{\Phi}_{\mathcal{D}} = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}}^T\boldsymbol{H}_{\mathcal{DR}}^*\boldsymbol{C}^*$ and $\boldsymbol{\Phi}_{\mathcal{S}} = \boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}}^H\boldsymbol{H}_{\mathcal{SR}}\boldsymbol{C}$.

By contrast, for the cooperative communications systems using distributed relays, we derive from (14) that

$$\tilde{\boldsymbol{H}} = \sum_{n=1}^{N}\sqrt{\alpha_n}\boldsymbol{\Phi}_{\mathcal{D}n}\left(\boldsymbol{\Phi}_{\mathcal{D}n} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\left(\boldsymbol{\Phi}_{\mathcal{S}n} + \rho\lambda_{\mathcal{SR}n}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Phi}_{\mathcal{S}n}$$

and

$$\begin{aligned}\boldsymbol{R}_{\tilde{\boldsymbol{n}}} =& \sigma_{\mathcal{R}}^2\sum_{n=1}^{N}\alpha_n\boldsymbol{\Phi}_{\mathcal{D}n}\left(\boldsymbol{\Phi}_{\mathcal{D}n} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\\ &\times\left(\boldsymbol{\Phi}_{\mathcal{S}n} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Phi}_{\mathcal{S}n}\left(\boldsymbol{\Phi}_{\mathcal{S}n} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\\ &\times\left(\boldsymbol{\Phi}_{\mathcal{D}n} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{\Phi}_{\mathcal{D}n} + \sigma_{\mathcal{D}}^2\boldsymbol{I}_K\end{aligned}$$

where $\boldsymbol{\Phi}_{\mathcal{D}n} = \boldsymbol{C}^T\boldsymbol{H}_{\mathcal{DR}n}^T\boldsymbol{H}_{\mathcal{DR}n}^*\boldsymbol{C}^*$ and $\boldsymbol{\Phi}_{\mathcal{S}n} = \boldsymbol{C}^H\boldsymbol{H}_{\mathcal{SR}n}^H\boldsymbol{H}_{\mathcal{SR}n}\boldsymbol{C}$ for $n = 1, 2, \ldots, N$.

By substituting the corresponding $\tilde{\boldsymbol{H}}$ and $\boldsymbol{R}_{\tilde{\boldsymbol{n}}}$ into (17), we can evaluate the exact SINR γ_k in the context of the two-hop MC-CDMA systems employing either cooperative relays or distributed relays. Furthermore, the spectral-efficiency of the two-hop MC-CDMA systems can be evaluated using (16). Unfortunately, the SINR expression seen in (17) is too complex to analyze. Therefore, we below turn to consider the approximate analysis of the SINR γ_k.

B. Approximate Analysis of SINR γ_k

First, in the context of the scenario using cooperative relays, when approximating the kth user's decision variables at both the relays and destinations as independent Gaussian random variables, we can approximate the end-to-end SINR of the kth user as [12]

$$\gamma_k \approx \frac{\gamma_{\mathcal{S}k}\cdot\gamma_{\mathcal{D}k}}{\gamma_{\mathcal{S}k} + \gamma_{\mathcal{D}k} + 1},\ k = 1, 2, \ldots, K \tag{18}$$

where $\gamma_{\mathcal{S}k}$ and $\gamma_{\mathcal{D}k}$ are the SINRs corresponding to the source-relay and relay-destination channels, respectively.

When the MUD $\boldsymbol{W}_{\mathcal{SR}}$ of (8) is employed associated with the first hop, it can be readily shown that $\gamma_{\mathcal{S}k}$ is [13]

$$\gamma_{\mathcal{S}k} = \frac{P_{\mathcal{S}}/\sigma_{\mathcal{R}}^2}{\boldsymbol{e}_k^T\left(\boldsymbol{\Phi}_{\mathcal{S}} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \rho \tag{19}$$

where $\rho \in [0, 1]$. Specifically, (19) is the SINR of the MMSE-MUD when $\rho = 1$ and that of the ZF-MUD if $\rho = 0$.

For the MUT operated in the second hop associated with the pre-processing matrix $\boldsymbol{P}_{\mathcal{RD}}$, the closed-form expression of $\gamma_{\mathcal{D}k}$ is still too complicated to analyze. However, base on the equivalency between the MUT and MUD optimization as investigated in [8, 11], the SINR of the relay-destination link achieved by the pre-processing using $\boldsymbol{P}_{\mathcal{RD}}$ can be approximated by the SINR of the destination-relay link achieved by the MUD using $\boldsymbol{W}_{\mathcal{DR}} = \boldsymbol{P}_{\mathcal{RD}}^*$, when the noise variances of both the links are set the same. Based on this approximation, we can find that $\gamma_{\mathcal{D}k}$ in (18) can be expressed as

$$\gamma_{\mathcal{D}k} \approx \frac{P_{\mathcal{R}}/\sigma_{\mathcal{D}}^2}{\boldsymbol{e}_k^T \left(\boldsymbol{\Phi}_{\mathcal{D}} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \rho \tag{20}$$

where, again, $\rho = 0$ and $\rho = 1$ correspond to the ZF-MUT and MMSE-MUT, respectively.

Finally, upon substituting (19) and (20) into (18), we can obtain the simplified end-to-end SINR γ_k of the cooperative communications systems employing N cooperative relays. Furthermore, the spectral-efficiency can be evaluated by substituting $\gamma_k,\ k = 1, 2, \ldots, K$ of (18) into (16).

Second, for the two-hop MC-CDMA systems using distributed relays, by approximating the MUI at each relay as Gaussian noise, the end-to-end SINR $\gamma_{k,n}$ of the kth user in terms of the nth relay can be approximated as

$$\gamma_{k,n} \approx \frac{\gamma_{\mathcal{S}}^{k,n} \cdot \gamma_{\mathcal{D}}^{k,n}}{\gamma_{\mathcal{S}}^{k,n} + \gamma_{\mathcal{D}}^{k,n} + 1} \tag{21}$$

where $\gamma_{\mathcal{S}}^{k,n}$ and $\gamma_{\mathcal{D}}^{k,n}$, denoting the SINR values of the source-relay and relay-destination links, can be derived by the similar approaches as for (19) and (20), and can be expressed as

$$\gamma_{\mathcal{S}}^{k,n} = \frac{P_{\mathcal{S}}/\sigma_{\mathcal{R}}^2}{\boldsymbol{e}_k^T \left(\boldsymbol{\Phi}_{\mathcal{S}n} + \rho\lambda_{\mathcal{S}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \rho \tag{22}$$

$$\gamma_{\mathcal{D}}^{k,n} \approx \frac{P_{\mathcal{R}}/\sigma_{\mathcal{D}}^2}{\boldsymbol{e}_k^T \left(\boldsymbol{\Phi}_{\mathcal{D}n} + \rho\lambda_{\mathcal{D}}\boldsymbol{I}_K\right)^{-1}\boldsymbol{e}_k} - \rho \tag{23}$$

Since the destinations combine the signals received from all the N relays, the final SINR γ_k is the weighted sum of $\gamma_{k,1}, \gamma_{k,2}, \cdots, \gamma_{k,N}$, which can be found to be

$$\gamma_k \approx \frac{\left(\sum_{n=1}^{N} |h_{k,n}|\right)^2}{\sum_{n=1}^{N} |h_{k,n}|^2/\gamma_{k,n}},\ k = 1, 2, \ldots, K \tag{24}$$

where $h_{k,n}$ is defined as

$$h_{k,n} = \frac{\sqrt{\alpha_n}\gamma_{\mathcal{S}}^{k,n} \cdot \gamma_{\mathcal{D}}^{k,n}}{(\gamma_{\mathcal{S}}^{k,n} + \rho)(\gamma_{\mathcal{D}}^{k,n} + \rho)} \tag{25}$$

Finally, the spectral-efficiency for the two-hop MC-CDMA systems using distributed relays can be evaluated by substituting $\gamma_k,\ k = 1, 2, \ldots, K$ of (24) into (16).

V. Performance Results

In this section we illustrate the spectral-efficiency performance of the TDD multiuser two-hop MC-CDMA systems employing the proposed E-A optimization for relays. The MC-CDMA system utilizing $M = 16$ subcarriers is considered and $\beta = K/M$ is defined as the system load factor. In our simulations we assumed that the MC-CDMA employed random spreading sequences. The MC-CDMA signals were transmitted over frequency-selective Rayleigh fading channels having L iid time-domain resolvable multiple paths. We assumed that at all the nodes the noise had the same variance of $\sigma_{\mathcal{R}}^2 = \sigma_{\mathcal{D}}^2 = \sigma^2$. The total transmit power of the K sources was assumed to equal that of the N relays, being expressed as $P_{\mathcal{S}} = P_{\mathcal{R}} = 1/2P$. The SNR per symbol is P/σ^2. Note that, the above-mentioned as well as other related parameters used in our simulations are displayed associated with the corresponding figures.

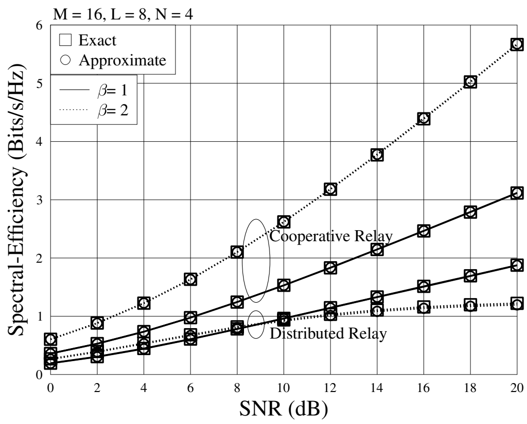

Fig. 2. Spectral-efficiency versus SNR per symbol performance of the two-hop MC-CDMA systems employing MMSE-assisted E-A relay optimization, when communicating over frequency-selective fading channels associated with $L = 8$ time-domain resolvable multipaths.

Fig. 2 shows the spectral-efficiency of the two-hop MC-CDMA systems employing cooperative relays or distributed relays. Both the exact approaches seen in Section IV-A and the approximate approaches discussed in Section IV-B are considered. Explicitly, the results obtained from the approximate approaches are very accurate, which cannot be distinguished from that obtained from the exact approaches. The same conclusions can also be drawn from Fig. 3. From the results of Fig. 2 we can have the following observations. First, given the same set of parameters, the spectral-efficiency of a two-hop MC-CDMA system using cooperative relays is significantly higher than that of a system using distributed relays, owing to the fact that under the cooperative relays the K users are capable of sharing a total of MN degrees-of-freedom provided both by the M subcarriers and by the N relays. By contrast, since the distributed relays do not cooperate, the K users at each relay can only share M degrees-of-freedom contributed by the MC-CDMA alone.

Second, as shown in Fig. 2, if the relays are in cooperation, the spectral-efficiency increases significantly either when the load factor β increases or when the SNR per bit increases. For using the distributed relays, the spectral-efficiency with $\beta = 1$ improves explicitly when the SNR per bit increases, but that with $\beta = 2$ only improves slightly when increasing the SNR per bit value. Furthermore, as seen in Fig. 2, in the low SNR region, the spectral-efficiency of $\beta = 2$ may be higher than that of $\beta = 1$. This observation is inverted in the relatively high SNR region. According to the analysis in Section III-A and Section III-B, we can know that a two-hop MC-CDMA system using cooperative relays becomes overloaded only when $\beta > N$, while the system using distributed relays becomes overloaded provided that $\beta > 1$. Once the system is overloaded, the residual MUI after the E-A optimization becomes significant, which will degrade the achievable performance.

Figure 3 illustrates the impact of system load factor β on the spectral-efficiency performance of the two-hop MC-CDMA systems. Due to the above-mentioned residual MUI after the E-A optimization, the spectral-efficiency does not monotonously increase with β. For each case considered, there is an optimal system load factor β, which results in the highest spectral-efficiency achievable. Specifically, when using cooperative relays, the optimum β value is in the range of $0.8N \sim 0.9N$. When using distributed relays, the optimum β value ranges within $1 \sim 1.2$.

In Figs. 4 and 5 we study the impact of the number of time-domain resolvable multipaths L as well as the system load factor β on the spectral-efficiency of the two-hop MC-CDMA systems us-

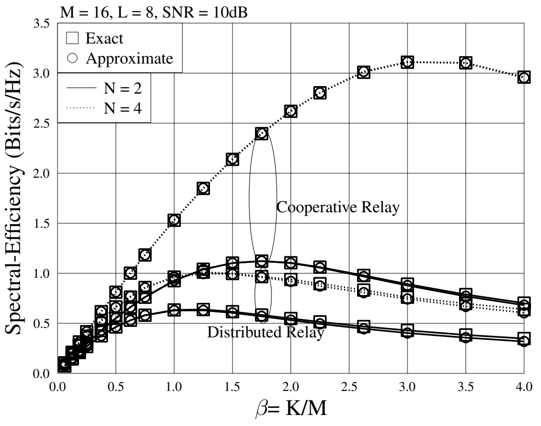

Fig. 3. Impact of system load factor β on the spectral-efficiency of the two-hop MC-CDMA systems employing MMSE-assisted E-A relay optimization, when communicating over frequency-selective fading channels associated with $L = 8$ time-domain resolvable paths.

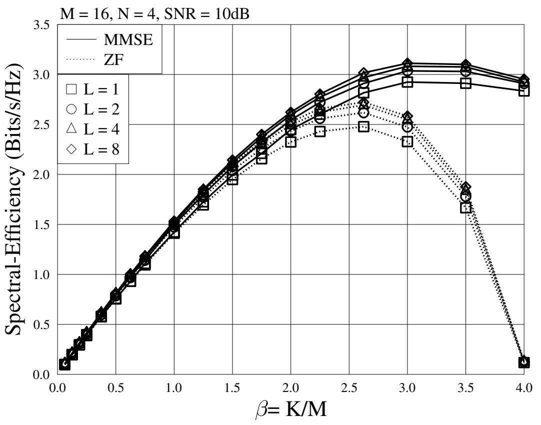

Fig. 4. Spectral-efficiency versus system load factor β for the two-hop MC-CDMA systems employing cooperative relays.

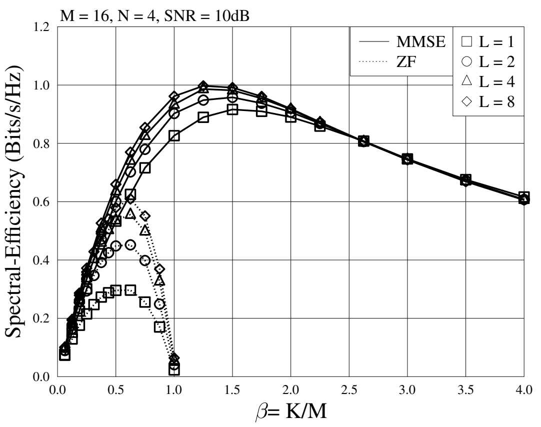

Fig. 5. Spectral-efficiency versus system load factor β for the two-hop MC-CDMA systems employing distributed relays.

ing either cooperative relays (Fig. 4) or distributed relays (Fig. 5). Both the MMSE- and ZF-assisted E-A optimization approaches are investigated. The results of Figs. 4 and 5 show that, within certain range of β, the number of time-domain resolvable multipaths affects noticeably the achievable spectral-efficiency of the two-hop MC-CDMA systems. Generally, the spectral-efficiency increases, as the number of time-domain resolvable multipaths L increases. As can be seen in both figures, the MMSE-assisted E-A relay optimization results in significantly higher spectral-efficiency than the ZF-assisted E-A relay optimization. The MMSE-assisted E-A relay optimization is more robust to the frequency-selective fading than the ZF-assisted E-A relay optimization. Furthermore, as shown in Figs. 4 and 5, when a two-hop MC-CDMA system is full-load corresponding to $\beta = N = 4$ in Fig. 4 and $\beta = 1$ in Fig. 5, the spectral-efficiency of the two-hop MC-CDMA system tends to zero, when using the ZF scheme. Additionally, when comparing Fig. 4 with Fig. 5, we can find that a two-hop MC-CDMA system using cooperative relays achieves higher spectral-efficiency than a corresponding two-hop MC-CDMA system employing distributed relays.

In **conclusion**, our studies in this paper show that the E-A relay optimization employs the freedom to choose separately the optimization schemes for detection and transmission at the relays. The joint detection/transmission relay optimization proposed in literature constitutes one special case of our proposed E-A relay optimization. In our investigation we have assumed that the relays are either in full cooperation (cooperative relays) or without cooperation at all (distributed). The studies show that using cooperative relays is capable of providing significantly higher spectral-efficiency than using distributed relays. Furthermore, our performance results show that the MMSE-assisted relay optimization results in higher spectral-efficiency than the ZF-assisted relay optimization. In general, the spectral-efficiency of a two-hop MC-CDMA system increases, as the fading channel becomes more frequency-selective.

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