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A Symbol-level FDE and Spread-Spectrum Mode Design for Multi-code Multiple Access Systems

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***Abstract*— Design of equalizers and spread-spectrum modes or code allocation schemes is crucial for achieving good performance multi-code multiple access systems. In this paper, a symbol-level frequency domain equalizer and spread-spectrum mode are jointly designed to maximize the diversity gain with low complexity. Since the proposed transceiver scheme is independent of the orthogonal codes, it can be applied to direct sequence code division multiple access (DS-CDMA), multi-carrier CDMA (MC-CDMA) and orthogonal frequency division multiple access (OFDMA) systems. Simulation results are provided which validate the theoretical analysis.**

I. Introduction

Orthogonal multi-code multiple access systems are widely applied in mobile and personal communication networks thanks to their plentiful merits, such as high spectral efficiency, high flexibility to tradeoff the diversity and multiplexing gain, less multiple access interference (MAI) by using the orthogonal codes, and so on.

A multi-code system with M orthogonal codes can support M active users. When the system is not fully loaded, *i.e.*, the number of active users $K < M$, the system will have the spreading gain $N = M/K$. When different codes are selected for transmitting, different level of MAI will be introduced in the received signal.

The system performance can be improved either by allocating the spreading codes in the transmitter [1–3] or by designing multi-user detection algorithms in the receiver [4,5], which are often referred to as equalizers in downlink systems [6–9].

In the transmission scheme design, [1] gives a code allocation searching procedure to minimize the maximal MAI, [2] proposes a code allocation scheme for MC-CDMA systems employing Walsh codes, and [3] selects codes with known channel state information. These approaches need to calculate the MAI or error probability. Moreover, their results are obtained by exhausting search, which depend on the features of orthogonal codes and the equalization algorithms. In contrast, we optimize the code allocation scheme by investigating the diversity gain provided by the spread-spectrum mode, which can reflect the relationship between the channel power spectra before and after de-spreading in this paper. As a result, the designed scheme is independent of codes and easy to implement.

In the receiver side, since linear chip-level equalizers equalize multipath channel (or equivalently whiten the received signal) directly, they will degrade the performance due to neglecting the correlation existed among chips. Linear symbol-level equalizers exploit this correlation by de-spreading before whitening filtering thus can achieve more diversity gain and outperform the chip-level equalizers [4]. These equalizers can be implemented in time or frequency domain, which is respectively referred to as time domain equalizers (TDEs) or frequency domain equalizers (FDEs). Symbol-level TDEs are available for both single-code systems (such as DSSS systems) and multi-code systems, but their huge complexity is unacceptable with the increasing bandwidth and date rate [5]. FDEs are well-known to be implemented with one-tap equalization using Fourier transforming [6–9]. If a one-tap symbol-level FDE is available, it can achieve good performance meanwhile and meet the low-cost and low-power demands for mobile terminals in downlink systems. However, existing symbol-level FDEs can only be applied to single-code spread-spectrum systems [6]. How to design a one-tap symbol-level FDE is still unsolved for multi-code multiple access systems. This is why many systems have to employ chip-level FDEs to trade-off complexity with performance [8, 9].

The main contributions of this paper are presenting a linear symbol-level FDE with weighted minimum mean square error (WMMSE) and designing a spread-spectrum mode for orthogonal multi-code multiple access systems. The analysis and simulation show that the proposed spread-spectrum mode can provide the maximal diversity gain. With this spread-spectrum mode, the presented symbol-level FDE can outperform the chip-level FDE with the same complexity. By choosing a key parameter in the WMMSE symbol-level equalizer, which is the interference weighting factor ρ, the WMMSE-FDE with the optimal ρ can achieve the maximal output weighted signal to noise plus interference ratio (WSINR) under equal power control, while the WMMSE-FDE with $\rho \to \infty$ becomes a zero forcing (ZF) FDE and can provide a good trade-off between performance and complexity under unequal power control.

This paper is organized as follows. Section II describes the signal models. Section III presents the designed symbol-level FDE and the spread-spectrum mode. Section IV analyzes the performance of the presented transceiver scheme. Simulation results are shown in Section V, and conclusions are provided in the last section.

II. Signal Models

Consider a downlink multi-code system using M orthogonal codes $\mathbf{X} = [\mathbf{x}_0, \cdots, \mathbf{x}_{M-1}]$, each code is M-length. For transmitting the symbols for K users $\mathbf{d} = [d_0, \cdots, d_{K-1}]^T$ in each symbol duration T_s, the transmit signal from the base station is

$$s(t) = \sum_{i=0}^{M-1} s_i g(t - iT_c) = \sum_{k=0}^{K-1} \sqrt{E_k} d_k \sum_{i=0}^{M-1} c_{i,k} g(t - iT_c), \quad (1)$$

where $T_c = T_s/M$ is the chip duration, $g(t)$ is the pulse waveform. The chip vector $\mathbf{s} = [s_0, \cdots, s_{M-1}]^T$ can also be represented as

$$\mathbf{s} = \mathbf{CAd}, \quad (2)$$

where $\mathbf{A} = \text{diag}\{\sqrt{E_0}, \cdots, \sqrt{E_{K-1}}\}$ denotes the energy of the symbols, and the $M \times K$ matrix $\mathbf{C} = [\mathbf{c}_0, \cdots, \mathbf{c}_{K-1}]^T$ represents the employed 'spreading codes' in the transmitter. We use an $M \times K$ matrix $\mathbf{Q}$ to describe the relationship between $\mathbf{C}$ and $\mathbf{X}$ as $\mathbf{C} = \mathbf{XQ}$, *i.e.*, $\mathbf{c}_i = \sum_{j=0}^{M-1} Q_{ij}\mathbf{x}_j$. The nonzero element Q_{ij} indicates that code $\mathbf{x}_j$ will be selected to transmit symbol d_i, so $\mathbf{Q}$ is called the code allocation matrix.

For a desired user k, after a pulse waveform matched filtering and chip-rate sampling in one symbol duration, the received vector $\mathbf{y}_k = [y_{k,0}, \cdots, y_{k,M-1}]^T$ can be obtained as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{s} + \mathbf{n}_k = \mathbf{H}_k\mathbf{CAd} + \mathbf{n}_k, \quad (3)$$

where $\mathbf{n}_k$ is the zero mean white Gaussian noise with variance $\mathbb{E}\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_n^2\mathbf{I}$, in which $\sigma_n^2 = N_0/2$ is the two-sided power spectrum density. $\mathbf{H}_k$ is the multipath channel matrix, and it is well known that $\mathbf{H}_k$ becomes a circulant matrix by inserting a cyclic prefix at the transmitter and removing it at the receiver,

$$\mathbf{H}_k = \begin{pmatrix} h_0 & 0 & h_L \cdots h_1 \\ \vdots & \ddots & \ddots \ddots \vdots \\ h_{L-1} \cdots h_0 & 0 & h_L \\ & \ddots & \ddots \ddots \\ 0 & & h_L \cdots h_0 \end{pmatrix}_{M \times M},$$

where the number of resolvable paths is $L + 1$ and h_l is the channel coefficient of the lth path.

$\mathbf{H}_k$ can be decomposed as $\mathbf{F}^H\mathbf{\Lambda}_k\mathbf{F}$, where $\mathbf{\Lambda}_k = \text{diag}\{\Lambda_{k,0}, \cdots, \Lambda_{k,M-1}\}$, the element of its diagonals $\Lambda_{k,i} = \sum_{l=0}^{L} h_l e^{-j2\pi il/M}$, $i \in [0, M-1]$, is the frequency domain channel response of the ith 'subcarrier'. $\mathbf{F}$ is the discrete Fourier transform matrix of size M and $F_{mn} = 1/\sqrt{M}e^{-j2\pi mn/M}, m, n \in [0, M-1], j = \sqrt{-1}$.

III. Symbol-level FDE and Spread-Spectrum Mode Design

In this section, a symbol-level FDE and the corresponding spread-spectrum mode for multi-code systems will be investigated. To make the basic idea of the proposed scheme be easily understood, conventional algorithms will be briefly reviewed.

A. Conventional Equalizers

For the kth user, the estimation of desired symbol d_k obtained by a linear equalization is $\hat{d}_k = \mathbf{w}^H\mathbf{y}_k$. By using the notation of noise-suppression factor [10], the symbol-level and chip-level TDEs with WMMSE can be obtained as

$$\mathbf{w}_S^H = \sqrt{E_k^{-1}}\mathbf{v}_k^H \left(\mathbf{\Psi}_S + \sigma_n^2/\rho\mathbf{A}^{-2}\right)^{-1} \mathbf{C}^H\mathbf{H}_k^H \quad (4)$$

and

$$\mathbf{w}_C^H = \sqrt{E_k^{-1}}\mathbf{v}_k^H\mathbf{C}^H \left(\mathbf{\Psi}_C + N\beta\mathbf{I}_K\right)^{-1} \mathbf{H}_k^H, \quad (5)$$

where $\mathbf{\Psi}_C = \mathbf{H}_k^H\mathbf{H}_k$ and $\mathbf{\Psi}_S = \mathbf{C}^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{C}$ are the correlation matrices of the 'multipath channel' and the 'equivalent channel' after de-spreading, respectively. $\mathbf{v}_k = [v_0, \cdots, v_{K-1}]^T$ is a vector of length K and $v_i = \delta(i-k)$, in which $\delta(n)$ is a Dirac function. $N = M/K$ is the spreading gain, *i.e.*, processing gain. $\rho > 0$ is the interference weighing factor which can adjust the capability of suppressing MAI in the equalizer. $\beta = \sigma_n^2/(E_k\rho)$ is the reciprocal of the weighted signal-to-noise ratio (SNR) of the received signals.

Substitute $\mathbf{H}_k = \mathbf{F}^H\mathbf{\Lambda}_k\mathbf{F}$ into (5), then we obtain a chip-level FDE as

$$\mathbf{w}_C^H = \sqrt{E_k^{-1}}\mathbf{v}_k^H\mathbf{C}^H\mathbf{F}^H \left(\mathbf{\Delta}_k + N\beta\mathbf{I}_M\right)^{-1} \mathbf{\Lambda}_k^H\mathbf{F}, \quad (6)$$

where $\mathbf{\Delta}_k = \mathbf{\Lambda}_k^H\mathbf{\Lambda}_k = \text{diag}\{\Delta_{k,0}, \cdots, \Delta_{k,M-1}\}$, and the diagonal elements represent the power spectrum of channel response.

It is shown by comparing (5) with (6) that the FDE avoids the complicated matrix inversion operation by Fourier transforming. The computational complexity can then be reduced from $o(M^3)$ to $o(MlogM)$.

By utilizing eigen-decomposition $\mathbf{\Psi}_C = \mathbf{F}^H\mathbf{\Delta}_k\mathbf{F}$, the chip-level FDE becomes a one-tap equalizer by Fourier transforming. However, due to the influence of codes $\mathbf{C}$ on the de-spreading, eigen-decomposition for $\mathbf{\Psi}_S = \mathbf{C}^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{C}$ is difficult except for single-code systems where $\mathbf{\Psi}_S$ keeps circulant [6]. Therefore, the key issue to develop a one-tap symbol-level FDE for multi-code systems is to find a simple way to decompose $\mathbf{\Psi}_S$.

B. One-tap Symbol-level FDE

The correlation matrix of the 'equivalent channel' after de-spreading can be represented as $\mathbf{\Psi}_S = \mathbf{G}^H\mathbf{\Delta}_k\mathbf{G}$, where $\mathbf{G} = \mathbf{FC}$ stands for the employed orthogonal codes in frequency domain. Since each $M \times K$ matrix $\mathbf{G}$ can be decomposed as $\mathbf{G} = \mathbf{PU}$, where $\mathbf{U}$ is a $K \times K$ unitary matrix and $\mathbf{P}$ is an arbitrary $M \times K$ matrix, the correlation matrix of the 'equivalent channel' becomes

$$\mathbf{\Psi}_S = \mathbf{U}^H\mathbf{P}^H\mathbf{\Delta}_k\mathbf{PU}. \quad (7)$$

If $\mathbf{\Pi}_k = \mathbf{P}^H\mathbf{\Delta}_k\mathbf{P}$ is a diagonal matrix, $\mathbf{\Pi}_k$ and $\mathbf{U}$ will be the eigenvalue and eigenvector matrices of $\mathbf{\Psi}_S$. As a result, a low-complexity symbol-level FDE can be obtained as

$$\mathbf{w}_S^H = \sqrt{E_k^{-1}}\mathbf{v}_k^H\mathbf{U}^H \left(\mathbf{\Pi}_k + \beta\mathbf{I}_K\right)^{-1} \mathbf{P}^H\mathbf{\Lambda}_k^H\mathbf{F}. \quad (8)$$

The element on ith row and jth column of $\mathbf{\Pi}_k$ is

$$\Pi_{k,ij} = \sum_{m=0}^{M-1} P_{mi}P_{mj}\Delta_{k,m} = \mathbf{p}^T\mathbf{b},$$

where $\mathbf{p}_{ij} = [P_{0i}P_{0j}, P_{1i}P_{1j}, \cdots, P_{M-1i}P_{M-1j}]^T$ and $\mathbf{b} =$

$[\Delta_{k,0}, \Delta_{k,1}, \cdots, \Delta_{k,M-1}]^T$. Keeping $\mathbf{\Pi}_k$ diagonal for each $\mathbf{b}$ is equivalent to ensuring $\mathbf{p}_{ij} = \mathbf{0}, \forall i \neq j, i, j \in [0, K-1]$. Therefore, $\mathbf{\Pi}_k$ will become diagonal if η_i, the number of nonzero elements in the ith row of $\mathbf{P}$, satisfies

$$\eta_i \leq 1, i \in [0, M-1]. \quad (9)$$

It is very difficult to decompose $\mathbf{G}$ into a unitary matrix $\mathbf{U}$ and a matrix $\mathbf{P}$ satisfying (9). However, we can implement such a decomposition by designing the code allocation matrix as $\mathbf{Q} = \mathbf{\Gamma}^H\mathbf{P}\mathbf{U}$, where the $M \times M$ matrix $\mathbf{\Gamma} = \mathbf{F}\mathbf{X}$ denotes the available codes in frequency domain. When we select $\mathbf{U} = \mathbf{\Gamma}_B$ as a $K \times K$ subset of orthogonal codes $\mathbf{\Gamma}$, the code allocation matrix becomes $\mathbf{Q} = \mathbf{\Gamma}^H\mathbf{P}\mathbf{\Gamma}_B$. This means that the one-tap symbol-level FDE can be obtained by a joint design of the transmitter and receiver.

By utilizing $\mathbf{G} = \mathbf{P}\mathbf{\Gamma}_B$, the correlation matrix of the 'equivalent channel' is decomposed as $\mathbf{\Psi}_S = \mathbf{\Gamma}_B^H\mathbf{P}^H\mathbf{\Delta}_k\mathbf{P}\mathbf{\Gamma}_B$, where $\mathbf{\Gamma}_B$ represents the code set after de-spreading and $\mathbf{\Pi}_k = \mathbf{P}^H\mathbf{\Delta}_k\mathbf{P}$ is the power spectrum of the 'equivalent channel' after de-spreading. Since $\mathbf{P}$ reflects the relationship between the channel power spectra before and after de-spreading, it is called the spread-spectrum mode matrix. Moreover, since $\mathbf{Q} = \mathbf{\Gamma}^H\mathbf{P}\mathbf{\Gamma}_B$, we can see that allocating codes is equivalent to selecting the spread-spectrum mode.

It's shown by comparing (8) with (6) that the complexity of symbol-level FDE is the same as that of the chip-level FDE. By exploiting the correlation between chips, the symbol-level FDE combines the power spectrum of the frequency selective fading channel directly, while the chip-level FDE combines that of the 'equivalent channel' after the whitening filter. Therefore, the symbol-level FDE can obtain more diversity gain and outperform the chip-level FDE.

C. Spread-Spectrum Mode Design

In the previous subsection, we have designed a spread-spectrum mode $\mathbf{P}$ to reduce the complexity of the symbol-level equalization. Now we will study how to improve the system performance through optimizing the mode $\mathbf{P}$.

According to (8), we know that the spread-spectrum system with the orthogonal codes $\mathbf{G} = \mathbf{P}\mathbf{\Gamma}_B$ and channel power spectrum $\mathbf{\Delta}_k$ is equivalent to the nonspread-spectrum system with the orthogonal codes $\mathbf{\Gamma}_B$ and channel power spectrum $\mathbf{\Pi}_k = \mathbf{P}^H\mathbf{\Delta}_k\mathbf{P}$. Since $\mathbf{\Pi}_k$ is independent of $\mathbf{\Gamma}_B$, it can reflect the provided diversity gain of each spread-spectrum mode sufficiently. Thus, we can design the optimal mode by analyzing and comparing $\mathbf{\Pi}_k$, and determine the corresponding code allocation scheme by $\mathbf{Q} = \mathbf{\Gamma}^H\mathbf{P}\mathbf{\Gamma}_B$ in the next step.

To simplify the analysis and description, several typical modes are compared in the case that spreading gain $N = M/K$ is an integer. They can be expressed as follows,

$$\mathbf{\Xi} = \mathbf{I}_K \bigotimes \{[1, 0, \cdots, 0]^T\}_{N\times 1},$$
$$\mathbf{\Sigma} = \{\sqrt{N^{-1}}[1, 1, \cdots, 1]^T\}_{N\times 1} \bigotimes \mathbf{I}_K,$$
$$\mathbf{\Xi}' = \{[1, 0, \cdots, 0]^T\}_{N\times 1} \bigotimes \mathbf{I}_K$$

and

$$\mathbf{\Sigma}' = \mathbf{I}_K \bigotimes \{\sqrt{N^{-1}}[1, 1, \cdots, 1]^T\}_{N\times 1},$$

where the operator $\bigotimes$ denotes the Kronecker tensor product.

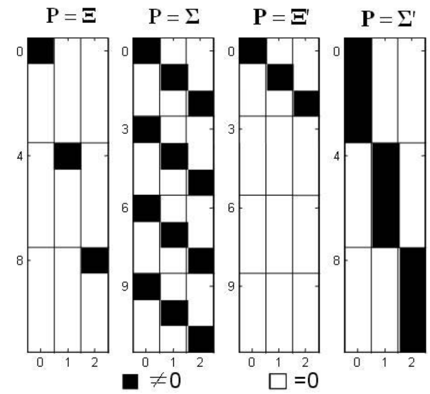

Fig. 1. Several typical spread-spectrum Modes ($N = 4, K = 3$).

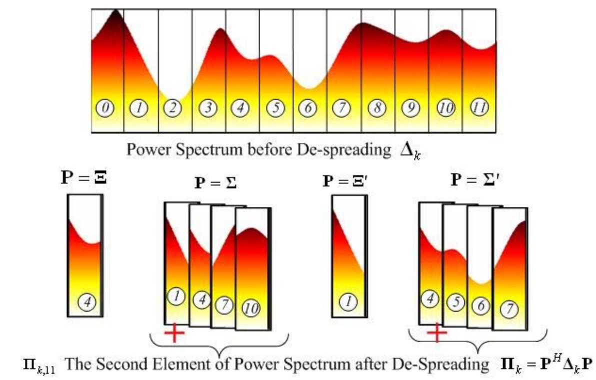

Fig. 2. The process of de-spreading ($N = 4, K = 3$).

After de-spreading, the power spectra of the corresponding 'equivalent channel' become

$$\mathbf{\Pi}_{\mathbf{\Xi}} = \text{diag}\left\{\Delta_{k,0}, \Delta_{k,N}, \cdots, \Delta_{k,N(K-1)}\right\},$$
$$\mathbf{\Pi}_{\mathbf{\Sigma}} = \text{diag}\left\{\frac{1}{N}\sum_{i=0}^{N-1}\Delta_{k,iK}, \cdots, \frac{1}{N}\sum_{i=0}^{N-1}\Delta_{k,iK+K-1}\right\},$$
$$\mathbf{\Pi}_{\mathbf{\Xi}'} = \text{diag}\left\{\Delta_{k,0}, \Delta_{k,1}, \cdots, \Delta_{k,K-1}\right\}$$

and

$$\mathbf{\Pi}_{\mathbf{\Sigma}'} = \text{diag}\left\{\frac{1}{N}\sum_{i=0}^{N-1}\Delta_{k,i}, \cdots, \frac{1}{N}\sum_{i=0}^{N-1}\Delta_{k,(K-1)N+i}\right\},$$

respectively.

Figure 1 displays the value of these matrices with $N = 4, K = 3$. Figure 2 illustrates the relationship between the channel power spectra before and after de-spreading with these modes. As shown in Fig. 2, the modes $\mathbf{P} = \mathbf{\Xi}$ and $\mathbf{P} = \mathbf{\Xi}'$ only directly extract the $1/N$ of the spectrum of $\mathbf{\Delta}_k$, hence there is no diversity gain. However, the modes $\mathbf{P} = \mathbf{\Sigma}$ and $\mathbf{P} = \mathbf{\Sigma}'$ combine the whole spectrum of $\mathbf{\Delta}_k$ by N times 'overlapping', thus they can provide more diversity gain than the two former modes. The mode $\mathbf{P} = \mathbf{\Sigma}'$ combines the adjacent 'subcarriers', while the mode $\mathbf{P} = \mathbf{\Sigma}$ combines the 'subcarriers' with the maximal distance. Due to the correlation of adjacent 'subcarriers', the mode $\mathbf{P} = \mathbf{\Sigma}$ provide more diversity gain than the mode $\mathbf{P} = \mathbf{\Sigma}'$. The mode '$\mathbf{P} = \mathbf{\Sigma}$' utilizes the whole bandwidth and maximizes the distance between the combined 'subcarriers', so it can provide the maximal diversity gain.

Base on $\mathbf{Q} = \mathbf{\Gamma}^H\mathbf{P}\mathbf{\Gamma}_B$ and $\mathbf{P} = \mathbf{\Sigma}$, we can design the

TABLE I
CODE ALLOCATION MATRICES $\mathbf{Q}$ FOR DIFFERENT ORTHOGONAL CODES $\mathbf{X}$

Codes $\mathbf{X}/\mathbf{\Gamma}$ $M \times M$	Sub-codes $\mathbf{\Gamma}_B$ $K \times K$	$\mathbf{P} = \mathbf{\Sigma}$ $M \times K$	Examples
$\mathbf{X} = \mathbf{I}$, $\mathbf{\Gamma} = \mathbf{F}$	$\mathbf{\Gamma}_B = \mathbf{\Sigma}^H \mathbf{\Gamma} \mathbf{\Xi}$	$\mathbf{Q} = \mathbf{\Xi}$	DSSS [9]
$\mathbf{X} = \mathbf{F}^H \mathbf{T}$, $\mathbf{\Gamma} = \mathbf{T}$	$\mathbf{\Gamma}_B = \mathbf{\Sigma}^H \mathbf{\Gamma} \mathbf{\Xi}$	$\mathbf{Q} = \mathbf{\Xi}$	MC-CDMA [2]
$\mathbf{X} = \mathbf{T}$, $\mathbf{\Gamma} = \mathbf{F}\mathbf{T}$	$\mathbf{\Gamma}_B = \mathbf{\Sigma}^H \mathbf{\Gamma} \mathbf{\Sigma}$	$\mathbf{Q} = \mathbf{\Sigma}$	DS-CDMA [4]
$\mathbf{X} = \mathbf{F}^H$, $\mathbf{\Gamma} = \mathbf{I}$	$\mathbf{\Gamma}_B = \mathbf{\Sigma}^H \mathbf{\Gamma} \mathbf{\Sigma}$	$\mathbf{Q} = \mathbf{\Sigma}$	OFDMA

code allocation scheme with maximal diversity gain easily. In Table I, we take several typical orthogonal codes as examples and list the corresponding code allocation schemes, where $\mathbf{T}$ denotes the 'Paley-order' Walsh code.

IV. Performance Analysis

In this section, we will analyze the performance of the presented transceiver scheme with the metric of WSINR. Since the impact of interference and noise on performance is different, by introducing a weighting factor ρ^* on the interference, the WSINR can reflect the system performance more accurately than SINR.

Rewrite the linear estimation of the desired signal as

$$\hat{d}_k = \mathbf{z}^H \mathbf{d} + \mathbf{w}^H \mathbf{n}_k,$$

where $\mathbf{z}^H = \mathbf{w}^H \mathbf{H}_k \mathbf{C} \mathbf{A}$. The ith element of $\mathbf{z}$ and $\mathbf{w}$, z_i and w_i can reflect the influence of d_i and n_i on d_k. Hence the power of signal, interference and noise of user k are $P_{s,k} = |z_k|^2$, $P_{I,k} = \sum_{i=0,i\neq k}^{K-1} |z_i|^2 = \mathbf{z}^H \mathbf{z} - |z_k|^2$ and $P_{n,k} = \sigma_n^2 \mathbf{w}^H \mathbf{w}$, respectively. Then the output WSINR after equalization is

$$\gamma_k = \frac{P_{s,k}}{\rho^* P_{I,k} + P_{n,k}} = \frac{|z_k|^2}{\rho^*(\mathbf{z}^H \mathbf{z} - |z_k|^2) + \sigma_n^2 \mathbf{w}^H \mathbf{w}}. \quad (10)$$

The performance of systems with various orthogonal codes $\mathbf{X}$, spread-spectrum modes $\mathbf{P}$ and equalizers $\mathbf{w}$ can be evaluated and compared with (10). We can show after some manipulations that the symbol-level TDE with $\rho = \rho^*$ can achieve the maximal WSINR. We omit the proof due to the lack of space.

Under equal power control, it is shown from (8) that the symbol-level FDE is equivalent to the symbol-level WMMSE-TDE and also can achieve the maximal WSINR by choosing $\rho = \rho^*$. The maximal WSINR is

$$\gamma_k = \frac{E_k}{\sigma_n^2} \frac{\mathbf{v}_k^H \mathbf{\Gamma}_B^H \left(\mathbf{P}^H \mathbf{\Delta}_k \mathbf{P} + \beta^* \mathbf{I}_K\right)^{-1} \mathbf{P}^H \mathbf{\Delta}_k \mathbf{P} \mathbf{\Gamma}_B \mathbf{v}_k}{\mathbf{v}_k^H \mathbf{\Gamma}_B^H \left(\mathbf{P}^H \mathbf{\Delta}_k \mathbf{P} + \beta^* \mathbf{I}_K\right)^{-1} \mathbf{\Gamma}_B \mathbf{v}_k}, \quad (11)$$

where $\beta^* = \sigma_n^2/(E_k \rho^*)$.

Under unequal power control, the symbol-level FDE can not achieve the maximal WSINR any more. With $\rho \to \infty$, the WMMSE-FDE becomes a ZF-FDE and its WSINR is

$$\gamma_k = \frac{E_k}{\sigma_n^2} \frac{1}{\mathbf{v}_k^H \mathbf{\Gamma}_B^H \left(\mathbf{P}^H \mathbf{\Delta}_k \mathbf{P}\right)^{-1} \mathbf{\Gamma}_B \mathbf{v}_k}, \quad (12)$$

so it is obvious that the symbol-level ZF-FDE mitigates the influence of noise enhancement by de-spreading and eliminates MAI thoroughly. Furthermore, it doesn't require the power estimation of other users. The symbol-level ZF-FDE can achieve a good performance and complexity trade-off in this case. Consequently, it can be widely applied, especially in the systems with different requirement for the quality of service.

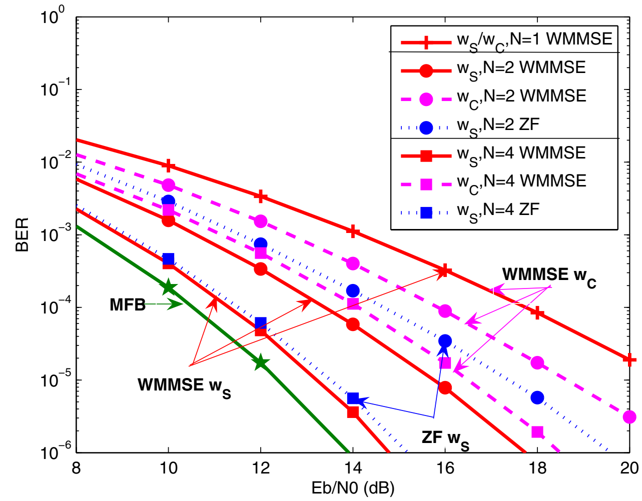

Fig. 3. BER versus E_b/N_0 ($E_b = E_k/2$).

V. Simulation Results

In the simulations, we consider $M = 32$, equal power control, quadrature phase-shift keying (QPSK) modulation and independent and identically-distributed (i.i.d.) 8-path Rayleigh fading channels with the flat power delay profile. The results are obtained with 10000 Monte Carlo tests.

Figure 3 shows the bit error rate (BER) versus E_b/N_0 with the codes $\mathbf{X} = \mathbf{F}^H \mathbf{T}$ and the mode $\mathbf{P} = \mathbf{\Sigma}$. Under equal power control, the FDE is equivalent to the TDE and the optimal value of interference weighting factor $\rho = \rho^* = 1.2$, which is obtained by simulations. The matched filter bound (MFB), which is the performance of RAKE receiver without interference, is also provided as reference.

As Fig. 3 shown, all the performance can be improved by increasing the spreading gain N. When $N > 1$, the symbol-level WMMSE-FDE can outperform the chip-level WMMSE-FDE under the same N due to obtaining more diversity gain. Though the symbol-level ZF-FDE couldn't perform as well as the symbol-level WMMSE-FDE with the optimal factor, the performance gap between them decreases significantly when N increases.

Figure 4 compares the required E_b/N_0 to achieve BER=10^{-3} when using different spread-spectrum modes and orthogonal codes. The curve with the legend of 'Flat Fading' represents the performance obtained in the flat Rayleigh fading channel.

The performance gap between different codes with the same spread-spectrum mode reflects the diversity gain provided by the orthogonal codes $\mathbf{X}$, which can be evaluated from the spectra of the 'spreading signals' with these codes (shown

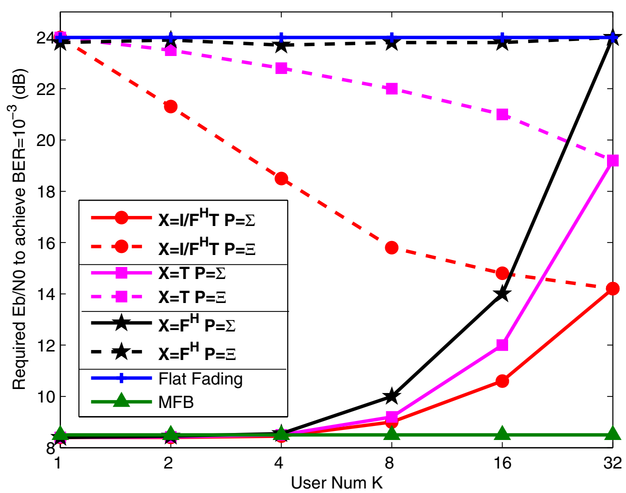

Fig. 4. The requried E_b/N_0 to achieve BER=10^{-3} versus K.

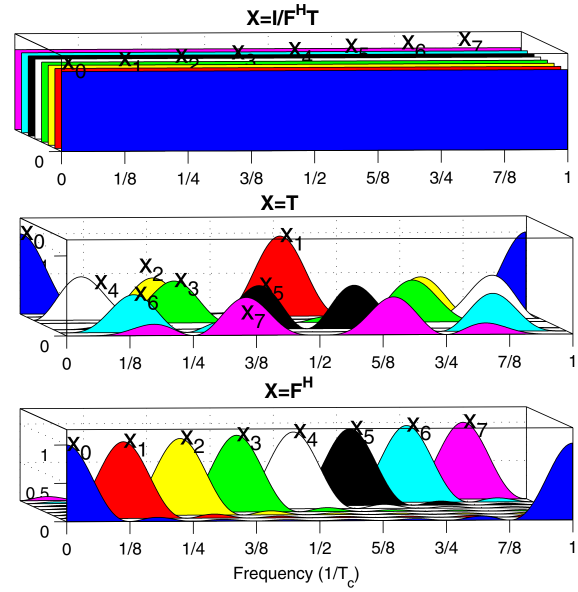

Fig. 5. Spectra of the 'spreading signals' with several typical orthogonal codes ($M = 8$).

in Fig . 5). Each 'spreading signal' of systems with the codes $\mathbf{X} = \mathbf{F}^H$ is a single subcarrier, so there is no diversity gain. However, in systems employing the codes $\mathbf{X} = \mathbf{I}/\mathbf{F}^H\mathbf{T}$, each 'spreading signal' covers the whole bandwidth. Hence the larger the bandwidth is, the larger diversity gain will be. Even if there is no spreading gain, the diversity gain can still be obtained by multi-code de-spreading for the orthogonal codes [11]. As for systems with the codes $\mathbf{X} = \mathbf{T}$, each 'spreading signal' covers only part of the signal bandwidth, so the diversity gain is between those of the two former codes.

The performance difference between the modes $\mathbf{P} = \mathbf{\Sigma}$ and $\mathbf{P} = \mathbf{\Xi}$ with the same code illustrates the diversity gain of spectrum-spread modes. The mode $\mathbf{P} = \mathbf{\Sigma}$ provides the maximal diversity gain, so the performance with each kind of codes can be improved by increasing $N = 32/K$. However, the mode $\mathbf{P} = \mathbf{\Xi}$ itself can not provide any gain. It will even reduce the diversity gain provided by codes. Consequently, when $N = 32/K$ increase, the performance will degrade gradually and approach to that obtained in the flat fading channel.

VI. Conclusion

This paper presents a spread-spectrum mode to provide the maximal diversity gain and a one-tap symbol-level FDE for downlink orthogonal multi-code spread-spectrum systems.

It's shown that the spread-spectrum mode design is very crucial for spread-spectrum systems. A good mode can improve their performance, while a bad mode will even result in a worse performance than the nonspread-spectrum systems.

With the designed spread-spectrum mode, the proposed symbol-level FDE can outperform the chip-level FDE with the same complexity and approach to the matched filter bound with the increasing of spreading gain. Under the equal power control, the symbol-level WMMSE-FDE can achieve the maximal WSINR with proper interference weighting factor. With unequal power control, the suboptimal symbol-level ZF-FDE can trade off performance with complexity very well.

The designed transceiver scheme is independent of the orthogonal codes, so it can be applied to all kinds of orthogonal multiple access systems, such as DS-CDMA, MC-CDMA and OFDMA.

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