ACHIEVING THE DEGREES OF FREEDOM OF RELAY-AIDED INTERFERENCE BROADCAST CHANNELS

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ABSTRACT

This paper studies the degrees of freedom (DoF) of relay-aided interference broadcast channels and the relay resource to achieve them. We show that, with the aid of half-duplex relays, a G-cell system can achieve $GM_{BS}/2$ DoF, where M_{BS} is the number of transmit antennas in each cell. By studying the interference-free constraints, we obtain a lower bound of the relay resource that ensures interference-free transmission. Instead of directly solving the complicated multivariate problem with cubic interference-free constraints, we propose to relax the problem to linear equations by randomly initiating certain variables, and derive an achievable bound on relay resource. Numerical results show that the relay-aided interference broadcast channels require less relay resource than relay-aided interference channels and transmission protocol has large impact on the required resources.

Index Terms— relay-aided interference broadcast channel, degrees of freedom, relay resource

1. INTRODUCTION

Interference channels (ICs) or interference broadcast channels (IBCs) refer to setups where multiple transmit and receive pairs communicate over a shared common resource (time or frequency) and there is no data sharing among transmitters. In ICs, each transmitter communicates with only one user, while in IBCs, each transmitter communicates with multiple users.

The degrees of freedom (DoF), which is a first order approximation of the sum-rate capacity in the high SNR regime, has been studied to approximate the capacity of ICs. It has been shown that for a G-cell MIMO IC, where each transmitter and receiver is equipped with M antennas, a total DoF of $GM/2$ can be achieved through interference alignment (IA) with infinite symbol extensions (time or frequency) [1]. However, when only finite or no symbol extension is allowed, the achievable DoF is limited by the sum of transmit and receive antennas [2, 3], indicating that the DoF cannot grow linearly with the cell number G.

It was noted in [4] that employing a half-duplex relay in IC makes IA feasible with limited symbol extensions. It has been verified that relays can help achieve $GM/2$ DoF in ICs with the cost of multiple relays or additional antennas at the relay [5, 6]. Then a key question is the minimum requirement of relay resources including the number of relays and relay antennas to achieve interference-free transmission. The minimum number of single-antenna relays was obtained in [5] for the two-hop IC. In [6], the authors found the minimum number of relay antennas to achieve interference-free transmission in the IC with a single relay. In [7] and [8], a novel idea referred to as *aligned interference neutralization (IN)* was introduced in a two-cell IC with full-duplex relay. Interference are eliminated over-the-air at the the user side so that the information theoretic outer bound of the two-cell IC channel can be achieved.

Compared to the widely studied ICs, little is known about the more generalized IBCs. Prior research on the DoF of IBCs only considered several special cases (e.g., the two-cell scenario [9, 10]). It was found in [11] that the DoF of IBCs is also limited by the sum of transmit and receive antennas. The natural questions to ask are: Is it also possible to achieve $GM/2$ DoF with the help of relays in IBCs? What is the cost required at the relay nodes and how does this cost compare to that in the ICs? In this paper, we intend to answer these questions.

We limit ourselves to half-duplex relays because they are practical. We consider different transmission protocols because they have a great impact on the required relay antennas. We show that a total DoF of $GM/2$ is achievable as well in IBCs with the help of relays. We study the required relay resource in IBCs and compare it with that in ICs.

2. SYSTEM MODEL AND TRANSMISSION PROTOCOLS

We consider a system with G coordinating cells and K users in each cell. The BS and each user is, respectively, equipped with M_{BS} and M_{UE} antennas. Each BS transmits d data streams for each user. We assume $M_{BS} = Kd$ and $M_{UE} = d$. Every BS and user have the channel state information (CSI) of all links. Assume there are N_R half-duplex amplify-and-forward (AF) relays in the system, and the number of antennas at each relay is M_R. We consider the symmetric case here for simplicity although the following work can be extended to asymmetric network.

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We assume all links among the BSs, users and relays are Rayleigh block fading channels without channel extensions. Denote the channel between the ith BS and the rth relay as $\mathbf{F}_{r,i}$ (of size $M_r \times M_{BS}$), the channel between user i_k and relay r as $\mathbf{G}_{i_k,r}$ (of size $M_{UE} \times M_r$) and the direct link between user i_k and BS j as $\mathbf{H}_{i_k,j}$ (of size $M_{UE} \times M_{BS}$). In the cellular network where users and relays are located at cell edge, inter-cell interference is a major bottle-neck, which is a suitable scenario for the considered setting.

Since the relay is half duplex, downlink transmission is divided into two time slots. In the first time slot, the BS i employs an $M_{BS} \times d$ transmit matrix $\mathbf{V}_{i_k}^{(1)}$ to convey the symbol $\mathbf{x}_{i_k} \in \mathbb{C}^{d\times 1}$ to its kth user i_k. The transmitted signal at BS i can be written as $\mathbf{V}_i^{(1)}\mathbf{X}_i = \sum_{k=1}^{K} \mathbf{V}_{i_k}^{(1)}\mathbf{x}_{i_k}$, where $\mathbf{V}_i^{(1)} = \begin{bmatrix} \mathbf{V}_{i_1}^{(1)} & \cdots & \mathbf{V}_{i_K}^{(1)} \end{bmatrix}$ and $\mathbf{X}_i = \begin{bmatrix} \mathbf{x}_{i_1}^H & \cdots & \mathbf{x}_{i_K}^H \end{bmatrix}^H$ are the precoder and data matrix satisfying $E\{||\mathbf{x}_{i_k}||^2\} = 1$ and $(\cdot)^H$ is the conjugate transpose.

In the second time slot, the relays forward a processed version of the received signal. Depending on whether the BSs retransmit[1] or stay idle in this phase, there are two transmission protocols. In the sequel, we use η to denote the state of BS in the second time slot. When $\eta = 1$, the BSs retransmit data using a different precoder $\mathbf{V}_{i_k}^{(2)}$ [4]; when $\eta = 0$, the BSs stay idle and only the relays transmit [6].

At the end the the second time slot, the received signal at user i_k can be expressed as follows, assuming there is a storage unit at the user.

$$
\begin{aligned}
\mathbf{Y}_{i_k} &= \begin{bmatrix} \mathbf{U}_{i_k}^{(1)H} & \mathbf{U}_{i_k}^{(2)H} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{i_k}^{(1)} \\ \mathbf{Y}_{i_k}^{(2)} \end{bmatrix} \\
&= \underbrace{\begin{bmatrix} \mathbf{U}_{i_k}^{(1)} \\ \mathbf{U}_{i_k}^{(2)} \end{bmatrix}^H \begin{bmatrix} \mathbf{H}_{i_k,i} & \mathbf{0} \\ \sum_{r=1}^{N_R} \mathbf{G}_{i_k,r}\mathbf{\Gamma}_r\mathbf{F}_{r,i} & \mathbf{H}_{i_k,i} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{i_k}^{(1)} \\ \eta\mathbf{V}_{i_k}^{(2)} \end{bmatrix} \mathbf{x}_{i_k}}_{\text{desired signal}} + \\
&\quad \underbrace{\sum_{j'_k \neq i_k} \begin{bmatrix} \mathbf{U}_{i_k}^{(1)} \\ \mathbf{U}_{i_k}^{(2)} \end{bmatrix}^H \begin{bmatrix} \mathbf{H}_{i_k,j} & \mathbf{0} \\ \sum_{r=1}^{N_R} \mathbf{G}_{i_k,r}\mathbf{\Gamma}_r\mathbf{F}_{r,j} & \mathbf{H}_{i_k,j} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{j_{k'}}^{(1)} \\ \eta\mathbf{V}_{j_{k'}}^{(2)} \end{bmatrix} \mathbf{x}_{j_{k'}}}_{\text{Interference}} \\
&\quad + \underbrace{\begin{bmatrix} \mathbf{U}_{i_k}^{(1)} \\ \mathbf{U}_{i_k}^{(2)} \end{bmatrix}^H \begin{bmatrix} \mathbf{n}_{i_k}^{(1)} \\ \tilde{\mathbf{n}}_{i_k}^{(2)} \end{bmatrix}}_{\text{effective noise}}. \qquad (1)
\end{aligned}
$$

where $\mathbf{\Gamma}_r$ is the relay processing matrix of relay r, $\mathbf{U}_{i_k}^{(1)}$ and $\mathbf{U}_{i_k}^{(2)}$ (of size $M_{UE} \times d$) are receiving matrices. With half-duplex transmission, the size of the effective channel from the transmitter to a receiver is extended to $2M_{UE} \times ((1+\eta)M_{BS})$.

[1]In conventional single-cell three-node cooperation, the BS sends new data in the second time slot to achieve higher rate. In IBCs, however, the situation is more complicated if the BSs send new data. We thus do not consider this case here and leave the topic for future research.

3. INTERFERENCE-FREE TRANSMISSION FOR RELAY-AIDED IBCS

In this section, we study the interference-free constraint of the G-cell relay-aided IBCs and show that the maximum achievable DoF is $GM_{BS}/2$. The relay resource to achieve the DoF is investigated. By analyzing the necessary condition for interference-free transmission we obtain a lower bound on the required relay resource. In addition, we linearize the interference-free problem by randomly selecting certain processing matrices to obtain an achievable bound of relay resource that ensures interference-free transmission.

We enforce the interference-free constraint by setting the coefficients of the interference term in (1) to zero, i.e.,

$$
\begin{bmatrix} \mathbf{U}_{i_k}^{(1)} \\ \mathbf{U}_{i_k}^{(2)} \end{bmatrix}^H \begin{bmatrix} \mathbf{H}_{i_k,j} & \mathbf{0} \\ \sum_{r=1}^{N_R} \mathbf{G}_{i_k,r}\mathbf{\Gamma}_r\mathbf{F}_{r,j} & \mathbf{H}_{i_k,j} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{j_{k'}}^{(1)} \\ \eta\mathbf{V}_{j_{k'}}^{(2)} \end{bmatrix} \mathbf{x}_{j_{k'}} = 0. \qquad (2)
$$

The DoF represents the total number of interference-free data streams that can be transmitted through the system. When (2) is satisfied, a total number of GKd data steams can be transmitted through an IBC within two time slots. Consequently, the maximum DoF of the IBCs is $GM_{BS}/2$. In the following, we obtain two bounds of the required relay resource by examining both the necessary and sufficient conditions for interference-free transmission.

3.1. Lower bound on the required relay resource

In [2, 3], the authors stated that for the interference-free constraints described by a group of multivariate high-order equations, when the number of independent variables is less than that of equations, the interference-free constraints are infeasible. In the relay-aided IBC channel, the interference-free constraint of (2) is given via a set of multivariate cubic equations, so the same method can be applied. Following the same logic, we obtain a lower bound on the relay resource for relay-aided IBCs, which means that when the relay resource is less than this lower bound, (2) is infeasible.

We first count the number of equations in (2). To ensure that all the interference are canceled at each user, $(GK-1)d^2$ equations in total are needed. Since there are GK users, the total number of equations is $N_e = GK(GK-1)d^2$.

The number of independent variables in $\mathbf{\Gamma}_r$ is M_R^2. For the effective receive matrix $\mathbf{U}_{i_k} = \begin{bmatrix} \mathbf{U}_{i_k}^{(1)H} & \mathbf{U}_{i_k}^{(2)H} \end{bmatrix}^H$ and transmit precoder $\mathbf{V}_{i_k} = \begin{bmatrix} \mathbf{V}_{j_{k'}}^{(1)H} & \eta\mathbf{V}_{j_{k'}}^{(2)H} \end{bmatrix}^H$ with size $2M_{UE}\times d$ and $((\eta+1)M_{BS}) \times d$, respectively, the number of independent variables can be counted as $d(2M_{UE}-d)$ and $d((\eta+1)M_{BS}-d)$ as in [2], which ensures that d data streams can be transmitted at the BS and detected at the user. Since there are GK users and N_R relays, the total number of variables is $N_v = GKd(2M_{UE}-d) + GKd((\eta+1)M_{BS}-d) + N_R M_R^2$.

To ensure $N_v \geq N_e$, the required relay resources satisfy

$$
N_R M_R^2 \geq GK(GK-1)d^2 - GKd(2M_{UE} + (\eta+1)M_{BS} - 2d), \qquad (3)
$$

i.e., (2) is unsolvable when the inequality does not hold. However, this necessary condition does not tell whether the interference-free equation (2) is feasible and how to solve the multivariate cubic equations. From prior studies in IA, we know that it is already rather difficult to solve a set of multivariate quadratic equations. We thus propose a linearization method in next section to solve this problem. The price paid is the increase of relay resource, so the result should be viewed as an achievable bound on relay resources for interference-free transmission.

3.2. Achievable bound on the relay resource

We first rewrite the interference-free equation (2) as

$$\mathbf{U}_{i_k}^{(1)H}\mathbf{H}_{i_k,j}\mathbf{V}_{j_{k'}}^{(1)} + \eta\mathbf{U}_{i_k}^{(2)H}\mathbf{H}_{i_k,j}\mathbf{V}_{j_{k'}}^{(2)} + \sum_{r=1}^{N_R}\mathbf{U}_{i_k}^{(2)H}\mathbf{G}_{i_k,r}\mathbf{\Gamma}_r\mathbf{F}_{r,j}\mathbf{V}_{j_{k'}}^{(1)} = \mathbf{0}, j_{k'} \neq i_k. \quad (4)$$

Suppose the precoder $\mathbf{V}_{j_{k'}}^{(1)}$ and receiving matrix $\mathbf{U}_{i_k}^{(2)H}$ are known before transmission, then (4) becomes linear because each of its terms has only one unknown variable: $\mathbf{U}_{i_k}^{(1)H}$ in the first term, $\mathbf{V}_{j_{k'}}^{(2)}$ in the second term, and $\mathbf{\Gamma}_r$ in the third. The cubic equations in (2) then degenerate to linear equations. After a Kronecker product transform $\overrightarrow{\mathbf{AXB}} = (\mathbf{B}^T \otimes \mathbf{A})\overrightarrow{\mathbf{X}}$, (4) can be rewritten in the following homogeneous form

$$\left(\left(\mathbf{H}_{i_k,j}\mathbf{V}_{j_{k'}}^{(1)}\right)^T \otimes \mathbf{I}_d\right)\overrightarrow{\mathbf{U}}_{i_k}^{(1)H} + \left(\mathbf{I}_d \otimes \left(\mathbf{U}_{i_k}^{(2)H}\mathbf{H}_{i_k,j}\right)\right)\eta\overrightarrow{\mathbf{V}}_{j_{k'}}^{(2)} + \sum_{r=1}^{N_R}\left(\left(\mathbf{F}_{r,j}\mathbf{V}_{j_{k'}}^{(1)}\right)^T \otimes \left(\mathbf{U}_{i_k}^{(2)H}\mathbf{G}_{i_k,r}\right)\right)\overrightarrow{\mathbf{\Gamma}}_r = \mathbf{0}, \quad (5)$$

where $j_{k'} \neq i_k$ and $\overrightarrow{\mathbf{X}}$ denotes the column vector that consists of all columns of $\mathbf{X}$.

These equations with independent coefficients have nontrivial solutions if and only if the number of equations is less than that of variables [12].

Randomly selecting $\mathbf{V}_{j'_k}^{(1)}$ and $\mathbf{U}_{i_k}^{(2)H}$ can ensure d data steams to be transmitted, so $\mathbf{U}_{i_k}^{(1)H}$, $\mathbf{V}_{i_k}^{(2)}$ and $\mathbf{\Gamma}_r$ are all used for interference coordination, resulting in the total number of variables as $N_v = GKM_{UE}d + \eta GKM_{BS}d + N_R M_r^2$.

To have $N_v \geq N_e + 1$, the required number of relay resource must satisfy the following relationship,

$$N_R M_R^2 \geq GK\left(GK-1\right)d^2 + 1 - GKd\left(M_{UE} + \eta M_{BS}\right). \quad (6)$$

We now show that the maximum of $GM_{BS}/2$ DoF can be achieved by proposing a centralized algorithm that jointly designs the BS precoder, the relay forwarding matrix and the user receiving matrix. In order to solve the equations in (5), we first randomly initialize $\mathbf{V}_{i_k}^{(1)}$ and $\mathbf{U}_{i_k}^{(2)}$. Then equation (5) can be rewritten as $\mathbf{H}_e\mathbf{p}_e = \mathbf{0}$, where $\mathbf{H}_e$ is composed of the coefficient matrices in (5) and $\mathbf{p}_e = \left[\overrightarrow{\mathbf{U}}_{1_1}^{(1)H}, \cdots, \overrightarrow{\mathbf{U}}_{G_K}^{(1)H}, \overrightarrow{\mathbf{V}}_{1_1}^{(2)}, \cdots, \overrightarrow{\mathbf{V}}_{G_K}^{(2)}, \overrightarrow{\mathbf{\Gamma}}_1, \cdots, \overrightarrow{\mathbf{\Gamma}}_{N_R}\right]$.

Let $\mathbf{H}_e = \mathbf{U}_e\mathbf{\Sigma}_e\mathbf{V}_e^H$ be the singular value decomposition of $\mathbf{H}_e$. Set $\mathbf{p}_e$ as the last column of $\mathbf{V}_e$ so that the desired matrices $\mathbf{U}_{i_k}^{(1)H}$, $\mathbf{V}_{j_{k'}}^{(2)}$ and $\mathbf{\Gamma}_r$ are obtained by rewriting the vector $\mathbf{p}_e$ into original matrix form.

As long as (6) is satisfied, (4) can be solved, i.e., (6) is a sufficient condition for interference-free transmission. In next section, we will show the gap between the achievable bound and the lower bound obtained by the necessary condition which indicates a possible future work of how to achieve the latter one.

4. SIMULATION AND NUMERICAL RESULTS

In this section, we first show by simulations that the maximum DoF of $GM_{BS}/2$ can be achieved when the relay resource satisfies (6). We then compare the gap between the two bounds, and study the impact of transmission protocols numerically. Finally, we compare ICs and IBCs in terms of required relay resources to achieve the same DoF.

Figure 1 shows the sum rate achieved by using the centralized algorithm with different system parameters. We set $N_R = 1$, $\eta = 1$ and the number of relay antennas M_R is obtained from (6). The maximum DoF of $GM_{BS}/2$ under each configuration is depicted as the dashed lines. We can see from the slope of the sum rate that the maximal DoF is achievable using our linearized algorithm.

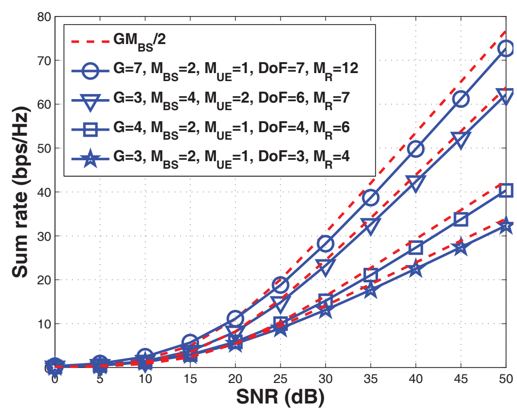

Fig. 1. Achievability of $GM_{BS}/2$ DoF.

Comparing (3) and (6), we can see the impact of different transmission protocols. When $\eta = 1$, there are more variables to be designed so that less relay resource is required. Since the direct link in the second time slot provides additional information, to save relay resources, it is better for the BSs to transmit in the relay-forwarding phase rather than being idle.

In Fig. 2, both bounds of the required relay antennas of the two protocols are presented. We consider the G-cell IBC with $K = 2$ users in each cell and a single relay in the system. Each user is equipped with 2 antennas for receiving 2 streams. The number of antennas at the BS is $M_{BS} = 4$.

The top line with legend "GM_{BS}" represents the total number of data streams transmitted within two time slots. When $M_R = GM_{BS}$, the relay is able to receive and forward all the data streams without interference. The proposed linearization algorithm reduces the relay antennas because part of the transmit precoders and receive matrices are jointly designed with the relay-forwarding matrix to coordinate the interference. The gap between the two bounds is due to the linearization of the interference-free constraints, but the gap is no more than two antennas in the considered setting.

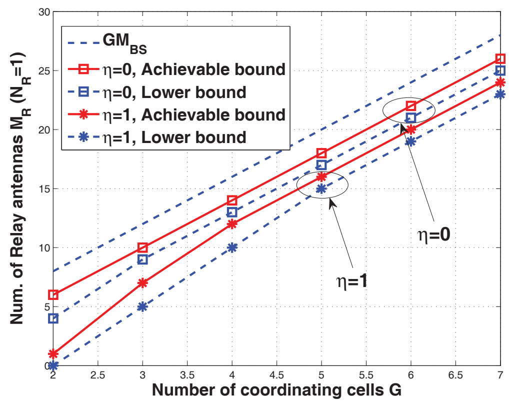

Fig. 2. Lower and achivable bounds with different transmission protocols.

The relay-aided IBC reduces to an IC when $K = 1$ and all the above results are still valid. The required number of relay antennas using linearized method in the IBC and IC are presented in Fig. 3, we compare them under the same DoF $D = G_{IC}M_{IC}/2 = G_{IBC}M_{IBC}/2$ and the same number of user antennas $M_{UE,IC} = M_{UE,IBC} = d$, where $N_R = G$. The result shows that the IBC requires fewer relay antennas than the IC. The curves are not smooth because the number of relays should be integers. Since $M_{BS} = Kd$, the number of transmit antennas in the IBC is greater than that in the IC. As a result, to achieve the same DoF, IC requires more coordinating cells than IBC.

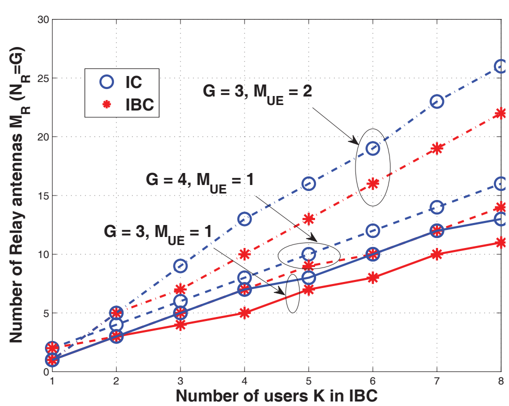

Fig. 3. IC versus IBC with the same DoF and M_{UE}.

5. CONCLUSION

In this paper, we studied the achievable DoF in relay-aided interference broadcast channel and the required relay resource. We showed that $GM_{BS}/2$ DoF can be achieved with the aid of AF half-duplex relays. Two bounds of the required relay resources in the IBC channel based on the necessary and sufficient conditions of interference-free transmission were obtained. We also showed that transmission protocols has a great impact on the number of relay antennas. When the total DoF and user antennas are given, interference broadcast channels need less relay antennas than interference channels.

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