Necessity of Phase Ambiguity Quantization for Limited Feedback Coordinated Multi-point Transmission

Di Su and Chenyang Yang
School of Electronics and Information Engineering, Beihang University, Beijing 100191, P. R. China
Email: disu@ee.buaa.edu.cn and cyyang@buaa.edu.cn

***Abstract*— Per-cell codebook based limited feedback is desirable for coordinated multi-point (CoMP) transmission system due to its flexibility and scalability. In this paper, we study if and when the quantization performance of CoMP channel direction information will benefit from the phase ambiguity (PA) feedback. To this end, we analyze the average quantization performance of the feedback strategies with or without PA feedback, respectively. By deriving the approximated bounds, we show that when the number of coordinated base stations is large and the number of antennas at each base station is small, the PA feedback will introduce evident performance gain especially for cell-edge users. Simulation results validate our analysis.**

I. Introduction

Coordinated multi-point (CoMP) transmission has drawn abroad attention recently [1–3]. When all the coordinated base stations (BSs) share both data and channel state information (CSI) and the CSI acquired at the coordinated BSs is of high quality, coherent cooperative transmission[1] can provide high spectral efficiency for cellular systems. This however leads to considerable feedback overhead for frequency division duplexing (FDD) systems.

Limited feedback technique is widely applied for reporting CSI to the BS in multi-input-multi-output (MIMO) systems and has been extensively studied [4]. Applying vector quantization theory to design a global codebook for CoMP channel simply by treating it as a large MIMO channel is optimal. However, such a global codebook suffers from the lack of flexibility and compatibility [5]. In fact, CoMP channel is an aggregation of multiple single cell channels between each BS and each mobile station (MS). Moreover, the cooperative cluster may be dynamic [3]. Therefore, it is more desirable to design per-cell codebook based feedback scheme, where each single cell channel is quantized with a single codebook.

When we employ per-cell codebooks to quantize CoMP channels, there are various issues to optimize the quantization performance [6–8], although we can simply reuse the codebooks designed for single-cell systems. Depending on the information fed back to the BS, CoMP channel reconstruction method can be optimized as in single cell MIMO systems [9]. Considering the fact that the contribution of each per-cell channel direction information (CDI) to the global CDI differs due to the channel asymmetry, bit allocation for the per-cell codebooks can provides significant performance gain [7]. Codeword selection is another critical issue. Per-cell codewords can be selected independently to minimize the quantization error of each per-cell CDI, or jointly to minimize the quantization error of the global CDI. Independent codeword selection is of low complexity, but is inferior to the joint codeword selection owing to a phase ambiguity (PA) led by the per-cell quantization [6]. Intuitively, its performance can be enhanced by co-phasing the multi-cell channels with the feedback information of PA. Given the same overall bits for feedback, it has been shown that the independent codeword selection with PA feedback outperforms the joint selection without PA feedback when the bit is allocated to maximize the instantaneous quantization accuracy of the global CDI [6]. However, this kind of bit allocation is not feasible in practice since it will introduce more signaling overhead. When PA is fed back, joint codeword selection performs the best.

In this paper, we try to find out if and when the PA feedback will have benefit, when bit allocation is optimized for average quantization performance. To focus on the necessity of the PA quantization, we simply use the large scale fading factors to re-construct the CoMP channel and employ the joint codeword selection. We will analyze the quantization performance of the per-cell codebook based feedback strategies with or without PA quantization.

II. System Model

We consider a coherent CoMP system, where N BSs each equipped with n_t antennas cooperatively serve multiple single antenna MSs in a frequency, time or spatial division manner.

The downlink CoMP channel of each MS is comprised of all the per-cell channels from each coordinated BS to the MS, which can be represented by

$$\mathbf{h}_{\mathrm{C}} = [\alpha_1 \mathbf{h}_1, \cdots, \alpha_N \mathbf{h}_N] = \mathbf{h}\mathbf{R}^{\frac{1}{2}}, \tag{1}$$

where α_k is the large scale fading factor including the path loss and shadowing, $\mathbf{h}_k \in \mathbb{C}^{1\times n_t}$ is the small scale fading

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[1]For simplicity, we refer coherent cooperative transmission as CoMP in the rest of the paper, though CoMP has various forms.

channel vector from the kth coordinated BS to the MS. To simplify the analysis and highlight the feature of CoMP channels, we assume that the per-cell channels are uncorrelated, and each entries in $\mathbf{h}_k$ subjects to $\mathcal{CN}(0,1)$. Then, $\mathbf{R} = diag\left\{\alpha_1^2\mathbf{I}_{n_t}, \cdots, \alpha_N^2\mathbf{I}_{n_t}\right\}$, which is a diagonal matrix.

The global CDI vector of CoMP channel is denoted as

$$\mathbf{g} \triangleq \frac{\mathbf{h}}{\| \mathbf{h} \|} = [g_1\bar{\mathbf{h}}_1, \cdots, g_N\bar{\mathbf{h}}_N], \tag{2}$$

where $\bar{\mathbf{h}}_k \triangleq \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ is the kth per-cell CDI vector and $g_k = \frac{\alpha_k\|\mathbf{h}_k\|}{\sqrt{\sum_{i=1}^{N}\alpha_i^2\|\mathbf{h}_i\|^2}}$ is the weighting factor, which reflects the contribution of the per-cell CDI to the global CDI.

We quantize the global CDI vector based on per-cell codebooks. As in [6], the BS re-constructs the global CDI vector as $\hat{\mathbf{g}} \triangleq [\hat{g}_1\hat{\mathbf{h}}_1, \cdots, \hat{g}_N\hat{\mathbf{h}}_N]$, where $\hat{g}_k \triangleq \frac{\alpha_k}{\sqrt{\sum_{i=1}^{N}\alpha_i^2}}$, i.e., only large scale fading factors are employed. $\hat{\mathbf{h}}_k = \mathbf{c}_{i_k}^H$ is the kth quantized per-cell CDI vector, which is selected from the corresponding codebook with unitary codewords $\mathbf{c}_{i_k} \in \mathcal{C}_k$ according to some criteria.

The quantization error of global CDI is defined as

$$d(\mathbf{g}, \hat{\mathbf{g}}) \triangleq \sqrt{1 - |\mathbf{g}\cdot\hat{\mathbf{g}}^H|^2} = \sqrt{1-\cos^2\theta}, \tag{3}$$

where $\cos^2\theta \triangleq |\mathbf{g}\cdot\hat{\mathbf{g}}^H|^2$. For simplicity, we will use $\cos^2\theta$ as the performance metric in the following.

When the PA is not fed back, the quantization accuracy is

$$\cos^2\theta|_{\mathrm{NPA}} = \Big|\sum_{k=1}^{N} g_k\hat{g}_k\cdot\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\Big|^2 = \Big|\sum_{k=1}^{N} g_k\hat{g}_k\cdot|\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H|e^{j\xi_k}\Big|^2, \tag{4}$$

where $e^{j\xi_k} \triangleq \frac{\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H}{|\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H|}$ is the kth per-cell PA induced by per-cell codebook based quantization.

When the PA is quantized and fed back to the BS, it can be compensated during the global CDI re-constriction. The correction will not be perfect since PA has quantization error, which is denoted as $e^{j\Delta\xi_k} \triangleq e^{j(\xi_k - \hat{\xi}_k)}$. The quantization accuracy of the global CDI becomes

$$\begin{aligned}\cos^2\theta|_{\mathrm{PA}} &= \Big|\sum_{k=1}^{N} g_k\hat{g}_k\cdot|\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H|\cdot e^{j(\Delta\xi_k)}\Big|^2 \\ &= \left[\left(\sum_{k=1}^{N} g_k\hat{g}_k\cdot|\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H|\right)^2 - \Phi_{\mathrm{PA}}\right],\end{aligned} \tag{5}$$

where $e^{j\hat{\xi}_k} = c_{j_k}^{\xi}$ is the quantized PA, which is selected from a scalar codebook $\mathcal{C}_k^{\xi}$. $\Phi_{\mathrm{PA}} \triangleq 2\sum_{k=1}^{N}\sum_{m>k}^{N} g_k\hat{g}_k|\bar{\mathbf{h}}_k\hat{\mathbf{h}}_k^H|\cdot g_m\hat{g}_m|\bar{\mathbf{h}}_m\hat{\mathbf{h}}_m^H|\cdot(1-\cos(\Delta\xi_k - \Delta\xi_m))$ is the impact of the PA quantization error on the quantization accuracy.

III. Average Quantization Performance with or without PA Feedback

In the following, we will analyze the average quantization accuracy $\mathbb{E}[\cos^2\theta]$ of the feedback strategies with and without PA quantization, respectively.

A. Quantization Performance without PA Feedback

Denote the codebook for the kth per-cell CDI quantization as $\mathcal{C}_k$, which is of size B_k and $\sum_{k=1}^{N} B_k = B_{\mathrm{sum}}$. The per-cell codewords are selected jointly to minimize the quantization error of the global CDI, which is

$$[i_1, \cdots, i_N] = \arg\max_{\mathbf{c}_{i_k}\in\mathcal{C}_k}\Big|\sum_{k=1}^{N} g_k\hat{g}_k\cdot|\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|e^{j\xi_k}\Big|^2. \tag{6}$$

The closed-form expression of $\mathbb{E}[\cos\theta|_{\mathrm{NPA}}]$ is very difficult to derive. To gain useful insight into the problem, we consider a special case, where $\alpha_1 = \cdots = \alpha_N$, which is the worst case in terms of the negative impact of PA [6,8]. Then the CoMP channel can be viewed as a channel of single-cell MIMO system with a "super BS" having Nn_t transmit antennas. When we quantize this equivalent "MIMO channel" with a globally generated codebook with size of B_{sum}, the averaged quantization accuracy is upper bounded by $1-2^{-\frac{B_{\mathrm{sum}}}{Nn_t-1}}$, which is tight when the global codebook is well designed [10].

For the considered special case, the per-cell codebook can achieve the performance of the global codebook with sufficiently large Nn_t, as proved in [5, lemma 1]. Therefore, we can approximate the averaged quantization accuracy of global CDI quantized with per-cell codebook as follows,

$$A_{\mathrm{NPA}} \triangleq \mathbb{E}[\cos\theta|_{\mathrm{NPA}}] \approx 1 - 2^{-\frac{B_{\mathrm{sum}}}{Nn_t-1}}. \tag{7}$$

B. Quantization Performance with PA Feedback

Denote the number of bits for PA quantization as B_{PA}, then the total bits for CDI quantization are $B_{\mathrm{CDI}} = B_{\mathrm{sum}} - B_{\mathrm{PA}}$ and $\sum_{k=1}^{N} B_k = B_{\mathrm{CDI}}$. The codewords for each per-cell CDI and PA are selected jointly to minimize the quantization error of the global CDI, which is

$$\begin{aligned}&[i_1, \cdots, i_N; j_1, \cdots, j_N] \\ &= \arg\max_{\mathbf{c}_{i_k}\in\mathcal{C}_k, c_{j_k}^{\xi}\in\mathcal{C}_k^{\xi}}\left[\left(\sum_{k=1}^{N} g_k\hat{g}_k\cdot|\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|\right)^2 - \Phi_{\mathrm{PA}}\right].\end{aligned} \tag{8}$$

The average quantization performance $\mathbb{E}[\cos^2\theta|_{\mathrm{PA}}]$ is also very hard to develop due to the joint codeword selection. Instead, we derive its upper bound and lower bound in the following, where $\mathbb{E}[\cos^2\theta|_{\mathrm{PA}}]$ will be denoted as A_{PA} for concise.

1) Upper Bound of A_{PA}: When the PA can be perfectly quantized with B_{PA} bits, i.e., $e^{j\Delta\xi_k} = 1, k = 1, \cdots, N$, A_{PA} will achieve its upper bound. With this assumption, the quantization accuracy of the global CDI is the sum of the quantization accuracy of the per-cell CDI, as shown in (5). Then to maximize the quantization accuracy of the global CDI, the codeword for each per-cell CDI can be selected independently to maximize the quantization accuracy of the CDI of each cell. The upper bound of the averaged quantization accuracy of the global CDI can be derived as

$$\begin{aligned}A_{\rm PA}^{\rm UB} &= \mathbb{E}[\cos^2\theta|_{\rm PA}^{\rm UB}] \\ &= \mathbb{E}\Big[\sum_{k=1}^{N} g_k^2\hat{g}_k^2|\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|^2 \\ &\quad + 2\sum_{k=1}^{N}\sum_{m>k}^{N} g_k\hat{g}_k g_m\hat{g}_m|\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|\cdot|\bar{\mathbf{h}}_m\mathbf{c}_{i_m}|\Big] \\ &= \mathbb{E}\Big[\sum_{k=1}^{N} g_k^2\hat{g}_k^2\cos^2\theta_k + 2\sum_{k=1}^{N}\sum_{m>k}^{N} g_k\hat{g}_k g_m\hat{g}_m\cos\theta_k\cos\theta_m\Big],\end{aligned} \tag{9}$$

where the codeword $\mathbf{c}_{i_k}$ for the kth per-cell CDI is selected independently based on its own quantization performance, i.e., $i_k = \arg\max_{\mathbf{c}_{i_k}\in\mathcal{C}_k} |\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|$, and $\cos\theta_k \triangleq |\bar{\mathbf{h}}_k\mathbf{c}_{i_k}|$ is the quantization accuracy of the kth per-cell CDI.

The norm of the per-cell channel is independent with its CDI vector, since the per-cell channel is assumed as complex Gaussian. Therefore, g_k is independent from $\cos^2\theta_k$ and $\cos\theta_k$, where g_k depends on the norms of per-cell channels and $\cos^2\theta_k$ and $\cos\theta_k$ are associated with the per-cell CDI. As a result, (9) can be derived as

$$\begin{aligned}A_{\rm PA}^{\rm UB} &= \sum_{k=1}^{N}\hat{g}_k^2\mathbb{E}[g_k^2]\mathbb{E}[\cos^2\theta_k] \\ &\quad + 2\sum_{k=1}^{N}\sum_{m>k}^{N}\hat{g}_k\hat{g}_m\cdot\mathbb{E}[g_k]\mathbb{E}[g_m]\cdot\mathbb{E}[\cos\theta_k]\mathbb{E}[\cos\theta_m].\end{aligned} \tag{10}$$

The closed-form expressions of $\mathbb{E}[g_k]$ and $\mathbb{E}[g_k^2]$ are still hard to derive if not possible. Fortunately, we have

$$\mathbb{E}[g_k^2] \approx \hat{g}_k^2, \tag{11}$$

which can be derived according to [8, Appendix B].

Considering that the variance $\sigma_{g_k}^2 = \mathbb{E}[g_k^2] - (\mathbb{E}[g_k])^2 > 0$, we have $\mathbb{E}[g_k] \le \sqrt{\mathbb{E}[g_k^2]}$. When the MS locates at the cell edge, i.e., $\alpha_i \approx \alpha_j, i \neq j$, $\sigma_{g_k}^2$ is small with sufficiently large n_t. As a result, $\mathbb{E}[g_k] \approx \sqrt{\mathbb{E}[g_k^2]}$. Upon further substituting (11), we have

$$\mathbb{E}[g_k] \approx \hat{g}_k. \tag{12}$$

Substituting (11) and (12) into (10), we can obtain

$$A_{\rm PA}^{\rm UB} \approx \sum_{k=1}^{N}\hat{g}_k^4\mathbb{E}[\cos^2\theta_k] + 2\sum_{k=1}^{N}\sum_{m>k}^{N}\hat{g}_k^2\hat{g}_m^2\cdot\mathbb{E}[\cos\theta_k]\mathbb{E}[\cos\theta_m]. \tag{13}$$

When the per-cell codebook is well designed, e.g., using the Grassmannian codebook [11], $\mathbb{E}[\cos^2\theta_k]$ can be accurately approximated as

$$\mathbb{E}[\cos^2\theta_k] \approx 1 - 2^{-B_k/(n_t-1)}. \tag{14}$$

In this case, the variance of quantization accuracy $\sigma_{\cos\theta_k}^2 = \mathbb{E}[\cos\theta_k^2] - (\mathbb{E}[\cos\theta_k])^2$ will be small. Therefore, $\mathbb{E}[\cos\theta_k]$ can be approximated by $\sqrt{\mathbb{E}[\cos\theta_k^2]}$, i.e.,

$$\mathbb{E}[\cos\theta_k] \approx \sqrt{1 - 2^{-B_k/(n_t-1)}}. \tag{15}$$

Upon substituting (14) and (15), (13) becomes

$$A_{\rm PA}^{\rm UB} \approx \left(\sum_{k=1}^{N}\hat{g}_k^2\cdot\sqrt{1 - 2^{-B_k/(n_t-1)}}\right)^2. \tag{16}$$

For the special case where $\alpha_1 = \cdots = \alpha_N$, the weighting factor is $\hat{g}_k = \frac{1}{\sqrt{N}}$ and the bits allocated for the each per-cell codebook are $B_k = \frac{B_{\rm CDI}}{N}, k = 1, \cdots, N$ [2]. Then the upper bound of the average quantization accuracy of global CDI can be approximated as

$$A_{\rm PA}^{\rm UB} \approx 1 - 2^{-\frac{B_{\rm CDI}}{N(n_t-1)}}. \tag{17}$$

2) Lower Bound of $A_{\rm PA}$: When the quantization error of PA is taken into account, the performance of the independent codeword selection for both per-cell CDI and PA can serve as a lower bound for the quantization performance of global CDI. Then the lower bound of the averaged quantization accuracy can be written as

$$\begin{aligned}A_{\rm PA}^{\rm LB} &= \mathbb{E}[\cos^2\theta|_{\rm PA}^{\rm LB}] = \sum_{k=1}^{N}\hat{g}_k^2\mathbb{E}[g_k^2]\cdot\mathbb{E}[\cos^2\theta_k] \\ &\quad + 2\sum_{k=1}^{N}\sum_{m>k}^{N}\hat{g}_k\hat{g}_m\mathbb{E}[g_k]\mathbb{E}[g_m]\cdot\mathbb{E}[\cos\theta_k]\mathbb{E}[\cos\theta_m] \\ &\quad \cdot\mathbb{E}[\cos(\Delta\xi_k - \Delta\xi_m)].\end{aligned} \tag{18}$$

Substituting (11), (12), (14) and (15) into (18), we have

$$A_{\rm PA}^{\rm LB} \approx \sum_{k=1}^{N}\hat{g}_k^4\cdot(1 - 2^{-B_k/(n_t-1)}) + \Psi_{\rm PA}^{\rm LB}, \tag{19}$$

where $\Psi_{\rm PA}^{\rm LB} \triangleq 2\sum_{k=1}^{N}\sum_{m>k}^{N}\hat{g}_k^2\hat{g}_m^2 \cdot \sqrt{1 - 2^{-\frac{B_k}{n_t-1}}} \cdot \sqrt{1 - 2^{-\frac{B_m}{n_t-1}}}\cdot\mathbb{E}[\cos(\Delta\xi_k - \Delta\xi_m)]$.

For mathematical tractability, again we consider the special case where $\alpha_1 = \cdots = \alpha_N$. Then the lower bound can be derived as

$$\begin{aligned}A_{\rm PA}^{\rm LB} &\approx \sum_{k=1}^{N}\frac{1 - 2^{\frac{B_{\rm CDI}}{N}}_{n_t-1}}{N^2} \\ &\quad + \frac{2(1 - 2^{\frac{B_{\rm CDI}}{N}}_{n_t-1})}{N^2}\underbrace{\sum_{k=1}^{N}\sum_{m>k}^{N}\mathbb{E}[\cos(\Delta\xi_k - \Delta\xi_m)]}_{\eta}.\end{aligned} \tag{20}$$

In the following we derive the closed-form expression of η. According to (4), if we normalize all the per-cell PA with a certain per-cell PA, say 1st per-cell PA $e^{j\xi_1}$, it does not affect the quantization accuracy of the global CDI. Then equivalently the 1st per-cell PA can be viewed as perfectly quantized, i.e., $\Delta\xi_1 = 0$. Hence we have

$$\eta = \sum_{k=2}^{N}\sum_{m>k}^{N}\mathbb{E}[\cos(\Delta\xi_k - \Delta\xi_m)] + \sum_{k=2}^{N}\mathbb{E}[\cos\Delta\xi_k]. \tag{21}$$

[2]Here we consider that $\frac{B_{\rm CDI}}{N}$ is an integer. Otherwise, we can round it to the nearest integer, which does not affect our analysis.

For the considered special case, the quantization of different per-cell PA are equally important [6], i.e., the number of bit for each PA quantization is $b_k = \frac{B_{\rm PA}}{N-1}, k = 2, \cdots, N$.

It is reasonable to assume that the per-cell PA is uniformly distributed in $[0, 2\pi]$, since the per-cell channel is isotropical. This indicates that it is optimal to quantize PA with a uniform scalar quantizer, and the quantization error of the kth per-cell PA is uniformly distributed within $[-\frac{\pi}{2^{b_k}}, \frac{\pi}{2^{b_k}}]$. Then we have

$$\begin{aligned}\mathbb{E}[\cos(\Delta\xi_k)] &= \int_{-\pi/2^{b_k}}^{\pi/2^{b_k}} \frac{\cos x \cdot 2^{b_k}}{2\pi} {\rm d}x \\ &= \frac{2^{\frac{B_{\rm PA}}{N-1}}}{\pi} \sin(\frac{\pi}{2^{\frac{B_{\rm PA}}{N-1}}}), k = 2, \cdots, N.\end{aligned} \tag{22}$$

In the same manner, we obtain $\mathbb{E}[\sin(\Delta\xi_k)] = 0$. As a result, we have

$$\begin{aligned}&\mathbb{E}[\cos(\Delta\xi_k - \Delta\xi_m)] \\ =&\mathbb{E}[\cos\Delta\xi_k]\mathbb{E}[\cos\Delta\xi_m] - \mathbb{E}[\sin\Delta\xi_k]\mathbb{E}[\sin\Delta\xi_m] \\ =&\frac{2^{\frac{2B_{\rm PA}}{N-1}}}{\pi^2} \sin^2(\frac{\pi}{2^{\frac{B_{\rm PA}}{N-1}}}), k \neq m \neq 1.\end{aligned} \tag{23}$$

Upon substituting (23), (21) becomes

$$\begin{aligned}\eta &= \sum_{k=2}^{N} \left(\frac{2^{B_{\rm PA}}}{\pi} \sin(\frac{\pi}{2^{B_{\rm PA}}}) + \sum_{m>k}^{N} \frac{2^{2B_{\rm PA}}}{\pi^2} \sin^2(\frac{\pi}{2^{B_{\rm PA}}}) \right) \\ &= \frac{(N-1)2^{B_{\rm PA}} \sin(\frac{\pi}{2^{B_{\rm PA}}})}{\pi} \cdot \left(1 + \frac{(N-2)2^{B_{\rm PA}} \sin(\frac{\pi}{2^{B_{\rm PA}}})}{2\pi}\right).\end{aligned} \tag{24}$$

Substituted (24) into (20), we obtain the approximated lower bound of the average quantization accuracy as

$$A_{\rm PA}^{\rm LB} \approx (\frac{1 - 2^{-\frac{B_{\rm CDI}}{N(n_t-1)}}}{N}) \cdot (1 + \frac{2\eta}{N}). \tag{25}$$

3) Relationship between the Upper Bound and the Lower Bound: When the number of bits allocated for PA quantization $B_{\rm PA}$ is sufficient large, we can derive that $\lim_{B_{\rm PA}\to\infty} \frac{2^{\frac{B_{\rm PA}}{N-1}}}{\pi} \sin(\frac{\pi}{2^{\frac{B_{\rm PA}}{N-1}}}) = 1$. Consequently we have

$$\lim_{B_{\rm PA}\to\infty} \eta = \frac{(N-1)(N-2)}{2} + (N-1) = \frac{N^2 - N}{2}. \tag{26}$$

Substituting (26) into (25), we can obtain

$$\lim_{B_{\rm PA}\to\infty} A_{\rm PA}^{\rm LB} = 1 - 2^{-\frac{B_{\rm CDI}}{N(n_t-1)}} = A_{\rm PA}^{\rm UB}. \tag{27}$$

Note that this does not mean we should allocate as many bits as possible for PA quantization. Otherwise, the bits allocated for quantizing per-cell CDI will reduce and finally the quantization performance of the global CDI will degrade.

IV. Numerical and Simulation Results

In this section, we will compare the quantization performance of the two feedback strategies with or without PA quantization via numerical and simulation results.

First, we evaluate the accuracy of the approximations and the tightness of the bounds. Considering the prohibitive complexity, we choose a simulation scenario with 2 coordinated BSs, each equipped with 4 antennas. We consider an exact cell edge MS, i.e., $\alpha_1 = \alpha_2$. The overall number of bits for the MS, $B_{\rm sum}$, is set to 8 bits. The simulation and numerical results are presented in Fig. 1.

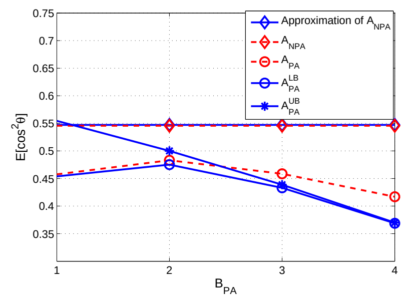

Fig. 1. The accuracy of the approximated $A_{\rm NPA}$, the tightness of the upper and lower bounds of $A_{\rm PA}$. Note that the simulated result of $A_{\rm NPA}$ overlaps with that of the approximation of $A_{\rm NPA}$.

For the strategy without PA feedback, n_t needs to be large enough to ensure an accurate approximation of $A_{\rm NPA}$. As shown in the figure, however, $n_t = 4$ is enough. For the strategy with PA feedback, the simulated result of $A_{\rm PA}$ is close to $A_{\rm PA}^{\rm LB}$ when $B_{\rm PA}$ is small, while $A_{\rm PA}^{\rm LB}$ and $A_{\rm PA}^{\rm UB}$ tend to overlap for large $B_{\rm PA}$, e.g., 2 or 3 bits. This can be explained as follows. When we allocate more bits for PA quantization, the bits for per-cell CDI quantization will reduce. For example, when $B_{\rm PA} = 4$ bits, $B_{\rm CDI} = B_{\rm sum} - B_{\rm PA} = 4$ bits. This means that only 2 bits is employed for quantizing each per-cell CDI. It makes the approximation in (14) and (15) to be not accurate, which leads to the looser upper and lower bounds. Nonetheless, this does not affect the performance comparison of the two strategies, since $A_{\rm PA}$, $A_{\rm PA}^{\rm LB}$ and $A_{\rm PA}^{\rm UB}$ are all much less than $A_{\rm NPA}$ in this case.

Then we compare the quantization performance of two strategies for the cell-edge MS by numerical results[3], as shown in Fig. 2. For a fair comparison, we fix the total antenna number of each CoMP cluster, i.e., $N \times n_t = 8$.

It shows that given the same $B_{\rm sum}$, the system with more coordinated BSs and fewer antennas at each BS benefits from the quantization of PA. To achieve the performance gain by PA quantization for the system with 4 coordinated BSs and 2 antennas at each BS, 5 bits are used for each cell on average.

We can find from the figure that there is an optimal bit allocation to maximize $A_{\rm PA}^{\rm LB}$, e.g., $B_{\rm PA}^{\rm op} = 5$ bits and $B_{\rm CDI}^{\rm op} = 15$ bits for the system with 4 BSs each equipped with 2 antennas. In the following, we will show the normalized performance gain brought by PA quantization for various $B_{\rm sum}$, where $\frac{A_{\rm PA}^{\rm LB} - A_{\rm NPA}}{A_{\rm NPA}}$ is used as the metric. This is a pessimistic estimation for the benefit of quantizing the PA, because we consider the lower bound of the performance. For

[3]Though $n_t = 2$ is not sufficient large to ensure the approximation accuracy, it does not affect the comparison results.

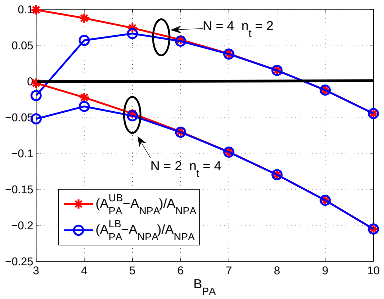

Fig. 2. The performance of the exact cell-edge MS with various number of BSs and number of antennas at each BS, where $B_{\rm sum}$=20 bits.

each $B_{\rm sum}$, the performance gain is obtained based on the optimal bit allocation between $B_{\rm PA}$ and $B_{\rm CDI}$, which is found by exhaustive searching. The numerical results are presented in Fig. 3. It shows that when there are fewer antennas at each BS the PA quantization will bring a performance gain. In contrast, for the system with more antennas at each BS, the PA quantization will have gain only when $B_{\rm sum}$ is considerably large, e.g., for a system with $N = 3$ and n_t =4, $B_{\rm sum}$ should be more than 30 bits, i.e., 10 bits for each cell on average. This is because more bits have to be used to improve the CDI quantization accuracy as the number of antennas increases. Though when $B_{\rm sum}$ is large enough, the global CDI quantization still benefits from PA quantization, however, the performance gain is minor in all cases. This is because the quantized global CDI is already very close to the perfect CDI with large $B_{\rm sum}$.

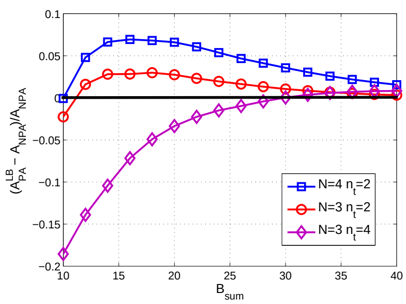

Fig. 3. The performance gain introduced by the PA quantization.

To show if our previous analysis for cell-edge uses is also valid for other users, we compare the performance of the two strategies for the MSs with different locations by simulations, as shown in Fig. 4. In the simulation, the codebook for per-cell CDI is random vector codebook. The optimal number of bits for each codebook are found by exhaustive searching to maximize the averaged quantization accuracy of the global CDI. The total bits for each MS $B_{\rm sum} = 8$ bit. We consider a simple CoMP system, with 2 coordinated BSs and each BS equipped with 1 or 2 antennas.[4] The MS is located in the 1st cell, where $\frac{\alpha_1}{\alpha_2}$ reflects its location. A large $\frac{\alpha_1}{\alpha_2}$ indicates that the MS is close to the cell center, and $\frac{\alpha_1}{\alpha_2} = 1$ means that the MS is at the exact cell edge. All the results are obtained over 1000 small scale fading channel realizations. It shows that in the case of 2 coordinated BSs each with single antenna, most of the MSs benefits from the PA quantization, especially when the MS located close to the cell edge.

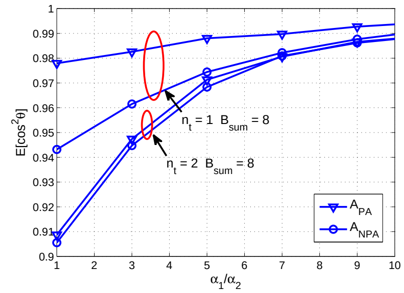

Fig. 4. The comparison of the two strategies when the location of MS differs.

[4]Our analysis can be applied for more general cases, but the simulation will be too slow for more bits, more BSs and more antennas due to the exhaustive searching for the joint codeword selection and bit allocation.

V. Conclusions

In this paper, we analyzed the average quantization performance of two per-cell codebook based limited feedback strategies with or without phase ambiguity quantization for CoMP systems. By deriving the approximated bounds of the averaged quantization accuracy of global CDI, we show that when the number of BSs is large and the number of antennas at each BS is small, the PA feedback can bring significant performance gain especially for cell-edge MSs. Simulation results validate our analysis.

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