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SINR Estimation in Limited Feedback Coordinated Multi-point Systems

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***Abstract*— Coordinated multi-point (CoMP) transmission can provide high spectral efficiency for cellular systems if perfect channels are available. In limited feedback systems, except for channel direction, signal to noise and interference ratio (SINR) is necessary to assist user scheduling and adaptive transmission. When the methods to estimate SINR in single cell systems is applied to CoMP systems, the performance will degrade significantly. In this paper, we estimate the SINR for downlink CoMP systems. We start by showing that the quantization error vector is no longer isotropic when quantizing CoMP channels. This implies that the vector has an inherent structure. We proceed to formulate an optimization problem to find the quantization error vector that maximizes the multiuser interference power under the structure constraint. In this way we can exploit the non-isotropic nature of the quantization error vector and avoid overestimating the SINR. Simulation results show that the proposed method provides high throughput and low outage probability with negligible extra feedback overhead.**

***Index Terms*— Coordinated multi-point, signal to noise and interference ratio estimation, downlink, limited feedback**

I. Introduction

Coordinated multi-point transmission (CoMP) has drawn broad attention recently for its potential to support high throughput [1, 2], which has various forms depending on the information shared among the coordinated base stations (BSs). When all the BSs share both data and channel information, CoMP joint transmission (CoMP-JT) can provide high spectral efficiency for cellular systems. This however requires large feedback overhead for frequency division duplexing (FDD) systems [3].

Limited feedback techniques are widely applied for reporting channel information to the BS in multi-input-multi-output (MIMO) systems, which has been extensively studied [4]. To assist downlink beamforming for multi-user MIMO (MU-MIMO) systems, each user needs to feed back the quantized channel direction information (CDI). To facilitate user scheduling and modulation and coding scheme (MCS) selection, the signal to noise and interference ratio (SINR) needs to be available at the BS [5].

Despite that CoMP-JT can be viewed as a large MIMO system with a super BS, there are many subtle but critical differences in terms of system setting and channel feature. As a result, the well-explored techniques developed for single-cell MIMO systems can not be extended to CoMP straightforwardly. To feed back CDI in CoMP-JT systems, the per-cell codebook based limited feedback scheme is preferred due to its flexibility and compatibility [6]. Though such a scheme is suboptimal in nature, its performance can be improved significantly by judiciously designing the methods for codeword selection [6, 7], global CDI reconstruction [8], and bit allocation among the per-cell codebooks [3, 7, 9].

The quality of the SINR estimation is critical to explore the potential of MU-MIMO as well as CoMP systems, which is however largely overlooked in the literature. An optimistic estimation will lead to the outage in the downlink transmission, while a conservative estimation will cause the rate loss. To help each user estimate the SINR accurately, full multiplexing spatial scheduling and equal power allocation among users are usually assumed [5, 10]. When the number of scheduled users is small, the transmit power of each user will be underestimated, which leads to severe performance degradation. To cope with this drawback, a method was proposed in [11] to improve the SINR fed back from each user with the number of co-scheduled users at the BS. A performance loss will be unavoidable if these methods are applied to CoMP-JT systems, because the powers allocated to the users are not necessarily equal to meet the per-BS power constraint (PBPC) and hence are unknown to the users.

In this paper we address the problem of downlink SINR estimation for CoMP-JT systems. We first analyze the impact of CoMP channel on the per-cell codebook based quantization, and show that the quantization error vector is no longer isotropic. By exploiting such a property, we propose a SINR estimation method, which improves the throughput significantly with negligible extra feedback overhead compared with the directly extended method from that for single cell systems. For simplicity, we refer CoMP-JT as CoMP in the rest of the paper.

II. System Model

A. System and Channel Model

Consider a downlink CoMP system consisting $\tilde{N}_b$ BSs each equipped with n_t antennas and M single antenna MSs. These BSs are connected to a central unit (CU) via backhaul links with unlimited capacity and zero latency, which completes all the pre-processing. Since different users have different receive powers from different BSs, each user selects several preferred BSs as a cooperative cluster from the $\tilde{N}_b$ BSs for joint transmission. In other words, the sets of cooperative BSs of different users may differ. Define

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the BS selection matrix of each user, say the mth user, as $\mathbf{S}_m = \mathrm{diag}\{\omega_{m,1}\mathbf{I}_{n_t}, \cdots, \omega_{m,\tilde{N}_b}\mathbf{I}_{n_t}\}$, where $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix, and $\mathbf{I}_{n_t}$ is the identity matrix with dimension n_t; $\omega_{m,i} = 1$ if the user selects the ith BS, otherwise $\omega_{m,i} = 0$.

For notational simplicity and without loss of generality, in the derivations we assume that the mth user selects the BSs with indices $\{1, \cdots, N_b\}$ for downlink transmission, i.e., $\omega_{m,i} = 1, i = 1, \cdots, N_b$ and $\omega_{m,j} = 0, j = N_b + 1, \cdots, \tilde{N}_b$, where $N_b = 1$ suggests the user chooses single cell transmission and $1 < N_b \leq \tilde{N}_b$ represents CoMP transmission. The downlink global channel of the mth user can be expressed as follows, which is comprised of all the per-cell channels from each cooperative BS to the user,

$$\begin{aligned} \mathbf{g}_m &= \mathbf{S}_m \tilde{\mathbf{g}}_m \\ &= \left[\alpha_{m,1}\mathbf{h}_{m,1}^H, \cdots, \alpha_{m,N_b}\mathbf{h}_{m,N_b}^H, \mathbf{0}_{(\tilde{N}_b - N_b)n_t \times 1}^H\right]^H, \end{aligned} \tag{1}$$

where $\alpha_{m,k}$ is the large scale fading gain including the path loss and shadowing, $\mathbf{h}_{m,k} \in \mathbb{C}^{n_t \times 1}$ is the small scale fading channel vector from the kth BS to the user, and $\tilde{\mathbf{g}}_m = [\alpha_{m,1}\mathbf{h}_{m,1}^H, \cdots, \alpha_{m,\tilde{N}_b}\mathbf{h}_{m,\tilde{N}_b}^H]$ is the global channel comprised of all the per-cell channels from all candidate BSs in the system; $\mathbf{0}_{m \times n}$ is the all-zero matrix with dimension $m \times n$. To simplify the analysis and highlight the feature of CoMP channels, we assume that the per-cell channels $\mathbf{h}_{m,k}, k = 1, \ldots, N_b$ are spatially uncorrelated, and each entry of them is subject to complex Gaussian distribution of zero mean with unit variance.

The CDI of the global channel of the mth user is

$$\begin{aligned} \bar{\mathbf{g}}_m &\triangleq \frac{\mathbf{g}_m}{\| \mathbf{g}_m \|} \\ &= \left[g_{m,1}\bar{\mathbf{h}}_{m,1}^H, \cdots, g_{m,N_b}\bar{\mathbf{h}}_{m,N_b}^H, \mathbf{0}_{(\tilde{N}_b - N_b)n_t \times 1}^H\right]^H, \end{aligned} \tag{2}$$

where $\bar{\mathbf{h}}_{m,k} \triangleq \frac{\mathbf{h}_{m,k}}{\|\mathbf{h}_{m,k}\|}$ is the kth per-cell CDI vector, $g_{m,k} = \frac{\alpha_{m,k}\|\mathbf{h}_{m,k}\|}{\|\mathbf{g}_m\|} = \frac{\alpha_{m,k}\|\mathbf{h}_{m,k}\|}{\sqrt{\sum_{k=1}^{N_b}\alpha_{m,k}^2\|\mathbf{h}_{m,k}\|^2}}$ is the weighting factor, which reflects the contribution of the kth per-cell CDI to the global CDI, and $\| \mathbf{h}_{m,k} \|$ is the per-cell small scale fading channel norm.

In FDD systems the global CDI needs to be fed back for beamforming. When per-cell codebook based quantization is used, which is practical and popular [6], each user selects the codeword from a given codebook to quantize each per-cell CDI from the cooperative BSs, and then feeds back the codeword indices to its master BS. Then each BS forwards all the indices from users to the CU. The required feedback overhead depends on the number of selected cooperative BSs, N_b. Considering that the large scale fading gains are semi-static and no need frequent feedback, we assume that they are available at the BS. The global CDI can be reconstructed at the CU only with the large scale fading gains as [6, 8]

$$\hat{\mathbf{g}}_m = [\hat{g}_{m,1}\hat{\mathbf{h}}_{m,1}^H, \cdots, \hat{g}_{m,N_b}\hat{\mathbf{h}}_{m,N_b}^H, \mathbf{0}_{(\tilde{N}_b - N_b)n_t \times 1}^H]^H, \tag{3}$$

which is applicable when the user chooses single cell or CoMP transmission. $\hat{\mathbf{h}}_{m,k}$ is the selected codeword for quantizing the kth per-cell CDI, and $\hat{g}_{m,i} = \frac{\alpha_{m,i}}{\sum_{k=1}^{N_b}\alpha_{m,k}^2}$ is the estimated weighting factor.

Remark 1: The users can select the per-cell codewords based on different criterions, e.g., independent codeword selection [8] and serial codeword selection [7]. With the independent codeword selection, the phase ambiguity of each per-cell channel, $e^{j\xi_{m,k}} \triangleq \frac{\bar{\mathbf{h}}_{m,k}^H\hat{\mathbf{h}}_{m,k}}{|\bar{\mathbf{h}}_{m,k}^H\hat{\mathbf{h}}_{m,k}|}$, may need to be fed back, since it has large impact on the quantization performance of the global CDI [8]. With the fed back phase ambiguity, denoted as $e^{j\hat{\xi}_{m,k}}$, the global CDI is reconstructed at the CU as $\hat{\mathbf{g}}_m = [\hat{g}_{m,1}\hat{\mathbf{h}}_{m,1}^H e^{-j\hat{\xi}_{m,1}}, \cdots, \hat{g}_{m,N_b}\hat{\mathbf{h}}_{m,N_b}^H e^{-j\hat{\xi}_{m,N_b}}, \mathbf{0}_{(\tilde{N}_b - N_b)n_t \times 1}^H]^H$.

B. Downlink SINR

To facilitate power allocation, user scheduling and MCS selection, the CU needs to know the downlink SINR of each user. The SINR experienced at the mth user is

$$\mathrm{SINR}_m = \frac{p_m||\mathbf{g}_m||^2|\bar{\mathbf{g}}_m^H\mathbf{w}_m|^2}{\sigma_m^2 + ||\mathbf{g}_m||^2\sum_{k\neq m}^M p_k|\bar{\mathbf{g}}_m^H\mathbf{w}_k|^2}, \tag{4}$$

where p_m is the transmit power to the user, $\mathbf{w}_m$ is the beamforming vector of the user[1], and σ_m^2 is the power of noise and inter-cluster interference from other BSs not in the cooperative cluster of the mth user, i.e., BS i, $i = N_b + 1, \cdots, \tilde{N}_b$.

The downlink SINR can be estimated either at the CU, or at the user (and then fed back to the BS who forwards to the CU). It is shown from (4) that to compute the SINR, the CU or the user needs the following information: (1) global channel norm $\| \mathbf{g}_m \|$, (2) σ_m^2, (3) powers allocated to all scheduled users $p_1, \cdots, p_M$, (4) global CDIs of all scheduled users $\bar{\mathbf{g}}_1, \cdots, \bar{\mathbf{g}}_M$, and (5) beamforming for all scheduled users $\mathbf{w}_1, \cdots, \mathbf{w}_M$.

III. SINR Estimation at the CU

In limited feedback MU-MIMO systems, each user only knows its own channel and the noise and interference power but is unaware of the information for other users, while the BS knows all the beamforming vectors and allocated powers but does not know the true values of the downlink channels. As a result, neither the user nor the BS has all the required information to compute the downlink SINR.

One popular approach to circumvent this problem is orthogonal beamforming based limited feedback, e.g., per user unitary and rate control (PU2RC) [10], where the beamforming vectors are selected from an orthonormal codebook. Since both the BS and each user are aware that the beamforming vectors of co-scheduled users are orthogonal and with unit norm, the SINR can be estimated at each user accurately if the number of selected users equals to the number of antennas at the BS. An implicit assumption behind the PU2RC is that the transmit power is equally allocated to multiple users.

[1] If user m does not choose some BSs for CoMP transmission, we set the corresponding elements in $\mathbf{w}_m$ as zero.

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PU2RC is proposed for single-cell systems, which cannot be extended into CoMP without performance loss. This is because to meet the PBPC inherent in CoMP systems the equal power allocation is usually not efficient. When considering any non-equal power allocation, each user does not know the power allocated to it.

Fortunately, we can borrow the basic idea of the PU2RC in using orthogonal beamforming to improve the quality of the SINR estimation. To this end, we consider a transmission scheme using semi-orthogonal user scheduling (SUS) [5] and zero-forcing beamforming (ZFBF) [12], which is asymptotically optimal in terms of sum rate in single cell systems and has been widely applied in practical systems. Since the powers allocated among the selected users are known at the CU, the SINR should be estimated at the CU.

When the orthogonal threshold of SUS is zero, i.e., the CU schedules the users with orthogonal global CDIs, we have $\hat{\mathbf{g}}_k^H \hat{\mathbf{g}}_m = 0, \forall k \neq m$ and the ZFBF vector for each user is $\mathbf{w}_m = \hat{\mathbf{g}}_m$. Then, (4) becomes

$$\mathrm{SINR}_m = \frac{p_m ||\mathbf{g}_m||^2 |\bar{\mathbf{g}}_m^H \hat{\mathbf{g}}_m|^2}{\sigma_m^2 + p_m ||\mathbf{g}_m||^2 \sum_{k\neq m}^M \rho_k |\bar{\mathbf{g}}_m^H \hat{\mathbf{g}}_k|^2}, \tag{5}$$

where $\rho_k \triangleq \frac{p_k}{p_m}$ reflects the possible difference in powers allocated to multiple users when various power constraints such as PBPC are considered.

Similar to single cell systems [13], the global CDI of the mth user can be modeled as

$$\bar{\mathbf{g}}_m = \cos\theta_m e^{j\xi_m} \hat{\mathbf{g}}_m + \sin\theta_m \mathbf{s}_m, \tag{6}$$

where $\cos\theta_m \triangleq |\bar{\mathbf{g}}_m^H \hat{\mathbf{g}}_m|$ is the quantization accuracy of the global CDI, $\sin\theta_m = \sqrt{1-\cos^2\theta_m}$, $e^{j\xi_m} \triangleq \frac{\bar{\mathbf{g}}_m^H \hat{\mathbf{g}}_m}{|\bar{\mathbf{g}}_m^H \hat{\mathbf{g}}_m|}$ is the phase difference between the reconstructed and actual global CDIs, and $\mathbf{s}_m$ is a quantization error vector with $\mathbf{s}_m^H \hat{\mathbf{g}}_m = 0$ and $||\mathbf{s}_m|| = 1$.[2] Since the reconstructed and actual CDIs have zero elements in the same position as in (2) and (3), $\mathbf{s}_m$ is in the similar form as the CDI, i.e., $\mathbf{s}_m = [(\mathbf{s}_m^{'})^H, \mathbf{0}^H_{(\tilde{N}_b - N_b)n_t \times 1}]^H$, where $\mathbf{s}_m^{'} \in \mathbb{C}^{N_b n_t \times 1}$ is a sub-vector in $\mathbf{s}_m$ with non-zero elements.

Substituting (6) into (5), the SINR at the mth user is

$$\mathrm{SINR}_m = \frac{p_m (||\mathbf{g}_m||^2/\sigma_m^2) \cos^2\theta_m}{1 + p_m (||\mathbf{g}_m||^2/\sigma_m^2) \sin^2\theta_m \sum_{k\neq m}^M \rho_k |\mathbf{s}_m^H \hat{\mathbf{g}}_k|^2}. \tag{7}$$

To compute the SINR, the CU needs the following information: (1) the value of $\frac{||\mathbf{g}_m||}{\sigma_m^2}$, (2) the transmit powers to all the users $p_1, \cdots, p_M$, (3) $\cos\theta_m$, and (4) non-zero sub-vector in $\mathbf{s}_m$, $\mathbf{s}_m^{'}$.

All the information except for the transmit powers is unknown to the CU thus needs to be fed back, where $||\mathbf{g}_m||^2$ and σ_m^2 can be fed back in the form of $\frac{||\mathbf{g}_m||}{\sigma_m^2}$. After the scalars $\frac{||\mathbf{g}_m||}{\sigma_m^2}$ and $\cos\theta_m$ are available at the CU, if the CU only schedules single user, i.e., $M = 1$, multiuser interference power is zero and the SINR can be computed accurately. Otherwise the only term left to compute the SINR is multiuser interference power, i.e.,

$$I_m = \sum_{k\neq m}^M \rho_k |\mathbf{s}_m^H \hat{\mathbf{g}}_k|^2, \tag{8}$$

which depends on the quantization error vector $\mathbf{s}_m$, more specifically, its non-zero sub-vector $\mathbf{s}_m^{'}$. In the following we discuss how to estimate SINR in the multiuser case.

Because $\mathbf{s}_m^{'}$ has the dimension of $N_b n_t \times 1$ and needs the same number of bits for feedback as the global CDI, it may not be affordable to feed back such a vector. Of course, the CU can estimate I_m by assuming an isotropic $\mathbf{s}_m^{'}$, i.e., any direction is possible for $\mathbf{s}_m^{'}$, and simply extending the method in single cell systems [10]. However, this will inevitably lead to a conservative estimation of the SINR. To deal with this problem, we estimate the vector $\mathbf{s}_m^{'}$ that maximizes the interference power under a structure constraint of the vector shown as follows, and then use the corresponding maximal value of I_m to compute the SINR.

From (1) we see that the large scale fading gains $\alpha_{m,i} \neq \alpha_{m,j}$ for $i \neq j$, i.e., the CoMP channel $\mathbf{g}_m$ is asymmetric. In the sequel we show that $\mathbf{s}_m^{'}$ is also asymmetric, i.e., not isotropic. We write $\mathbf{s}_m^{'}$ as

$$\mathbf{s}_m^{'} = [\breve{g}_{m,1} \breve{\mathbf{s}}_{m,1}^H, \cdots, \breve{g}_{m,N_b} \breve{\mathbf{s}}_{m,N_b}^H]^H, \tag{9}$$

where $\breve{\mathbf{s}}_{m,i}$ is the per-cell quantization error vector with unit norm, $\breve{g}_{m,i}$ reflects the contribution of the per-cell error vector to the global quantization error vector. Then we have $\mathbf{s}_m = [\breve{g}_{m,1} \breve{\mathbf{s}}_{m,1}^H, \cdots, \breve{g}_{m,N_b} \breve{\mathbf{s}}_{m,N_b}^H, \mathbf{0}^H_{(\tilde{N}_b - N_b)n_t \times 1}]^H$.

Proposition 1: When each user independently selects codewords for different per-cell CDIs, we have $\mathbb{E}[\breve{g}_{m,i}^2] \approx \hat{g}_{m,i}, \forall i = 1, \cdots, N_b$, which is accurate when $n_t \to \infty$.

Proof: See Appendix. ■

Because $\hat{g}_{m,i}$ depends on the large scale fading gains, it follows that $\hat{g}_{m,i} \neq \hat{g}_{m,j}$ for $i \neq j$, i.e., $\mathbf{s}_m^{'}$ is asymmetric. In fact, n_t is unnecessarily too large to ensure that the approximation is accurate. Simulation results show that n_t = 4 is enough. Though the closed-form expression of $\mathbb{E}[\breve{g}_{m,i}^2]$ is derived under the assumption of using the independent codeword selection, the asymmetry feature of $\mathbf{s}_m^{'}$ still exists as indicated in (A.4) of the Appendix, no matter which codeword selection method is used.

Considering that $\breve{\mathbf{s}}_{m,i}$ in (9) is with unit norm and is isotropic with the independent codeword selection, we define a structure vector of $\mathbf{s}_m^{'}$ (referred to as "error structure vector") as $\mathbf{f}_m = [\breve{g}_{m,1}^2, \cdots, \breve{g}_{m,N_b}^2]^H$, which is able to reflect the "direction" of the global quantization error vector. By feeding back $\mathbf{f}_m$ instead of $\mathbf{s}_m^{'}$, we can also estimate the interference power as shown soon, but with much less overhead.

In practical systems, hybrid automatic repeat request (HARQ) will be employed when outage occurs. However, retransmission requires extra time, which leads to the reduction of spectral efficiency. Consequently, the SINR estimation needs to be conservative. To reduce the outage probability in transmission led by overestimating the SINR and exploit the

[2]Note that there are infinite candidates of $\mathbf{s}_m$, whose difference only lies in the different phases. The phase of $\mathbf{s}_m$ does not affect the SINR estimation. For conciseness, in (6) we choose the one having the same phase with $\bar{\mathbf{g}}_m$.

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"direction" information of $\mathbf{s}_m^{'}$, we formulate the optimization problem as follows to maximize the interference power I_m,

$$\begin{aligned} &\max_{\mathbf{s}_m^{'}} I_m \\ &s.t. ||\mathbf{B}_i^{'}\mathbf{s}_m^{'}||^2 = \breve{g}_{m,i}^2, i = 1, \cdots, N_b, \end{aligned} \tag{10}$$

where $\mathbf{B}_i^{'} = \mathrm{diag}\{\mathbf{0}_{(i-1)n_t}, \mathbf{I}_{n_t}, \mathbf{0}_{(N_b-i)n_t \times N_b-i)n_t}\}$ and $\breve{g}_{m,i}$ in $\mathbf{f}_m$ are known to the CU after feedback. The feasible region of (10) is not convex, therefore the optimization problem is not convex. In the sequel, we transform the original optimization problem (10) into another problem.

From (6), we know that the quantization error vector $\mathbf{s}_m$ lies in the null space of the reconstructed global CDI $\hat{\mathbf{g}}_m$. With the SUS and ZFBF, the beamforming vectors of other scheduled users $\hat{\mathbf{g}}_k, k \neq m$ are also in the null space of $\hat{\mathbf{g}}_m$. As a result, we can express $\mathbf{s}_m$ as

$$\mathbf{s}_m = \sum_{k \neq m}^{M} \tau_{m,k}\hat{\mathbf{g}}_k + \tau_{m,m}\mathbf{v}_m, \tag{11}$$

where $\tau_{m,k}, \forall k \neq m$ are the inner product of $\mathbf{s}_m$ and $\hat{\mathbf{g}}_k$, $\tau_{m,m}$ is the inner product of $\mathbf{s}_m$ and $\mathbf{v}_m$, and $\sum_{k=1}^{M} |\tau_{m,k}|^2 = 1$. $\mathbf{v}_m \in \mathbb{C}^{N_b n_t \times 1}$ is a unitary vector that meets $\mathbf{v}_m^H \hat{\mathbf{g}}_k = 0$, which is not unique. If there exists a vector $\mathbf{v}_m$ meets the condition, its rotated version $\mathbf{v}_m \cdot e^{j\varepsilon}$ also meets the condition with any phase $\varepsilon \in [0, 2\pi]$.

Substituting (11) into (8), the multiuser interference can be rewritten as

$$I_m = \sum_{k \neq m}^{M} \rho_k |\tau_{m,k}|^2, \tag{12}$$

which is a quadratic function of $|\tau_{m,k}|, \forall k = 1, \cdots, N_b$.

With (11), the equality constraint of problem (10) can be derived as

$$||\mathbf{B}_i^{'}\mathbf{s}_m^{'}|| = ||\mathbf{B}_i\mathbf{s}_m|| = ||\sum_{k \neq m}^{M} \tau_{m,k}\mathbf{B}_i\hat{\mathbf{g}}_k + \tau_{m,m}\mathbf{B}_i\mathbf{v}_m||, \tag{13}$$

where $\mathbf{B}_i = \mathrm{diag}\{\mathbf{B}_i^{'}, \mathbf{0}_{(\tilde{N}_b - N_b)n_t \times (\tilde{N}_b - N_b)n_t}\}$, and $i \in \{1, \cdots, N_b\}$. Because the phase ε of $\mathbf{v}_m$ does not affect the expression in (11), we assume that $\mathbf{v}_m$ has a proper phase to ensure that $\sum_{k \neq m}^{M} \tau_{m,k}\mathbf{B}_i\hat{\mathbf{g}}_k$ has the same phase with $\tau_{m,m}\mathbf{B}_i\mathbf{v}_m$. Consequently, (13) can be further written as $||\mathbf{B}_i\mathbf{s}_m|| = ||\sum_{k \neq m}^{M} \tau_{m,k}\mathbf{B}_i\hat{\mathbf{g}}_k|| + ||\tau_{m,m}\mathbf{B}_i\mathbf{v}_m||$. By ignoring the second term, we can loosen the constraint as $||\mathbf{B}_i\mathbf{s}_m|| \geq || \sum_{k \neq m}^{M} \tau_{m,k}\mathbf{B}_i\hat{\mathbf{g}}_k || = || \sum_{k \neq m}^{M} \tau_{m,k}\hat{g}_{k,i}\hat{\mathbf{h}}_{k,i} ||$, where the equality holds if $\mathbf{s}_m$ can be linearly expressed with $\hat{\mathbf{g}}_k, \forall k \neq m$, i.e., $\mathbf{v}_m = \mathbf{0}$ or $\tau_{m,m} = 0$. This happens when the number of scheduled users is $N_b n_t$, because all the reconstructed CDIs of other users except for user m become a set of basis for the null space of $\hat{\mathbf{g}}_m$, and $\mathbf{s}_m$ is a vector in the null space.

Since $||\mathbf{B}_i\mathbf{s}_m||^2 = \breve{g}_{m,i}^2$, $i \in \{1, \cdots, N_b\}$, we have $|| \sum_{k \neq m}^{M} \tau_{m,k}\hat{g}_{k,i}\hat{\mathbf{h}}_{k,i} ||^2 \leq \breve{g}_{m,i}^2$. The left hand side of this inequation meets $|| \sum_{k \neq m}^{M} \tau_{m,k}\hat{g}_{k,i}\hat{\mathbf{h}}_{k,i} ||^2 \leq \left(\sum_{k \neq m}^{M} \hat{g}_{k,i}|\tau_{m,k}|\right)^2$, the equality constraint of problem (10) is now turned into an inequality constraint

$$\left(\sum_{k \neq m}^{M} \hat{g}_{k,i}|\tau_{m,k}|\right)^2 \leq \breve{g}_{m,i}^2, i = 1, \cdots, N_b. \tag{14}$$

Remember that $\sum_{k=1}^{M} |\tau_{m,k}|^2 = 1$, we have

$$\sum_{k \neq m}^{M} |\tau_{m,k}|^2 \leq 1. \tag{15}$$

Both the constraints in (14) and (15) are convex. Then, the original problem in (10) is converted into the following convex optimization problem

$$\begin{aligned} &\max_{\{|\tau_{m,k}|, k \neq m\}} \sum_{k \neq m}^{M} \rho_k |\tau_{m,k}|^2 \\ &s.t. \quad (14), \quad (15) \end{aligned} \tag{16}$$

The solution of this problem can be found efficiently using convex optimization tools [14]. Then the maximal power of the multi-user interference, I_m^{max}, can be computed with (12). To find the solution of (16), the CU only needs to know the estimated weighting factors define in (3), allocated powers and the error structure vector. This solution is not the optimal solution of problem (10), which may optimistically estimate the downlink SINR and lead to outage in transmission.

With I_m^{max}, the allocated powers to multiple users $p_1, \cdots, p_M$, the fed back values of $||\mathbf{g}_m||^2/\sigma_m^2$, as well as the global quantization accuracy $\cos\theta_m$, the CU can estimate the SINR experienced at the mth user from (7).

IV. Simulation Results

In this section we evaluate the proposed SINR estimation method via simulations. We consider a homogeneous cellular network and show the performance of a reference set of candidate cooperative cells, which consists of seven hexagonal cells, i.e., $\tilde{N}_b = 7$. The simulation parameters are listed in Table 1. To reduce the complexity of simulation, instead of generating multiple interfering cells, we model the interference from the surrounding cells of the reference set as white noise, which is the worst-case interference [15]. To show the impact of channel quantization, the cell-edge SNR is set as 10 dB, such that the system operating in multi-user interference limited region. The users are distributed within a cell-edge region satisfying $\alpha_{m,i_L}^2 / \max_{j \neq i_L} \alpha_{m,j}^2 < 6$ dB unless otherwise specified, called "6 dB CoMP region", where α_{m,i_L} is the large scale fading gain of the mth user from its master BS and $\alpha_{m,j}$, $j \neq i_L$ are those from cooperative BSs. The cooperative cluster of the mth user is formed as $\Omega_m = \{j | \alpha_{m,j}^2 / \alpha_{m,i_L}^2 \geq \mu, j = 1, \cdots, \bar{N}_b\}$, where $\mu = -11$ dB ensures the cluster size no more than 3. Note that the sizes of the cooperative clusters for different users may differ in the simulation. The SUS [5] is performed with quantized global norms and global CDIs, and the ZFBF is computed with reconstructed global CDIs in (3). The transmit power is allocated among users to maximize the sum

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rate under PBPC [16]. All the results are obtained from 1000 trails of Monte-Carlo simulations, where both user locations and small scale fading channels are randomly generated.

TABLE I
SIMULATION PARAMETERS [17].

Number of transmit antennas at each BS	4
Number of receive antennas per user	1
Number and distribution of users	10 users uniformly distributed in CoMP region
Cell radius	250 m
Path loss model, in dB	$36.3 + 37.6\log(D)$, where D is the distance from the user to the BS
Per-cell small scale fading channel	Independent and identically distributed Rayleigh channel
Quantization strategy for channel direction information	Per-cell codebook based quantization
Per-cell codebook	4-bit RVQ codebook [13]
Feedback error and delay	None
Beamformer	ZFBF
Scheduling method	SUS with the orthogonality threshold 0.3, i.e., the scheduled users meet $\hat{\mathbf{g}}_m\hat{\mathbf{g}}_k^H < 0.3$, which is selected to balance the cell-edge and cell average throughputs

The global quantization accuracy $\cos\theta_i$ is quantized with a uniform scalar codebook of size b bits. Its quantized version, $\hat{\cos}\,\theta_i$, is defined as the lower end of the quantization interval that includes $\cos\theta_i$. In this way we can ensure $\hat{\cos}\theta_m \leq \cos\theta_m$, such that the outage caused by the quantization error of $\cos\theta_m$ can be avoided.

The simulation results of using the independent codeword selection and the serial codeword selection are similar, therefore we only show the results with the independent codeword selection, where each phase ambiguity is fed back with 2 bits. Considering that the feedback for $||\mathbf{g}_m||/\sigma_m^2$ are necessary for all kinds of SINR estimation methods whose impact is well-understood, in the simulations we assume that it is perfectly known at the CU.

To show the impact of overestimating SINR on the outage probability, we first evaluate the system without HARQ, i.e., if the estimated SINR exceeds the actual SINR, an outage occurs and the data rate is 0 bps/Hz.

We start by showing whether large overhead is needed for feeding back the error structure vector $\mathbf{f}_m$. To this end, we compare the performance when the CU knows either perfect $\mathbf{f}_m$ or $\mathbb{E}[\mathbf{f}_m]$. In this simulation, the global quantization accuracy $\cos\theta_m$ is assumed perfectly known at the CU. As shown in Fig. 1, the performance of using the instantaneous vector $\mathbf{f}_m$ and average vector $\mathbb{E}[\mathbf{f}_m]$ only have slight difference. This is because we transform the equality constraint in problem (10) into the inequality constraint in problem (16), which makes the new optimization problem in (16) robust to the quantization error of $\mathbf{f}_m$. Another reason is because the global quantization error vector is asymmetric. According to *Proposition* 1, when each user independently selects per-cell codewords, $\mathbb{E}[\mathbf{f}_m]$ only depends on the large scale fading gains. Since large scale fading gains are necessary for reconstructing the global CDI, no extra overhead is required for the BS to obtain $\mathbb{E}[\mathbf{f}_m]$.

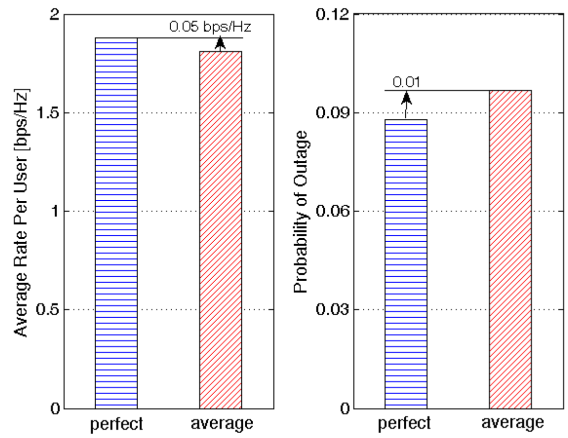

Fig. 1. Comparison in average data rate per-user and outage probability of each user with perfect $\mathbf{f}_m$ (with legend "perfect") and with $\mathbb{E}[\mathbf{f}_m]$ (with legend "average").

Next, we show how many bits are required for feeding back the global quantization accuracy $\cos\theta_m$. Considering the outage that may occur, we use two metrics: average data rate and average effective rate, which are respectively calculated by averaging the rates of all the scheduled users and averaging the rates only for the users without outage. We also compare with a directly extended PU2RC from that designed for single cell systems [10], where the SINR is estimated at the CU by using the same formula in [10] with the feedback of $\cos\theta_m$.[3] To meet the PBPC in CoMP systems meanwhile provide equal power allocation required by the PU2RC in [10], the power allocation in [1] is considered. As shown in Fig. 2, the extended PU2RC is too conservative and leads to low average data rate, since it does not take advantage of the asymmetry in CoMP channels. In contrary, the proposed method has significant performance gain in data rate over the extended PU2RC, although the probability of outage is no longer 0. When 4 bits are used for feeding back $\cos\theta_m$, for both methods the average rate of each user is close to that with perfect $\cos\theta_m$, and for the proposed method the probability of outage will be less than 10%.

Finally, we evaluate the proposed SINR estimation methods with more practical system setting, where the MCS selection in [18] and the HARQ with chasing combining [19] are used. With the HARQ, the data to a user will be retransmitted if an outage occurs. The retransmission occupies extra time slots, which has been taken into account when computing the data rate. The throughput is the sum rate of all the co-scheduled users, which is averaged over all the user locations

[3]Since the result of power allocation is perfectly known at the CU, this method can achieve the same performance as that of [11], which enhances the PU2RC in [10] by adjusting the transmit powers in the estimated SINR with the number of scheduled users.

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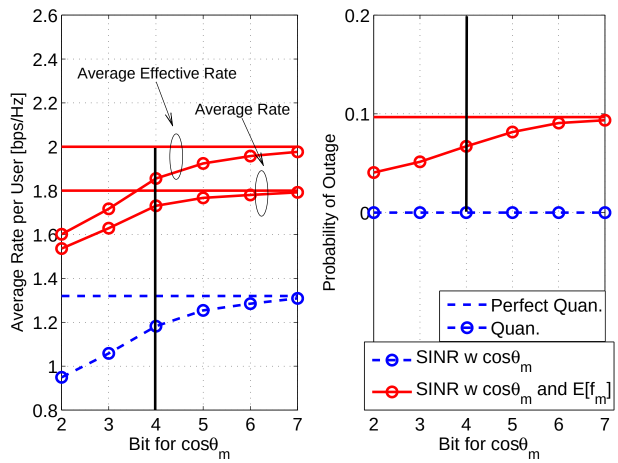

Fig. 2. Comparison in average data rate per-user and outage probability of different SINR estimation methods. The legend "SINR w $\cos\theta_m$ and $\mathbb{E}[\mathbf{f}_m]$" represent the performance of the proposed SINR estimation method with quantized $\cos\theta_m$ and $\mathbb{E}[\mathbf{f}_m]$. A method directly extended from the PU2RC designed for single cell systems [10] is with the legend "SINR w $\cos\theta_m$"

and channel realizations. The global quantization accuracy $\cos\theta_m$ is quantized with 4 bits. Different CoMP regions are considered, where "Full Region" is a case where the users are uniformly located in the whole CoMP cluster. As shown in Fig. 3, the proposed method outperforms the extended PU2RC, and the performance gain is more significant for large CoMP region. This is because the proposed method takes advantage of the asymmetry in CoMP channel, and the users in the cell center have more asymmetric channels.

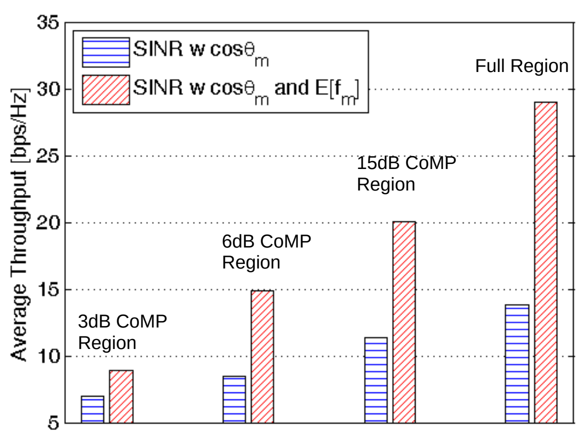

Fig. 3. Comparison in average throughput of different SINR estimation methods with MCS selection and HARQ.

V. Conclusions

In this paper we studied SINR estimation for downlink CoMP systems. To reduce the outage in transmission due to an optimistic SINR estimate and exploit the fact that the global quantization error vector is no longer isotropic for CoMP channels, we proposed a method to estimate the quantization error vector maximizing the interference power under a structure constraint, with which the SINR can be obtained. Simulation results show that with the same feedback overhead and even when considering the re-transmission mechanism, the proposed method outperforms existing method and the performance gain increases with the size of the CoMP region.

Appendix

Substituting (2) and (3) into (6), for the ith per-cell CDI of the mth user, $i \in \{1, \cdots, N_b\}$, we have

$$g_{m,i}\bar{\mathbf{h}}_{m,i} = \cos\theta_m e^{j\xi_m} \cdot \hat{g}_{m,i}\hat{\mathbf{h}}_{m,i} + \sin\theta_m \breve{g}_{m,i}\breve{\mathbf{s}}_{m,i}. \quad \text{(A.1)}$$

The quantized per-cell CDI can be modeled as $\hat{\mathbf{h}}_{m,i} = \cos\theta_{m,i}e^{j\xi_{m,i}}\bar{\mathbf{h}}_{m,i} + \sin\theta_{m,i}\mathbf{e}_{m,i}$, where $\cos\theta_{m,i} \triangleq |\bar{\mathbf{h}}_{m,i}^H\hat{\mathbf{h}}_{m,i}|$ is the quantization accuracy of the per-cell CDI, $\sin\theta_{m,i} = \sqrt{1-\cos^2\theta_{m,i}}$, $e^{j\xi_{m,i}}$ is the phase ambiguity, and $\mathbf{e}_{m,i} \in \mathbb{C}^{n_t\times 1}$ is a unit vector with $\mathbf{e}_{m,i}^H\bar{\mathbf{h}}_{m,i} = 0$.

Substituting $\hat{\mathbf{h}}_{m,i}$ into (A.1), we obtain

$$\begin{aligned}\sin\theta_m\breve{g}_{m,i}\breve{\mathbf{s}}_{m,i} =& (g_{m,i} - \hat{g}_{m,i}\cos\theta_m\cos\theta_{m,i}e^{j(\xi_m+\xi_{m,i})})\bar{\mathbf{h}}_{m,i} \\ &- \hat{g}_{m,i}\cos\theta_m\sin\theta_{m,i}e^{j\xi_m}\mathbf{e}_{m,i}.\end{aligned} \quad \text{(A.2)}$$

By taking the two norm on both sides of (A.2), we have

$$\sin^2\theta_m\breve{g}_{m,i}^2 = g_{m,i}^2 X_{m,i}, \quad \text{(A.3)}$$

where $X_{m,i} = 1 + \frac{\hat{g}_{m,i}^2}{g_{m,i}^2}\cos^2\theta_m - \frac{2\hat{g}_{m,i}^2}{g_{m,i}^2}\cos\theta_m\cos\theta_{m,i}\cos(\xi_{m,i}+\xi_m)$.

It is not hard to show that $\lim_{n_t\to\infty} g_{m,i}^2 = \hat{g}_{m,i}^2$ according to the law of large numbers. Then we can derive $\lim_{n_t\to\infty} X_{m,i} = 1 + \cos^2\theta_m - \cos\theta_m\cos\theta_{m,i}\cos(\xi_{m,i}+\xi_m) \triangleq \bar{X}_{m,i}$.

As a result, when $n_t \to \infty$ (A.3) can be further written as $\sin^2\theta_m\breve{g}_{m,i}^2 = \hat{g}_{m,i}^2\bar{X}_{m,i}$. Taking the expectation over both sides of it, we obtain $\mathbb{E}[\sin^2\theta_m\breve{g}_{m,i}^2] = \hat{g}_{m,i}^2\mathbb{E}[\bar{X}_{m,i}]$. Since the quantization error vector $\mathbf{s}_m$ is independent of the quantization error $\sin\theta_m$ [13], $\breve{g}_{m,i}$ is independent of $\sin\theta_m$. Therefore,

$$\mathbb{E}[\sin^2\theta_m\breve{g}_{m,i}^2] = \mathbb{E}[\sin^2\theta_m]\mathbb{E}[\breve{g}_{m,i}^2] = \hat{g}_{m,i}^2\mathbb{E}[\bar{X}_{m,i}], \quad \text{(A.4)}$$

from which we have

$$\mathbb{E}[\sin^2\theta_m]\cdot\sum_{i=1}^{N_b}\mathbb{E}[\breve{g}_{m,i}^2] = \sum_{i=1}^{N_b}\hat{g}_{m,i}^2\mathbb{E}[\bar{X}_{m,i}]. \quad \text{(A.5)}$$

Since $\sum_{i=1}^{N_b}\mathbb{E}[\breve{g}_{m,i}^2] = \mathbb{E}[\|\ \mathbf{s}_m\ \|^2] = 1$, (A.5) can be further written as

$$\mathbb{E}[\sin^2\theta_m] = \sum_{i=1}^{N_b}\hat{g}_{m,i}^2\mathbb{E}[\bar{X}_{m,i}]. \quad \text{(A.6)}$$

When each user independently selects codewords for the per-cell CDIs from the per-cell codebook, the quantization accuracy $\cos\theta_{m,i}$ and the phase ambiguity $e^{j\xi_{m,i}}$ of different per-cell channels have the same probability distribution. It means that $\bar{X}_{m,i}, i = 1, \cdots, N_b$ have the same probability distribution, therefore $\mathbb{E}[\bar{X}_{m,i}] = \mathbb{E}[\bar{X}_{m,j}], \forall i \neq j$. Upon substituting into (A.6) we have $\mathbb{E}[\bar{X}_{m,i}] = \mathbb{E}[\sin^2\theta_m]$.

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Substituting it into (A.4), we obtain $\mathbb{E}[\breve{g}_{m,i}^2] = \hat{g}_{m,i}^2$, which is derived under the condition that $n_t \rightarrow \infty$. In general cases we have $\mathbb{E}[\breve{g}_{m,i}^2] \approx \hat{g}_{m,i}^2$, which is accurate when n_t is sufficiently large.

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