Hybrid Precoder for Massive MIMO Systems with Coverage Constraint

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***Abstract*—Hybrid architecture has been shown to be cost effective for massive multi-input multi-out (MIMO) systems, where analog precoder is used to form narrow beams towards users for data transmission with reduced number of radio frequency chains. However, besides the user-specific data, a wideband massive MIMO system needs to broadcast control signaling intended for all users simultaneously, which requires wide beams to ensure the coverage. In this paper, we study the wideband hybrid precoder for downlink space-division multi-access and orthogonal frequency division multi-access (SDMA-OFDMA) massive MIMO systems, aimed at minimizing the total transmit power of the base station, subject to the coverage constraint of signaling broadcasting and data rate requirements of users. We first derive an upper bound of coverage probability under general spatially correlated channels, with which a hybrid precoder without requiring the iteration between analog and digital precoders is proposed. Simulation results show that under the same data rate requirement and coverage constraint, the proposed hybrid precoder can effectively reduce the total transmit power compared to the method with narrow-beam analog precoder only based on data rate requirements.**

I. Introduction

Massive multi-input multi-output (MIMO) is a promising technology of fifth-generation (5G) cellular systems, which can support very high throughput by using large-scale antenna array at base stations (BSs) [1]. However, the benefits of massive MIMO comes at the penalty of excessive hardware cost, transceiver complexity, and energy consumption when equipping every antenna element with a radio frequency (RF) chain. Hybrid architecture has been shown as a cost effective approach to reduce the number of required RF chains, where the analog precoder in RF domain and low-dimensional digital precoders in baseband are involved [2].

Existing studies on hybrid precoder optimization mainly focus on user-specific data transmission. In narrowband (i.e., single-subcarrier) systems, hybrid precoders were designed for single user MIMO case in [3–5] and for multi-user MIMO case in [6–9], respectively. The results demonstrated that the hybrid precoder with few number of RF chains performs close to the full digital precoder for massive MIMO systems operating in both millimeter wave frequency [3–7] and ultra-high frequency band (e.g., Long-term Evolution (LTE)) [8,9]. When applied to wideband orthogonal frequency division multiplexing (OFDM) systems, these narrowband hybrid precoders, in particular those with analog precoder designed based on spatial correlation information, can work well if every user occupies all subcarriers, i.e., frequency-domain scheduling is not employed as studied in [10]. When frequency-domain scheduling is considered, different users served over different subcarriers share the same analog precoder, so that existing narrowband hybrid precoders are no longer applicable. In [11] and [12], wideband hybrid precoders together with frequency-domain scheduling were designed to maximize the sum rate, which effectively improve the performance of wideband hybrid massive MIMO systems.

Besides user-specific data, a wideband massive MIMO system needs to simultaneously broadcast signaling intended for all user, such as control signal and common reference signal (CRS) in LTE systems. In OFDM systems user-specific data and signaling are multiplexed on different subcarriers, which can use different digital precoders in baseband but have to share the same analog precoder in RF domain. For user-specific data transmission, analog precoder needs to form multiple narrow beams towards the users. For signaling broadcasting, however, wide beams are preferred to ensure the cell coverage. These conflicting requirements on analog precoder make the joint design of wideband analog precoder and digital precoder very challenging. Precoder design for signaling broadcasting in massive MIMO systems was studied in [13] and [14], where full-digital systems equipping every antenna with a RF chain were considered and hence only digital wide beam on each signaling subcarrier was designed. To the best of the authors' knowledge, hybrid precoder design subject to both user-specific data rate requirements and signaling coverage constraint has not been investigated in the literature.

In this paper, we design wideband hybrid precoder for downlink space-division multiple-access and orthogonal frequency-division multiple-access (SDMA-OFDMA) massive MIMO systems, subject to both coverage constraint of signaling broadcasting and data rate requirements of users. Under the general spatially correlated channels, we first derive an upper bound of the coverage probability, based on which a wideband hybrid precoder is proposed to minimize the total transmit power of the BS. The proposed hybrid precoder has low complexity in the sense that iterative optimizations between analog and digital precoders are not required. Simulation results show that under the same data rate requirement and signaling coverage constraint, compared to the method that designs narrow-beam analog precoder only based on data rate requirements, the proposed method can effectively reduce the power for signaling broadcasting with negligible increase of the power for data transmission.

II. System model and Channel model

A. System model

Consider a single-cell downlink SDMA-OFDMA massive MIMO system, where the BS is equipped with M antennas and D RF chains and each user has a single antenna. The whole bandwidth is divided into N resource blocks (RBs), within each of which flat fading channels are assumed. Let $\mathcal{N}_D$ denote the set of RBs for user-specific data transmission, and let $\mathcal{N}_S$ denote the set of RBs for signaling broadcasting, on which control signaling such as system information and CRS is broadcast. Then, $|\mathcal{N}_D| + |\mathcal{N}_S| = N$, where $|\mathcal{X}|$ is the cardinality of set $\mathcal{X}$.

The received signal of the k-th user on the n-th data RB (denoted by UE_{nk}) for $n \in \mathcal{N}_D$ can be expressed as

$$y_{nk} = \mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nk} s_{nk} + \sum_{j=1, j\neq k}^{K} \mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nj} s_{nj} + z_{nk}, \quad (1)$$

where $\mathbf{h}_{nk} \in \mathbb{C}^{M\times 1}$ is the downlink channel of UE_{nk}, $\mathbf{V} \in \mathbb{C}^{M\times D}$ is the wideband analog precoder, $\mathbf{w}_{nk} \in \mathbb{C}^{D\times 1}$ is the digital precoder for UE_{nk}, s_{nk} is the data symbol of UE_{nk} with $\mathbb{E}\{|s_{nk}|^2\} = 1$, z_{nk} is the additive white Gaussian noise (AWGN) of UE_{nk} with zero mean and variance σ_{nk}^2, and K is the number of users served on each RB.

For signaling broadcasting, let UE_b be a user at cell edge with a random location. The received signal of UE_b on the n-th RB for $n \in \mathcal{N}_S$ can be expressed as

$$\tilde{y}_{nb} = \tilde{\mathbf{h}}_{nb}^H \mathbf{V} \tilde{\mathbf{w}}_{nS} \tilde{s}_{nS} + \tilde{z}_{nS}, \quad (2)$$

where $\tilde{\mathbf{h}}_{nb} \in \mathbb{C}^{M\times 1}$ is the channel of UE_b on the n-th RB, $\mathbf{V}$ is the wideband analog precoder the same as the one for user-specific data transmission in (1), $\tilde{\mathbf{w}}_{nS} \in \mathbb{C}^{D\times 1}$ is the digital precoder for signaling $\tilde{s}_{nS}$ on the n-th RB with $\mathbb{E}\{|\tilde{s}_{nS}|^2\} = 1$, and $\tilde{z}_{nS}$ is the AWGN with zero mean and variance σ_{nS}^2. Herein, digital precoder $\tilde{\mathbf{w}}_{nS}$ is used for signaling broadcasting and hence is common for all users.

B. Channel model

We consider general spatially correlated channels based on 3GPP massive MIMO channel model with uniform linear antenna array (ULA) [15]. Let L_b denote the length of time-domain multipath channels of UE_b. The l-th multipath component can be expressed as

$$\mathbf{h}_{bl}(t) = \sqrt{\frac{\alpha q_{bl}}{I_b}} \sum_{i=1}^{I_b} e^{j\phi_{bli}} \mathbf{a}(\theta_{bli}) \delta(t - \tau_{bl}), \quad (3)$$

where α is the large-scale fading gain of the cell-edge user, UE_b, q_{bl} is the power of the l-th path satisfying $\sum_{l=1}^{L_b} q_{bl} = 1$, I_b is the number of subpaths per path, ϕ_{bli} and θ_{bli} are he phase and angle of departure (AoD) of the i-th subpath of the l-th path, respectively, ϕ_{bli} follows uniform distribution $U(-\pi, \pi)$, τ_{bl} is the delay of the l-th path, and $\mathbf{a}(\theta_{bli})$ is the antenna array response vector at the BS, whose m-th element is $e^{-j\frac{2\pi d}{\lambda_c}(m-1)\cos(\theta_{bli})}$ with λ_c and d denoting the carrier wavelength and antenna spacing, respectively, $m = 1, \ldots, M$.

The AoD of the i-th subpath of the l-th path can be expressed as

$$\theta_{bli} = \bar{\theta}_b + \varphi_{bl} + \Delta_{bli}, \quad (4)$$

where $\bar{\theta}_b$ is the line-of-sight angle of UE_b, φ_{bl} is the angle offset of the l-th path, and Δ_{bli} is the angle offset of the i-th subpath. The statistical behaviors of $\bar{\theta}_b$, φ_{bl} and Δ_{bli} depend on user distribution as well as scenarios such as urban, suburban, etc. We consider that each of $\bar{\theta}_b$, φ_{bl} and Δ_{bli} is independent and identically distributed (i.i.d.), but no specific distributions are assumed.

Then, we can obtain the frequency-domain channel of UE_b on the n-th RB as

$$\begin{aligned} \tilde{\mathbf{h}}_{nb} &= \sum_{l=1}^{L_b} e^{-2\pi n f_0 \tau_{bl}} \sqrt{\frac{\alpha q_{bl}}{I_b}} \sum_{i=1}^{I_b} e^{j\phi_{bli}} \mathbf{a}(\theta_{bli}) \\ &\triangleq \sum_{l=1}^{L_b} e^{-2\pi n f_0 \tau_{bl}} \sqrt{\alpha q_{bl}} \mathbf{g}_{bl}, \end{aligned} \quad (5)$$

where $\mathbf{g}_{bl} \triangleq \frac{1}{\sqrt{I_b}} \sum_{i=1}^{I_b} e^{j\phi_{bli}} \mathbf{a}(\theta_{bli})$, and f_0 is the frequency spacing of RBs.

III. Hybrid Precoder Optimization

A. Problem Formulation

We optimize the hybrid precoder to minimize the total transmit power of the BS subject to both user-specific data rate requirements and signaling coverage constraints.

From (1), the signal-to-interference-plus noise ratio (SINR) of UE_{nk} can be expressed as

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nk}|^2}{\sum_{j=1, j\neq k}^{K} |\mathbf{h}_{nk}^H \mathbf{V} \mathbf{w}_{nj}|^2 + \sigma_{nk}^2}. \quad (6)$$

The signaling coverage probability on the n-th RB can be expressed as

$$P_{cov,n} = \texttt{Pr}\Big(\frac{|\tilde{\mathbf{h}}_{nb}^H \mathbf{V} \tilde{\mathbf{w}}_{nS}|^2}{\sigma_{nS}^2} \geq \gamma_S\Big), \quad (7)$$

where $\texttt{Pr}(\cdot)$ denotes the probability and γ_S is the required receive signal-to-noise ratio (SNR) for the reception of system information and CRS. Note that the probability in (7) is with respect to both small-scale channels and random locations of users.

We assume that perfect channel state information is available at the BS, which can be achieved in time-division duplex (TDD) systems via uplink training [16]. Then the hybrid precoder optimization problem can be formulated as

$$\min_{\mathbf{V}, \{\mathbf{w}_{nk}\}, \{\tilde{\mathbf{w}}_{nS}\}} \sum_{n\in\mathcal{N}_D} \sum_{k=1}^{K} \|\mathbf{V}\mathbf{w}_{nk}\|^2 + \sum_{n\in\mathcal{N}_S} \|\mathbf{V}\tilde{\mathbf{w}}_{nS}\|^2 \quad (8a)$$

$$s.t. \quad \text{SINR}_{nk} \geq \gamma_{nk}, \ n \in \mathcal{N}_D, k = 1, \ldots, K \quad (8b)$$

$$P_{cov,n} \geq \epsilon, \ n \in \mathcal{N}_S, \quad (8c)$$

where $\gamma_{nk} = 2^{R_{nk}} - 1$ is the target SINR of UE_{nk} with R_{nk} representing the required data rate, and ϵ is the minimum acceptable coverage probability.

B. Coverage Probability

To solve problem (8), we start with the derivation of coverage probability $P_{cov,n}$.

Considering that the number of subpaths I_b is generally large (e.g., $I_b = 20$ in 3GPP massive MIMO channel model [15]), based on central-limit theorem (CLT), we can obtain that $\mathbf{g}_{bl}$ in (5) follows complex Gaussian distribution $\mathcal{CN}(\boldsymbol{\nu}_{bl}, \boldsymbol{\Sigma}_{bl})$.

With $\phi_{bli} \sim U(-\pi, \pi)$, we can obtain that $\boldsymbol{\nu}_{bl} = \mathbf{0}$. The elements of covariance matrix $\boldsymbol{\Sigma}_{bl}$ can be derived as

$$\begin{aligned}[\boldsymbol{\Sigma}_{bl}]_{m_1 m_2} &= \mathbb{E}_{\Delta_{bli}}\big\{[\mathbf{a}(\theta_{bli})]_{m_1}[\mathbf{a}^H(\theta_{bli})]_{m_2}\big\} \\ &= \mathbb{E}_{\Delta_{bli}}\big\{e^{-j(m_1-m_2)\frac{2\pi d}{\lambda}\cos(\bar{\theta}_b+\varphi_{bl}+\Delta_{bli})}\big\}, \quad (9)\end{aligned}$$

where the expectation is taken over Δ_{bli} with given $\bar{\theta}_b$ and φ_{bl}. For $m_1 = m_2$, we have $[\boldsymbol{\Sigma}_{bl}]_{m_1 m_2} = 1$; otherwise, $[\boldsymbol{\Sigma}_{bl}]_{m_1 m_2}$ can be obtained numerically given the distribution of Δ_{bli}. Herein, $[\mathbf{X}]_{ij}$ denotes the element on the i-th row and j-th column of matrix $\mathbf{X}$.

Since the random variable ϕ_{bli} in $\mathbf{g}_{bl}$ follows independent uniform distribution $U(-\pi, \pi)$, we can find that the elements in set $\{\mathbf{g}_{bl}\}$ are independent complex Gaussian distributed vectors for different b and l. Thus, we can obtain that $\tilde{\mathbf{h}}_{nb} \sim \mathcal{CN}(\mathbf{0}, \alpha\sum_{l=1}^{L_b} q_{bl}\boldsymbol{\Sigma}_{bl})$, where $\boldsymbol{\Sigma}_{bl}$ is a function of $\bar{\theta}_b$ and φ_{bl} as shown in (9). Then, the conditional probability density function of $\tilde{\mathbf{h}}_{nb}^H\mathbf{V}\tilde{\mathbf{w}}_{nS}$ given $\bar{\theta}_b$ and φ_{bl} can be obtained as

$$\tilde{\mathbf{h}}_{nb}^H\mathbf{V}\tilde{\mathbf{w}}_{nS} \sim \mathcal{CN}\Big(\mathbf{0}, \alpha\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\big(\sum_{l=1}^{L_b} q_{bl}\boldsymbol{\Sigma}_{bl}\big)\mathbf{V}\tilde{\mathbf{w}}_{nS}\Big). \quad (10)$$

By using (10), the coverage probability can be derived as

$$\begin{aligned}P_{cov,n} &= \mathbb{E}_{\bar{\theta}_b,\varphi_{bl}}\Big\{\mathtt{Pr}\big(\frac{|\tilde{\mathbf{h}}_{nb}^H\mathbf{V}\tilde{\mathbf{w}}_{nS}|^2}{\sigma_{nS}^2} \geq \gamma_S \big| \bar{\theta}_b, \varphi_{bl}\big)\Big\} \\ &= \mathbb{E}_{\bar{\theta}_b,\varphi_{bl}}\Big\{\exp\big(\frac{-\sigma_{nS}^2\gamma_S}{\alpha\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\big(\sum_{l=1}^{L_b} q_{bl}\boldsymbol{\Sigma}_{bl}\big)\mathbf{V}\tilde{\mathbf{w}}_{nS}}\big)\Big\}. \quad (11)\end{aligned}$$

The coverage probability given in (11) is still complicated for the optimization over $\mathbf{V}$ and $\tilde{\mathbf{w}}_{nS}$. To simplify it, we examine the convexity of function $\exp(-\frac{1}{x})$. One can find that $\exp(-\frac{1}{x})$ is concave when $x > 0.5$, is approximately affine when $0.25 \leq x \leq 0.5$, and approaches to zero when $x < 0.25$. Therefore, based on Jensen's inequality, we can obtain $\mathbb{E}_x\{\exp(-\frac{1}{x})\} \leq \exp(-\frac{1}{\mathbb{E}_x\{x\}})$. Then, an upper bound of the coverage probability can be obtained from (11) as

$$\begin{aligned}P_{cov,n} &\leq \exp\big(-\frac{\sigma_{nS}^2\gamma_S}{\alpha\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\mathbb{E}_{\bar{\theta}_b,\varphi_{bl}}\{\sum_{l=1}^{L_b} q_{bl}\boldsymbol{\Sigma}_{bl}\}\mathbf{V}\tilde{\mathbf{w}}_{nS}}\big) \\ &\triangleq P_{cov,n}^{up}. \quad (12)\end{aligned}$$

It is not hard to find that when the channel is i.i.d. complex Gaussian distributed, the upper bound in (12) is tight, i.e., (12) holds with equality in this case.

Since φ_{bl} are i.i.d. variables, we have $\mathbb{E}_{\varphi_{b1}}\{\boldsymbol{\Sigma}_{b1}\} = \cdots = \mathbb{E}_{\varphi_{bL_b}}\{\boldsymbol{\Sigma}_{bL_b}\} \triangleq \bar{\boldsymbol{\Sigma}}_b$. Further recalling that $\sum_{l=1}^{L_b} q_{bl} = 1$, we can obtain from (12) that

$$P_{cov,n}^{up} = \exp\big(-\frac{\sigma_{nS}^2\gamma_S}{\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\mathbf{C}\mathbf{V}\tilde{\mathbf{w}}_{nS}}\big), \quad (13)$$

where $\mathbf{C} = \alpha\mathbb{E}_{\bar{\theta}_b}\{\bar{\boldsymbol{\Sigma}}_b\}$, which can be pre-determined numerically for a specific system with given channel model without the need of online computation.

With the upper bound of coverage probability in (13), we can rewrite the coverage constraint in (8c) as

$$\exp\big(-\frac{\sigma_{nS}^2\gamma_S}{\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\mathbf{C}\mathbf{V}\tilde{\mathbf{w}}_{nS}}\big) - \Delta\epsilon \geq \epsilon, \ n \in \mathcal{N}_S, \quad (14)$$

where $\Delta\epsilon$ is a back off margin to ensure the minimum coverage requirement because of the usage of coverage probability upper bound.

C. Wideband Hybrid Precoder

With (14), we can reformulate problem (8) as follows,

$$\min_{\substack{\mathbf{V},\{\mathbf{w}_{nk}\},\\ \{\tilde{\mathbf{w}}_{nS}\}}} \sum_{n\in\mathcal{N}_D}\sum_{k=1}^{K}\|\mathbf{V}\mathbf{w}_{nk}\|^2 + \sum_{n\in\mathcal{N}_S}\|\mathbf{V}\tilde{\mathbf{w}}_{nS}\|^2 \quad (15a)$$

$$s.t. \ \frac{|\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nk}|^2}{\sum_{j\neq k}|\mathbf{h}_{nk}^H\mathbf{V}\mathbf{w}_{nj}|^2 + \sigma_{nk}^2} \geq \gamma_{nk}, \ n \in \mathcal{N}_D, \forall k \quad (15b)$$

$$\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\mathbf{C}\mathbf{V}\tilde{\mathbf{w}}_{nS} \geq -\frac{\sigma_{nS}^2\gamma_S}{\ln(\epsilon+\Delta\epsilon)}, \ n \in \mathcal{N}_S, \quad (15c)$$

where constraint (15c) comes from (14).

Problem (15) is non-convex, making it difficult to find the global optimal solution. Next, we propose a suboptimal algorithm to solve (15) by decoupling the optimization of $\mathbf{V}$ and $\{\mathbf{w}_{nk}, \tilde{\mathbf{w}}_{nS}\}$. First, by assuming the dirty paper coding used as digital precoders, we optimize the analog precoder $\mathbf{V}$ to maximize sum rate of both equivalent data channels and equivalent signaling channels (as will be clear later), where the transmit powers for data and signaling are given, which are estimated from problem (15) based on users' data rate requirement and coverage constraint. Then, given $\mathbf{V}$, we optimize the digital precoder $\{\mathbf{w}_{nk}, \tilde{\mathbf{w}}_{nS}\}$ to minimize the total transmit power of the BS.

1) ***Optimization of*** $\mathbf{V}$*:* With analog precoder $\mathbf{V}$, the equivalent channel of UE_{nk} for data transmission is $\mathbf{h}_{nk}^H\mathbf{V}$. For the cell-edge user, UE_b, for signaling broadcasting, we define the equivalent channel as $\mathbf{C}^{\frac{H}{2}}\mathbf{V}$ by noting that the left-hand side of the coverage constraint (15c) can be written as $\tilde{\mathbf{w}}_{nS}^H\mathbf{V}^H\mathbf{C}^{\frac{1}{2}}\mathbf{C}^{\frac{H}{2}}\mathbf{V}\tilde{\mathbf{w}}_{nS}$,

Based on the uplink-downlink duality theory [17] and after some manipulations, we can obtain the optimization problem only with respect to analog precoder as

$$\begin{aligned}\max_{\mathbf{V}}\max_{\{\mathbf{D}_n\},D_S}&\sum_{n\in\mathcal{N}_D}\log\frac{\det(\mathbf{V}^H\mathbf{H}_n\boldsymbol{\Xi}_n^{\frac{1}{2}}\mathbf{D}_n\boldsymbol{\Xi}_n^{\frac{1}{2}}\mathbf{H}_n^H\mathbf{V}+\mathbf{V}^H\mathbf{V})}{\det(\mathbf{V}^H\mathbf{V})} \\ &+\sum_{n\in\mathcal{N}_S}\log\frac{\det(\frac{D_S}{\sigma_{nS}^2}\mathbf{V}^H\mathbf{C}\mathbf{V}+\mathbf{V}^H\mathbf{V})}{\det(\mathbf{V}^H\mathbf{V})} \quad (16a)\end{aligned}$$

$$s.t. \quad \textstyle\sum_{n\in\mathcal{N}_D}\mathtt{tr}(\mathbf{D}_n) + |\mathcal{N}_S|D_S \leq P_{total}, \quad (16b)$$

where P_{total} is the total downlink transmit power of the BS, i.e., the value of the objective function (15a), $\mathbf{D}_n \in \mathbb{R}^{K\times K}$ is a diagonal matrix whose k-th diagonal element is the transmit power of UE_{nk} in the dual uplink data channels, D_S is transmit power of the cell-edge user, UE$_b$, in the dual uplink signaling channels, $\mathbf{H}_n = [\mathbf{h}_{n1}, \ldots, \mathbf{h}_{nK}]$ is the channel matrix of the K data users served on the n-th RB, and $\boldsymbol{\Xi}_n$ is a diagonal matrix with $[\boldsymbol{\Xi}_n]_{kk} = \frac{1}{\sigma_{nk}^2}$.

By defining $\bar{\mathbf{H}}_n = \mathbf{H}_n \boldsymbol{\Xi}_n^{\frac{1}{2}}$ and based on the properties of matrix determinant, (16) can be rewritten as

$$\max_{\mathbf{V}} \max_{\{\mathbf{D}_n\}, D_S} \sum_{n \in \mathcal{N}_D} \operatorname{logdet}\big(\mathbf{D}_n \bar{\mathbf{H}}_n^H \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H \bar{\mathbf{H}}_n + \mathbf{I}\big)$$

$$+ \sum_{n \in \mathcal{N}_S} \log\det\big(\frac{D_S}{\sigma_{nS}^2}\mathbf{C}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H + \mathbf{I}\big) \quad (17a)$$

$$s.t. \quad \textstyle\sum_{n \in \mathcal{N}_D} \mathtt{tr}(\mathbf{D}_n) + |\mathcal{N}_S| D_S \leq P_{total}. \quad (17b)$$

In (17a) the term $\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H$ is a projection matrix, which can be expressed as $\mathbf{U}\mathbf{U}^H$ with $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Without loss of generality, we select the analog precoder satisfying $\mathbf{V}^H\mathbf{V} = \mathbf{I}$. Then, the problem (17) can be converted as

$$\max_{\mathbf{V}} \max_{\{\mathbf{D}_n\}, D_S} \sum_{n \in \mathcal{N}_D} \log\det\big(\mathbf{D}_n \bar{\mathbf{H}}_n^H \mathbf{V}\mathbf{V}^H \bar{\mathbf{H}}_n + \mathbf{I}\big)$$

$$+ \sum_{n \in \mathcal{N}_S} \log\det\big(\frac{D_S}{\sigma_{nS}^2}\mathbf{C}\mathbf{V}\mathbf{V}^H + \mathbf{I}\big) \quad (18a)$$

$$s.t. \quad \textstyle\sum_{n \in \mathcal{N}_D} \mathtt{tr}(\mathbf{D}_n) + |\mathcal{N}_S| D_S \leq P_{total} \quad (18b)$$

$$\mathbf{V}^H\mathbf{V} = \mathbf{I}. \quad (18c)$$

To obtain $\mathbf{V}$ from problem (18), we need to first find $\mathbf{D}_n$ and D_S, which however depend on the parameter P_{total} that equals to the value of objective function (15a) and is unknown. To tackle this difficulty, we next estimate $\mathbf{D}_n$, D_S and P_{total} from problem (15) by assuming that the system is full digital.

Under the full digital assumption, we have $\mathbf{V} = \mathbf{I}$ and problem (15) can be decoupled for $\{\mathbf{w}_{nk}\}$ and $\{\tilde{\mathbf{w}}_{nS}\}$. The resultant problem for $\{\mathbf{w}_{nk}\}$ is to minimize the total power for data transmission only subject to user-specific data rate requirements (15b), which can be converted into a second-cone constrained convex problem and can be solved efficiently [18], whose optimal solutions are denoted by $\{\hat{\mathbf{w}}_{nk}\}$. The decoupled problem for $\{\tilde{\mathbf{w}}_{nS}\}$ is to minimize the total power for signaling broadcasting only subject to coverage constraint (15c), whose optimal solution (denoted as $\{\hat{\tilde{\mathbf{w}}}_{nS}\}$) can be obtained as

$$\hat{\tilde{\mathbf{w}}}_{nS} = \sqrt{\frac{-\sigma_{nS}^2 \gamma_S}{\lambda_{max}(\mathbf{C}) \ln(\epsilon + \Delta\epsilon)}}\,\mathbf{u}_{max}(\mathbf{C}), \quad (19)$$

where $\lambda_{max}(\mathbf{X})$ and $\mathbf{u}_{max}(\mathbf{X})$ denote the maximal eigenvalue and the corresponding eigen-vector of matrix $\mathbf{X}$, respectively.

Then, we can obtain the estimate of P_{total} as $\hat{P}_{total} = \sum_{n \in \mathcal{N}_D} \sum_{k=1}^{K} \|\hat{\mathbf{w}}_{nk}\|^2 + \sum_{n \in \mathcal{N}_S} \|\hat{\tilde{\mathbf{w}}}_{nS}\|^2$. To estimate $\mathbf{D}_n$ and D_S, we exploit the duality between uplink and downlink SINR [19], which states that there exist uplink power $\mathbf{D}_n$ and D_S satisfying the identical user-specific data rate requirements and coverage constraints in uplink as downlink with the same total transmit power, i.e., $\sum_{n \in \mathcal{N}_D} \mathtt{tr}(\mathbf{D}_n) + |\mathcal{N}_S| D_S = \hat{P}_{total}$. According to the results in [19], we can estimate $\mathbf{D}_n$ as

$$\hat{\mathbf{D}}_n = \mathtt{diag}\big((\mathbf{B}_n - \mathbf{A}_n)^{-1}\mathbf{1}\big), \quad (20)$$

where $\mathbf{B}_n \in \mathbb{R}^{K \times K}$ is a diagonal matrix with $[\mathbf{B}_n]_{kk} = \frac{(1+\gamma_{nk})|\mathbf{h}_{nk}^H \hat{\mathbf{w}}_{nk}|^2}{\gamma_{nk}\|\hat{\mathbf{w}}_{nk}\|^2 \sigma_{nk}^2}$, $\mathbf{A}_n \in \mathbb{R}^{K \times K}$ is defined as $[\mathbf{A}_n]_{kj} = \frac{|\mathbf{h}_{nj} \hat{\mathbf{w}}_{nk}|^2}{\|\hat{\mathbf{w}}_{nk}\|^2 \sigma_{nj}^2}$, $\mathbf{1}$ is the all-one vector, and $\mathtt{diag}(\mathbf{x})$ denotes a diagonal matrix whose diagonal is given by the elements of vector $\mathbf{x}$. Similarly, we can estimate D_S as

$$\hat{D}_S = \frac{-\sigma_{nS}^2 \gamma_S}{\lambda_{max}(\mathbf{C}) \ln(\epsilon + \Delta\epsilon)}. \quad (21)$$

Given the estimated $\mathbf{D}_n$ and D_S, problem (18) is still not convex for $\mathbf{V}$. To solve the problem, we resort to semi-definite relaxation (SDR) method by defining $\mathbf{V}\mathbf{V}^H = \mathbf{Q}$ [20]. After obtaining the optimal $\mathbf{Q}$, we can employ the Gaussian randomization method to obtain the analog precoder $\mathbf{V}$ [20], where the generated random matrices need to be orthogonalized to ensure constraint (18c). The derivations are omitted due to the lack of space (for details refer to [11]).

*2) **Optimization of** $\mathbf{w}_{nk}$ **and** $\tilde{\mathbf{w}}_{nS}$:* Given $\mathbf{V}$, the optimization problem (15) for $\mathbf{w}_{nk}$ and $\tilde{\mathbf{w}}_{nS}$ can be decoupled. The decoupled problem with respect to $\mathbf{w}_{nk}$ is

$$\min_{\{\mathbf{w}_{nk}\}} \sum_{n \in \mathcal{N}_D} \sum_{k=1}^{K} \|\mathbf{V}\mathbf{w}_{nk}\|^2 \quad (22a)$$

$$s.t. \quad \frac{|\mathbf{h}_{nk}^H \mathbf{V}\mathbf{w}_{nk}|^2}{\sum_{j \neq k} |\mathbf{h}_{nk}^H \mathbf{V}\mathbf{w}_{nj}|^2 + \sigma_{nk}^2} \geq \gamma_{nk}, \; n \in \mathcal{N}_D, \forall k, \quad (22b)$$

which can be transformed into a second-cone constrained convex problem and can be solved with efficient algorithms [18].

The decoupled problem with respect to $\tilde{\mathbf{w}}_{nS}$ is

$$\min_{\{\tilde{\mathbf{w}}_{nS}\}} \sum_{n \in \mathcal{N}_S} \|\mathbf{V}\tilde{\mathbf{w}}_{nS}\|^2 \quad (23a)$$

$$s.t. \; \tilde{\mathbf{w}}_{nS}^H \mathbf{V}^H \mathbf{C}\mathbf{V}\tilde{\mathbf{w}}_{nS} \geq -\frac{\sigma_{nS}^2 \gamma_S}{\ln(\epsilon + \Delta\epsilon)}, \; n \in \mathcal{N}_S. \quad (23b)$$

Based on the first-order optimality condition, we can obtain the optimal solution to problem (23) as

$$\tilde{\mathbf{w}}_{nS} = \sqrt{\frac{-\sigma_{nS}^2 \gamma_S}{\lambda_{max}(\mathbf{V}^H\mathbf{C}\mathbf{V}) \ln(\epsilon + \Delta\epsilon)}}\,\mathbf{u}_{max}(\mathbf{V}^H\mathbf{C}\mathbf{V}), \quad (24)$$

where the constraint $\mathbf{V}^H\mathbf{V} = \mathbf{I}$ in (18c) is used.

*3) **Algorithm Summary**:* We summarize the proposed algorithm as follows.

1) Setting $D = M$ and $\mathbf{V} = \mathbf{I}$ (i.e., under full digital assumption), find the optimal solution of $\{\mathbf{w}_{nk}\}$ to problem (15), denoted by $\{\hat{\mathbf{w}}_{nk}\}$, with standard convex optimization algorithms.
2) Obtain $\{\hat{\tilde{\mathbf{w}}}_{nS}\}$ with (19).
3) Obtain $\{\hat{\mathbf{D}}_n\}$ and $\hat{D}_S$ with (20) and (21), respectively.
4) Given $\{\hat{\mathbf{D}}_n\}$ and $\hat{D}_S$, obtain $\mathbf{V}$ by solving problem (18) with SDR and Gaussian randomization method.
5) Given $\mathbf{V}$, obtain $\{\mathbf{w}_{nk}\}$ by solving problem (22) with standard convex optimization algorithms, and obtain $\{\tilde{\mathbf{w}}_{nS}\}$ with (24).

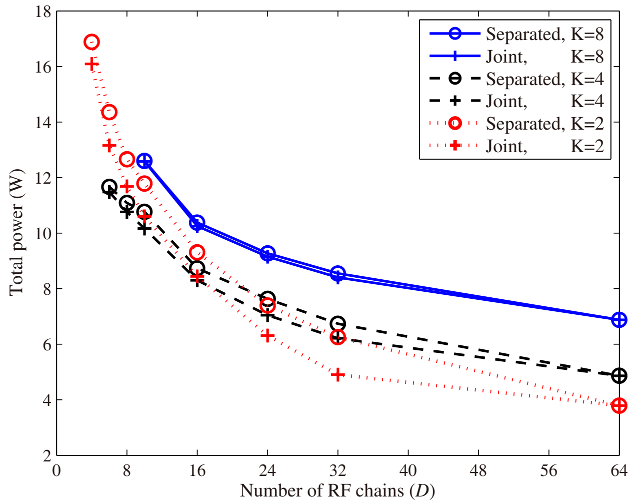

Fig. 1. Total transmit power v.s. the number of RF chains with $R_{nk} = 4$ bps/Hz.

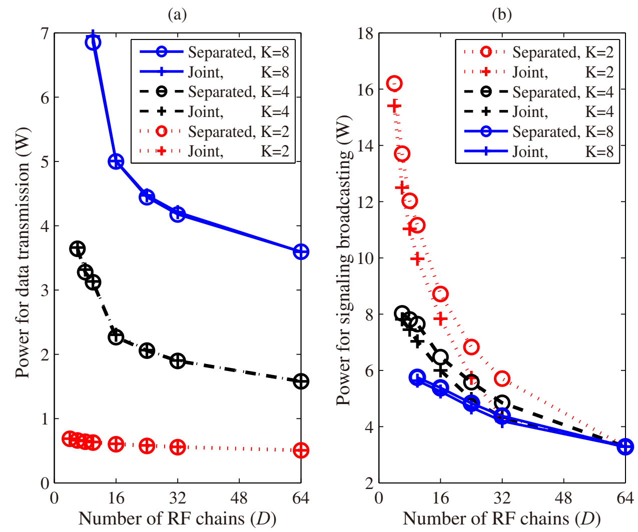

Fig. 2. Transmit power for data transmission and signaling broadcasting v.s. the number of RF chains with $R_{nk} = 4$ bps/Hz

IV. Simulation Results

In this section, we evaluate the performance of the proposed hybrid precoder under both data rate and coverage constraints. To show the impact of coverage constraint on hybrid precoder design, we also simulate a modified version of the proposed method, named "Separated method". It uses the same algorithm as described in Section III-C3, except that the analog precoder $\mathbf{V}$ is optimized to only minimize the power for data transmission in Step 4. The proposed method is called "Joint method" in the simulations, which optimizes the analog precoder to minimize the total power for both data and signaling transmission. After obtaining the analog precoder, both "Separated method" and "Joint method" use Step 5 of the proposed algorithm to compute digital precoders for data and signaling transmission, respectively. The same data rate requirement and coverage constraint are considered for the two methods.

Unless otherwise specified, the following parameters are used throughout the simulations. To speed up the simulations, we consider that the system has 10 MHz bandwidth, corresponding to $N = 50$ RBs in LTE systems, and the BS is equipped with $M = 64$ antennas. In order to highlight the impact of coverage constraint, frequency-domain user scheduling is not employed in simulations. The system operates in SDMA-OFDM fashion, where the BS serves K single-antenna data users located at cell edge and every user occupies the whole bandwidth. According to LTE specifications, 10% RBs of the system are used for signaling broadcasting and the others are for data transmission, i.e., $|\mathcal{N}_S| = 5$ and $|\mathcal{N}_D| = 45$. The maximal transmit power of the BS is $P_0 = 46$ dBm, the cell radius is $d_0 = 250$ m, and the average receive SNR of a user located at the cell edge when the BS transmits with the maximal power and one antenna, denoted by $\mathrm{SNR}_{\mathrm{edge}}$, is set as 10 dB. Then, given the path loss model $128.1+37.6\log_{10} d$ with d denoting the distance in km, the noise variance can be obtained as $P_0 - (128.1 + 37.6\log_{10} d_0) - \mathrm{SNR}_{\mathrm{edge}}$. The small-scale channels are generated based on 3GPP massive MIMO channel model in the urban macro-cell non-line of sight (UMa-NLoS) scenario, where the antenna array is set as ULA and the channel parameters to calculate the matrix $\mathbf{C}$ used in the proposed algorithm can be found in [15]. For coverage constraint, the required received SNR and the minimum acceptable coverage probability are set as $\gamma_S = 0$ dB and $\epsilon = 95\%$, respectively, and the back off margin is set as $\Delta\epsilon = 3\%$ to ensure the coverage requirement always satisfied.

In Fig. 1 we compare the total transmit power of the BS with "Joint method" and "Separated method" as a function of the number of RF chains, where different number of users K are considered and the data rate requirement is set as $R_{nk} = 4$ bps/Hz. It is shown that "Separated method" and "Joint method" achieve the same performance in full-digital systems, i.e., $D = 64$, because in this case analog precoder is not needed and the two methods use the same digital precoders. With hybrid architecture, i.e, $D < 64$, "Separated method" requires more transmit power compared to "Joint method" and the gap increases with decreasing the number of users K. This can be explained as follows. The analog precoder in "Separated method" only minimizing the power for data transmission results in narrow beams to K users, and the covered area by the analog precoder shrinks with the decrease of K. As a result, the power of digital precoder for signaling broadcasting needs to largely increase to ensure the coverage, which leads to the increase of total transmit power.

To observe the impact of coverage requirement on transmit power more clearly, we plot the power used for data transmission and signaling broadcasting, respectively, in Fig. 2. Theoretically, since the analog precoder of "Joint method" is wider than the one of "Separated method" in order to improve the coverage, "Joint method" requires a higher power for data transmission. This is verified by Fig. 2(a), but the power increase is very limited because the wider beams optimized

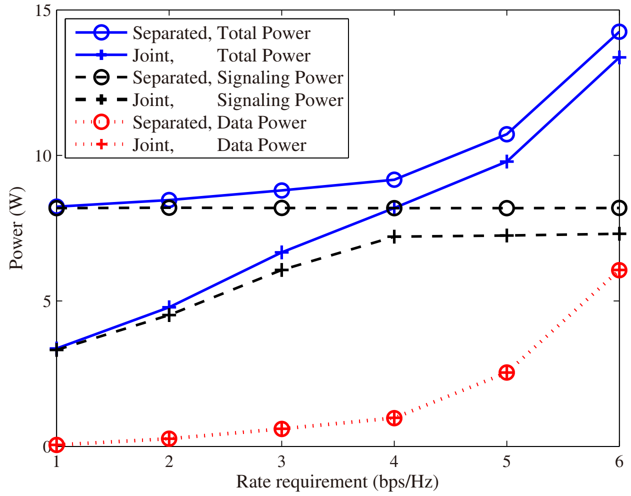

Fig. 3. Transmit power v.s. rate requirements with $K = 2$ and $D = 16$.

by the proposed "Joint method" still provide sufficiently large array gain. Yet, the benefits of "Joint method" on reducing the power for signaling broadcasting are evident. As explained before, the smaller the number of users, the larger the performance gap between the two methods.

In Fig. 3 the total transmit power and the power for data transmission and signaling broadcasting are depicted as a function of data rate requirement, where $K = 2$ and $D = 16$. We can see that "Joint method" can reduce the power for signaling broadcasting with negligible increase on the power for data transmission. The performance gap between "Joint method" and "Separated method" reduces with the increase of data rate requirement. This is because for high target data rate the power for data transmission increases so that "Joint method" will generate narrow-beam analog precoder and perform close to "Separated method".

V. Conclusions

In this paper we studied the optimization of wideband hybrid precoder for downlink SDMA-OFDMA massive MIMO systems, aimed at minimizing the total transmit power of the BS under both coverage constraint and data rate requirements. We first derived an upper bound of the coverage probability under a general spatially correlated channel model, then a low-complexity wideband hybrid precoder proposed. Simulation results showed that under the same data rate requirement and coverage constraint, compared to the method that generates narrow-beam analog precoder only based on data rate requirements, the proposed method can effectively reduce the power for signaling broadcasting with negligible increase of the power for data transmission.

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