

# Energy-efficient Uplink Training Design For Closed-loop MISO Systems

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**Abstract**—When the circuit power consumption and the overhead for channel estimation are taken into account, the system designed for maximizing the spectrum efficiency (SE) does not necessarily yield high energy efficiency (EE). In this paper, we strive to optimize the uplink training length towards maximizing the EE of closed-loop multi-antenna systems under the constraint of the SE. The upper bounds of the system EE and the net downlink SE with channel estimation errors are derived, based on which the optimization problem is proved as convex, and the impacts of signal-to-noise ratio (SNR) and circuit power consumption on the optimal training length are analyzed. Analytical and simulation results show that in general the EE-oriented optimization leads to a longer training length than the SE-oriented optimization, and it will reduce to the SE-oriented optimization at high SNR and very low SNR regime, or with very high circuit power consumption.

## I. INTRODUCTION

Energy efficiency (EE) is becoming an important design goal for future cellular networks [1,2]. When the circuit power consumption and the signaling overhead to support high data rate transmission are taken into account, a system with high spectrum efficiency (SE) does not necessarily provide high EE.

Closed-loop systems are widely deployed, where data transmission is adapted to the channel variation. To facilitate downlink closed-loop transmission, training symbols are sent in the uplink of time division duplex (TDD) systems in order to provide channel state information (CSI) at the base station (BS). Existing studies on the optimization of uplink training mainly focus on the SE maximization. In [3] and [4], the transmit power, position and the number of training symbols were optimized, respectively aimed at maximizing the capacity lower bound and minimizing the Cramer-Rao bound of the channel estimation errors. In [5], the optimal downlink training length was studied. It was shown that the optimal training length equals to the number of antennas at the transmitter, when the optimal power allocation between the training symbols and data is considered. In [6], the optimal resource allocation strategies towards maximizing SE were investigated for multiuser multi-antenna systems.

In general, the SE-oriented training design does not maximize the EE. When the circuit energy consumption is taken into account, the relationship between SE and EE becomes

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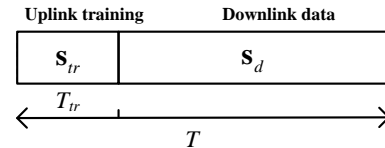


Fig. 1. Frame structure of the considered TDD system

complicated, depending on the network architectures and transmission schemes [2]. In [7] and [8], the training signals were optimized to maximize the EE for open-loop systems, either without or with the circuit power consumption. Yet the EE-oriented training design for closed-loop systems has not been addressed in literature.

In this paper, we study the EE-oriented training length design under the constraint of the SE requirement in closed-loop multi-antenna systems. We start with deriving the upper bounds of the net SE and the EE considering imperfect channel estimation and circuit power consumption, based on which the optimization problem is formulated, and the optimal training length under the two criteria of maximizing EE and SE is compared. The impacts of signal-to-noise ratio (SNR) and circuit power consumption on the optimal training length are investigated. Simulation results validate our theoretical analysis.

*Notations:*  $\mathbf{I}$  denotes the identity matrix.  $|\cdot|$  and  $\|\cdot\|$  represent the magnitude and two-norm, respectively.  $(\cdot)^H$  denotes conjugate transpose,  $\mathbb{E}[\cdot]$  denotes expectation operation,  $\otimes$  is the Kronecker product, and  $\mathbb{N}$  denotes the set of non-negative integers.

## II. SYSTEM MODEL

Consider a TDD multi-input single-output (MISO) system, where one BS with  $N_t$  antennas serves one single-antenna user. We assume that the user undergoes block fading channel, which remains constant during each uplink-downlink frame but is independent among different frames. The structure of the frame is shown in Fig. 1. Each frame contains  $T$  symbols, among which  $T_{tr}$  symbols are for uplink training.

Let  $\mathbf{s}_{tr}$  denote the uplink training symbols, where  $\mathbf{s}_{tr}\mathbf{s}_{tr}^H = P_u T_{tr}$  and  $P_u$  denotes the transmit power of training symbols. Then the received training symbols at the BS are

$$\mathbf{y}_u = \mathbf{h}\mathbf{S}_{tr} + \mathbf{n}_u, \quad (1)$$

where  $\mathbf{h} = [h_1, \dots, h_{N_t}] \in \mathbb{C}^{1 \times N_t}$  denotes the complex Gaussian channel vector with zero mean and correlation matrix  $\sigma_h^2 \mathbf{I}$ ,  $\mathbf{S}_{tr} = \mathbf{I} \otimes \mathbf{s}_{tr} \in \mathbb{C}^{N_t \times T_{tr} N_t}$ , and  $\mathbf{n}_u$  is the additive white Gaussian noise (AWGN) at the BS with zero mean and variance  $\sigma_u^2$ .

With the minimum mean-square error (MMSE) criterion, the uplink channel can be estimated at the BS as

$$\hat{\mathbf{h}} = \mathbf{y}_u \times (\mathbf{S}_{tr}^H \mathbf{S}_{tr} + \frac{\sigma_u^2}{\sigma_h^2} \mathbf{I})^{-1} \times \mathbf{S}_{tr}^H, \quad (2)$$

and the relationship between the estimated channel and the true value of the channel satisfies [9]

$$\mathbf{h} = \hat{\mathbf{h}} + \mathbf{e}, \quad (3)$$

where  $\mathbf{e}$  is the channel estimation error with zero mean and correlation matrix  $\sigma_e^2 \mathbf{I}$ , the channel estimate  $\hat{\mathbf{h}}$  is with zero mean and correlation matrix  $\sigma_{\hat{\mathbf{h}}}^2 \mathbf{I}$ , and  $\hat{\mathbf{h}}$  and  $\mathbf{e}$  are uncorrelated. Because  $\mathbf{S}_{tr} \mathbf{S}_{tr}^H = P_u T_{tr} \mathbf{I}$ , it is easy to show that

$$\sigma_e^2 = \frac{\sigma_u^2 \sigma_h^2}{\sigma_u^2 + \sigma_h^2 P_u T_{tr}}, \quad (4)$$

$$\sigma_{\hat{\mathbf{h}}}^2 = \frac{\sigma_h^4 P_u T_{tr}}{\sigma_u^2 + \sigma_h^2 P_u T_{tr}}. \quad (5)$$

By exploiting the channel reciprocity, the uplink channel estimate is used as downlink channel for precoding. With CSI at the BS, maximum ratio transmission (MRT) [10] can be employed for downlink transmission. Then, the precoding vector is

$$\mathbf{w} = \sqrt{P_d} \frac{\hat{\mathbf{h}}}{\|\hat{\mathbf{h}}\|}, \quad (6)$$

where  $P_d$  is the downlink transmit power. The received signal at the user can be expressed as

$$y_d = \mathbf{h} \mathbf{w}^H s_d + n_d = \sqrt{P_d} \frac{\mathbf{h} \hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|} s_d + n_d, \quad (7)$$

where  $s_d$  is the data symbol, and  $n_d$  is the AWGN at the user with zero mean and variance  $\sigma_d^2$ .

### III. PROBLEM FORMULATION

#### A. Net Spectrum Efficiency

To focus on analyzing the impact of uplink training, we assume that the user has perfect knowledge of downlink channels for coherent detection. From (7), the receive SNR at the user can be expressed as

$$\text{SNR} = \frac{P_d}{\sigma_d^2} \left| \frac{\mathbf{h} \hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|} \right|^2 \triangleq \frac{P_d}{\sigma_d^2} |\varphi|^2, \quad (8)$$

where  $\varphi = \frac{\mathbf{h} \hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|} = \frac{(\hat{\mathbf{h}} + \mathbf{e}) \hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|} \triangleq \|\hat{\mathbf{h}}\| + \alpha$ , and  $\alpha = \frac{\mathbf{e} \hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|}$ .

It is easy to show that  $\mathbb{E}[\alpha] = 0$ ,  $\mathbb{E}[\alpha^2] = \sigma_e^2$ , and  $\mathbb{E}[\|\hat{\mathbf{h}}\| \alpha^H] = 0$ . Then, we can obtain

$$\begin{aligned} \mathbb{E}[|\varphi|^2] &= N_t \sigma_h^2 + \sigma_e^2 \\ &= \frac{\sigma_h^2 \sigma_u^2 + N_t \sigma_h^4 P_u T_{tr}}{\sigma_u^2 + \sigma_h^2 P_u T_{tr}}. \end{aligned} \quad (9)$$

The downlink ergodic capacity using the imperfect CSI with unit bandwidth is

$$C = \mathbb{E}[\log_2(1 + \text{SNR})] = \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_d}{\sigma_d^2} |\varphi|^2 \right) \right], \quad (10)$$

whose closed-form expression is hard to derive.

To facilitate the optimization, we alternatively consider an upper bound of the ergodic capacity using Jensen's inequality. Further considering (9), we have

$$\begin{aligned} C_{up} &= \log_2 \left( 1 + \frac{P_d}{\sigma_d^2} \frac{\sigma_h^2 \sigma_u^2 + N_t \sigma_h^4 P_u T_{tr}}{\sigma_u^2 + \sigma_h^2 P_u T_{tr}} \right) \\ &\triangleq \log_2 \left( 1 + \frac{\gamma_d + N_t \gamma_d \gamma_u T_{tr}}{1 + \gamma_u T_{tr}} \right), \end{aligned} \quad (11)$$

where  $\gamma_u = \frac{P_u}{\sigma_u^2} \sigma_h^2$  and  $\gamma_d = \frac{P_d}{\sigma_d^2} \sigma_h^2$  are the average uplink and downlink receive SNR, respectively.

The net spectrum efficiency is defined as

$$\text{SE}(T_{tr}) = \frac{C(T - T_{tr})}{T}, \quad (12)$$

and the upper bound of the net SE is

$$\text{SE}_{up}(T_{tr}) = (1 - \frac{T_{tr}}{T}) \log_2 \left( 1 + \frac{\gamma_d + N_t \gamma_d \gamma_u T_{tr}}{1 + \gamma_u T_{tr}} \right). \quad (13)$$

#### B. Energy Efficiency

Considering that the power consumption at the user side is far less than that at the BS,<sup>1</sup> we define the EE of the system as the ratio of the net spectrum efficiency to the total power consumed at the BS during an uplink-downlink frame.

The power consumption of the BS during the uplink training phase does not include the transmit power, i.e.,

$$P_{bu} = P_{RXC}, \quad (14)$$

where  $P_{RXC}$  is the circuit power when the BS operates in receive mode, which includes the power consumed by the RF links, the baseband signal processing (i.e., synchronization and channel estimation), and the power supply and cooling.

The power consumption of the BS during the downlink transmission phase includes both the transmit power and the circuit power. Based on the linear power consumption model in [11], the power consumed during the downlink phase is

$$P_{bd} = \frac{P_d}{\eta} + P_{TXC}, \quad (15)$$

where  $\eta$  is the efficiency of power amplifier, and  $P_{TXC}$  is the circuit power when the BS operates in transmit mode, which includes the power consumed by the RF links, the baseband signal processing (e.g., synchronization, precoding, modulation and coding), and the power supply and cooling.

From (14) and (15), the total power consumption during an uplink-downlink frame can be obtained as

$$P_{tot} = \frac{P_{bu} T_{tr} + P_{bd}(T - T_{tr})}{T}. \quad (16)$$

<sup>1</sup>Note that the energy efficient design for the user is also important, which is however beyond the scope of this paper.

With (16) and (12), the EE can be obtained as

$$EE(T_{tr}) = \frac{SE(T_{tr})}{P_{tot}} = \frac{C(T - T_{tr})}{P_{bu}T_{tr} + P_{bd}(T - T_{tr})}, \quad (17)$$

and further considering (11), the upper bound of the EE is

$$EE_{up}(T_{tr}) = \frac{T - T_{tr}}{P_{bu}T_{tr} + P_{bd}(T - T_{tr})} \cdot \log_2 \left( 1 + \frac{\gamma_d + N_t \gamma_d \gamma_u T_{tr}}{1 + \gamma_u T_{tr}} \right). \quad (18)$$

With the upper bounds in (13) and (18), the optimization problem of uplink training length aimed at maximizing the EE under the SE constraint can be formulated as

$$\max_{T_{tr}} EE_{up}(T_{tr}) \quad (19a)$$

$$\text{s.t. } SE_{up}(T_{tr}) - \Delta SE_0 \geq SE_0 \quad (19b)$$

$$0 \leq T_{tr} \leq T \quad (19c)$$

$$T_{tr} \in \mathbb{N}, \quad (19d)$$

where  $SE_0$  is the minimal SE requirement of the user, and  $\Delta SE_0$  is a back off margin to ensure an acceptable outage probability due to the usage of the SE upper bound. We will evaluate the impact of using the upper bounds on the design in Section V.

#### IV. ANALYSIS OF THE OPTIMAL TRAINING LENGTH

##### A. Solution of the Optimal Training Length

To solve the optimization problem (19), we first relax (19) by regarding  $T_{tr}$  as a continuous variable within  $[0, T]$ , and then round the optimal solution to the nearest integer. In the sequel, we first show that the relaxed version of the problem (19) is a convex optimization problem.

*Theorem 1:* Both the upper bounds of the EE and the SE are concave functions with respect to  $T_{tr}$ , and their first-order derivatives are monotonically decreasing functions of  $T_{tr}$ .

*Proof:* See Appendix A. ■

It follows that the relaxed version of (19) omitting the constraint in (19d) is a convex problem, whose solution can be numerically found with efficient algorithms [12]. Though we cannot find its closed-form solution, we are able to compare the difference in the optimal training lengths towards maximizing the EE and towards maximizing the SE.

##### B. Optimal Training Length Difference Under Two Criteria

Define  $T_{trSE}^*$  and  $T_{trEE}^*$  as the optimal uplink training length of the SE-oriented optimization and the EE-oriented optimization, respectively, where  $T_{trEE}^*$  is the solution of the relaxed version of problem (19). Denote  $\Delta T_{tr}^* = T_{trEE}^* - T_{trSE}^*$  as the difference of the optimal training length under the two criteria.

*Theorem 2:*  $\Delta T_{tr}^* \geq 0$  always holds. If  $EE'_{up}(0) \leq 0$ , then  $\Delta T_{tr}^* = 0$  and  $T_{trEE}^* = T_{trSE}^* = 0$ . If  $EE'_{up}(0) > 0$ , then  $\Delta T_{tr}^* \geq 0$  and the equality holds if and only if  $SE_0 = SE_{max}$ , where  $SE_{max}$  is the maximum achievable net SE.

*Proof:* See Appendix B. ■

Theorem 2 indicates that the difference between the optimal training length of the SE-oriented and the EE-oriented optimization depends on the value of  $EE'_{up}(0)$ , and the EE-oriented optimization usually leads to a longer training length. We will analyze the impact of SNR on  $EE'_{up}(0)$  in next subsection.

##### C. Impact of SNR on the Difference in Optimal Training Length

Since the average downlink receive SNR is usually larger than the average uplink receive SNR, we set  $\gamma_d = \beta\gamma_u$  with  $\beta > 1$ . One can find from (B.1) in Appendix B that  $EE'_{up}(0)$  is a function of SNR. For notational simplicity, in the following we use  $G(\gamma_u)$  to denote  $EE'_{up}(0)$ , and then rewrite (B.1) as

$$G(\gamma_u) = EE'_{up}(0) = \frac{1}{P_{bd}} \left[ \frac{\beta(N_t - 1)}{\ln 2} \frac{\gamma_u^2}{1 + \beta\gamma_u} - \frac{P_{bu}}{P_{bd}T} \log_2(1 + \beta\gamma_u) \right], \quad (20)$$

whose first-order derivative is

$$G'(\gamma_u) = \frac{\beta(N_t - 1)}{P_{bd} \ln 2} \frac{\beta\gamma_u^2 + 2\gamma_u}{1 + \beta\gamma_u} - \frac{P_{bu}}{P_{bd}^2 T \ln 2} \frac{\beta}{1 + \beta\gamma_u}. \quad (21)$$

It is easy to show that  $G''(\gamma_u) > 0$  always holds, therefore  $G'(\gamma_u)$  is a monotonically increasing function. We can further show that  $G'(0) < 0$  and  $G'(\infty) > 0$ , hence  $G(\gamma_u)$  first decreases and then increases.

Based on L'Hospital's rule, we can derive that

$$\lim_{\gamma_u \rightarrow \infty} \frac{\beta(N_t - 1)}{\ln 2} \frac{\gamma_u^2}{1 + \beta\gamma_u} / \left( \frac{P_{bu}}{P_{bd}T} \log_2(1 + \beta\gamma_u) \right) = \infty. \quad (22)$$

Then from (20) and (22), we know  $G(\infty) > 0$ . Further considering  $G(0) = 0$  from (20) and the fact that  $G(\gamma_u)$  first decreases and then increases, there must be a positive value of  $\gamma_u^o$  that meets  $G(\gamma_u^o) = 0$ .

- When  $0 < \gamma_u \leq \gamma_u^o$ , we have  $G(\gamma_u) \leq 0$ , i.e.,  $EE'_{up}(0) \leq 0$ . According to Theorem 2 and the proof, in this case  $EE_{up}(T_{tr})$  is monotonically decreasing,  $T_{trEE}^* = T_{trSE}^* = 0$  and  $\Delta T_{tr}^* = 0$ .
- When  $\gamma_u > \gamma_u^o$ ,  $G(\gamma_u)$  is an increasing function and  $G(\gamma_u) > 0$ , i.e.,  $EE'_{up}(0) > 0$ . Now from Theorem 2  $EE_{up}(T_{tr})$  first increases and then decreases,  $T_{trEE}^* \geq T_{trSE}^*$  and  $\Delta T_{tr}^* \geq 0$ .

In the extreme case where  $\gamma_u \rightarrow \infty$ , we can show that  $T_{trEE}^* = T_{trSE}^* = 1$ . To see this, we consider two cases.

- When  $T_{tr} \geq 1$ , we have  $SE_{up}(T_{tr}) \rightarrow (1 - \frac{T_{tr}}{T}) \log_2(1 + N_t \beta \gamma_u)$  from (13) and  $EE_{up}(T_{tr}) \rightarrow \frac{T - T_{tr}}{P_{bu}T_{tr} + P_{bd}(T - T_{tr})} \log_2(1 + N_t \beta \gamma_u)$  from (18). The asymptotic results of both  $EE_{up}(T_{tr})$  and  $SE_{up}(T_{tr})$  are monotonically decreasing functions of  $T_{tr}$ . Therefore,  $T_{trEE}^* = T_{trSE}^* = 1$  in this case.
- When  $T_{tr} = 0$ ,  $EE_{up}(0) \rightarrow \frac{T}{P_{bd}} \log_2(1 + \beta\gamma_u)$  and  $SE_{up}(0) \rightarrow \log_2(1 + \beta\gamma_u)$ .

It is not hard to show that  $EE_{up}(1) \geq EE_{up}(0)$  and  $SE_{up}(1) \geq SE_{up}(0)$ . Therefore, we have  $T_{trEE}^* = T_{trSE}^* = 1$  when  $\gamma_u \rightarrow \infty$ .

TABLE I  
LIST OF SIMULATION PARAMETERS

Simulation parameters	Values
Number of antennas at the BS, $N_t$	4
Total number of symbols in a frame, $T$	200
Transmit power of uplink training, $P_u$	23 dBm
Transmit power of downlink transmission, $P_d$	46 dBm
Uplink/downlink SNR difference, $\beta$	23 dB
Power amplifier efficiency, $\eta$	31.1%
Circuit power of BS in transmit mode, $P_{TXc}$	364 W
Minimal SE requirement, $SE_0$	1 bps/Hz

In summary, we show that  $T_{trEE}^* = T_{trSE}^* = 0$  at low SNR regime when  $\gamma_u \leq \gamma_u^o$ , and  $T_{trEE}^* = T_{trSE}^* = 1$  at high SNR regime. In general scenarios,  $T_{trEE}^*$  is greater than  $T_{trSE}^*$ .

### V. SIMULATION RESULTS

In this section, we evaluate the impact of uplink training length on the EE and the SE of the system via simulations. The simulation parameters listed in Table I will be used in the following unless otherwise specified. The maximum transmit powers for uplink training and downlink transmission, the circuit power, and the power amplifier efficiency are configured as in [1]. According to the results in [1], the circuit power ranges from 10 W to 500 W depending on the types of the BS, and the circuit power consumed of BS in uplink receive mode approximately equals to that in downlink transmit mode. Therefore, we set  $P_{RXc} = P_{TXc}$  in the simulations. The average uplink receive SNR of the user at the cell edge,  $SNR_{edge}^{UL}$ , is set as -20 dB. The average uplink receive SNR of the user from the BS with distance  $d$  is computed as  $SNR^{UL} = SNR_{edge}^{UL} + 37.6 \log_{10}(r/d)$ , where the cell radius  $r = 250$  m. In the simulations, the back off margin,  $\Delta SE_0$ , is set as 0.25 bps/Hz to ensure  $SE_0$  always satisfied.

We first evaluate the impact of using the upper bounds on the training length optimization. We simulate the EEs achieved by the optimization based on the upper bound (11) and based on the true value (10) as a function of average uplink SNR, and find that their results almost overlap. For example, when the average uplink SNR is 30 dB, the achieved EE is 0.039 bit/Joule/Hz, and the gap between the two results is about  $2 \times 10^{-5}$  bit/Joule/Hz. This suggests that the training length optimization with the upper bounds has little impact on the performance. Due to the lack of space, we do not show the figure.

Figure 2 compares the training length optimized towards the SE and the EE. As shown in the figure, when the SNR exceeds 15 dB,  $T_{trEE}^* = T_{trSE}^* = 1$  and  $\Delta T_{tr}^* = 0$ . In general cases, the EE-oriented optimization requires more training symbols than the SE-oriented optimization. These results agree with our analytical analysis.

In Fig.3, we show the impact of circuit power consumption on the difference of the optimal training length  $\Delta T_{tr}^*$

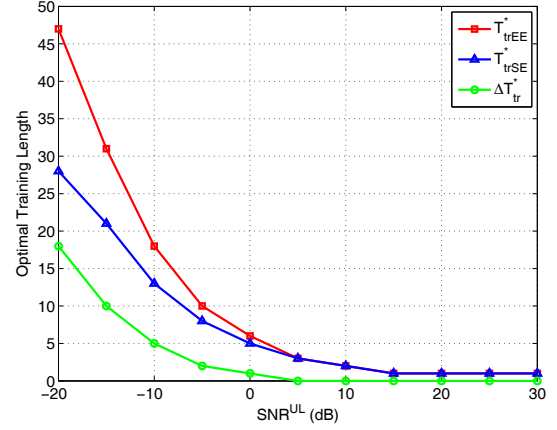


Fig. 2. Optimal training length as a function of uplink SNR.

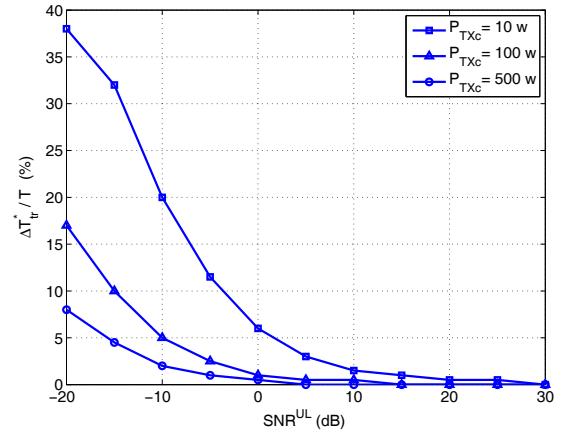


Fig. 3. Optimal training length difference with different circuit power consumption.

under the two criteria. It is shown that when the circuit power consumption is small, there is an obvious difference of training length between the EE-oriented and the SE-oriented optimization. As the circuit power consumption increases, the difference reduces. This is because in this case the total power consumed at the BS in receive mode and in transmit mode gradually become identical with the fixed transmit power, i.e.,  $P_{bd} \approx P_{bu}$ , and then the EE-oriented optimization will reduce to the SE-oriented optimization according to (B.5) in Appendix B.

Figure 4 shows the EE gain of the EE-oriented optimization over the SE-oriented optimization. The EE gain is defined as  $(EE(T_{trEE}^*) - EE(T_{trSE}^*)) / EE(T_{trSE}^*)$ , where  $EE(T_{trEE}^*)$  and  $EE(T_{trSE}^*)$  are respectively the EEs achieved by the EE-oriented and the SE-oriented optimization, as defined in Section IV. It is shown that the EE gain decreases with the growing of the circuit power. This is because the optimal training length difference under the two criteria decreases with the increase of the circuit power, as shown in Fig. 3. When the SNR is sufficiently high, the EE gain tends to zero, i.e.

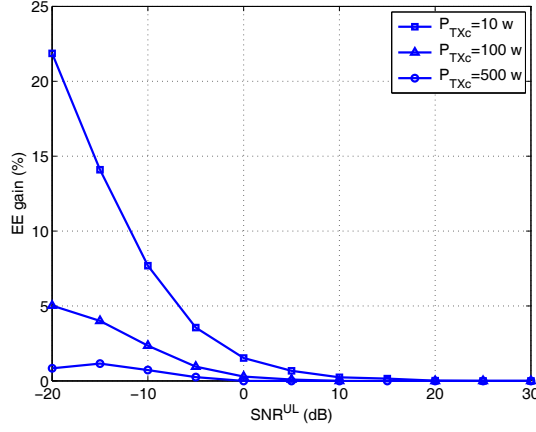


Fig. 4. EE gain of the EE-oriented optimization over the SE-oriented optimization.

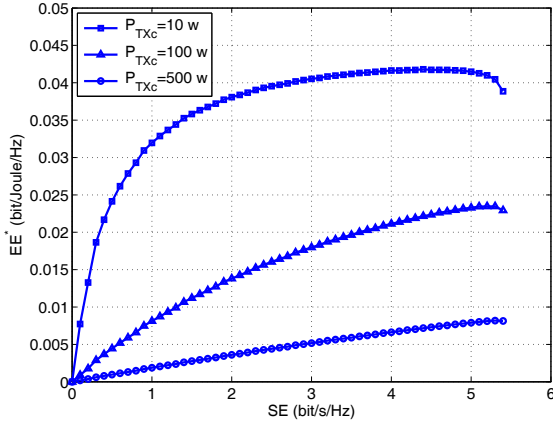


Fig. 5. EE vs. SE ( $\text{SNR}^{\text{UL}} = -10$  dB).

the EE-oriented optimization will reduce to the SE-oriented optimization, which agrees with our previous analysis.

Figure 5 shows the relationship between the EE and the net SE achieved with different values of downlink circuit power consumption. Because the maximal transmit power is given, the maximal achievable SE is bounded. When the circuit power consumption is high, the EE approximately grows with the SE monotonically, because in this case the value of  $\Delta T_{tr}^*$  is very small. When the circuit power consumption is low, on the other hand, the EE first increases and then finally reduces with the increase of the SE, which is led by the difference in the optimal training lengths under the two criteria. According to the trade-off, we can configure the training length reasonably.

## VI. CONCLUSIONS

In this paper, we investigated the design of uplink training length aimed at maximizing the EE for TDD closed-loop MISO systems under the constraint on the minimal SE. To facilitate the optimization and analysis, we derived the upper bounds of the EE and the SE considering imperfect channel

estimation, both of which were proved as concave functions of the uplink training length. Both theoretical analysis and simulation results showed that a longer training length is necessary to maximize the EE compared to that maximizing the SE in general cases, while the EE-oriented optimization will be equivalent to the SE-oriented optimization if the uplink SNR is high and very low or the the circuit power consumption is high.

## APPENDIX A PROOF OF THEOREM 1

To simplify the notation, we rewrite (18) as

$$EE_{up}(T_{tr}) = g_1(T_{tr}) \cdot g_2(T_{tr}), \quad (\text{A.1})$$

where  $g_1(T_{tr}) = \frac{T - T_{tr}}{P_{bu}T_{tr} + P_{bd}(T - T_{tr})} \geq 0$  and  $g_2(T_{tr}) = \log_2(1 + N_t\gamma_d - \frac{(N_t-1)\gamma_d}{1+\gamma_u T_{tr}}) > 0$ .

It is easy to show that

$$\begin{aligned} g_1'(T_{tr}) &= -\frac{P_{bu}T}{[P_{bu}T_{tr} + P_{bd}(T - T_{tr})]^2} < 0, \\ g_2'(T_{tr}) &= \frac{(N_t - 1)\gamma_d\gamma_u}{\ln 2(1 + \gamma_u T_{tr})} \frac{1}{1 + \gamma_d + \gamma_u T_{tr} + N_t\gamma_d\gamma_u T_{tr}} > 0, \\ g_1''(T_{tr}) &= \frac{-2P_{bu}T(P_{bd} - P_{bu})}{[P_{bd}T - (P_{bd} - P_{bu})T_{tr}]^3} < 0, \text{ and } g_2''(T_{tr}) < 0. \end{aligned}$$

Then, we can obtain the derivatives of  $EE_{up}(T_{tr})$  as

$$EE'_{up}(T_{tr}) = g_1'(T_{tr})g_2(T_{tr}) + g_1(T_{tr})g_2'(T_{tr}), \quad (\text{A.2})$$

$$\begin{aligned} EE''_{up}(T_{tr}) &= g_1''(T_{tr})g_2(T_{tr}) + 2g_1'(T_{tr})g_2'(T_{tr}) \\ &\quad + g_1(T_{tr})g_2''(T_{tr}). \end{aligned} \quad (\text{A.3})$$

We can see that  $EE''_{up}(T_{tr}) < 0$  always holds. Therefore,  $EE_{up}(T_{tr})$  is a concave function of  $T_{tr}$ , and  $EE'_{up}(T_{tr})$  is a monotonically decreasing function of  $T_{tr}$ .

Similarly, we rewrite (13) as

$$SE_{up}(T_{tr}) = f_1(T_{tr}) \cdot f_2(T_{tr}), \quad (\text{A.4})$$

where  $f_1(T_{tr}) = 1 - \frac{T_{tr}}{T} \geq 0$  and  $f_2(T_{tr}) = \log_2(1 + N_t\gamma_d - \frac{(N_t-1)\gamma_d}{1+\gamma_u T_{tr}}) > 0$ , whose derivatives are as follows.

$$\begin{aligned} f_1'(T_{tr}) &= -\frac{1}{T_{tr}} < 0, \\ f_2'(T_{tr}) &= \frac{(N_t - 1)\gamma_d\gamma_u}{\ln 2(1 + \gamma_u T_{tr})} \frac{1}{1 + \gamma_d + \gamma_u T_{tr} + N_t\gamma_d\gamma_u T_{tr}} > 0, \\ f_1''(T_{tr}) &= 0, \text{ and } f_2''(T_{tr}) = g_2''(T_{tr}) < 0. \end{aligned}$$

Then we have

$$SE'_{up}(T_{tr}) = f_1'(T_{tr})f_2(T_{tr}) + f_1(T_{tr})f_2'(T_{tr}), \quad (\text{A.5})$$

$$\begin{aligned} SE''_{up}(T_{tr}) &= f_1''(T_{tr})f_2(T_{tr}) + 2f_1'(T_{tr})f_2'(T_{tr}) \\ &\quad + f_1(T_{tr})f_2''(T_{tr}). \end{aligned} \quad (\text{A.6})$$

Since  $SE''_{up}(T_{tr}) < 0$  always holds,  $SE_{up}(T_{tr})$  is a concave function of  $T_{tr}$ , and  $SE'_{up}(T_{tr})$  is monotonically decreasing.

APPENDIX B  
PROOF OF THEOREM 2

We begin with providing the expressions of  $EE'_{up}(T_{tr})$  and  $SE'_{up}(T_{tr})$  at two extreme cases of the training length, i.e.,  $T_{tr} = 0$  and  $T_{tr} = T$ .

From (A.2) and (A.5), we have

$$EE'_{up}(0) = \frac{1}{P_{bd}} \left[ \frac{(N_t - 1)\gamma_d\gamma_u}{\ln 2(1 + \gamma_d)} - \frac{P_{bu}}{P_{bd}} \frac{1}{T} \log_2(1 + \gamma_d) \right], \quad (\text{B.1})$$

$$EE'_{up}(T) = -\frac{1}{P_{bu}} g_2(T), \quad (\text{B.2})$$

$$SE'_{up}(0) = \frac{(N_t - 1)\gamma_d\gamma_u}{\ln 2(1 + \gamma_d)} - \frac{1}{T} \log_2(1 + \gamma_d), \quad (\text{B.3})$$

$$SE'_{up}(T) = -\frac{1}{f_2(T)}. \quad (\text{B.4})$$

In the following, we consider two cases.

1)  $EE'_{up}(0) \leq 0$ :

Because the power consumption of the BS in uplink receive mode is less than that in downlink transmit mode, i.e.,  $\frac{P_{bu}}{P_{bd}} < 1$ , we know from (B.1) and (B.3) that  $SE'_{up}(0) \leq 0$  always holds when  $EE'_{up}(0) \leq 0$ . Further considering Theorem 1, which shows that  $EE'_{up}(T_{tr})$  and  $SE'_{up}(T_{tr})$  are monotonically decreasing functions of  $T_{tr}$ , we have  $EE'_{up}(T_{tr}) \leq 0$  and  $SE'_{up}(T_{tr}) \leq 0$  for any  $T_{tr} \in [0, T]$ . This means that both  $EE_{up}(T_{tr})$  and  $SE_{up}(T_{tr})$  are monotonically decreasing functions of  $T_{tr}$ . As a result,  $T_{trSE}^* = 0$  is optimal to maximize the  $SE_{up}$ , and  $T_{trEE}^* = 0$  is the optimal solution of the relaxed version of problem (19) since it maximizes  $EE_{up}$  and satisfies the constraint in (19b).

The result implies that uplink training is unnecessary for both the SE-oriented and the EE-oriented optimization in this case, and  $\Delta T_{tr}^* = 0$ .

2)  $EE'_{up}(0) > 0$ :

Considering  $EE'_{up}(T) < 0$  from (B.2), and the fact that  $EE'_{up}(T_{tr})$  is monotonically decreasing from Theorem 1, in this case  $EE_{up}(T_{tr})$  first increases and then decreases.

Let  $T_{trEE}^o$  denote the training length that only maximizes  $EE_{up}$ , i.e., without the constraint on the minimal SE. Since  $EE_{up}(T_{tr})$  is a concave function, we know that  $EE'_{up}(T_{trEE}^o) = 0$ . In the following, we prove that  $T_{trEE}^o > T_{trSE}^*$  no matter  $SE'_{up}(0) \leq 0$  or  $SE'_{up}(0) > 0$ .

- When  $SE'_{up}(0) \leq 0$ , since  $SE'_{up}(T_{tr})$  is monotonically decreasing with  $T_{tr}$ ,  $SE'_{up}(T_{tr}) < 0$  and  $SE_{up}(T_{tr})$  is monotonically decreasing. Therefore, we have  $T_{trSE}^* = 0$ , and  $T_{trEE}^o > T_{trSE}^*$ .
- When  $SE'_{up}(0) > 0$ , considering that  $SE'_{up}(T) < 0$  from (B.4) and the fact that  $SE'_{up}(T_{tr})$  is monotonically decreasing,  $SE_{up}(T_{tr})$  first increases and then decreases with  $T_{tr}$ . Since  $SE_{up}(T_{tr})$  is concave,  $T_{trSE}^*$  can be found from  $SE'_{up}(T_{trSE}^*) = 0$ . From (13) and (18), we can obtain the relationship between the upper bounds of the EE and the SE as

$$EE_{up}(T_{tr}) = \frac{T \cdot SE_{up}(T_{tr})}{P_{bu}(T + (\frac{P_{bd}}{P_{bu}} - 1)(T - T_{tr}))}. \quad (\text{B.5})$$

By taking the derivative with respect to  $T_{tr}$  respectively to the left and right hand sides of (B.5) and considering that  $EE'_{up}(T_{trEE}^o) = 0$ , we obtain

$$SE'_{up}(T_{trEE}^o) = -\frac{SE_{up}(T_{trEE}^o)}{\frac{T}{\frac{P_{bd}}{P_{bu}} - 1} + T - T_{trEE}^o}. \quad (\text{B.6})$$

Obviously,  $SE'_{up}(T_{trEE}^o) < 0$ . Since  $SE'_{up}(T_{trSE}^*) = 0$  and  $SE'_{up}(T_{tr})$  is monotonically decreasing,  $T_{trEE}^o > T_{trSE}^*$  holds.

If  $T_{trEE}^o$  satisfies constraint (19b), we know that  $T_{trEE}^* = T_{trEE}^o > T_{trSE}^*$ .

Now we see what will happen if  $T_{trEE}^o$  does not satisfy (19b). Denote  $[T_{tr}^{min}, T_{tr}^{max}]$  as the feasible set of  $T_{tr}$ . It's easy to see that  $T_{trSE}^* \in [T_{tr}^{min}, T_{tr}^{max}]$  since  $T_{trSE}^*$  maximizes the SE so that the constraint in (19b) is satisfied. Further considering the fact  $T_{trEE}^o > T_{trSE}^*$ , if  $T_{trEE}^o$  is outside of the feasible set, there must be  $T_{trEE}^o > T_{tr}^{max}$ . As we have analyzed,  $EE_{up}(T_{tr})$  first increases and then decreases with  $T_{tr}$ , and the transition point is at  $T_{trEE}^o$ . Therefore, we have  $EE_{up}(T_{tr}^{min}) \leq EE_{up}(T_{trSE}^*) \leq EE_{up}(T_{tr}^{max}) < EE_{up}(T_{trEE}^o)$ . Consequently,  $T_{trEE}^* = T_{tr}^{max} \geq T_{trSE}^*$  in this case. The equality  $T_{trEE}^* = T_{trSE}^*$  holds if and only if  $T_{tr}^{max} = T_{trSE}^*$ , i.e., if  $SE_0$  equals to the maximum achievable net SE,  $SE_{max}$ .

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