

Cross-layer Transmission Design for Tactile Internet

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Abstract—To ensure the low end-to-end (E2E) delay for tactile internet, short frame structures will be used in 5G systems. As such, transmission errors with finite blocklength channel codes should be considered to guarantee the high reliability requirement. In this paper, we study cross-layer transmission optimization for tactile internet, where both queueing delay and transmission delay are accounted for in the E2E delay, and different packet loss/error probabilities are considered to characterize the reliability. We show that the required transmit power becomes unbounded when the allowed maximal queueing delay is shorter than the channel coherence time. To satisfy quality-of-service requirement with finite transmit power, we introduce a proactive packet dropping mechanism, and optimize a queue state information and channel state information dependent transmission policy. Since the resource and policy for transmission and the packet dropping policy are related to the *packet error probability, queueing delay violation probability, and packet dropping probability*, we optimize the three probabilities and obtain the policies related to these probabilities. We start from single-user scenario and then extend our framework to the multi-user scenario. Simulation results show that the optimized three probabilities are in the same order of magnitude. Therefore, we have to take into account all these factors when we design systems for tactile internet applications.

I. INTRODUCTION

Tactile internet enables unprecedented mobile applications such as autonomous vehicles, mobile robots, augmented reality and factory automation [1], which calls for ultra-low latency (say 1 ms) and ultra-high reliability (say 99.99999%). In the fifth generation (5G) communication systems, achieving such an extremely stringent quality-of-service (QoS) has become one of the major goals [2]. Ensuring the short end-to-end (E2E) delay and low packet loss/error probability calls for new air interface for 5G systems. By introducing short frame structure and short transmit time interval (TTI), transmission delay can be reduced [3]. With short frame, the channel coding is then performed with a finite block of symbols under ultra-high reliability requirement, and hence the transmission error probability should be considered. Compared with the channel capacity, the maximal achievable rate with finite blocklength channel codes under given transmission error probability requirement is more relevant to our current problem [4].

Though important, the queueing delay is largely overlooked in most of the existing literatures that study ultra-short delay and ultra-high reliability transmissions. For applications with medium delay requirement, the throughput with finite blocklength codes under statistical queueing constraint was studied in [5], where the delay is much larger than the channel

coherence time. However, in tactile internet applications, the ultra-short delay could be shorter than the coherence time. When the average delay approaches the coherence time, the average transmit power may become unbounded [6].

In this paper, we study cross-layer optimization for tactile internet. To ensure the QoS requirement of ultra-low E2E delay and ultra-high reliability, the transmission delay and error probability as well as the statistical queueing delay requirement (characterized by a delay bound and a small delay violation probability) are considered. To satisfy the QoS with finite transmit power, a proactive packet dropping mechanism is introduced. Since the packet error probability, queueing delay violation probability, and packet dropping probability depend on the transmission resource and policy as well as the packet dropping policy, we optimize them together to minimize the transmit power of the base station (BS). We first optimize the power allocation and packet dropping policy in each TTI in single-user scenario, then extend our framework to multi-user scenario and optimize the bandwidth allocation among users. Simulation results show that the optimized three probabilities are in the same order of magnitude.

II. SYSTEM MODEL

Consider a time division duplexing cellular system, which consists of a BS with N_t antennas and $K + M$ single-antenna nodes, as shown in Fig. 1. The nodes are divided into two types. The first type of nodes are K users, which need to upload and download packets to the BS. The second type of nodes are M sensors, which only upload packets. Time is discretized into frames. As illustrated in Fig. 2, each frame has duration T_f and consists of a downlink (DL) transmission phase with duration ϕ and an uplink (UL) transmission phase with duration φ . In the UL phase, all the nodes upload their own messages in short packets to the BS. In the DL phase, the BS processes the UL received messages from the nodes that lie in the concerned area of each user, and then transmits the relevant messages to the target users. Since the interference among nodes cause severe deterioration in QoS, we assume a frequency division multiple access system.

A. Reliability and Delay Metrics

For tactile service, the QoS can be characterized by a maximal E2E delay for each packet, D_{\max} , and a maximal packet loss/error probability, ε_D . The E2E delay is very short, say 1 ms [7], which includes UL and DL transmission delay

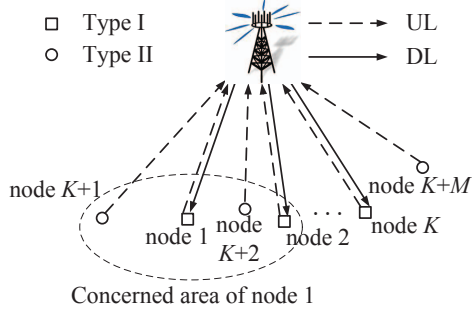


Fig. 1. System model.

and queuing delay in the buffer of BS. Denote the size of each packet as u bits. We assume u is small enough such that it can be transmitted within one UL phase. Then, we focus on the DL transmission and investigate how to achieve the stringent QoS requirement.

To ensure ultra-low transmission delay, we consider short frame structure proposed in [3], where the TTI is the same as the frame duration and $T_f \ll D_{\max}$. Moreover, we assume that the DL transmission can be finished within the duration of ϕ . Then, the queuing delay for every packet should be bounded as $D_{\max}^q \triangleq D_{\max} - T_f$ with a small violation probability ε_k^q . With finite blocklength channel codes, the transmission of each packet can be finished within one frame with a small error probability ε_k^c . As detailed later, to ensure the statistical requirement imposed on the queuing delay for each packet ($D_{\max}^q, \varepsilon_k^q$), the required transmit power may become unbounded in deep fading. To guarantee the E2E delay and reliability with finite transmit power, we proactively discard some packets from the head of the queue under deep fading, and control the overall E2E reliability as follows:

$$1 - (1 - \varepsilon_k^c)(1 - \varepsilon_k^q)(1 - \varepsilon_k^h) \approx \varepsilon_k^c + \varepsilon_k^q + \varepsilon_k^h \leq \varepsilon_D, \quad (1)$$

where ε_k^h is the proactive packet dropping probability for the k th user. Note that the above approximation is valid since $\varepsilon_k^c, \varepsilon_k^q$ and ε_k^h are extremely small.

B. Channel Model

Denote the coherence time of the channel as T_c . Since the E2E delay is extremely short, we assume that $T_c > D_{\max} > D_{\max}^q$ as illustrated in Fig. 2.¹

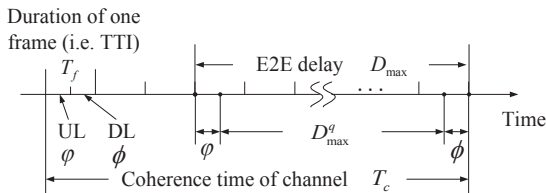


Fig. 2. Illustration of TTI, E2E delay and channel coherence time.

When the number of users is large or the overall bandwidth is small, each user will be allocated with bandwidth that is

¹For instance, for users with velocities less than 120 km/h and the system operating in carrier frequency of 2 GHz, the channel coherence time is larger than 1 ms, which exceeds the delay bound of each packet.

smaller than the coherence bandwidth of the channel. When channel coding is performed within each frame, which is shorter than the coherence time, the channel is referred to as *quasi-static fading channel* [4]. Denote the average channel gain and channel vector of the k th user in a certain coherence block as α_k and $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$, whose elements are independent and identically complex Gaussian distributed with zero mean and unit variance. According to the normal approximation in [4], when α_k and \mathbf{h}_k are perfectly known at the BS and the user, the maximal number of packets that *can be* transmitted to the k th user in the n th frame (i.e., in the n th TTI) can be accurately approximated as

$$s_k(n) \approx \frac{\phi W_k}{u \ln 2} \left\{ \ln \left[1 + \frac{\alpha_k P_k(n) g_k}{N_0 W_k} \right] - \sqrt{\frac{V}{\phi W_k}} f_G^{-1}(\varepsilon_k^c) \right\}, \quad (2)$$

where W_k is the bandwidth allocated to the k th user, $P_k(n)$ is the transmit power allocated to the k th user according to its queue length and channel state in the n th frame, $g_k = \mathbf{h}_k^H \mathbf{h}_k$, $[\cdot]^H$ denotes the conjugate transpose, N_0 is the single-sided noise spectral density, $f_G^{-1}(x)$ is the inverse of the Gaussian Q-function, and V is the channel dispersion given by [4]

$$V = 1 - \frac{1}{\left[1 + \frac{\alpha_k P_k(n) g_k}{N_0 W_k} \right]^2}. \quad (3)$$

Then, the number of symbols transmitted in the DL transmission phase of one frame, n_k^s , is determined by the bandwidth and transmission time, i.e., $n_k^s = \phi W_k$.

C. Queue Model

In the n th frame, the k th user requests packets that are uploaded from its nearby nodes. The indices of the nodes that lie in the concerned area of the k th user constitute a set \mathcal{A}_k with cardinality $|\mathcal{A}_k|$. As illustrated in Fig. 1, the index set of the nearby nodes of node 1 is $\mathcal{A}_1 = \{2, K+1, K+2\}$. Then, the number of packets waiting in the queue for the k th user at the beginning of the $(n+1)$ th frame can be expressed as

$$Q_k(n+1) = \max \{Q_k(n) - s_k(n), 0\} + \sum_{i \in \mathcal{A}_k} a_i(n), \quad (4)$$

where $a_i(n)$, $i \in \mathcal{A}_k$ is the number of packets uploaded to the BS from the i th nearby node of the k th user. Denote the number of packets departed from the k th queue in the n th frame as $b_k(n)$. If all the packets in the queue can be completely transmitted in the n th frame, then $b_k(n) = Q_k(n)$. Otherwise, $b_k(n) = s_k(n)$. Hence, we have

$$b_k(n) = \min \{Q_k(n), s_k(n)\}. \quad (5)$$

III. ENSURING THE QOS REQUIREMENT

In this section, we first employ effective bandwidth, a widely used design tool [8], to represent the statistical queuing delay requirement. Then, we show that the required transmit power to ensure $(D_{\max}^q, \varepsilon_k^q)$ for some packets may become unbounded. To guarantee the QoS on D_{\max} and ε_D with

finite transmit power, we propose a proactive packet dropping mechanism. For notational simplicity, we consider a single-user scenario herein, where $K = 1$ and $M > 1$. As such, the index k can be omitted.

A. Queuing Delay Requirement

For stationary packet arrival process $\{\sum_{i \in \mathcal{A}} a_i(n), n = 1, 2, \dots\}$, the effective bandwidth is defined as [8]

$$E^B(\theta) = \lim_{N \rightarrow \infty} \frac{1}{NT_f\theta} \ln \left\{ \mathbb{E} \left[\exp \left(\theta \sum_{n=1}^N \sum_{i \in \mathcal{A}} a_i(n) \right) \right] \right\}, \quad (6)$$

where N is the number of frames, and θ is the QoS exponent. A large value of θ indicates a stringent delay requirement. According to [9], when the delay bound approaches the coherence time, the power allocation over fading channel is channel inversion. In other words, the service rate becomes a constant. When the constant service rate equals to $E^B(\theta)$, the steady state queuing delay bound violation probability can be approximated as [10]

$$\Pr\{D(\infty) > D_{\max}^q\} \approx \eta \exp\{-\theta E^B(\theta) D_{\max}^q\}, \quad (7)$$

where η is the buffer non-empty probability. Since $\eta \leq 1$, we have

$$\Pr\{D(\infty) > D_{\max}^q\} \leq \exp\{-\theta E^B(\theta) D_{\max}^q\} \triangleq P_D^{\text{UB}}. \quad (8)$$

If the upper bound in (8) satisfies

$$P_D^{\text{UB}} = \exp\{-\theta E^B(\theta) D_{\max}^q\} = \varepsilon^q, \quad (9)$$

then the requirement $(D_{\max}^q, \varepsilon^q)$ can be satisfied.

Remark 1: Note that (8) is not a strict upper bound for all kinds of arrival processes since (7) is accurate when the queue length is large enough. However, (8) is still a valid upper bound for Poisson arrival process even in very short delay regime [11]. If the packet arrival process is more bursty than Poisson, then (8) will still be an upper bound as shown from the results in [12].

The aggregation of the packet arrival processes from the $|\mathcal{A}|$ nodes of the user (i.e., $\sum_{i \in \mathcal{A}} a_i(n)$ in (4)) can be modeled as a Poisson process in vehicle communication scenarios as well as other machine type communication scenarios [13, 14]. For a Poisson arrival process, the effective bandwidth is

$$E^B(\theta) = \frac{\lambda}{T_f\theta} (e^\theta - 1), \quad (10)$$

where λ is the average number of the packets arrived at the queue during one frame, which is identical for all frames.

Substituting (10) into (9), we can obtain that $\theta = \ln \left[\frac{T_f \ln(1/\varepsilon^q)}{\lambda D_{\max}^q} + 1 \right]$, such that (10) can be rewritten as,

$$E^B(\theta) = \frac{\ln(1/\varepsilon^q)}{D_{\max}^q \ln \left[\frac{T_f \ln(1/\varepsilon^q)}{\lambda D_{\max}^q} + 1 \right]}. \quad (11)$$

If the number of packets transmitted to the user is a constant among frames that satisfies

$$s(n) = T_f E^B(\theta), \quad (12)$$

then the queuing delay requirement $(D_{\max}^q, \varepsilon^q)$ can be ensured and the related departure process in (5) becomes

$$b(n) = \min\{Q(n), T_f E^B(\theta)\}. \quad (13)$$

B. Transmit Power

In what follows, we show that the required transmit power to guarantee the queuing delay requirement for some packets may become unbounded for any given values of W and N_t . We consider the case where $Q(n) \geq T_f E^B(\theta)$, then $b(n) = T_f E^B(\theta)$. If the transmit power can guarantee such a departure rate, then for the other case where $Q(n) < T_f E^B(\theta)$, $b(n) < T_f E^B(\theta)$ can also be supported.

Substituting $s(n)$ in (2) and $E^B(\theta)$ in (11) into (12), we can derive the required SNR γ to ensure $(D_{\max}^q, \varepsilon^q)$ and ε^c for all packets using the following equation,

$$\ln(1 + \gamma) = \frac{T_f u \ln 2 \ln(1/\varepsilon^q)}{\phi W D_{\max}^q \ln \left[1 + \frac{T_f \ln(1/\varepsilon^q)}{D_{\max}^q \lambda} \right]} + \sqrt{\frac{V}{\phi W}} f_G^{-1}(\varepsilon^c). \quad (14)$$

Since the elements of \mathbf{h} whose elements are independent and identically complex Gaussian distributed with zero mean and unit variance, the instantaneous channel gain g follows the Wishart distribution [15], whose probability density function is $f_g(x) = \frac{1}{(N_t - 1)!} x^{N_t - 1} e^{-x}$. In the considered *quasi-static fading channel*, some packets may experience deep fading with channel gain g that is arbitrarily close to zero. Then, the required transmit power in the corresponding frame to achieve γ , $P(n) \triangleq \frac{N_0 W \gamma}{\alpha g}$, is unbounded.

C. Proactive Packet Dropping Mechanism

To ensure the reliability ε_D with a finite transmit power, we introduce a proactive packet dropping mechanism. Denote the maximal transmit power of the BS as P^{\max} . We allow some packets in deep fading to be dropped before transmission since the required SNR γ cannot be achieved with $P(n) \leq P^{\max}$.

From the approximation in (2), we can obtain the number of packets that can be transmitted with $P(n) = P^{\max}$ as follows:

$$s^{\max} \approx \frac{\phi W}{u \ln 2} \left\{ \ln \left[1 + \frac{\alpha P^{\max} g}{N_0 W} \right] - \sqrt{\frac{V}{\phi W}} f_G^{-1}(\varepsilon^c) \right\}. \quad (15)$$

When $g < \frac{N_0 W \gamma}{\alpha P^{\max}}$, $s^{\max} < T_f E^B(\theta)$. Since $b(n) = \min\{Q(n), T_f E^B(\theta)\}$ should be satisfied to ensure $(D_{\max}^q, \varepsilon^q)$, we should discard some packets from the head of the queue. Otherwise, $(D_{\max}^q, \varepsilon^q)$ cannot be satisfied. Denote the number of packets dropped in the n th frame as $b^d(n) = \max\{b(n) - s^{\max}, 0\}$. Then, we have

$$b^d(n) = \begin{cases} \max(T_f E^B(\theta) - s^{\max}, 0), & \text{if } Q(n) \geq T_f E^B(\theta), \\ \max(Q(n) - s^{\max}, 0), & \text{if } Q(n) < T_f E^B(\theta). \end{cases} \quad (16)$$

Similar to the time averaged delivery ratio in [16], we define the time average packet dropping ratio as follows:

$$\varepsilon^h \triangleq \lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N b^d(n)}{\sum_{n=1}^N \sum_{i \in \mathcal{A}} a_i(n)} = \frac{\mathbb{E}[b^d(n)]}{\lambda}, \quad (17)$$

where the second equality is obtained under the assumption that the queueing system is stationary and ergodic, and the average is taken over both channel gain and queue length. To obtain ε^h , we consider an upper bound of $b^d(n)$ as follows:

$$b^U(n) = \begin{cases} \max(T_f E^B(\theta) - s^{\max}, 0), & \text{if } Q(n) > 0, \\ 0, & \text{if } Q(n) = 0, \end{cases}$$

When $Q(n) \geq T_f E^B(\theta)$ or $Q(n) = 0$, $b^U(n) = b^d(n)$. When $0 < Q(n) < T_f E^B(\theta)$, $b^U(n) > b^d(n)$. Then, we can derive an upper bound of $\mathbb{E}[b^d(n)]$ as follows:

$$\mathbb{E}[b^U(n)] = \eta \int_0^{\frac{N_0 W \gamma}{\alpha P^{\max}}} (T_f E^B(\theta) - s^{\max}) f_g(g) dg.$$

Upon substituting $\mathbb{E}[b^U(n)]$ into (17), we can derive an upper bound of the packet dropping ratio as follows,

$$\varepsilon^h < \int_0^{\frac{N_0 W \gamma}{\alpha P^{\max}}} \left[1 - \frac{s^{\max}}{T_f E^B(\theta)} \right] f_g(g) dg, \quad (18)$$

where $\eta = \Pr\{Q(n) > 0\} = \mathbb{E}\{\sum_{i \in \mathcal{A}} a_i(n)\} / \mathbb{E}[s(n)] = \frac{\lambda}{T_f E^B(\theta)}$ is applied.²

By substituting s^{\max} in (15), and considering (11) and (14), we have

$$\frac{s^{\max}}{T_f E^B(\theta)} = \frac{\ln \left[1 + \frac{\alpha P^{\max} g}{N_0 W} \right] - \sqrt{\frac{V}{\phi W}} f_G^{-1}(\varepsilon^c)}{\ln(1 + \gamma) - \sqrt{\frac{V}{\phi W}} f_G^{-1}(\varepsilon^c)}. \quad (19)$$

Note that a packet is dropped only if it is transmitted in deep fading. When $g \rightarrow 0$, V in (3) approaches 0, and hence (19) can be accurately approximated by

$$\frac{s^{\max}}{T_f E^B(\theta)} \approx \frac{\ln \left[1 + \frac{\alpha P^{\max} g}{N_0 W} \right]}{\ln(1 + \gamma)} \approx \frac{\alpha P^{\max} g}{N_0 W \ln(1 + \gamma)}. \quad (20)$$

Substituting (20) into (18), we obtain

$$\varepsilon^h < \int_0^{\frac{N_0 W \gamma}{\alpha P^{\max}}} \left[1 - \frac{\alpha P^{\max} g}{N_0 W \ln(1 + \gamma)} \right] f_g(g) dg. \quad (21)$$

IV. CROSS-LAYER TRANSMISSION DESIGN

In this section, we find the optimal resource allocation policy and packet dropping policy that minimize the required maximal transmit power. We consider the cases that $Q(n) > 0$. For the other case $Q(n) = 0$, $P(n) = 0$.

²Simulation results show that $Q(n) = 0$ or $Q(n) \geq T_f E^B(\theta)$ with more than 90 % probability (the results are not provided due to the lack of space). Hence, in the most cases $b^d(n) = b^U(n)$. This suggests that the upper bound $\mathbb{E}[b^U(n)]$ is tight, and hence (18) is also tight.

A. Single-user Scenario

Since the values of ε^q , ε^c and ε^h are related to the resource allocation and packet dropping policy, we use the following framework to determine their optimal combination to ensure the reliability $\varepsilon^q + \varepsilon^c + \varepsilon^h \leq \varepsilon_D$,

$$\begin{aligned} & \min_{\varepsilon^q, \varepsilon^c, \varepsilon^h} P^{\max}, \\ & \text{s.t.} \quad (1), (14) \text{ and } (21). \end{aligned} \quad (22)$$

From the solution of this problem, we can obtain the power allocation policy $P(n)$ and packet dropping policy $b^d(n)$ that minimizes P^{\max} . Specifically, with the values of ε^q and ε^c as well as D_{\max}^q , we can obtain the required SNR γ from (14). Given the values of γ and ε^h , P^{\max} can be obtained from the right hand side of (21). The optimal power allocation policy to the TTIs when $Q(n) > 0$ is given by:

$$P^*(n) = \begin{cases} P^{\max}, & \text{if } g < \frac{N_0 W \gamma}{\alpha P^{\max}}, \\ \frac{N_0 W \gamma}{\alpha g}, & \text{if } g > \frac{N_0 W \gamma}{\alpha P^{\max}}. \end{cases} \quad (23)$$

Furthermore, by substituting P^{\max} into s^{\max} in (16), the optimal packet dropping policy is obtained.

In the following, we propose a two-step method to find the optimal solution of problem (22). In the first step, the upper bound of the proactive packet dropping probability is fixed as $\varepsilon_0^h \in (0, \varepsilon_D)$. Given ε_0^h , P^{\max} in (21) increases with γ . Hence, minimizing P^{\max} is equivalent to minimizing γ . The optimal values of ε^q and ε^c that minimize the required γ can be obtained by solving the following problem,

$$\begin{aligned} & \min_{\varepsilon^q, \varepsilon^c} (14) \\ & \text{s.t.} \quad \varepsilon^q + \varepsilon^c \leq \varepsilon_D - \varepsilon_0^h. \end{aligned} \quad (24)$$

As proved in Appendix A, (14) is strictly convex in ε^q and ε^c . As shown in (3), when the required SNR γ is high, $V \approx 1$ and does not depend on SNR.³ Then, there is a unique optimal solution of ε^q and ε^c that minimizes γ . Denote the minimal SNR obtained from problem (24) as γ^* . Since the right hand side of (21) decreases with P^{\max} , for any given ε_0^h , P^{\max} can be obtained numerically from (21). Then, we obtain the relation between the minimal P^{\max} and ε_0^h and denote it as $P^{\max}(\varepsilon_0^h)$.

In the second step, we find the optimal $\varepsilon_0^h \in (0, \varepsilon_D)$ that minimizes $P^{\max}(\varepsilon_0^h)$. Since there is no closed-form expression of $P^{\max}(\varepsilon_0^h)$, the exhaustive search method is needed to obtain the optimal ε_0^h in general. However, numerical results indicate that $P^{\max}(\varepsilon_0^h)$ first decreases and then increases with ε_0^h (which are omitted due to the lack of space). With this property, we can find the optimal ε_0^h and the required transmit power to ensure ε_D via binary searching [17].

In summary, we can obtain the optimal solution and the minimal transmit power from the two-step method, denoted as $(\varepsilon^{q*}, \varepsilon^{c*}, \varepsilon^{h*})$ and $P^{\max*}$, respectively. As proved in

³Different from the cases in (19) that packets are dropping, the required SNR for successful transmission is high.

Appendix B, $(\varepsilon^{q*}, \varepsilon^{c*}, \varepsilon^{h*})$ is the global optimal solution of problem (22).

B. Multi-user Scenario

Denote the total bandwidth as W^{\max} , we jointly allocate W_k , $P_k(n)$ and $b_k^d(n)$ by optimizing W_k , ε^q , ε^c , and ε^h . In multi-user scenario, packet dropping only happens when $\sum_{k=1}^K \frac{N_0 W_k \gamma_k}{\alpha_k g_k} > P^{\max}$. However, since the packet dropping ratio depends on the channel states of all the users, it is hard to derive. To overcome this difficulty, we introduce the maximal transmit power to each user, i.e., when $\frac{N_0 W_k \gamma_k}{\alpha_k g_k} \geq P_k^{\text{th}}$ some packets are dropped. Then, the upper bound of the packet dropping ratio of the k th user can be expressed as

$$\varepsilon_k^h = \int_0^{\frac{N_0 W_k \gamma_k}{\alpha_k P_k^{\text{th}}}} \left[1 - \frac{\alpha_k P_k^{\text{th}} g}{N_0 W_k \ln(1 + \gamma_k)} \right] f_g(g) dg. \quad (25)$$

The optimization framework in the multi-user scenario is:

$$\begin{aligned} \min_{W_k, \varepsilon_k^q, \varepsilon_k^c, \varepsilon_k^h} P^{\max} &= \sum_{k=1}^K P_k^{\text{th}} \\ \text{s.t. } \sum_{k=1}^K W_k &\leq W^{\max}, (1), (14) \text{ and } (25). \end{aligned} \quad (26)$$

Since the event $\left\{ \sum_{k=1}^K \frac{N_0 W_k \gamma_k}{\alpha_k g_k} > P^{\max} \right\}$ is a subset of $\bigcup_k \left\{ \frac{N_0 W_k \gamma_k}{\alpha_k g_k} \geq P_k^{\text{th}} \right\}$, the packet dropping ratio is overestimated. Hence, P^{\max} obtained from (26) is an upper bound of the minimal transmit power that is required to ensure the QoS. Given W_k , the transmit power allocation policy among subsequent TTIs and packet dropping policy is similar to that in single-user scenario, i.e., (23) and (16).

Note that the number of symbols transmitted in each DL phase $n_k^s = \phi W_k$ is an integer. Thus, W_k , $k = 1, \dots, K$ are discrete variables, and (26) is a mixed-integer programming problem. To reduce the complexity, a heuristic algorithm is proposed in Table I. The basic idea is similar to the steepest descent method [17]. Given the values of the discrete variables W_k , the problem can be decomposed into K single-user problems similar to (22), which can be solved by the two-step method. We refer to the K single-user problems as *subproblem I*. The bandwidth allocation algorithm includes $\phi W^{\max} - K$ steps. In each step, $1/\phi$ bandwidth is allocated to one of the K users that leads to the steepest total transmit power descent. The proposed algorithm only needs to solve subproblem I $K(\phi W^{\max} - K)$ times, and hence the complexity is $O(K(\phi W^{\max} - K))$.

V. SIMULATION RESULTS

In this section, we first validate the optimality of the proposed policy. Then, we show the effect of ε_k^q , ε_k^c and ε_k^h on the system performance. The three factors that lead to packet loss have not been considered in existing literatures, and hence we do not compare our policy with existing policies.

TABLE I
BANDWIDTH ALLOCATION ALGORITHM

Input: Number of users K , total bandwidth W^{\max} , duration of each DL phase ϕ , packet size u , noise spectral density N_0 , number of transmit antennas N_t , average channel gains of users α_k , $k = 1, \dots, K$.

Output: Bandwidth allocation W_k^* , $k = 1, \dots, K$.

- 1: Set $n_k^s(0) = 1$, $k = 1, \dots, K$. Set $l = 1$.
- 2: Solve subproblem I with given $W_k(0) = n_k^s(0)/\phi$, and obtain the total transmit power $P^{\max}(0)$.
- 3: **while** $l \leq \phi W^{\max} - K$ **do**
- 4: Set $\hat{k} = 1$
- 5: **while** $\hat{k} \leq K$ **do**
- 6: $n_{\hat{k}}^s(l) := n_{\hat{k}}^s(l-1) + 1$; $n_{\hat{k}^*}^s(l) := n_{\hat{k}^*}^s(l-1)$, $k \neq \hat{k}$.
- 7: Solve subproblem I with $W_k(l) = n_k^s(l)/\phi$, and obtain $\hat{P}_k^{\max}(l)$.
- 8: $\hat{k} := \hat{k} + 1$.
- 9: **end while**
- 10: $k^* := \arg \min_{\hat{k}} \hat{P}_k^{\max}(l)$.
- 11: $n_{k^*}^s(l) := n_{k^*}^s(l-1) + 1$; $n_k^s(l) := n_k^s(l-1)$, $k \neq k^*$.
- 12: $l := l + 1$.
- 13: **end while**
- 14: **return** $W_k^* = n_k^s(l-1)/\phi$, $k = 1, \dots, K$.

The users are uniformly distributed with distances from the BS 50 m \sim 200 m. The concerned area of each user is a circle region with diameter $d_c = 50$ m. The sensors are uniformly distributed with density 0.01 user/m². Each node uploads packets with rate 10 packets/s, and each user needs to download packets that are uploaded to the BS from the nodes in the concerned areas of it. Other parameters are listed in Table II, unless otherwise specified.

TABLE II
PARAMETERS [2]

Reliability requirement ε_D	1 - 99.99999%
Queueing delay requirement D_{\max}^q	0.9 ms
Duration of each frame (equals to TTI)	0.1 ms
Duration of downlink phase	0.05 ms
Single-sided noise spectral density N_0	-173 dBm/Hz
Packet size u	20 bytes
Path loss model $10 \lg(\alpha_k)$	35.3 + 37.6 $\lg(d_k)$

The required P^{\max} obtained by the proposed algorithm and exhaustive search method are provided in Table III, which illustrate that the proposed algorithm is near-optimal. Because the complexity of exhaustive search method is extremely high with large W^{\max} , we only provide results with small values of W^{\max} and K . However, the proposed algorithm can be applied to systems with large W^{\max} and K , as shown in Table IV.

The optimal values of ε_k^c , ε_k^q and ε_k^h that minimize the transmit power are illustrated in Fig. 3, which are obtained by (22) in single-user scenario. The results show that ε_k^c , ε_k^q and

TABLE III
REQUIRED TRANSMIT POWER, $W^{\max} = 1$ MHz, $N_t = 2$

Number of users K	2	4	6
Proposed Algorithm	0.0216 W	0.155 W	5.26 W
Exhaustive Search	0.0216 W	0.155 W	5.26 W

TABLE IV
REQUIRED TRANSMIT POWER, $K = 40$, $N_t = 2$

Bandwidth W^{\max}	6 MHz	7 MHz	8 MHz
Proposed Algorithm	35.7 W	6.92 W	3.28 W

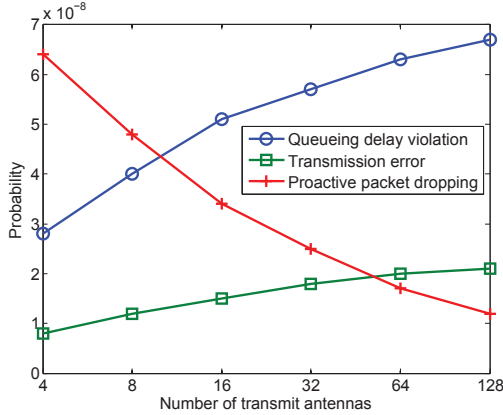


Fig. 3. $K = 1$, the user-BS distance is 200 m, the bandwidth is 0.5 Mhz, and the number of nodes in the concerned area is 100.

ε_k^h are in the same order of magnitude. Hence, when design transmission policy for tactile internet, we conclude that we can not ignore any of these factors.

VI. CONCLUSION

In this paper, we showed how to transmit short packets in tactile internet under ultra-low E2E delay and ultra-high reliability requirement. Both queuing delay and transmission delay were considered in the E2E delay, and the transmission error probability, queuing delay violation probability, and packet dropping probability were taken into account in the reliability. The queue state and channel state information dependent transmission policy was optimized to minimize the required maximal transmit power of the BS. Bandwidth allocation was also optimized in multi-user scenario. Simulation results showed that the transmission error probability, queuing delay violation probability, and packet dropping probability are in the same order of magnitude.

APPENDIX A

PROOF OF THE CONVEXITY OF (14)

Proof. The Gaussian Q-function is $f_G(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{\tau^2}{2}\right) d\tau$. Then, $f'_G(x) \triangleq -\frac{1}{\sqrt{2\pi}} e^{-x^2/2} < 0$ and $f''_G(x) = \frac{x}{\sqrt{2\pi}} e^{-x^2/2}$, which is positive when $x > 0$, (i.e., $f_G(x) < 0.5$). Thus, $f_G(x)$ is a decreasing and strictly convex function when $x > 0$. Since $\varepsilon^c < 0.5$, and the inverse function of a decreasing and strictly convex function is also strictly convex [17], $f_G^{-1}(\varepsilon^c)$ is strictly convex in ε^c . Hence, the second term of (14) is strictly convex in ε^c .

To prove that the first term of (14) is strictly convex in ε^q , we derive the second order derivative of it. Denote $y = -\ln(\varepsilon^q) > 0$ and $z = \frac{T_i}{D_{\max}^q \lambda} > 0$. Then, by removing the constants that are not relevant to ε^q , the first term of (14) can be expressed as follows,

$$f(y) = \frac{y}{\ln(1+zy)}. \quad (\text{A.1})$$

After some regular derivations, we can obtain that

$$\frac{d^2 f}{d(\varepsilon^q)^2} = \frac{f_{\text{num}}(y, z)}{[\ln(1+zy)]^3 (1+zy)^2 (\varepsilon^q)^2}, \quad (\text{A.2})$$

where $f_{\text{num}}(y, z) = (1+zy)^2 [\ln(1+zy)]^2 + 2z^2 y - (2z+zy+z^2 y+z^2 y^2) \ln(1+zy)$. It is not hard to prove that $f_{\text{num}}(y, z)$ is positive for all $y > 16.1$ (i.e., $\varepsilon^q < 10^{-7}$), $z > 0$ (The computing details are omitted due to lack of space). Hence, the first term of (14) is strictly convex in ε^q .

From the above results, we can obtain that the Hessian matrix of (14) is positive definite. This completes the proof. \square

APPENDIX B

PROOF OF THE OPTIMALITY OF THE TWO-STEP METHOD

Proof. Denote an arbitrary feasible solution of problem (22) and the related transmit power as $(\tilde{\varepsilon}^q, \tilde{\varepsilon}^c, \tilde{\varepsilon}^h)$ and \tilde{P}^{\max} , respectively. Given $\tilde{\varepsilon}^h$, we can obtain the minimal transmit power $P^{\max}(\tilde{\varepsilon}^h) \leq \tilde{P}^{\max}$ via the first step of the two-step method. In the second step, the optimal ε^{h*} is obtained such that $P^{\max*} \leq P^{\max}(\tilde{\varepsilon}^h)$. Therefore, $P^{\max*} \leq \tilde{P}^{\max}$. The proof follows. \square

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