Macro-Pico Amplitude-Space Sharing with Optimized Han-Kobayashi Coding

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***Abstract*—Heterogeneous network is a new paradigm in next generation cellular systems, which is promised to significantly improve the spatial spectrum efficiency through overlapped coverage. This however calls for efficient interference management techniques. In this paper, we propose an amplitude-space sharing strategy among the macro-cell user and pico-cell users, where different users occupy different levels in the signal amplitude-space. By optimizing the space sharing scheme, different layers of signal and interference are separable at each receiver and the network sum-rate can be maximized. We start from the single pico-cell scenario, where we employ Han-Kobayashi coding and derive the optimal transmit powers allocated to the private and common information of the users. With a unified framework, we derive the achievable sum-rates for various interference scenarios ranging from very strong to very weak cases. We then illustrate how the amplitude-space sharing strategy can be applied to the multiple pico-cell scenarios by developing a simple transmission scheme. Simulation results show the superiority of the proposed scheme over other interference management schemes.**

***Index Terms*—Amplitude-space sharing, Han-Kobayashi coding, heterogeneous network, interference channel, power allocation.**

I. Introduction

With penetration of smartphones and tablets, the crushing demands of wireless data traffic will soon exceed the capability of current homogeneous cellular networks. To deliver high-rate transmission and continuous coverage, the multi-tier heterogeneous networks are emerging as a promising and economically sustainable solution [1], [2]. While the macro-cell base station (MBS) provides basic coverage and supports high mobility, the low power nodes like pico-cell base stations (PBSs) support high-capacity transmission for hotspot zones. By shrinking the transmission range and intensifying spatial reuse of the spectrum, the heterogeneous infrastructure can achieve significant areal capacity gain.

Operated in the same frequency band, macro- and pico-cells may generate complicated interference to each other [3]. The pico-cells might be deployed at anywhere in the macro-cell, and the transmit powers of MBS are much stronger than the PBS. In turn, the pico-user has high probability encountering strong interference from the MBS, but sometimes the macro-user might encounter strong interference from the PBS as well.

With one pico-cell coexisted with the macro-cell, the simultaneous transmission of the macro-user and pico-user forms a two-user interference channel. To find the capacity of interference channel and propose capacity-achieving transmission schemes, the society have worked for several decades. The capacity region of two-user Gaussian interference channel in very strong interference was obtained in 1975 [4], and the capacity region in strong interference was obtained in 1981 [5]. The sum-capacities in mixed interference and very weak interference scenarios were found recently in [6]–[9]. However, in the weak interference scenario, although several outer bounds were developed [9]–[11], the capacity region and sum-capacity are still unknown.

The best known transmission scheme for two-user Gaussian interference channel is Han-Kobayashi (H-K) coding [12], where each user divides its transmit information into private and common portions. The private information is only decoded at the intended receiver, and the common information is decoded at both receivers. H-K coding allows power allocation between the private and common information, but does not specify the allocation method. In [13], Etkin, Tse and Wang proposed a fixed method that let the INR of the private information be 1 at the unintended receiver, and proved that such a scheme achieves the capacity region outer bound within one bit.

H-K coding is actually an amplitude-space sharing method with each user transmitting multiple layers at different signal level [14]. At each receiver, the layers of desired signal and interference are intercrossed and occupy different signal levels. To decode one layer of the signal, the upper layers should be decoded and canceled first no matter they are signal or interference, and the lower layers can be treated as noise. Interference cancelation is a necessary technique in the amplitude-space sharing method, but compared with the passive interference cancelation techniques [15], the amplitude-space sharing scheme proactively creates opportunities for effective interference cancelation and optimizes the occupied signal levels of different users to maximize the sum-rate. With this idea in mind, the amplitude-space sharing scheme can be extended to multiple-user interference channels.

Of course, the signals and interference can also be separated through multi-antenna beamforming or interference alignment approaches [16]–[19]. Nonetheless, the amplitude-space and vector-space are two kinds of complementary degrees of freedom of signals. We concentrate on exploiting the amplitude-space in this paper.

We start from optimizing the H-K coding for a network

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with one MBS and one PBS, each serving one user. Then, totally four layers of signals and interference will share the amplitude-space at each receiver. To maximize the sum-rate of the two users, we establish a unified optimization framework, from which not only the available capacity results under strong and mixed interference are obtained, but also a new achievable sum-rate under weak interference is derived. The rate-splitting problem was also studied in [20] for one macro-user and one pico-user in heterogeneous networks, but there only mixed interference scenario was considered.

Based on the insight gained from the H-K coding with optimized power allocation, we proceed to design a simple amplitude-space sharing scheme for the network with one MBS and multiple PBSs. Since each pico-cell can be randomly deployed, the interference scenarios may be different between the macro-user and each pico-user. In this setting, we only require each user transmits one layer, and construct appropriate amplitude-space sharing relations at every receiver. Simulation results show that the proposed scheme offers an approximately 50% throughput gain over the conventional time- or frequency-division schemes.

The rest of this paper is organized as follows. In Section II, we derive the optimal power allocation and achievable sum-rate of H-K coding. In Section III, we propose an amplitude-space sharing transmission scheme for heterogeneous networks with multiple pico-cells. In Section IV, we first show the connection between interference scenario and network deployment geometry, and then evaluate the performance gains of the optimized H-K coding and the amplitude-space sharing scheme over other interference coordination schemes. Finally, Section V concludes the paper.

II. H-K Coding with Optimized Power Allocation

H-K coding defines private and common layers of transmission for each user, but does not specify the power allocated to each layer. We can optimize the power allocation to maximize the sum-rate of the two users.

A. Problem Formulation

The nominal model of the two-user Gaussian interference channel is shown in Fig. 1, where h_{ii} denotes the channel of the direct link from Tx_i to Rx_i and h_{ij} denotes the channel of the cross link from Tx_j to Rx_i, $i, j \in \{1, 2\}$. The received symbols of the two users are respectively

$$y_1 = h_{11}s_1 + h_{12}s_2 + z_1 \tag{1}$$
$$y_2 = h_{21}s_1 + h_{22}s_2 + z_2 \tag{2}$$

where s_i is the symbol transmitted from Tx_i, which is complex Gaussian and $\mathbb{E}\{|s_i|^2\} = P_i$, and z_i is the circular symmetric complex Gaussian noise with zero mean and variance N_0.

The private and common information of each user, $s_{i,p}$, $s_{i,c}$, $i = 1, 2$, are coded separately and superimposed before transmission, i.e., $s_1 = s_{1,p} + s_{1,c}$, $s_2 = s_{2,p} + s_{2,c}$. The word 'common' here does not mean any data-sharing between the two users. For user i, assume that the power allocated to the private information is $P_{i,p}$ and the power to the common information is $P_{i,c}$, where $P_{i,p} + P_{i,c} = P_i$.

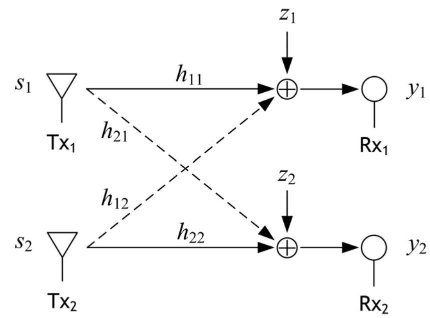

Fig. 1. Two user Gaussian interference channel model.

At each receiver, the common information from the intended and interference users and the private information from the intended user are jointly decoded, while the private information from the interference user is treated as noise [21]. The achievable rate region with arbitrary power allocation is thus given as follows,

$$R_{1,c}^{(1)} \le \log\left(1 + \frac{|h_{11}|^2 P_{1,c}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{3}$$
$$R_{2,c}^{(1)} \le \log\left(1 + \frac{|h_{12}|^2 P_{2,c}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{4}$$
$$R_{1,p}^{(1)} \le \log\left(1 + \frac{|h_{11}|^2 P_{1,p}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{5}$$
$$R_{1,c}^{(1)} + R_{2,c}^{(1)} \le \log\left(1 + \frac{|h_{11}|^2 P_{1,c} + |h_{12}|^2 P_{2,c}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{6}$$
$$R_{1,c}^{(1)} + R_{1,p}^{(1)} \le \log\left(1 + \frac{|h_{11}|^2 P_{1,c} + |h_{11}|^2 P_{1,p}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{7}$$
$$R_{2,c}^{(1)} + R_{1,p}^{(1)} \le \log\left(1 + \frac{|h_{12}|^2 P_{2,c} + |h_{11}|^2 P_{1,p}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{8}$$
$$R_{1,c}^{(1)} + R_{2,c}^{(1)} + R_{1,p}^{(1)} \le \log\left(1 + \frac{|h_{11}|^2 P_{1,c} + |h_{12}|^2 P_{2,c} + |h_{11}|^2 P_{1,p}}{|h_{12}|^2 P_{2,p} + N_0}\right) \tag{9}$$

$$R_{1,c}^{(2)} \le \log\left(1 + \frac{|h_{21}|^2 P_{1,c}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{10}$$
$$R_{2,c}^{(2)} \le \log\left(1 + \frac{|h_{22}|^2 P_{2,c}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{11}$$
$$R_{2,p}^{(2)} \le \log\left(1 + \frac{|h_{22}|^2 P_{2,p}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{12}$$
$$R_{1,c}^{(2)} + R_{2,c}^{(2)} \le \log\left(1 + \frac{|h_{21}|^2 P_{1,c} + |h_{22}|^2 P_{2,c}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{13}$$
$$R_{2,c}^{(2)} + R_{2,p}^{(2)} \le \log\left(1 + \frac{|h_{22}|^2 P_{2,c} + |h_{22}|^2 P_{2,p}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{14}$$
$$R_{1,c}^{(2)} + R_{2,p}^{(2)} \le \log\left(1 + \frac{|h_{21}|^2 P_{1,c} + |h_{22}|^2 P_{2,p}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{15}$$
$$R_{1,c}^{(2)} + R_{2,c}^{(2)} + R_{2,p}^{(2)} \le \log\left(1 + \frac{|h_{21}|^2 P_{1,c} + |h_{22}|^2 P_{2,c} + |h_{22}|^2 P_{2,p}}{|h_{21}|^2 P_{1,p} + N_0}\right) \tag{16}$$

where (3)-(9) are the multiple-access channel capacity region constraints at Rx_1, (10)-(16) are the multiple-access channel capacity region constraints at Rx_2, and $R_{i,c}^{(j)}$, $R_{i,p}^{(j)}$ are the common and private data rate constraints of user i at Rx_j, respectively. The transmission rates of one user should satisfy the constraints at both receivers simultaneously. Since the two common information and the intended private information are jointly decoded, the multiple-access channel capacity region at Rx_1 is the intersection of the individual data rate constraints of $R_{1,c}^{(1)}$, $R_{2,c}^{(1)}$, $R_{1,p}^{(1)}$, and their sums of different combinations. The capacity region at Rx_2 is similar.

The optimization problem of power allocation to maximize the sum-rate can be formulated as follows,

$$\begin{aligned} \max_{P_{1,c},P_{1,p},P_{2,c},P_{2,p}} \quad & R_{1,c}+R_{1,p}+R_{2,c}+R_{2,p} \\ \text{s.t.} \quad & P_{1,c}+P_{1,p}\le P_1 \\ & P_{2,c}+P_{2,p}\le P_2 \\ & (3)-(16). \end{aligned} \tag{17}$$

B. Sum-Rate Optimization

According to the achievable rate region (3)-(16), the achievable sum-rate can be expressed as the minimum of four possible combinations of the data rate constraints, i.e.,

$$\begin{aligned} R_{\mathsf{sum}} &= R_{1,c}+R_{1,p}+R_{2,c}+R_{2,p} \\ &= \min\left\{ \begin{array}{l} \underbrace{R_{1,c}^{(1)}+R_{2,c}^{(1)}+R_{1,p}^{(1)}}_{(9)}+\underbrace{R_{2,p}^{(2)}}_{(12)}, \\ \underbrace{R_{1,c}^{(2)}+R_{2,c}^{(2)}+R_{2,p}^{(2)}}_{(16)}+\underbrace{R_{1,p}^{(1)}}_{(5)}, \\ \underbrace{R_{1,c}^{(1)}+R_{1,p}^{(1)}}_{(7)}+\underbrace{R_{2,c}^{(2)}+R_{2,p}^{(2)}}_{(14)}, \\ \underbrace{R_{2,c}^{(1)}+R_{1,p}^{(1)}}_{(8)}+\underbrace{R_{1,c}^{(2)}+R_{2,p}^{(2)}}_{(15)} \end{array} \right\}. \end{aligned} \tag{18}$$

In terms of the sum-rate, the other constraints in (3)-(16) are redundant. This is because, for example, the sum of (3), (4) and (5) is larger than (9), the sum of (3) and (5) is larger than (7), etc.

According to (17) and (18), we can see that the sum-rate optimization problem is a max-min problem. To show how to optimize problem (17), we rewrite the objective function by different possible minimum cases in (18). Considering the first possible sum-rate in (18), the objective function of problem (17) is

$$\begin{aligned} R_{\mathsf{sum},1} =& R_{1,c}^{(1)}+R_{2,c}^{(1)}+R_{1,p}^{(1)}+R_{2,p}^{(2)} \\ \le& \log\left(C_1\frac{|h_{21}|^2P_{1,p}+|h_{22}|^2P_{2,p}+N_0}{(|h_{21}|^2P_{1,p}+N_0)\,(|h_{12}|^2P_{2,p}+N_0)}\right) \\ =& R_{\mathsf{constr1}} \end{aligned} \tag{19}$$

where

$$\begin{aligned} C_1 &= |h_{11}|^2(P_{1,c}+P_{1,p})+|h_{12}|^2(P_{2,c}+P_{2,p})+N_0 \\ &\le |h_{11}|^2P_1+|h_{12}|^2P_2+N_0. \end{aligned} \tag{20}$$

From (19) and (20) we can see that, although the power allocation between the common and private portions is not determined, the full use of the transmit power will always make the sum-rate constraint maximized. In the following, we will use C_1 as a constant that fulfills the equality constraint in (20).

Considering the second possible sum-rate, the objective function is

$$\begin{aligned} R_{\mathsf{sum},2} &= R_{1,c}^{(2)}+R_{2,c}^{(2)}+R_{2,p}^{(2)}+R_{1,p}^{(1)} \\ &\le \log\left(C_2\frac{|h_{11}|^2P_{1,p}+|h_{12}|^2P_{2,p}+N_0}{(|h_{21}|^2P_{1,p}+N_0)\,(|h_{12}|^2P_{2,p}+N_0)}\right) \\ &= R_{\mathsf{constr2}}, \end{aligned} \tag{21}$$

where the constant $C_2=|h_{21}|^2P_1+|h_{22}|^2P_2+N_0$.

Considering the third and fourth possible sum-rates in (18), the objective functions of problem (17) are respectively

$$\begin{aligned} &R_{\mathsf{sum},3} = R_{1,c}^{(1)}+R_{1,p}^{(1)}+R_{2,c}^{(2)}+R_{2,p}^{(2)} \\ &\le \log\left(1+\frac{|h_{11}|^2P_1}{|h_{12}|^2P_{2,p}+N_0}\right)+\log\left(1+\frac{|h_{22}|^2P_2}{|h_{21}|^2P_{1,p}+N_0}\right) \\ &= R_{\mathsf{constr3}} \end{aligned} \tag{22}$$

and

$$\begin{aligned} R_{\mathsf{sum},4} &= R_{2,c}^{(1)}+R_{1,p}^{(1)}+R_{1,c}^{(2)}+R_{2,p}^{(2)} \\ &\le \log\left(\frac{|h_{11}|^2P_{1,p}+|h_{12}|^2P_2+N_0}{|h_{12}|^2P_{2,p}+N_0}\right) \\ &\quad+\log\left(\frac{|h_{21}|^2P_1+|h_{22}|^2P_{2,p}+N_0}{|h_{21}|^2P_{1,p}+N_0}\right) \\ &= R_{\mathsf{constr4}}. \end{aligned} \tag{23}$$

Then, the original optimization problem (17) can be reformulated as follows,

$$\max_{P_{1,p},P_{2,p}} \{\min\{R_{\mathsf{constr1}},R_{\mathsf{constr2}},R_{\mathsf{constr3}},R_{\mathsf{constr4}}\}\} \tag{24}$$

$$\text{s.t. } 0\le P_{1,p}\le P_1,\quad 0\le P_{2,p}\le P_2. \tag{25}$$

The four sum-rate constraints are active in different channel conditions. In most cases, R_{constr1} or R_{constr2} is the minimum. But R_{constr3} will be the minimum when the direct links are too weak, and R_{constr4} will be the minimum when the cross links are too weak.

If R_{constr3} is the minimum, i.e.,

$$\begin{cases} R_{\mathsf{constr3}} < R_{\mathsf{constr1}} \\ R_{\mathsf{constr3}} < R_{\mathsf{constr2}} \\ R_{\mathsf{constr3}} < R_{\mathsf{constr4}} \end{cases} \tag{26}$$

then using the expressions (19)-(23) we can obtain

$$\begin{cases} |h_{11}h_{22}|^2P_1-|h_{12}h_{21}|^2P_{1,p}+(|h_{22}|^2-|h_{12}|^2)N_0<0 \\ |h_{11}h_{22}|^2P_2-|h_{12}h_{21}|^2P_{2,p}+(|h_{11}|^2-|h_{21}|^2)N_0<0 \\ |h_{11}|^2<|h_{21}|^2 \quad\text{and}\quad |h_{22}|^2<|h_{12}|^2. \end{cases} \tag{27}$$

Similarly, if R_{constr4} is the minimum, we can obtain

$$\begin{cases} |h_{12}h_{21}|^2P_1-|h_{11}h_{22}|^2P_{1,p}+(|h_{12}|^2-|h_{22}|^2)N_0<0 \\ |h_{12}h_{21}|^2P_2-|h_{11}h_{22}|^2P_{2,p}+(|h_{21}|^2-|h_{11}|^2)N_0<0 \\ |h_{21}|^2<|h_{11}|^2 \quad\text{and}\quad |h_{12}|^2<|h_{22}|^2. \end{cases} \tag{28}$$

These two conditions will be used later to help the optimization of R_{constr3} and R_{constr4}.

In the sequel, we first find the solution to maximize each of the four terms in (24), analyze the corresponding channel conditions in terms of SNR and INR, and then summarize the solution of the max-min optimization problem.

Define $\mathsf{SNR}_1 = |h_{11}|^2 P_1/N_0$ and $\mathsf{SNR}_2 = |h_{22}|^2 P_2/N_0$ as the received SNRs of users 1 and 2, and $\mathsf{INR}_1 = |h_{12}|^2 P_2/N_0$ and $\mathsf{INR}_2 = |h_{21}|^2 P_1/N_0$ as the received INRs of users 1 and 2, respectively.

1) Maximization of R_{constr1}: From (19) we can find that the term inside the logarithm is a function of $P_{1,p}$ and $P_{2,p}$, which is in a fractional form as

$$\frac{a_1 x_1 + b_1 x_2 + c}{(dx_1 + c)(fx_2 + c)} \tag{29}$$

if we let $a_1 = |h_{21}|^2$, $b_1 = |h_{22}|^2$, $c = N_0$, $d = |h_{21}|^2$, $f = |h_{12}|^2$, and $x_1 = P_{1,p}$, $x_2 = P_{2,p}$. The constant C_1 in (19) does not affect the maximization of R_{constr1}, and the logarithm function is monotonic. Therefore, the problem that maximize R_{constr1} under the constraint (25) is equivalent to the problem that maximizes (29) subject to $0 \le x_1 \le P_1$ and $0 \le x_2 \le P_2$.

The objective function (29) is non-convex and is hard to be optimized directly. Nonetheless, such a two-variable quadratic-fractional problem can be transformed to a four-variable quadratic polynomial problem using the perspective transformation technique introduced in [22]. According to the multiplication factors in the denominator of (29), we define four variables as follows

$$x_3 = \frac{x_1}{dx_1 + c},\ x_4 = \frac{1}{dx_1 + c},\ x_5 = \frac{x_2}{fx_2 + c},\ x_6 = \frac{1}{fx_2 + c}.$$

Then, the optimization problem is transformed into

$$\max_{x_3,x_4,x_5,x_6} \quad a_1 x_3 x_6 + (b_1 x_5 + c x_6) x_4 \tag{30}$$

$$\text{s.t. } x_3 - P_1 x_4 \le 0 \tag{31}$$

$$x_5 - P_2 x_6 \le 0 \tag{32}$$

$$dx_3 + cx_4 = 1 \tag{33}$$

$$fx_5 + cx_6 = 1 \tag{34}$$

$$x_3 \ge 0,\ x_4 \ge 0,\ x_5 \ge 0,\ x_6 \ge 0 \tag{35}$$

where constraints (31) and (32) come from $x_1 \le P_1$ and $x_2 \le P_2$, and constraints (33) and (34) come from the relationships of the four variables.

Substituting (33) and (34) into the objective function and other constraints and using the relation $a_1 = d$, the optimization problem can be simplified as

$$\max_{x_4,x_6} \quad \left(-\frac{b_1}{f} x_4 + \frac{1}{c}\right)(cx_6 - 1) \tag{36}$$

$$\text{s.t.} \quad \frac{1}{a_1 P_1 + c} \le x_4 \le \frac{1}{c} \tag{37}$$

$$\frac{1}{fP_2 + c} \le x_6 \le \frac{1}{c}. \tag{38}$$

Since (36) is a multiplication of two linear factors, the maximal value of the objective function depends on the maximal value of each factor as well as their signs.

As shown in Fig. 2, we can represent the two linear factors of (36) with two lines, $F_1(x_4) = -\frac{b_1}{f} x_4 + \frac{1}{c}$ and $F_2(x_6) = cx_6 - 1$. We can see that $F_2(x_6)$ is always less than or equal to 0 in the feasible region of x_6, and the sign of $F_1(x_4)$ depends on the slope of the line, i.e., the value of $-\frac{b_1}{f}$.

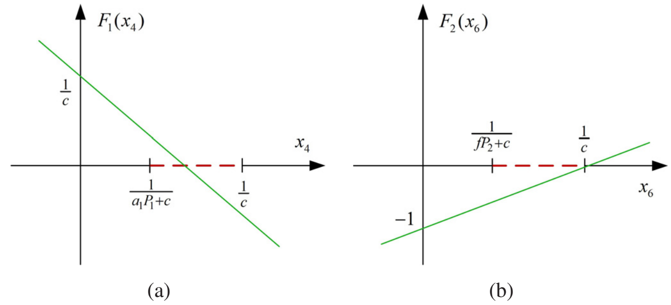

Fig. 2. Discussion of optimal solutions of problem (36), where the dashed lines denote feasible regions of x_4 and x_6.

When $F_1(\frac{1}{c}) \ge 0$, i.e., the slope $-\frac{b_1}{f} \ge -1$, the maximum value of R_{constr1} is achieved when $F_2(x_6) = 0$ where $x_6 = \frac{1}{c}$. In this scenario, the value of x_4 does not affect the maximum value of R_{constr1}. Correspondingly, $P_{2,p} = x_2 = 0$ and $P_{1,p} = x_1$ can be any value between 0 and P_1.

When $F_1(\frac{1}{c}) < 0$, the maximum value of R_{constr1} is achieved when $x_4 = \frac{1}{c}$ and $x_6 = \frac{1}{fP_2+c}$. Correspondingly, $P_{1,p} = 0$ and $P_{2,p} = P_2$.

The slope condition can be expressed as a function of SNR and INR. Since $b_1 = |h_{22}|^2$, $f = |h_{12}|^2$, the condition $-\frac{b_1}{f} \ge -1$ means $|h_{22}|^2 \le |h_{12}|^2$, which can be equivalently written as $\frac{|h_{22}|^2 P_2}{N_0} \le \frac{|h_{12}|^2 P_2}{N_0}$, i.e.,

$$\mathsf{SNR}_2 \le \mathsf{INR}_1. \tag{39}$$

2) Maximization of R_{constr2}: The optimization for R_{constr2} can be similarly obtained. Let $a_2 = |h_{11}|^2$, $b_2 = |h_{12}|^2$, and c, d, f are the same values defined in last section. With the relation $b_2 = f$, the optimization problem becomes

$$\max_{x_4,x_6} \quad \left(-\frac{a_2}{d} x_6 + \frac{1}{c}\right)(cx_4 - 1) \tag{40}$$

$$\text{s.t.} \quad \frac{1}{dP_1 + c} \le x_4 \le \frac{1}{c} \tag{41}$$

$$\frac{1}{b_2 P_2 + c} \le x_6 \le \frac{1}{c}. \tag{42}$$

Since $a_2 = |h_{11}|^2$, $d = |h_{21}|^2$, when the slope $-\frac{a_2}{d} \ge -1$, i.e., $\mathsf{SNR}_1 \le \mathsf{INR}_2$, the maximum value of R_{constr2} is achieved when $x_4 = \frac{1}{c}$ and x_6 is arbitrary in the feasible region. Correspondingly, $P_{1,p} = 0$ and $P_{2,p}$ can be any value between 0 and P_2. Otherwise, when $\mathsf{SNR}_1 > \mathsf{INR}_2$, the maximum value of R_{constr2} is achieved when $x_6 = \frac{1}{c}$ and $x_4 = \frac{1}{dP_1+c}$. Correspondingly, $P_{1,p} = P_1$ and $P_{2,p} = 0$.

3) Maximization of R_{constr3}: From (22) it is easy to see that R_{constr3} is maximized when $P_{1,p} = 0$ and $P_{2,p} = 0$. In this case, the conditions (27) become

$$\mathsf{SNR}_1 \le \frac{\mathsf{INR}_2}{1 + \mathsf{SNR}_2} \quad \text{and} \quad \mathsf{SNR}_2 \le \frac{\mathsf{INR}_1}{1 + \mathsf{SNR}_1}. \tag{43}$$

4) Maximization of R_{constr4}: The optimization of R_{constr4} is a little harder than that of R_{constr3}. For the convenience of readers, we rewrite (28) here,

$$R_{\mathsf{constr4}} = \log\left(\frac{|h_{11}|^2 P_{1,p} + |h_{12}|^2 P_2 + N_0}{|h_{21}|^2 P_{1,p} + N_0}\right) + \log\left(\frac{|h_{21}|^2 P_1 + |h_{22}|^2 P_{2,p} + N_0}{|h_{12}|^2 P_{2,p} + N_0}\right)$$

TABLE I
SUMMARY OF THE OPTIMIZATION RESULTS FOR R_{CONSTR1} AND R_{CONSTR2}

max R_{constr1}		max R_{constr2}	
$\mathsf{SNR}_2 \leq \mathsf{INR}_1$	$\mathsf{SNR}_2 \geq \mathsf{INR}_1$	$\mathsf{SNR}_1 \leq \mathsf{INR}_2$	$\mathsf{SNR}_1 \geq \mathsf{INR}_2$
$\forall P_{1,p},\, P_{2,p} = 0$	$P_{1,p} = 0,\, P_{2,p} = P_2$	$P_{1,p} = 0,\, \forall P_{2,p}$	$P_{1,p} = P_1,\, P_{2,p} = 0$

where the variables $P_{1,p}$ and $P_{2,p}$ appear both in the numerators and denominators. The term inside the first logarithm can be reformulated as,

$$\frac{|h_{11}|^2 P_{1,p} + |h_{12}|^2 P_2 + N_0}{|h_{21}|^2 P_{1,p} + N_0} = \frac{|h_{11}|^2}{|h_{21}|^2} + \frac{|h_{12}|^2 P_2 - \left(\frac{|h_{11}|^2}{|h_{21}|^2} - 1\right) N_0}{|h_{21}|^2 P_{1,p} + N_0}.$$

Apparently, if $|h_{12}|^2 P_2 \geq (|h_{11}|^2/|h_{21}|^2 - 1)N_0$, R_{constr4} achieves maximum when $P_{1,p} = 0$. Otherwise, R_{constr4} is maximal when $P_{1,p} = P_1$.

Similarly, for the term inside the second logarithm, if $|h_{21}|^2 P_1 \geq (|h_{22}|^2/|h_{12}|^2 - 1)N_0$, R_{constr4} is maximal when $P_{2,p} = 0$. Otherwise, R_{constr4} is maximal when $P_{2,p} = P_2$.

According to (28), the conditions that R_{constr4} is the minimal sum-rate among the four cases should satisfy $|h_{12}|^2 P_2 < (|h_{11}|^2/|h_{21}|^2 - 1)N_0$ and $|h_{21}|^2 P_1 < (|h_{22}|^2/|h_{12}|^2 - 1)N_0$. Therefore, R_{constr4} achieves its maximal value when $P_{1,p} = P_1$ and $P_{2,p} = P_2$. The corresponding SNR and INR conditions can be expressed as

$$\frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_1} > \mathsf{INR}_2 \quad \text{and} \quad \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_2} > \mathsf{INR}_1. \tag{44}$$

C. Optimization Results

The optimal power allocation to maximize R_{constr1} and R_{constr2} and the corresponding conditions are listed in Table I. For the max-min problem (24), there are four scenarios in terms of the SNR and INR conditions to maximize R_{constr1} or R_{constr2}, which are defined as

$$\begin{aligned}
&\text{Strong interference:} && \mathsf{SNR}_2 < \mathsf{INR}_1 \text{ and } \mathsf{SNR}_1 < \mathsf{INR}_2,\\
&\text{Mixed interference 1:} && \mathsf{SNR}_2 < \mathsf{INR}_1 \text{ and } \mathsf{SNR}_1 \geq \mathsf{INR}_2,\\
&\text{Mixed interference 2:} && \mathsf{SNR}_2 \geq \mathsf{INR}_1 \text{ and } \mathsf{SNR}_1 < \mathsf{INR}_2,\\
&\text{Weak interference:} && \mathsf{SNR}_2 \geq \mathsf{INR}_1 \text{ and } \mathsf{SNR}_1 \geq \mathsf{INR}_2.
\end{aligned}$$

Furthermore, we can define the scenario where R_{constr3} is the minimum value of R_{constr1}, R_{constr2} R_{constr3}, and R_{constr4} as the case of very strong interference, the scenario where R_{constr4} is the minimum value as the case of very weak interference. The SNR and INR conditions that these two scenarios will happen are listed as follows,

$$\begin{aligned}
&\text{Very strong interference:} && (43),\\
&\text{Very weak interference:} && (44).
\end{aligned}$$

Therefore, we have totally six scenarios, where the optimal power allocation and achievable sum-rate will be analyzed respectively in the following.

1) Very Strong Interference: In this scenario, the optimal power allocation solution is $P_{1,p} = 0$ and $P_{2,p} = 0$. That is to say, both users only transmit common information. The achievable sum-rate with the optimal power allocation can be obtained from (22) as

$$\begin{aligned}
R_{\mathsf{sum}_{\max}} &= \log\left(1 + \frac{|h_{11}|^2 P_1}{N_0}\right) + \log\left(1 + \frac{|h_{22}|^2 P_2}{N_0}\right)\\
&= \log(1 + \mathsf{SNR}_1) + \log(1 + \mathsf{SNR}_2).
\end{aligned} \tag{45}$$

2) Very Weak Interference: In this scenario, the optimal power allocation solution is $P_{1,p} = P_1$ and $P_{2,p} = P_2$. Both users only transmit private information. The corresponding achievable sum-rate can be obtained from (23) as

$$\begin{aligned}
R_{\mathsf{sum}_{\max}} &= \log\left(1 + \frac{|h_{11}|^2 P_1}{|h_{12}|^2 P_2 + N_0}\right)\left(1 + \frac{|h_{22}|^2 P_2}{|h_{21}|^2 P_1 + N_0}\right)\\
&= \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_1}\right) + \log\left(1 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_2}\right)
\end{aligned} \tag{46}$$

3) Strong Interference: As listed in Table I, when $\mathsf{SNR}_2 < \mathsf{INR}_1$ and $\mathsf{SNR}_1 < \mathsf{INR}_2$, R_{constr1} and R_{constr2} can be simultaneously maximized given $P_{1,p} = 0$ and $P_{2,p} = 0$. No matter which one of them is the minimum, maximizing the minimum yields the same power allocation solution. Again, both users only transmit common information, with which (19) and (21) become

$$R_{\mathsf{constr1}} = \log\left(1 + \frac{|h_{11}|^2 P_1 + |h_{12}|^2 P_2}{N_0}\right) \tag{47}$$

$$R_{\mathsf{constr2}} = \log\left(1 + \frac{|h_{21}|^2 P_1 + |h_{22}|^2 P_2}{N_0}\right). \tag{48}$$

If $R_{\mathsf{constr1}} > R_{\mathsf{constr2}}$, which happens when $\mathsf{SNR}_1 + \mathsf{INR}_1 > \mathsf{INR}_2 + \mathsf{SNR}_2$, we obtain the achievable sum-rate with the optimal power allocation as

$$R_{\mathsf{sum}_{\max}} = \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_2) \tag{49}$$

otherwise,

$$R_{\mathsf{sum}_{\max}} = \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1). \tag{50}$$

4) Mixed Interference 1: As shown in Table I, when $\mathsf{SNR}_2 < \mathsf{INR}_1$ and $\mathsf{SNR}_1 \geq \mathsf{INR}_2$, R_{constr1} and R_{constr2} can be simultaneously maximized as well given $P_{1,p} = P_1$ and $P_{2,p} = 0$. In this scenario, the first user only transmits private information and the second user only transmits common information, with which (19) and (21) become

$$R_{\mathsf{constr1}} = \log\left(\frac{|h_{11}|^2 P_1 + |h_{12}|^2 P_2 + N_0}{N_0}\right) \tag{51}$$

$$R_{\mathsf{constr2}} = \log\left(\frac{|h_{21}|^2 P_1 + |h_{22}|^2 P_2 + N_0}{N_0} \cdot \frac{|h_{11}|^2 P_1 + N_0}{|h_{21}|^2 P_1 + N_0}\right). \tag{52}$$

If $R_{\mathsf{constr1}} > R_{\mathsf{constr2}}$, which happens when $\mathsf{INR}_1\mathsf{INR}_2 + \mathsf{INR}_1 > \mathsf{SNR}_1\mathsf{SNR}_2 + \mathsf{SNR}_2$, we obtain the achievable sum-rate with the optimal power allocation as

$$\begin{aligned} R_{\mathsf{sum}_{\max}} &= \log\left(1+\frac{|h_{22}|^2P_2}{|h_{21}|^2P_1+N_0}\right)\left(\frac{|h_{11}|^2P_1+N_0}{N_0}\right) \\ &= \log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2}\right)+\log\left(1+\mathsf{SNR}_1\right) \end{aligned} \tag{53}$$

otherwise,

$$R_{\mathsf{sum}_{\max}} = \log(1+\mathsf{SNR}_1+\mathsf{INR}_1). \tag{54}$$

5) Mixed Interference 2: When $\mathsf{SNR}_2 \geq \mathsf{INR}_1$ and $\mathsf{SNR}_1 \leq \mathsf{INR}_2$, we have $P_{1,p} = 0$ and $P_{2,p} = P_2$, with which (19) and (21) become

$$R_{\mathsf{constr1}} = \log\left(\frac{|h_{11}|^2P_1+|h_{12}|^2P_2+N_0}{N_0}\cdot\frac{|h_{22}|^2P_2+N_0}{|h_{12}|^2P_2+N_0}\right) \tag{55}$$

$$R_{\mathsf{constr2}} = \log\left(\frac{|h_{21}|^2P_1+|h_{22}|^2P_2+N_0}{N_0}\right). \tag{56}$$

If $R_{\mathsf{constr1}} > R_{\mathsf{constr2}}$, which happens when $\mathsf{SNR}_1\mathsf{SNR}_2 + \mathsf{SNR}_1 > \mathsf{INR}_1\mathsf{INR}_2 + \mathsf{INR}_2$, we obtain the achievable sum-rate with the optimal power allocation as

$$R_{\mathsf{sum}_{\max}} = \log\left(1+\mathsf{INR}_2+\mathsf{SNR}_2\right) \tag{57}$$

otherwise,

$$R_{\mathsf{sum}_{\max}} = \log\left(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1}\right)+\log\left(1+\mathsf{SNR}_2\right). \tag{58}$$

6) Weak Interference: In this scenario, the conditions for maximizing R_{constr1} and maximizing R_{constr2} are conflicting, as shown in Table I. Observing from Fig. 2 and the functions in (36) and (40), in the weak interference scenario as x_4 increasing, the value of R_{constr1} increases but the value of R_{constr2} decreases. Similar trend happens to x_6. This suggests that the max-min optimization of R_{constr1} and R_{constr2} is achieved when $R_{\mathsf{constr1}} = R_{\mathsf{constr2}}$.

When (19) and (21) are equal, we obtain a linear relationship between $P_{1,p}$ and $P_{2,p}$ as follows,

$$P_{2,p} = \alpha P_{1,p} + \beta \tag{60}$$

where

$$\alpha = \frac{(|h_{11}|^2|h_{22}|^2-|h_{12}|^2|h_{21}|^2)P_2+(|h_{11}|^2-|h_{21}|^2)N_0}{(|h_{11}|^2|h_{22}|^2-|h_{12}|^2|h_{21}|^2)P_1+(|h_{22}|^2-|h_{12}|^2)N_0}$$

$$\beta = \frac{\left[(|h_{22}|^2-|h_{12}|^2)P_2+(|h_{21}|^2-|h_{11}|^2)P_1\right]N_0}{(|h_{11}|^2|h_{22}|^2-|h_{12}|^2|h_{21}|^2)P_1+(|h_{22}|^2-|h_{12}|^2)N_0}.$$

Since in the weak interference scenario $|h_{11}|^2 > |h_{21}|^2$ and $|h_{22}|^2 > |h_{12}|^2$, we can see that $\alpha > 0$ and β can be any value. In addition, when $P_{1,p} = P_1$ in (60), we can obtain $P_{2,p} = P_2$.

Substituting (60) into (19), we obtain a new optimization problem that maximizes R_{constr1} with respect to $P_{1,p}$, i.e.,

$$\max_{P_{1,p}} \ \mathcal{L} \triangleq \frac{(|h_{21}|^2+\alpha|h_{22}|^2)P_{1,p}+\beta|h_{22}|^2+N_0}{(\alpha|h_{12}|^2P_{1,p}+\beta|h_{12}|^2+N_0)(|h_{21}|^2P_{1,p}+N_0)} \tag{61}$$

$$\text{s.t.} \ \ \max\{0,-\frac{\beta}{\alpha}\} \leq P_{1,p} \leq P_1$$

where $P_{1,p} \geq -\beta/\alpha$ comes from the requirement $P_{2,p} \geq 0$.

The numerator of the objective function of (61) is a linear function of $P_{1,p}$, and the denominator is a quadratic function of $P_{1,p}$. As proved in Appendix I, the objective function is quasi-concave in the feasible region. By letting $\frac{d\mathcal{L}}{dP_{1,p}} = 0$, we can find the maximal value of $P_{1,p}$, defined as ρ with the expression shown in (59) under Table II.

If $\max\{0,-\frac{\beta}{\alpha}\} < \rho < P_1$, the optimal power allocation of $P_{1,p}$ is exactly ρ, and correspondingly $P_{2,p} = \alpha\rho + \beta$ from (60). If $\rho \leq \max\{0,-\frac{\beta}{\alpha}\}$, $\mathcal{L}$ is a decreasing function in the feasible region. Then, for $\beta \geq 0$ the optimal power allocation is $P_{1,p} = 0$ and $P_{2,p} = \beta$; for $\beta < 0$ the optimal power allocation is $P_{1,p} = -\frac{\beta}{\alpha}$ and $P_{2,p} = 0$. It indicates that except for the case where $\beta = 0$ there is at least one user transmits both private and common information.

If $\rho \geq P_1$, $\mathcal{L}$ is an increasing function in the feasible region. Then the optimal power allocation is $P_{1,p} = P_1$, $P_{2,p} = P_2$, which is the same as in the very weak interference scenario. Actually, the condition $\rho > P_1$ can be simplified to

$$\begin{aligned} &\left(1+\frac{|h_{11}|^2|h_{22}|^2-|h_{21}|^2|h_{12}|^2}{|h_{22}|^2-|h_{12}|^2}\frac{P_1}{N_0}\right) \\ &\cdot\left(1+\frac{|h_{11}|^2|h_{22}|^2-|h_{21}|^2|h_{12}|^2}{|h_{11}|^2-|h_{21}|^2}\frac{P_2}{N_0}\right) < \frac{|h_{11}|^2|h_{22}|^2}{|h_{21}|^2|h_{12}|^2}. \end{aligned} \tag{62}$$

Since in the weak interference scenario $|h_{21}|^2 < |h_{11}|^2$ and $|h_{12}|^2 < |h_{22}|^2$, if (62) is satisfied, we have

$$\left(1+|h_{21}|^2\frac{P_1}{N_0}\right)\left(1+|h_{12}|^2\frac{P_2}{N_0}\right) < \frac{|h_{11}|^2|h_{22}|^2}{|h_{21}|^2|h_{12}|^2}. \tag{63}$$

Compare (63) with (44), we can find that this condition is consistent with that in the very weak interference scenario. This means that the condition $\rho \geq P_1$ can only happen in very weak interference but not in weak interference scenario.

Consequently, the optimal power allocation in weak interference scenario is

$$P_{1,p}^* = \max\{0,-\frac{\beta}{\alpha},\rho\}, \quad P_{2,p}^* = \alpha P_{1,p}^* + \beta. \tag{64}$$

Substituting (64) into (19), the achievable sum-rate with the optimal power allocation can be derived as

$$R_{\mathsf{sum}}^* = \log\left(\frac{C_1C_2\frac{(|h_{11}|^2|h_{22}|^2-|h_{12}|^2|h_{21}|^2)P_{1,p}+(|h_{22}|^2-|h_{12}|^2)N_0}{(|h_{11}|^2|h_{22}|^2-|h_{21}|^2|h_{12}|^2)P_1+(|h_{22}|^2-|h_{12}|^2)N_0}}{(\alpha|h_{12}|^2P_{1,p}+\beta|h_{12}|^2+N_0)(|h_{21}|^2P_{1,p}+N_0)}\right).$$

7) Summary and Remarks: The optimal power allocation solutions and the corresponding achievable sum-rates in all the six interference scenarios are summarized in Table II. From the table we can find that, except for the weak interference scenario, only one layer (either the private or common layer) is required to achieve the maximal sum-rates in all other scenarios. In the very strong and strong interference scenarios, each user only transmits the common information. In the mixed interference scenario, one user transmits the common information and the other user transmits the private information. In the very weak interference scenario, each user only transmits the private information. In general weak interference scenario, both users may transmit two layers of information.

TABLE II
SUMMARY OF OPTIMAL POWER ALLOCATION SCHEMES AND ACHIEVABLE SUM-RATES

Interference Scenarios	Conditions	$P_{1,p}$	$P_{2,p}$	R_{sum}
Very Strong	$\mathsf{SNR}_1 < \frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}$ and $\mathsf{SNR}_2 < \frac{\mathsf{INR}_1}{1+\mathsf{SNR}_1}$	0	0	$\log(1+\mathsf{SNR}_1)+\log(1+\mathsf{SNR}_2)$
Strong	$\mathsf{SNR}_1 < \mathsf{INR}_2$ and $\mathsf{SNR}_2 < \mathsf{INR}_1$	0	0	$\min\left\{\begin{array}{c}\log(1+\mathsf{SNR}_2+\mathsf{INR}_2),\\ \log(1+\mathsf{SNR}_1+\mathsf{INR}_1)\end{array}\right\}$
Mixed 1	$\mathsf{SNR}_1 \geq \mathsf{INR}_2$ and $\mathsf{SNR}_2 < \mathsf{INR}_1$	P_1	0	$\min\left\{\begin{array}{c}\log(1+\mathsf{SNR}_1+\mathsf{INR}_1),\\ \log(1+\mathsf{SNR}_1)+\log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2}\right)\end{array}\right\}$
Mixed 2	$\mathsf{SNR}_1 < \mathsf{INR}_2$ and $\mathsf{SNR}_2 \geq \mathsf{INR}_1$	0	P_2	$\min\left\{\begin{array}{c}\log(1+\mathsf{SNR}_2+\mathsf{INR}_2),\\ \log\left(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1}\right)+\log(1+\mathsf{SNR}_2)\end{array}\right\}$
Weak	$\mathsf{SNR}_1 \geq \mathsf{INR}_2$ and $\mathsf{SNR}_2 \geq \mathsf{INR}_1$	$P_{1,p}^*$	$P_{2,p}^*$	R_{sum}^*
Very Weak	$\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1} > \mathsf{INR}_2$ and $\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2} > \mathsf{INR}_1$	P_1	P_2	$\log\left(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1}\right)+\log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2}\right)$

$$\rho = N_0\left(\frac{\sqrt{\frac{|h_{11}|^2|h_{22}|^2}{|h_{21}|^2|h_{12}|^2}(|h_{12}|^2-|h_{22}|^2)(|h_{21}|^2-|h_{11}|^2)\frac{1}{\alpha}}}{|h_{11}|^2|h_{22}|^2-|h_{21}|^2|h_{12}|^2} - \frac{|h_{22}|^2-|h_{12}|^2}{|h_{11}|^2|h_{22}|^2-|h_{21}|^2|h_{12}|^2}\right). \tag{59}$$

Remind that in the weak interference scenario both the capacity region and the sum-capacity are still unknown in the literature. The optimal power allocation results we derived in a unified framework achieve the sum-capacities in the very strong, strong, mixed, and very weak interference scenarios. Moreover, we have found a new achievable sum-rate in the weak interference scenario.

III. AMPLITUDE-SPACE SHARING WITH MULTIPLE PICO-CELLS

When K pico-cells coexist with one macro-cell, it is a $(K+1)$-user interference channel problem, where no capacity result is available for the general case. Practically, the pico-cells do not overlap with each other, such that the interference among the pico-cells can be treated as background noise due to the low power of PBS [23], then the interferences are mainly from MBS and the network can be regarded as partially connected.

As shown in Fig. 3, consider that one user is scheduled by each BS on the given time or frequency resource. Then, the macro-user link and each of the pico-user links constitute a two-user interference channel. Depending on the relative distances among the BSs and users, the interference scenarios are different for different link pairs. For example, in Fig. 3, the macro-user is located in the coverage of pico-cell 1, the link pair of macro-user and pico-user 1 may experience strong interference[1]. Pico-cell 2 is closer to the MBS than the macro-user, the link pair of macro-user and pico-user 2 may experience mixed interference. Pico-cell 3 has a longer distance to the MBS than the macro-user, thus the link pair of macro-user and pico-user 3 may experience weak interference.

Consider one of the link pairs, such as the macro-user and the k-th pico-user. Define the received SNRs and INRs at the macro-user and pico-user as

$$\mathsf{SNR}_{\mathrm{M},k} = \frac{|h_{00}|^2 P_{\mathrm{M}}}{\sum_{j=1,j\neq k}^{K}|h_{0j}|^2 P_{\mathrm{P},j}+N_0}$$
$$\mathsf{INR}_{\mathrm{M},k} = \frac{|h_{0k}|^2 P_{\mathrm{P},k}}{\sum_{j=1,j\neq k}^{K}|h_{0j}|^2 P_{\mathrm{P},j}+N_0}$$
$$\mathsf{SNR}_{\mathrm{P},k} = \frac{|h_{kk}|^2 P_{\mathrm{P},k}}{\sum_{j=1,j\neq k}^{K}|h_{kj}|^2 P_{\mathrm{P},j}+N_0}$$
$$\mathsf{INR}_{\mathrm{P},k} = \frac{|h_{k0}|^2 P_{\mathrm{M}}}{\sum_{j=1,j\neq k}^{K}|h_{kj}|^2 P_{\mathrm{P},j}+N_0}$$

where P_{M} is the transmit power of the MBS, $P_{\mathrm{P},j}$ is the transmit power of the j-th PBS, $h_{k,j}$ is the channel gain from the j-th PBS to the k-th pico-user for $\{j,k\} \neq 0$, $h_{k,0}$ denotes the channel from MBS, and $h_{0,j}$ denotes the

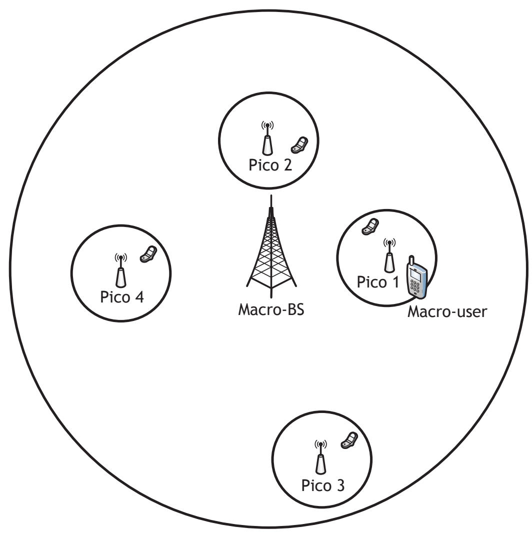

Fig. 3. The coexistence of one macro-cell and multiple pico-cells.

[1] In this paper, we use "pico-cell" to represent various small cells, including the operator-deployed pico-cells and user-deployed femto-cells. When a femto-cell is configured in the closed subscriber group (CSG) mode, the macro-user can only access to the macro-BS even it is close to the femto-BS.

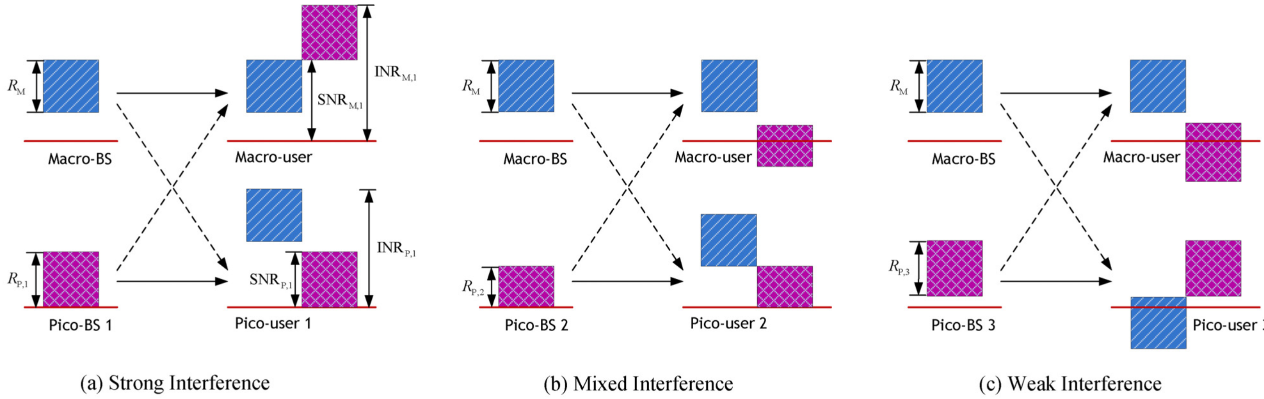

Fig. 4. The illustration of amplitude space sharing in different interference scenarios, where the SNRs and INRs are in logarithm scale.

channel to macro-user. Compared with the separated two-user interference channel case, here the interference from other pico-BSs serve as background noise.

According to Table II, the achievable sum-rate of link pair k can be expressed as a function of the SNRs and INRs, i.e.,

$$R_{\mathsf{sum},k} = \mathcal{F}(\mathsf{SNR}_{\mathrm{M},k}, \mathsf{INR}_{\mathrm{M},k}, \mathsf{SNR}_{\mathrm{P},k}, \mathsf{INR}_{\mathrm{P},k}). \quad (65)$$

To achieve this sum-rate, the optimal power allocation between the common and private layers are required for each user.

However, in the considered heterogeneous network, the K link pairs share the same MBS to macro-user link. As a result, only one transmission scheme and one achievable rate are allowed for the macro-user. From previous analysis for the optimal transmission schemes in various cases, we have known that only in the weak interference scenario each user transmits two layers, and in other interference scenarios it is enough for each user to transmit one layer to achieve the sum-capacity. In the network with one macro-cell and multiple pico-cells, to accommodate various possible interference scenarios, we study the case where both the macro-BS and pico-BS only transmit one layer. If the weak interference scenario is encountered, we will use the transmission scheme in the very weak interference scenario instead, i.e., treating the interference from the other user as noise.

With such a simplification, there is no power allocation among the common and private layers of each user, and a single layer employs the full transmit power. However, the transmit data rate of the macro-user should satisfy the sum-rate constraints of all the K link pairs. Fig. 4 explains this kind of constraints by illustrating the amplitude-space sharing scheme in different interference scenarios. For the two-user interference channel, at each receiver, the desired signal and interference signal share the same amplitude space and without overlapping. The height of the bars denotes the SNR or INR at each receiver, and the length of the bars denotes the transmission data rate. The graphical representations in different interference scenarios satisfy the sum-rate constraints in Table II.

For example, in the strong interference scenario, as shown in Fig. 4(a), at both receivers the interference signal occupies the upper layer and the desired signal occupies the lower layer. In the mixed interference scenario, as shown in Fig. 4(b), at both receivers the signal from macro-BS occupies the upper layer and the signal from pico-BS occupies the lower layer. In the weak interference scenario, as shown in Fig. 4(c), at both receivers the desired signal occupies the upper layer and the interference signal occupies the lower layer. If these three cases coexist in one network, then the transmit layer of the macro-BS behaves as common information in the former two cases and behaves as private information in the latter case.

Since the transmit data rate of the macro-BS should satisfy all the sum-rate constraints of K link pairs, the total throughput of the network is

$$R_{\mathsf{total}} = \min_k\{R_{\mathrm{M},k}\} + \sum_k R_{\mathrm{P},k} = R_{\mathrm{M}} + \sum_k R_{\mathrm{P},k} \quad (66)$$

where $R_{M,k}$ and $R_{P,k}$ denote the achievable rates of the macro-user and pico-user in the k-th link pair, respectively, and $R_{M,k} + R_{P,k} = R_{\mathsf{sum},k}$. The practical transmission rate of the macro-user R_{M} is the minimum of K possible rates $R_{M,k}$.

In Table II we have provided the achieved sum-rate of the optimized H-K coding, where the data rate of each user is not specified. In fact, when the sum-rate expression is composed by two logarithm functions, the first logarithm function is the achievable data rate of macro-user and the second logarithm function is the achievable data rate of pico-user. Otherwise, when the sum-rate expression is composed only by one logarithm function, the data rates between macro-user and pico-user should be compromised, i.e., the increase of one user implies the decrease of the other. For example, in the strong interference scenario, if the sum-rate is constrained by

$$R_{\mathsf{sum},k} = \log(1 + \mathsf{SNR}_{\mathrm{M},k} + \mathsf{INR}_{\mathrm{M},k}) \quad (67)$$

the data rate of the macro-user has a region, i.e.,

$$\log\left(1 + \frac{\mathsf{SNR}_{\mathrm{M},k}}{1 + \mathsf{INR}_{\mathrm{M},k}}\right) \le R_{\mathrm{M},k} \le \log\left(1 + \mathsf{SNR}_{\mathrm{M},k}\right) \quad (68)$$

and the data rate of the pico-user is

$$R_{\mathrm{P},k} = R_{\mathsf{sum},k} - R_{\mathrm{M},k}. \quad (69)$$

If the macro-user has higher priority, we should choose $R_{\mathrm{M},k}$ as large as possible, so that R_{M} has possibility to be large. On the contrary, if we want to maximize the network throughput, $R_{\mathrm{M},k}$ should be chosen as small as possible, since the data rate reduction of one macro-user can increase the data rates of multiple pico-users.

IV. Simulation Results

In this section, we evaluate the performance of the optimized H-K coding and the amplitude-space sharing transmission scheme in heterogeneous networks, and compare the achieved network throughput with other schemes. We start from a one pico-cell scenario. The relationship between the interference scenario and network deployment geometry is first shown, and then the achievable sum-rates versus the PBS positions are demonstrated using the results of Table II. Finally, we show the throughput of a K pico-cells network versus K.

The considered network configurations are as follows. The transmit power of the MBS is 46 dBm, the transmit power of the PBS is 30 dBm, and the transmit powers of the macro-user and pico-user are 23 dBm. To show the performance gain purely provided by the amplitude-space sharing scheme, single antenna is considered both in the BS and in the user. The MBS behaves as a central coordinator, which has channel state information of all links and allocate the power and data rate for all the users. The coverage of the macro-cell is 500 m, where the SNR at the cell edge is 5 dB. The radius of the pico-cell is set as 60 m. The path loss models for the MBS and PBS are from 3GPP channel models [24], which are

$$PL_{\text{MBS-UE}} = 15.3 + 37.6 \log_{10}(D),$$
$$PL_{\text{PBS-UE}} = 30.6 + 36.7 \log_{10}(D),$$

where D is the distance between a BS and a user, $PL_{\text{MBS-UE}}$ applies to the path loss of the MBS/macro-user link and MBS/pico-user link, and $PL_{\text{PBS-UE}}$ applies to the path loss of the PBS/macro-user link and PBS/pico-user link. To avoid near-field effect, the PBS, macro-user and pico-user are not allowed to be close to the MBS within 35 m.

A. Network Deployment Geometry and Interference Scenarios

The MBS is located in the center of a circular area. The macro-user and PBS can be located anywhere within the macro-cell. The pico-user is located in the pico-cell. The position distribution of these BSs and users and their relative distances are called network deployment geometry.

The interference scenarios are determined by the relationship of the SNRs and INRs.

In downlink, the values of SNR_1 and INR_2 only depend on the values of $|h_{11}|$ and $|h_{21}|$. Similar dependency happens to SNR_2 and INR_1. According to the path loss models, the reference power and path loss exponent are equal from one BS to different users, and the channel gains are inversely proportional to the distances between each BS and the two users. Therefore, the interference scenarios are determined by the relative distances of direct and cross links, D_{ij}, i.e.,

Strong Interference:	$D_{11} \geq D_{21}$	and	$D_{22} \geq D_{12}$,
Mixed Interference 1:	$D_{11} < D_{21}$	and	$D_{22} \geq D_{12}$,
Mixed Interference 2:	$D_{11} \geq D_{21}$	and	$D_{22} < D_{12}$,
Weak Interference:	$D_{11} < D_{21}$	and	$D_{22} < D_{12}$.

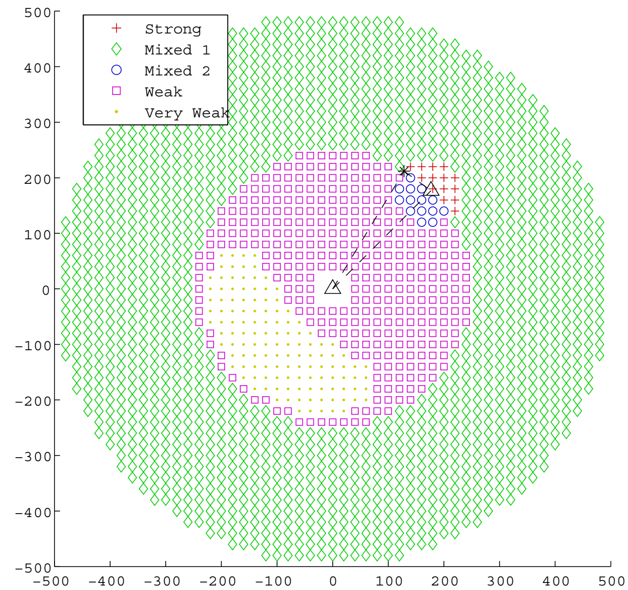

Fig. 5. Various interference scenarios in downlink, depending on the locations of the macro-user. '△' in the central of the circle area denotes the MBS, '△' in the upper right of the area denotes the PBS, and '∗' denotes the pico-user.

However, we cannot find a simple connection between the interference scenario and network deployment geometry under very strong interference and very weak interference.

In uplink, the users are transmitters and the BSs are receivers. Since the path loss from each user to different BSs subject to different path loss models, there is no simple connection between the interference scenario and network deployment geometry as well.

We illustrate such a connection through simulations. Fixing the positions of the PBS and pico-user, and changing the position of the macro-user, we can observe various interference scenarios both in the downlink and uplink transmissions.

In Fig. 5, we show the interference scenarios in downlink when the macro-user is located at different places of the macro-cell. We can see that overall five scenarios appear. The mixed, weak and very weak interference appear in most of the areas, but the strong interference only appears when the macro-user is located in a specific region of the pico-cell.

In Fig. 6, we illustrate the interference scenarios in uplink, which is quite different from that in downlink.

B. Comparisons with Other Schemes

We first compare the achievable rates of the optimized H-K scheme with other schemes in different interference scenarios for a heterogeneous network with one MBS and one PBS. As benchmarks, we show the sum rate upper bound developed in [13] (hereafter called "ETW upper bound"), and a tighter upper bound developed in [9] ("SKC upper bound"). We compare with the H-K coding scheme with a fixed power allocation strategy proposed in [13] ("ETW scheme"), where the INR of the private information is fixed as 1, i.e., $\mathsf{INR}_p = 1$. We also compare with two conventional schemes: orthogonal transmission and treating the interference as noise. The achievable sum-rate of orthogonal transmission

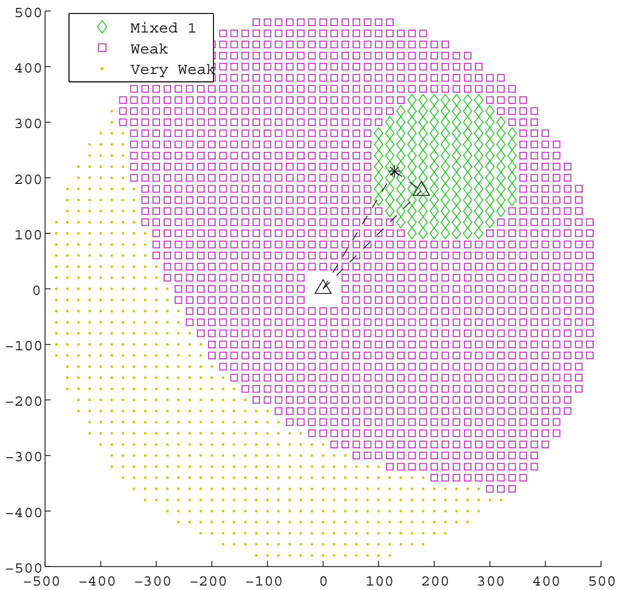

Fig. 6. Various interference scenarios in uplink, depending on the locations of the macro-user. '△' in the central of the circle area denotes the MBS, '△' in the upper right of the area denotes the PBS, and '∗' denotes the pico-user.

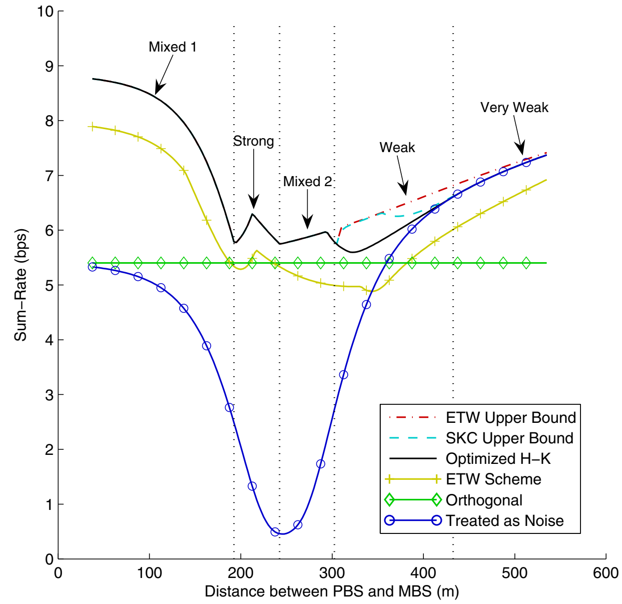

Fig. 7. The sum-rate of macro-user and pico-user when the PBS moves away from cell center to cell edge while the pico-user keeps relative position with the PBS.

is

$$R_{\mathsf{sum}} = \frac{1}{2}\log\left(1 + 2\cdot \mathsf{SNR}_1\right) + \frac{1}{2}\log\left(1 + 2\cdot \mathsf{SNR}_2\right). \quad (70)$$

In practical heterogeneous networks, orthogonal transmission can be implemented by the fractional frequency reuse (FFR) or the almost blank subframe (ABSF) scheme [3].

When the PBS moves from the cell center to the cell edge (35m - 500m) while keeping the relative positions of the MBS with the macro-user and the PBS with the pico-user, different interference scenarios will happen. In Fig. 7, we show the achievable sum-rates of the considered transmission schemes. We can see that as the PBS moves from the cell center to the cell edge, the two links successively experience the mixed 1, strong, mixed 2, weak and very weak interference scenarios. In all scenarios, the optimized H-K coding outperforms the other schemes, and achieves SKC upper bound in the strong, mixed, and very weak interference scenarios. Although the ETW scheme can achieve the capacity region outer bound within one bit, i.e., achieve the sum-capacity upper bound within two bits, its gap to the optimized H-K coding is obvious and it performs even worse than orthogonal transmission in some interference scenarios. Orthogonal transmission achieves a constant sum-rate as the PBS moves, since the channel gains of the MBS/macro-user link and the PBS/pico-user link do not change and there is no interference between these two links. Treating interference as noise performs better only under very weak interference, and degrades severely in other interference scenarios.

Next we compare the achievable sum-rates of the proposed amplitude-space sharing transmission scheme with the schemes of orthogonal transmission and treating interference as noise, for a heterogeneous network where K pico-cells are randomly deployed in the macro-cell. In this simulation, to deploy the PBSs with random locations while keep a minimum distance among them, we set a virtual grid in the macro-cell with 120m separation between the rows and the columns. Then, K intersection points are randomly selected as the locations of the PBSs, where $K = 1 \sim 20$. The macro-user is randomly located on a circle with a radius being $2/3$ of the cell radius, where the MBS is in the cell center. The pico-user is located randomly in each pico-cell with uniform distribution as well.

The results are shown in Fig. 8, where two criteria are respectively considered for the the amplitude-space sharing transmission scheme. One criterion is to maximize the network throughput ("Max Throughput"), and the other is to guarantee the priority of the macro-user ("Macro-User Prior"). As we have analyzed in Section III, keeping a large data rate of the macro-user will sacrifice the network throughput. But with this criterion we still can see 32% throughput gain over the orthogonal transmission scheme. With the criterion of throughput maximization, the proposed amplitude-space sharing scheme can have 47% throughput gain over the orthogonal transmission. Note that in the multiple pico-cells scenario, we only divide two time slots or frequency bands for orthogonal transmission, the MBS employs one slot or one band and all the PBSs employ another slot or band.

V. Conclusion

In this paper, we proposed an amplitude-space sharing strategy to coordinate the interference among macro-cell and pico-cells. We first established a unified framework to optimize H-K coding under various interference scenarios. Specifically, we optimize the transmit powers for private and common layers to maximize the network sum-rate. The optimized H-K coding achieves the sum-capacities in very strong, strong, mixed, and very weak interference scenarios, and achieves a best known sum-rate in weak interference scenario. Based on the insight gained from the optimized H-K coding, we

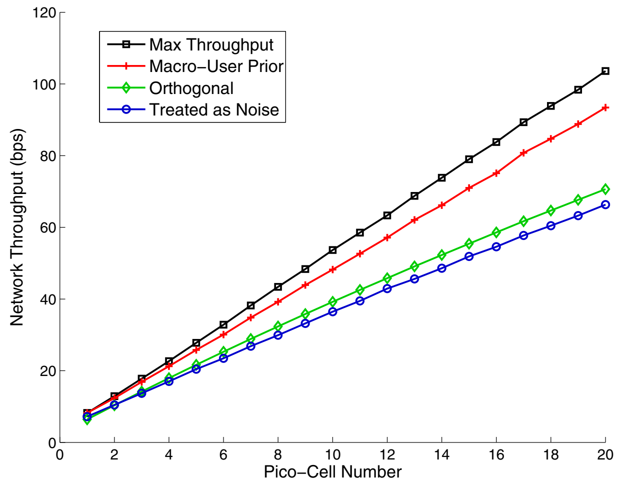

Fig. 8. The network throughput of heterogeneous networks with multiple PBSs randomly deployed in the macro-cell, where "Max Throughput" and "Macro-User Prior" are obtained using the amplitude-space sharing transmission scheme.

developed a simplified transmission scheme for the network with one macro-cell and multiple pico-cells, where significant throughput gain are observed over the time or frequency orthogonal transmissions. The principle of amplitude-space sharing can be extended to multiple-carrier and multiple-antenna systems.

Appendix I

The objective function in (61) can be rewritten as

$$\mathcal{L} = \frac{AP_{1,p} + B}{CP_{1,p}^2 + DP_{1,p} + E}, \tag{71}$$

where

$$\begin{aligned} A &= |h_{21}|^2 + \alpha |h_{22}|^2, \\ B &= \beta |h_{22}|^2 + N_0, \\ C &= \alpha |h_{12}|^2 |h_{21}|^2, \\ D &= \alpha |h_{12}|^2 N_0 + \beta |h_{12}|^2 |h_{21}|^2 + |h_{21}|^2 N_0, \\ E &= \left(\beta |h_{12}|^2 + N_0\right) N_0. \end{aligned}$$

Considering the constraint $P_{1,p} > \max\{0, -\frac{\beta}{\alpha}\}$ in (61), both the numerator and denominator are larger than zero, and thus $\mathcal{L} > 0$ in the constraint region.

Given an arbitrary constant $\delta > 0$, check the feasible region of $\mathcal{L} > \delta$ as follows

$$\frac{AP_{1,p} + B}{CP_{1,p}^2 + DP_{1,p} + E} > \delta, \tag{72}$$

which is equivalent to

$$\delta \left(CP_{1,p}^2 + DP_{1,p} + E\right) - \left(AP_{1,p} + B\right) < 0. \tag{73}$$

Since $\delta > 0$ and $C > 0$, the left-hand side of (73) is a quadratic convex function. This means that the feasible region defined in (73) is either a line segment or a null set.

The constraint $P_{1,p} > \max\{0, -\frac{\beta}{\alpha}\}$ constructs a ray. The intersection of a line segment and a ray is also a line segment. This means that the feasible region defined in (72) is a convex set. Therefore, $\mathcal{L}$ is quasi-concave over the feasible region defined by the constraint [22].

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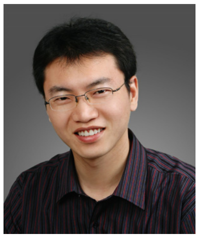

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