Distributed Precoding in Ultra-Dense Network with Dynamic User Access

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***Abstract*—In ultra-dense network with high-throughput applications, users arrive and depart the network randomly, and the inter-cell interference is severe and changes dramatically. To capture the essence of the problem, a network named two-plus-one network is considered, where a third new user is added into a two-user network. The distributed precoding is studied, where the precoding matrix of the third new user is designed and those of the two old users remain unchanged. Using the geometric interpretation of the covariance matrices, we find the principle on how precoding matrices affect the rate of the desired user and the interfered users. Based on the principle, we propose a criterion for precoding design and formulate an optimization problem. Then we solve this problem, and provide a geometric explanation of the optimization result, i.e., how to choose precoding directions and allocate power to the directions under certain constraints. Simulation results indicate that the proposed method achieves almost the same performance as that obtained from Weighted Minimum Mean Square Error criterion but with low complexity, and outperforms Maximum Signal-to-Leakage-plus-Noise Ratio and Block-Diagonal Zero-Forcing under certain circumstances.**

***Index Terms*—ultra-dense network, two-plus-one network, precoding design, interference coordination, semidefinite optimization.**

I. Introduction

To meet the explosively increasing traffic demands, ultra-dense network (UDN) is a promising technique in the fifth generation (5G) wireless communication systems [1]. However, interference problem becomes more severe due to the denser deployment of cells.

Many techniques are developed to deal with interference. Coordinated multiple point joint transmission [2] achieves good performance, but requires high capacity backhaul to share information among base stations. Therefore, the methods without data exchange between base stations such as interference avoidance [3], user-centric interference coordination [4] and interference alignment [5] [6], are more feasible. Various criterions have been used for precoding matrix design in the above techniques, such as Weighted Minimum Mean Square Error (WMMSE) [7], Block-Diagonal Zero-Forcing (ZF-BD) [8] and Maximum Signal-to-Leakage-plus-Noise Ratio (Max-SLNR) [9]. However, the WMMSE method proposed in [7] requires iterations between optimizing precoding matrices and detection matrices, and thus has high implementation complexity. ZF-BD criterion makes use of orthogonal subspaces, which is infeasible when the dimension condition in [8] is not satisfied. Max-SLNR criterion only minimizes the leakage at transmitter, which cannot guarantee that the interference is weak at interfered receivers.

In real-world networks, users arrive in a network sequentially rather than simultaneously. It is unreasonable to require a user to update its precoding matrix when it is communicating with others, since the update may cause delay or even disconnection. Therefore how to design the precoding matirx for a newly arrived user while keeping old users' precoding matrices unchanged is relevant. Besides, the distributed implementation of the precoding at each base station requires designing the precoding matrix for each user individually instead of jointly.

In this paper, for the convenience of analysis, we consider a simple but fundamental sequential access network named two-plus-one network, where a new user is added into a two-user network, while the precoding matrices for the existing users remain unchanged. Using the geometrical interpretation of the covariance matrices, we find the principle on how precoding matrices affect the rate of the desired user and the interfered users. Based on the principle, we propose a criterion for precoding matrix design, i.e., maximizing the newly added user's rate while keeping the interference to other users caused by it less than the interference and noise level of the original two-user network. Then we formulate an optimization problem, and provide a geometric explanation of the optimal solution, i.e., how to choose precoding directions and allocate power to the directions under certain constraints. Simulation results show that the precoding designed under the proposed criterion achieves similar performance as WMMSE, and outperforms Max-SLNR and ZF-BD under certain circumstances. With a similar sum-rate, the proposed method has lower complexity than that designed under the WMMSE criterion.

II. System Description

In UDNs, the number of base stations is about the same as that of users, thus we assume that one base station serves only one user in this paper. Though downlink is considered in this paper, the criterion and optimization problem also apply to uplink.

A. System Model

Consider a n cell downlink interfering network, where only one user exists in a cell. The base station in the ith cell is equipped with n_i antennas. The user in the ith cell is equipped with m_i antennas. d_i is the number of data streams of user i.

The signal received by user i is modeled as

$$\mathbf{y}_i = \mathbf{H}_{ii}\mathbf{V}_i\mathbf{s}_i + \sum_{j\neq i}\mathbf{H}_{ij}\mathbf{V}_j\mathbf{s}_j + \mathbf{n}_i, \tag{1}$$

where $\mathbf{H}_{ij} \in \mathbb{C}^{m_i \times n_j}$ represents the channel matrix from base station j to user i, $\mathbf{V}_i \in \mathbb{C}^{n_i \times d_i}$ denotes the precoding matrix of user i, $\mathbf{s}_i \in \mathbb{C}^{d_i}$ represents the signal vector of user i, satisfying $\mathbb{E}\left[\mathbf{s}_i\mathbf{s}_i^H\right] = \mathbf{I}$, $\mathbf{I}$ is unit matrix, and $\mathbf{n}_i \in \mathbb{C}^{m_i}$ denotes the noise at user i.

B. Non-convex Feature of Precoding Design Problem

The multi-cell precoding matrices design problem is non-convex. To illustrate this, we consider the simplest two-cell single-input single-output (SISO) downlink network, i.e., $n_i = 1$, $m_i = 1$ for $i = 1, 2$. Let s_i denote the signal for user i, h_{ij} denote the channel gain from base station j to user i, n_i denote the noise at user i with variance σ^2, and P_i denote the transmitting power allocation of base station i. The received signals of user 1 and user 2 are

$$\begin{aligned} y_1 &= h_{11}\sqrt{P_1}s_1 + h_{12}\sqrt{P_2}s_2 + n_1 \\ y_2 &= h_{22}\sqrt{P_2}s_2 + h_{21}\sqrt{P_1}s_1 + n_2, \end{aligned} \tag{2}$$

Note that the power allocations, P_1 and P_2, play the same role in SISO case as precoding matrices in multi-input multi-output (MIMO) case.

The sum-rate of the two user is [10]

$$\begin{aligned} R =& \log(1 + \frac{h_{11}^2 P_1}{h_{12}^2 P_2 + \sigma^2}) + \log(1 + \frac{h_{22}^2 P_2}{h_{21}^2 P_1 + \sigma^2}) \\ &\approx \max(\log(\frac{h_{11}^2 P_1}{h_{12}^2 P_2 + \sigma^2}), 0) + \max(\log(\frac{h_{22}^2 P_2}{h_{21}^2 P_1 + \sigma^2}), 0) \\ &\approx \max(g_{11} + x_1 - \max(g_{12} + x_2, 0), 0) + \\ &\quad \max(g_{22} + x_2 - \max(g_{21} + x_1, 0), 0), \end{aligned} \tag{3}$$

where $g_{ij} = \log(h_{ij}^2)$ and $x_i = \log(\frac{P_i}{\sigma^2})$. The approximation comes from the property of $\log(\cdot)$ function that $\log(1 + c) \approx \max(\log(c), 0)$, which is accurate when c is much larger than 1 or close to 0.

Obviously R is a piecewise function of variables $[x_1, x_2]$. An example is shown in Fig. 1.

We can see that even in the simplest case, i.e., two-cell SISO network, the sum-rate is non-convex. It can be expected that the problem to jointly design precoding matrices for all users in a multi-cell MIMO network is much more complicated. However, it is easier to design the precoding matrices for users sequentially. We call the user that newly arrives in the network as new user, and the users already in the network as existing users. Considering complexity and feasibility, we concentrate on designing the precoding matrix of the new user while keeping the precoding matrices of existing users unchanged.

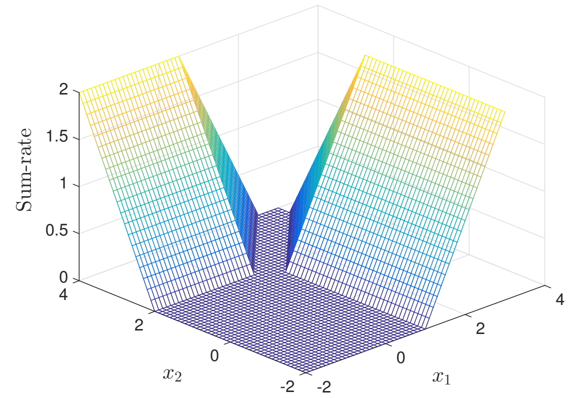


Fig. 1: An example of the sum-rate of two-cell SISO network, where $[g_{11}, g_{12}, g_{21}, g_{22}] = [-1, -1.7, -0.5, -2]$.

C. Two-plus-one Network

We refer to a network where a new user arrives with k existing users as k-plus-one network. For easy understanding, we consider a two-plus-one network, which can be extended to the k-plus-one network in a straightforward manner.

The received signals of the two existing users are

$$\begin{aligned} \mathbf{y}_1 = \mathbf{H}_{11}\mathbf{V}_1\mathbf{s}_1 + \mathbf{H}_{12}\mathbf{V}_2\mathbf{s}_2 + \mathbf{n}_1 \\ \mathbf{y}_2 = \mathbf{H}_{22}\mathbf{V}_2\mathbf{s}_2 + \mathbf{H}_{21}\mathbf{V}_1\mathbf{s}_1 + \mathbf{n}_2. \end{aligned} \tag{4}$$

The sum-rate of the two users is [10]

$$\begin{aligned} R =& R_1 + R_2 \\ =& \log\left|\mathbf{I} + \frac{\mathbf{S}_1}{\mathbf{F}_{12} + \mathbf{N}_1}\right| + \log\left|\mathbf{I} + \frac{\mathbf{S}_2}{\mathbf{F}_{21} + \mathbf{N}_2}\right|, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathbf{S}_i =& (\mathbf{H}_{ii}\mathbf{V}_i)(\mathbf{H}_{ii}\mathbf{V}_i)^H \\ \mathbf{F}_{ji} =& (\mathbf{H}_{ji}\mathbf{V}_i)(\mathbf{H}_{ji}\mathbf{V}_i)^H \\ \mathbf{N}_i =& \mathbb{E}\left[\mathbf{n}_i\mathbf{n}_i^H\right], \end{aligned} \tag{6}$$

representing the covariance matrices of desired signal of user i, interference from user i to user j and noise of user i, respectively, and $(\cdot)^H$ is conjugate transpose.

A third user arrives in this two-user network, whose rate is

$$R_3 = \log\left|\mathbf{I} + \frac{\mathbf{S}_3}{\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3}\right|. \tag{7}$$

Due to the interference caused by the new user to the existing users, the sum-rate of user 1 and user 2 decreases by

$$\begin{aligned} &\Delta R_{1,2} = \\ &(\log\left|\mathbf{I} + \frac{\mathbf{S}_1}{\mathbf{F}_{13} + \mathbf{F}_{12} + \mathbf{N}_1}\right| - \log\left|\mathbf{I} + \frac{\mathbf{S}_1}{\mathbf{F}_{12} + \mathbf{N}_1}\right|) + \\ &(\log\left|\mathbf{I} + \frac{\mathbf{S}_2}{\mathbf{F}_{23} + \mathbf{F}_{21} + \mathbf{N}_2}\right| - \log\left|\mathbf{I} + \frac{\mathbf{S}_2}{\mathbf{F}_{21} + \mathbf{N}_2}\right|). \end{aligned} \tag{8}$$

Thus the sum-rate variation of this network is

$$\Delta R = R_3 + \Delta R_{1,2}. \tag{9}$$

Note that $R_3 \geq 0$ and $\Delta R_{1,2} \leq 0$ always hold.

III. The proposed criterion

For easy understanding, we first analyze the two-plus-one SISO network. The rate of user i is

$$R_i = \log(1+\frac{S_i}{\sum_{j\neq i} F_{ij}+N_i}) \approx \log(1+\frac{S_i}{\max\limits_{j\neq i}(F_{ij}+N_i)}), \tag{10}$$

where S_i, F_{ij} and N_i are the degenerated expressions defined in (6) with $m_i = n_i = 1$. Again the approximation comes from the property of $\log(\cdot)$ that $\log(\sum\limits_i c_i) \approx \log(\max\limits_i(c_i))$. After the new user arrives, the data rate of the first existing user reduces by

$$\begin{aligned}\Delta R_1 =&\log(1+\frac{S_1}{F_{13}+F_{12}+N_1}) - \log(1+\frac{S_1}{F_{12}+N_1})\\ \approx&\log(1+\frac{S_1}{\max(F_{13},F_{12}+N_1)}) - \log(1+\frac{S_1}{F_{12}+N_1}).\end{aligned} \tag{11}$$

When $\max(F_{13}, F_{12}+N_1) = F_{12}+N_1$, $\Delta R_1 \approx 0$. Similarly, $\Delta R_2 \approx 0$ when $\max(F_{23}, F_{22}+N_2) = F_{22}+N_2$. If the above two conditions are satisfied, the sum-rate change of the three users is $\Delta R \approx R_3$. Since $R_3 \geq 0$ always holds, $\Delta R \geq 0$, which means that under these conditions the sum rate of the network always increases when the new user arrives.

For the two-plus-one MIMO network, the rate of user i can be approximated as

$$R_i = \log\left|\mathbf{I}+\frac{\mathbf{S}_i}{\sum\limits_{j\neq i}\mathbf{F}_{ij}+\mathbf{N}_i}\right| \approx \log\left|\mathbf{I}+\frac{\mathbf{S}_i}{\max\limits_{j\neq i}(\mathbf{F}_{ij}+\mathbf{N}_i)}\right|. \tag{12}$$

However, since matrices cannot be compared directly with each other, the function $\max(\cdot)$ in (12) has no definition. In the sequel, we briefly introduce the method to compare matrices.

A. Ellipsoid Interpretation of Covariance Matrix

Denote $\mathbf{S}^n_{++}$ as the set of Hermitian positive definite $n\times n$ matrices, each $\mathbf{A}\in\mathbf{S}^n_{++}$ can be associated with an ellipsoid centered at the origin, which is given by [11]

$$\Omega_{\mathbf{A}} = \{\mathbf{u}|\mathbf{u}^H\mathbf{A}^{-1}\mathbf{u}\leq 1\}. \tag{13}$$

The eigenvalue decomposition of $\mathbf{A}$ is

$$\mathbf{A} = \mathbf{U}_{\mathbf{A}}\mathbf{\Lambda}_{\mathbf{A}}\mathbf{U}_{\mathbf{A}}^H \tag{14}$$

where $\mathbf{U}_{\mathbf{A}} = [\mathbf{u}^{(1)}_{\mathbf{A}}\mathbf{u}^{(2)}_{\mathbf{A}},\cdots,\mathbf{u}^{(n)}_{\mathbf{A}}]$ is a unitary matrix satisfying $\mathbf{U}_{\mathbf{A}}\mathbf{U}_{\mathbf{A}}^H = \mathbf{U}_{\mathbf{A}}^H\mathbf{U}_{\mathbf{A}} = \mathbf{I}$, $\mathbf{\Lambda}_{\mathbf{A}}$ is a diagonal matrix with elements $\{\lambda_1,\lambda_2,\cdots,\lambda_n\}$, which are eigenvalues of $\mathbf{A}$.

$\Omega_{\mathbf{A}}$ has n axes orthogonal with each other. The ith axis of $\Omega_{\mathbf{A}}$ are in the direction parallel to $\mathbf{u}^{(i)}_{\mathbf{A}}$ with length of $\sqrt{\lambda_i}$. We say that $\Omega_{\mathbf{A}}$ has power of λ_i in the direction of $\mathbf{u}^{(i)}_{\mathbf{A}}$.

Consider two Hermitian positive definite matrices $\mathbf{A},\mathbf{B}\in\mathbf{S}^n_{++}$, which are associated with ellipsoids $\Omega_{\mathbf{A}}$ and $\Omega_{\mathbf{B}}$, respectively. Consider p as a point in $\mathbb{C}^n$, then we have the following definitions.

Definition 1 *If $\forall p\in\Omega_{\mathbf{A}}$ satisfies $p\in\Omega_{\mathbf{B}}$, then ellipsoid $\Omega_{\mathbf{A}}$ is called to be contained in $\Omega_{\mathbf{B}}$, which is denoted as $\Omega_{\mathbf{A}}\subseteq\Omega_{\mathbf{B}}$. Then $\mathbf{B}-\mathbf{A}$ is a positive semidefinite matrix, denoted as $\mathbf{A}\preceq\mathbf{B}$. If $\forall p\in\Omega_{\mathbf{A}}$ satisfies $p\in\Omega_{\mathbf{B}}$ and p is not at the boundary of $\Omega_{\mathbf{B}}$, then ellipsoid $\Omega_{\mathbf{A}}$ is called to be contained in $\Omega_{\mathbf{B}}$ completely, which is denoted as $\Omega_{\mathbf{A}}\subset\Omega_{\mathbf{B}}$. Then $\mathbf{B}-\mathbf{A}$ is a positive definite matrix, denoted as $\mathbf{A}\prec\mathbf{B}$. [11]*

An intuitive explanation of equivalent relation between $\Omega_{\mathbf{A}}\subset\Omega_{\mathbf{B}}$ and $\mathbf{A}\prec\mathbf{B}$ is that if $\mathbf{B}-\mathbf{A}$ is positive definite, then all the eigenvalues of $\mathbf{B}-\mathbf{A}$ are larger than 0, indicating that $\Omega_{\mathbf{B}}$ have more power than $\Omega_{\mathbf{A}}$ in all directions, which means $\Omega_{\mathbf{A}}\subset\Omega_{\mathbf{B}}$, and vice versa. Interested readers can read [11] for more information.

By associating a matrix with an ellipsoid, we can define the generalized function $\max(\cdot)$ with matrix variables.

Definition 2 *If $\mathbf{A}\preceq\mathbf{B}$ is satisfied, then $\mathbf{A}$ is smaller than $\mathbf{B}$, or equivalently $\mathbf{B}$ is larger than $\mathbf{A}$. If $\mathbf{B}$ is larger than $\mathbf{A}$, then $\mathbf{B}=\max(\mathbf{A},\mathbf{B})$.*

B. Optimization Problem

With the definition of the generalized function $\max(\cdot)$ with matrix variables, the rate variation of user 1 in two-plus-one MIMO network is

$$\begin{aligned}\Delta R_1 =&(\log\left|\mathbf{I}+\frac{\mathbf{S}_1}{\mathbf{F}_{13}+\mathbf{F}_{12}+\mathbf{N}_1}\right| - \log\left|\mathbf{I}+\frac{\mathbf{S}_1}{\mathbf{F}_{12}+\mathbf{N}_1}\right|)\\ \approx&(\log\left|\mathbf{I}+\frac{\mathbf{S}_1}{\max(\mathbf{F}_{13},\mathbf{F}_{12}+\mathbf{N}_1)}\right| - \log\left|\mathbf{I}+\frac{\mathbf{S}_1}{\mathbf{F}_{12}+\mathbf{N}_1}\right|).\end{aligned} \tag{15}$$

When $\max(\mathbf{F}_{13},\mathbf{F}_{12}+\mathbf{N}_1) = \mathbf{F}_{12}+\mathbf{N}_1$, i.e., $\mathbf{F}_{13}$ is less than $\mathbf{F}_{12}+\mathbf{N}_1$, $\Delta R_1\approx 0$. Similarly, when $\max(\mathbf{F}_{23},\mathbf{F}_{21}+\mathbf{N}_2) = \mathbf{F}_{21}+\mathbf{N}_2$, $\Delta R_2\approx 0$. If the above two conditions are satisfied, $\Delta R\geq 0$, implying that the sum rate of the network always increases when a new user arrives.

Denote existing interference as the interference experienced by existing users before the new user arrives in the network. Inspired by above observation, a precoding designing criterion is given as following.

Criterion: *Maximize the rate of the new user while keep the interference caused by the new user less than the existing interference plus noise at each existing user.*

Then, the precoding optimization problem for the two-plus-one network can be formulated as

$$\begin{aligned}\max_{\mathbf{V}_3}\quad & \left|\mathbf{H}_{33}\mathbf{V}_3\mathbf{V}_3^H\mathbf{H}_{33}^H+\mathbf{F}_{31}+\mathbf{F}_{32}+\mathbf{N}_3\right|\\ s.t.\quad & \tilde{\mathbf{U}}^H_{\mathbf{S}_1}(\mathbf{N}_1+\mathbf{F}_{12}-\mathbf{H}_{13}\mathbf{V}_3\mathbf{V}_3^H\mathbf{H}_{13}^H)\tilde{\mathbf{U}}_{\mathbf{S}_1}\succeq\mathbf{0}\\ & \tilde{\mathbf{U}}^H_{\mathbf{S}_2}(\mathbf{N}_2+\mathbf{F}_{21}-\mathbf{H}_{23}\mathbf{V}_3\mathbf{V}_3^H\mathbf{H}_{23}^H)\tilde{\mathbf{U}}_{\mathbf{S}_2}\succeq\mathbf{0}\\ & \mathrm{tr}(\mathbf{V}_3\mathbf{V}_3^{\mathrm{H}})\leq \mathrm{P}_{3,\max},\end{aligned} \tag{16}$$

where the columns of $\tilde{\mathbf{U}}_{\mathbf{S}_i}$ are the generalized eigenvectors corresponding to the significant generalized eigenvalues obtained from the generalized eigenvalue decomposition of $\mathbf{S}_i$

and $\sum_{j \neq i} \mathbf{F}_{ij} + \mathbf{N}_i$, which is

$$\mathbf{S}_i \mathbf{U}_{\mathbf{S}_i} = (\sum_{j \neq i} \mathbf{F}_{ij} + \mathbf{N}_i) \mathbf{U}_{\mathbf{S}_i} \mathbf{\Lambda}_{\mathbf{S}_i}, \tag{17}$$

where $\mathbf{U}_{\mathbf{S}_i}$ is a unitary matrix, and $\mathbf{\Lambda}_{\mathbf{S}_i}$ is a diagonal matrix. If in $\mathbf{\Lambda}_{\mathbf{S}_i}$ some elements are much larger than the others, these elements are called significant generalized eigenvalues. $\tilde{\mathbf{U}}_{\mathbf{S}_i}$ consists of a part of $\mathbf{U}_{\mathbf{S}_i}$'s columns.

The object function in (16) comes from the following equivalent relation

$$\begin{aligned}
&\max_{\mathbf{V}_3} \log(\left|\mathbf{I} + \frac{\mathbf{S}_3}{\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3}\right|) \\
\Leftrightarrow &\max_{\mathbf{V}_3} \left|\mathbf{I} + \frac{\mathbf{S}_3}{\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3}\right| \\
= &\max_{\mathbf{V}_3} \left|\frac{\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3 + \mathbf{S}_3}{\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3}\right| \\
\Leftrightarrow &\max_{\mathbf{V}_3} \left|\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3 + \mathbf{S}_3\right|,
\end{aligned} \tag{18}$$

where $\Leftrightarrow$ denotes equivalence. The first equivalence comes from the fact that $\log(\cdot)$ is a strictly monotonic increasing function. The second equivalence comes from the fact that for any matrix $\mathbf{M}_1$ and full rank matrix $\mathbf{M}_2$, $\left|\frac{\mathbf{M}_1}{\mathbf{M}_2}\right| = |\mathbf{M}_1| / |\mathbf{M}_2|$ always holds.

The first two constraints in (16) represent the requirement in the proposed criterion that the interference caused by the third new user is less than the existing interference and noise at the two existing users. Since the detection matrix at each existing user is designed to receive signal in the directions with more power, only in the directions corresponding to significant generalized eigenvalues, represented by the columns of $\tilde{\mathbf{U}}_{\mathbf{S}_i}$, the signal is received. Therefore, in the directions that are orthogonal to the columns of $\tilde{\mathbf{U}}_{\mathbf{S}_i}$, interference does not affect the rate. Thus only in the receiving directions the constraints exist.

The third constraint in (16) is power constraint.

The optimization variable is $\mathbf{V}_3$ only, which means that the precoding matrices of user 1 and user 2 remain unchanged.

The optimization problem in (16) can be generalized to a k-plus-one network easily according to the proposed criterion. In k-plus-one network, there are k existing users. The optimization problem can be formulated by adding $k-2$ more constraints to (16), which have the same form as the first two constraints in (16).

It is true that there are cases where ΔR_1 and ΔR_2 are negative but ΔR is still positive. However from this kind of cases, it is hard to find a criterion, from which an optimization problem as simple as (16) can be formulated.

Because in problem (16) the variable $\mathbf{V}_3$ appears in the quadratic form $\mathbf{V}_3 \mathbf{V}_3^H$, the optimization problem is convex, which can be solved by interior-point method [11].

IV. Geometric Interpretation for two-plus-one network

Optimization problem (16) is further simplified as

$$\begin{aligned}
\max_{\mathbf{V}_3} \quad & \left|\mathbf{H}_{33}\mathbf{X}\mathbf{H}_{33}^H + \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3\right| \\
s.t. \quad & \tilde{\mathbf{U}}_{\mathbf{S}_1}^H (\mathbf{N}_1 + \mathbf{F}_{12} - \mathbf{H}_{13}\mathbf{X}\mathbf{H}_{13}^H) \tilde{\mathbf{U}}_{\mathbf{S}_1} \succeq \mathbf{0} \\
& \tilde{\mathbf{U}}_{\mathbf{S}_2}^H (\mathbf{N}_2 + \mathbf{F}_{21} - \mathbf{H}_{23}\mathbf{X}\mathbf{H}_{23}^H) \tilde{\mathbf{U}}_{\mathbf{S}_2} \succeq \mathbf{0} \\
& \mathrm{tr}(\mathbf{X}) \le \mathrm{P}_{3,\max},
\end{aligned} \tag{19}$$

where $\mathbf{X} = \mathbf{V}_3 \mathbf{V}_3^H$.

For easy analysis, we first suppose that channel matrices are full rank, then

$$\begin{aligned}
&\max_{\mathbf{V}_3} \quad \left|\mathbf{H}_{33}\mathbf{X}\mathbf{H}_{33}^H + \mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3\right| \\
\Leftrightarrow &\max_{\mathbf{V}_3} \quad \left|\mathbf{X} + \mathbf{H}_{33}^{-1}(\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3)\mathbf{H}_{33}^{-H}\right|.
\end{aligned} \tag{20}$$

Also assume that $\tilde{\mathbf{U}}_{\mathbf{S}_1}$ and $\tilde{\mathbf{U}}_{\mathbf{S}_2}$ are full rank. Note the fact that for any matrix $\mathbf{D}$ and unitary matrix $\mathbf{U}$, $\mathbf{U}\mathbf{D}\mathbf{U}^H$ has the same semidefinite property with $\mathbf{D}$, because of the quadratic form $\mathbf{U}(\cdot)\mathbf{U}^H$ of $\mathbf{U}$. Therefore, we have

$$\begin{aligned}
\tilde{\mathbf{U}}_{\mathbf{S}_1}^H (\mathbf{N}_1 + \mathbf{F}_{12} - \mathbf{H}_{13}\mathbf{X}\mathbf{H}_{13}^H) \tilde{\mathbf{U}}_{\mathbf{S}_1} &\succeq \mathbf{0} \\
\Leftrightarrow \mathbf{H}_{13}^{-1}(\mathbf{N}_1 + \mathbf{F}_{12})\mathbf{H}_{13}^{-H} - \mathbf{X} &\succeq \mathbf{0} \\
\tilde{\mathbf{U}}_{\mathbf{S}_2}^H (\mathbf{N}_2 + \mathbf{F}_{21} - \mathbf{H}_{23}\mathbf{X}\mathbf{H}_{23}^H) \tilde{\mathbf{U}}_{\mathbf{S}_2} &\succeq \mathbf{0} \\
\Leftrightarrow \mathbf{H}_{23}^{-1}(\mathbf{N}_2 + \mathbf{F}_{21})\mathbf{H}_{23}^{-H} - \mathbf{X} &\succeq \mathbf{0}.
\end{aligned} \tag{21}$$

Due to the space limitation, the cases where $\tilde{\mathbf{U}}_{\mathbf{S}_1}$, $\tilde{\mathbf{U}}_{\mathbf{S}_2}$ or channel matrices are not full rank are not analyzed here, but similar results can be obtained.

When $\tilde{\mathbf{U}}_{\mathbf{S}_1}$, $\tilde{\mathbf{U}}_{\mathbf{S}_2}$ and channel matrices are full rank, optimization problem (16) is equivalent to the following optimization problem

$$\begin{aligned}
\underset{\mathbf{V}_3}{\text{maximize}} \quad & |\mathbf{X} + \mathbf{C}| \\
s.t. \quad & \mathbf{A} - \mathbf{X} \succeq \mathbf{0} \\
& \mathbf{B} - \mathbf{X} \succeq \mathbf{0} \\
& \mathrm{tr}(\mathbf{X}) \le \mathrm{P}_{3,\max},
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\mathbf{H}_{13}^{-1}(\mathbf{N}_1 + \mathbf{F}_{12})\mathbf{H}_{13}^{-H} &= \mathbf{A} \\
\mathbf{H}_{23}^{-1}(\mathbf{N}_2 + \mathbf{F}_{21})\mathbf{H}_{23}^{-H} &= \mathbf{B} \\
\mathbf{H}_{33}^{-1}(\mathbf{F}_{31} + \mathbf{F}_{32} + \mathbf{N}_3)\mathbf{H}_{33}^{-H} &= \mathbf{C}.
\end{aligned} \tag{23}$$

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{X}$ can be associated with some ellipsoids. Though the power constraint $\mathrm{tr}(\mathbf{X}) \le \mathrm{P}_{3,\max}$ is not in the form of semidefinite, it can also have a geometric interpretation, i.e., it represents that the ellipsoid associated with $\mathbf{X}$ is inside of a sphere whose axis length is $\sqrt{P_{3,\max}}$.

A. Optimization with Only Power Constraint

If $P_{3,\max}$ is small, in some cases when power constraint $\mathrm{tr}(\mathbf{X}) \le \mathrm{P}_{3,\max}$ is satisfied, the other two constraints in

(22) is satisfied automatically. In these cases, the optimization problem is equivalent to a simplified problem as

$$\begin{aligned} \underset{\mathbf{V}_3}{\text{maximize}} \quad & |\mathbf{X}+\mathbf{C}| \\ s.t. \;\; & \text{tr}(\mathbf{X}) \le \text{P}_{3,\max}. \end{aligned} \tag{24}$$

From the solution of optimization problem (24), we can explain the optimization solution of problem (22). Therefore we first consider the simplified problem (24).

The problem in (24) can be solved by Lagrange multiplier method. The Lagrangian associated with (24) is

$$L(\mathbf{X},\lambda) = |\mathbf{X}+\mathbf{C}| - \lambda(\text{tr}(\mathbf{X}) - \text{P}_{3,\max}), \tag{25}$$

where λ is the Lagrange multiplier.

If $\mathbf{X}^*$ maximizes (25) over $\mathbf{X}$, then it should satisfy

$$\frac{\nabla L(\mathbf{X}^*,\lambda)}{\nabla \mathbf{X}^*} = \text{cof}(\mathbf{X}^*+\mathbf{C})^{\text{T}} - \lambda \mathbf{I} = 0, \tag{26}$$

where $(\cdot)^T$ represents the transpose, and $\text{cof}(\mathbf{X}^*+\mathbf{C})$ is the cofactor of $\mathbf{X}^*+\mathbf{C}$, which is defined as

$$\text{cof}(\mathbf{X}^*+\mathbf{C}) = |\mathbf{X}^*+\mathbf{C}|\,(\mathbf{X}^*+\mathbf{C})^{-1}. \tag{27}$$

From (26) and (27) we have

$$|\mathbf{X}^*+\mathbf{C}|\,\mathbf{I} = \lambda(\mathbf{X}^*+\mathbf{C}), \tag{28}$$

which means that $\mathbf{X}^*+\mathbf{C}$ is a diagonal matrix, and the elements in diagonal are identical. Remember that $\mathbf{X}$ and $\mathbf{C}$ are associated with ellipsoids $\Omega_\mathbf{X}$ and $\Omega_\mathbf{C}$, and $\mathbf{I}$ is associated with the unit sphere in ellipsoid interpretation. Then (28) indicates that the superposition of $\Omega_\mathbf{X}$ and $\Omega_\mathbf{C}$ is also a sphere, which requires that $\Omega_\mathbf{X}$ and $\Omega_\mathbf{C}$ have the same axis directions. Also remember that the axes of $\Omega_\mathbf{X}$ and $\Omega_\mathbf{C}$ correspond to the eigenvectors of $\mathbf{X}$ and $\mathbf{C}$, respectively, thus $\mathbf{X}$ and $\mathbf{C}$ share the same eigenvectors.

The eigenvalue decomposition of $\mathbf{X}$ and $\mathbf{C}$ are respectively

$$\begin{aligned} \mathbf{X} &= \mathbf{U_C}\boldsymbol{\Lambda}_\mathbf{X}\mathbf{U}_\mathbf{C}^H \\ \mathbf{C} &= \mathbf{U_C}\boldsymbol{\Lambda}_\mathbf{C}\mathbf{U}_\mathbf{C}^H, \end{aligned} \tag{29}$$

where $\boldsymbol{\Lambda}_\mathbf{X} = \text{diag}(\lambda_{\mathbf{X},1},\lambda_{\mathbf{X},2},\cdots,\lambda_{\mathbf{X},\text{n}_3})$ and $\boldsymbol{\Lambda}_\mathbf{C} = \text{diag}(\lambda_{\mathbf{C},1},\lambda_{\mathbf{C},2},\cdots,\lambda_{\mathbf{C},\text{n}_3})$. Then the optimization problem (24) can be reformulated as

$$\begin{aligned} \underset{\lambda_{\mathbf{X},i}}{\text{maximize}} \quad & \prod_{i=1}^{n_3}(\lambda_{\mathbf{X},i}+\lambda_{\mathbf{C},i}) \\ s.t. \quad & \sum_{i=1}^{n_3}\lambda_{\mathbf{X},i} \le P_{3,\max}. \end{aligned} \tag{30}$$

The solution of problem (30) has a water-filling structure as

$$\lambda_{\mathbf{X},i} = (\gamma - \lambda_{\mathbf{C},i})^+, \tag{31}$$

where γ is chosen to meet the power constraint

$$\sum_{i=1}^{n_3}(\gamma - \lambda_{\mathbf{C},i})^+ = P_{3,\max}. \tag{32}$$

However, when extra constraints are added to (24), such as ellipsoid containing constraint by $\mathbf{A}$ and $\mathbf{B}$, the axes of $\Omega_\mathbf{X}$ do not coincide with the axes of $\Omega_\mathbf{C}$ in most cases. Some examples are provided in the following subsection.

B. Some Examples in Two-Dimension

For a better understanding of the proposed criterion, the two-dimension case of two-plus-one network, i.e., $m_i = n_i = 2$, is analyzed in this subsection.

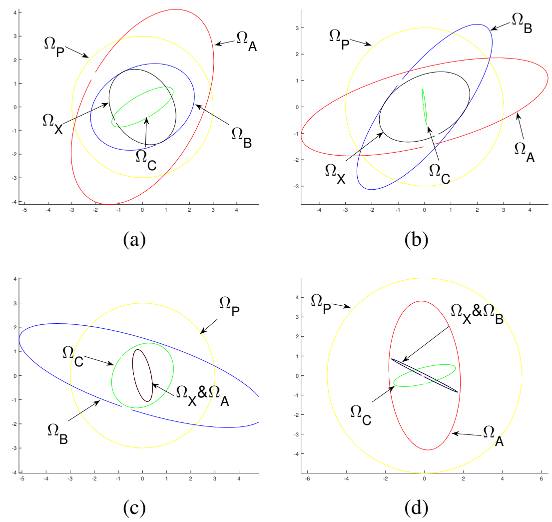


Fig. 2: Two-dimension full rank cases.

Fig. 2 presents a few two-dimension optimization results, where blue ellipsoid $\Omega_\mathbf{A}$, red ellipsoid $\Omega_\mathbf{B}$, green ellipsoid $\Omega_\mathbf{C}$ and black ellipsoid $\Omega_\mathbf{X}$ represent the ellipsoids associated with matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{X}$, respectively. Yellow sphere Ω_P represents the ellipsoid associated with power constraint. In all figures, $\Omega_\mathbf{X} \subset \Omega_P \cap \Omega_\mathbf{A} \cap \Omega_\mathbf{B}$, indicating that the ellipsoid containing constraints and power constraint are satisfied.

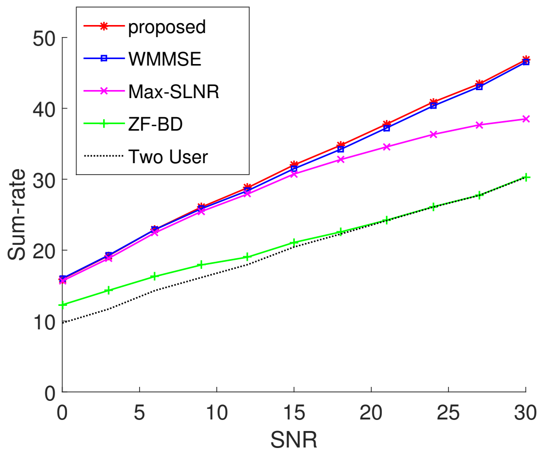
In Fig. 2(a), the axes of $\Omega_\mathbf{X}$ and $\Omega_\mathbf{C}$ coincide. Furthermore, the minor axis of $\Omega_\mathbf{X}$ coincides with the major axis of $\Omega_\mathbf{C}$, and the major axis of $\Omega_\mathbf{X}$ coincides with the minor axis of $\Omega_\mathbf{C}$. The result agrees to the analysis in Section IV-A. In Fig. 2(b), affected by $\Omega_\mathbf{A}$ and $\Omega_\mathbf{B}$, $\Omega_\mathbf{X}$ cannot allocate enough power along $\Omega_\mathbf{C}$'s minor axis. To maximize $|\mathbf{X}+\mathbf{C}|$, $\Omega_\mathbf{X}$ rotates its axes. Two more extreme examples are displayed in Fig. 2(c) and Fig. 2(d). In Fig. 2(c), $\Omega_\mathbf{A}$ is so small that $\Omega_\mathbf{A} \in \Omega_P \cap \Omega_\mathbf{B}$. To maximize $|\mathbf{X}+\mathbf{C}|$, $\Omega_\mathbf{X} = \Omega_\mathbf{A}$. Similarly, in Fig. 2(d), $\Omega_\mathbf{B} \in \Omega_P \cap \Omega_\mathbf{A}$, and $\Omega_\mathbf{X} = \Omega_\mathbf{B}$.

V. Simulation Results

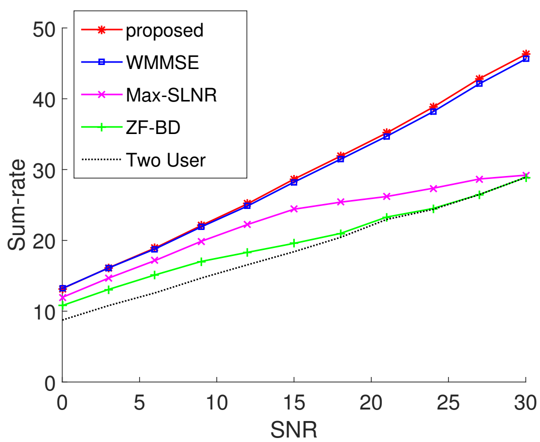
In this section, we analyze the performance of the proposed method by simulations. Three existing methods are simulated for comparison, namely the methods with WMMSE [7], ZF-BD [8] and Max-SLNR [9] criterion.

In this simulation, $m_i = n_i = 4$ in two-plus-one network, and 3GPP channel models [12] are used. As an initialization, the precoding matrices and detection matrices for the two existing users are designed under WMMSE criterion. Then different criterions are used to design the precoding matrix for the newly arrived user, while keeping the two existing users' precoding matrices unchanged. Both weak interference and moderate interference are considered. In weak interference case, SIR = 10 dB, which means that the distance between one user and each of its interfering base stations is about twice as far as the distance between it and its desired base station. In moderate interference case, SIR $= 0$ dB, which means that for each user, its desired base station and interfering base stations are in the same distance.

The sum-rate of the existing users is used as a benchmark to show the increased rate after the third user arrives.



(a) SIR = 10 dB.



(b) SIR = 0 dB.

Fig. 3: Performance comparison among different criterions with SIR = 10 dB and SIR = 0 dB.

The simulation results are shown in Fig. 3. It can be seen that the best performance is obtained by the proposed criterion and WMMSE. The result that the proposed criterion always outperforms ZF-BD comes from the fact that ZF-BD can only utilize the null subspace of the desired signal, while the proposed criterion can extra use part of the subspace of the desired signal without causing much interference.

As described in [7], the optimization problem under WMMSE criterion is non-convex even for a single precoding matrix $\mathbf{V}$, whose solution needs the iterations between $\mathbf{V}$ and detection matrix $\mathbf{U}$. By contrast, the optimization problem we formulated is convex for $\mathbf{V}$, whose optimal solution can be obtained by primal-dual interior-point method [11] without iterations between $\mathbf{V}$ and $\mathbf{U}$. Therefore, though the proposed criterion achieves almost the same performance with WMMSE, the precoder designed under the proposed criterion has lower complexity.

Moreover, based on the geometric interpretation, the proposed criterion can be extended to large-scale networks. In practice the users arrive in network sequentially, for each newly arrived user, the network is a k-plus-one network. The generalization from two-plus-one network to k-plus-one network has been mentioned in Section III-B.

VI. Conclusion

In this paper, we studied the distributed precoding design in a two-plus-one network. Using the geometrical interpretation of the covariance matrices, we discovered the principle that how precoding matrices affect the rate of the desired user and the interfered users. Then we proposed a criterion for precoding design, and formulate an optimization problem. Through the geometric explanation of the optimization result, we obtained insights on how to choose precoding directions and allocate powers under certain constraints. The proposed method achieves good performance with low complexity, and can be extended to large-scale networks.

Acknowledgements

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