Adaptive Channel Feedback for Coordinated Beamforming in Heterogeneous Networks

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***Abstract*—In this paper, a channel direction information (CDI) feedback strategy is proposed to assist coordinated multi-point beamforming (CoMP-CB) between a macro base station (MBS) and multiple pico BSs (PBSs) in heterogeneous networks, where the numbers of bits for feedback are adaptive to the average channel gains of multiple users. To maximize the average sum rate achieved by the users with given feedback resource, we optimize the overall number of feedback bits for each user and the codebook sizes for quantizing the per-cell CDIs of each user. To provide feasible feedback scheme for practice use, a low complexity two-step solution is developed from deriving the lower bounds of the per-user average rate. In the first step, we optimize the number of overall feedback bits for each user, which provides a tradeoff between the accuracy of CDI quantization and the reliability of feedback transmission. In the second step, we optimize the bit allocation for quantizing the per-cell CDIs, which accommodates the difference in transmit powers and antennas of the MBS and PBSs and in the channels of macro and pico cells. Two adaptive feedback schemes are developed from two lower bounds. By characterizing the degree of freedom (DoF) achieved by each user, we show that the systems using the proposed adaptive feedback schemes are no longer interference-limited. The maximum per-user DoFs achieved by the proposed schemes depend on the per-antenna feedback resource constraint for each user. Simulation results show significant performance gain of the proposed scheme over NonCoMP and CoMP-CB with fixed codebook size.**

***Index Terms*—CoMP, HetNet, CDI, limited feedback, DoF.**

I. INTRODUCTION

HETEROGENEOUS networks (HetNets) are promising in improving system spectral efficiency by off-loading heavy traffics [1–3], where a macro base station (MBS) and several inexpensive, low-power BSs are deployed in the same area. HetNet changes the topology and architecture of conventional cellular networks, where the interference among the cells with different coverage becomes more complicated.

Depending on the application scenarios, the low-power base stations (BSs) may include pico BS (PBS), remote radio head, home BS and relay [3, 4]. Some of them are placed by the operators and have open access to all users (e.g., PBSs), while the others are installed by the consumers and have close access only to specific users (e.g., home BSs). The different manners in deployment lead to the difference in interference environments and in the strategies to mitigate the inter-cell interference (ICI). Among all the design challenges arisen in the HetNets [5], one of the key challenges comes from the cross-tier interference between the macro cell and the co-channel deployed pico cells [1, 6–8]. The study in [6] reveals that the cross-tier interference is the bottleneck to improve the performance of the HetNets.

To provide high spectral efficiency, coordinated multi-point (CoMP) transmission [9] can be employed to avoid the cross-tier interference by using multiple antennas at the BSs [4, 5]. When the backhaul links among the coordinated BSs are able to share both the data and the channel information, we can apply CoMP joint processing (CoMP-JP). When the backhaul links are with limited capacity such that the data is not able to be shared, we can employ CoMP coordinated beamforming (CoMP-CB). In the HetNets consisting of MBSs and PBSs, low latency backhaul links could be available for sharing the required information to support CoMP transmission [4], either CoMP-JP or CoMP-CB.

It is well known that the knowledge of channel direction information (CDI) at the BS is essential for downlink beamforming [10]. In frequency division duplexing (FDD) systems, the CDI is first estimated at the users and then its quantized version can be fed back to the BS with finite number of bits via a codebook known at both the BS and each user. The limited feedback techniques have been extensively studied for various multi-input-multi-output (MIMO) systems [10–13], and have been extended to CoMP systems recently, e.g., [14–17]. Yet most existing works suppose fixed codebook size, which has been shown inadequate for multiuser MIMO systems [13]. This is because the codebook size should increase with the signal-to-noise ratio (SNR) to keep a constant rate loss [11], but the SNR varies from user to user.

In this paper, we study adaptive feedback strategy for downlink CoMP-CB in HetNets, where several PBSs are located in the coverage of a MBS to serve multiple users in the same time-frequency resource. By "adaptive", we mean that the number of bits for feedback to quantize the CDI of each user can be adjusted according to its location. In FDD systems, the distortion of the received CDI at the BSs depends on both the quantization error and the uplink transmission error. A large size codebook provides high quality quantization. However, the reliability of feedback transmission may degrade with the given uplink resource due to the increased feedback load. On

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the other hand, a user with high downlink SNR prefers a large size codebook [11], meanwhile the user may also have high uplink SNR and thus high uplink capacity. This indicates that the number of bits for CDI feedback of each user should be optimized according to its uplink and downlink SNRs.

In the considered HetNets, the PBSs are regular BSs with lower transmit powers and less antennas than the traditional MBSs [1]. Moreover, because of lower height of the PBSs, the large-scale fading including path loss and shadowing in the pico cells differ from the macro cells. Consequently, the receive SNRs at each user from the MBS and each PBS may be very different, and the spatial dimension of the channels from the MBS (macro channel) and the PBS (pico channel) to each user may also differ. As a result, equally allocating the bits for quantizing the CDIs of the macro and pico channels is not optimal. The bit allocation between different CDIs for CoMP-CB in homogenous networks was proposed in [14,15], where the imbalanced average channel gains were taken into account, and the BSs have equal number of antennas. In this paper, we optimize the bit allocation by exploiting the distinctive feature of HetNets, which not only lies in the heterogeneity of the SNRs but also in the unequal number of antennas.

The major contributions of this paper are in two-folds:

1) To maximize the average sum rate of the HetNet where each user has identical uplink resource for the CDI feedback, we optimize the overall number of bits for CDI feedback of each user and the number of bits allocated for quantizing the macro and pico CDIs. To reduce the computational complexity of the joint optimization, we propose a two-step solution from deriving the lower bounds of the per-user average rate. In the first step, we optimize the overall number of bits for each user. This achieves a trade off between the accuracy of the CDI quantization and the reliability of the feedback transmission. In the second step, we optimize the bit allocation for quantizing the macro and pico CDIs of each user. This accommodates the heterogeneous feature of the HetNets, i.e., different system settings and channel conditions of the macro and pico cells. Two adaptive feedback schemes are developed.
2) To demonstrate that the system with the proposed feedback strategy is no longer interference-limited, we derive the DoF of each user with identical feedback constraint. We show that the per-user DoF achieved by the optimal solution of the original joint optimization depends on the per-antenna feedback resource constraint. We demonstrate that one of the proposed adaptive feedback scheme achieves the same per-user DoF as the optimal solution and the other only has a minor DoF loss. As a by-product, we derive the number of resource blocks to achieve the per-user DoF of one, which provides a design guideline for the FDD HetNet using CoMP-CB.

The rest of the paper is organized as follows. In Section II, the system model of the considered HetNet is introduced. In Section III, the optimal feedback strategy is addressed. In Section IV, two adaptive feedback schemes are provided. The per-user DoFs of the proposed feedback schemes are analyzed in Section V. Simulation results are provided in Section VI, and the conclusions are given in the last section.

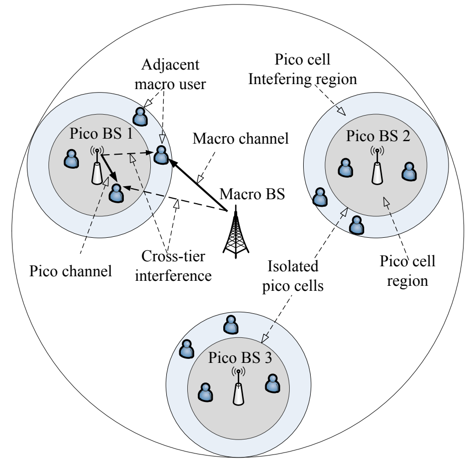

Fig. 1. Illustration of the HetNet: a macro cell covers three pico cells.

Notations: $(\cdot)^H$ is the transpose-conjugate operation, $\|\cdot\|$ is the norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $E\{\cdot\}$ is the expectation over a random variable or vector, $[x]^\dagger = \max(0, x)$, $A\backslash B$ and $A \cup B$ are respectively the set difference and set union operations between sets A and B, $x \perp y$ means $x^H y = 0$, and $\mathbb{G}(a, b)$ denotes a Gamma distribution with shape number a and scalar parameter b.

II. System Model

Consider a downlink HetNet, where a macro cell and L small pico cells serve multiple single antenna users in the same frequency band, as illustrated in Fig. 1. As shown in [1, 6–8], the cross-tier interference (i.e., the interference from a PBS to the macro users adjacent to the pico cell or from the MBS to the pico users) is the bottleneck to improve the spectral efficiency of the HetNets. To capture the essence of the capacity-limiting factor in real-world HetNets, we consider the following network model. Assume that the pico cells are geometrically separated and the interference among the PBSs are negligible. Assume that each PBS serves $K_{\rm L}$ pico users and there are $K_{\rm M}$ macro users in the vicinity of each pico cell suffering from strong interference from the PBS. Other macro users far from the pico cells treat the weak interference from the PBSs as additive noise.

For the nth macro user strongly interfered by the lth PBS, i.e., user $l_n^{\rm M}$, the received signal can be written as

$$y_{l_n^{\rm M}} = \alpha_{m,l_n^{\rm M}} \boldsymbol{h}_{m,l_n^{\rm M}}^H \boldsymbol{v}_{l_n^{\rm M}} x_{l_n^{\rm M}} + \sum_{b_z^{\rm M} \in \Phi_0 \backslash \{l_n^{\rm M}\}} \alpha_{m,l_n^{\rm M}} \boldsymbol{h}_{m,l_n^{\rm M}}^H \boldsymbol{v}_{b_z^{\rm M}} x_{b_z^{\rm M}} + \sum_{l_i^{\rm L} \in \Phi_l} \alpha_{l,l_n^{\rm M}} \boldsymbol{h}_{l,l_n^{\rm M}}^H \boldsymbol{w}_{l_i^{\rm L}} s_{l_i^{\rm L}} + z_{l_n^{\rm M}} \quad (1)$$

where $\boldsymbol{v}_{l_n^{\rm M}}$ and $x_{l_n^{\rm M}}$ are respectively the beamforming vector of unit norm and the data for the macro user at the MBS, $\boldsymbol{w}_{l_i^{\rm L}}$ and $s_{l_i^{\rm L}}$ are respectively the beamforming vector of unit norm and the data for the pico user $l_i^{\rm L}$ at the lth PBS, and $z_{l_n^{\rm M}}$ is the complex white Gaussian noise with variance σ_m^2 at the receiver.

For the ith pico user served by the lth PBS, i.e., user $l_i^{\rm L}$, which is strongly interfered by the MBS, the received signal

TABLE I
KEY PARAMETERS

L	Number of pico cells covered by the macro cell
$P_{\rm M}$ or $P_{\rm L}$	Total transmit power at the MBS or PBS
$N_{\rm M}$ or $N_{\rm L}$	Number of antennas at the MBS or PBS
$K_{\rm L}$	Number of pico users served by each PBS
$K_{\rm M}$	Number of macro users adjacent to each PBS
$l_i^{\rm L}$	The ith pico user served by the lth PBS
$l_n^{\rm M}$	The nth macro user adjacent to the lth PBS
Φ_0	The overall macro user set, which is $\{l_n^{\rm M} \mid 1 \le l \le L, 1 \le n \le K_{\rm M}\}$
Φ_l	The pico user set served by the lth PBS, which is $\{l_i^{\rm L} \mid 1 \le i \le K_{\rm M}\}$
$k \in \cup_{l=0}^{L} \Phi_l$	A user in the HetNet, which can either be a macro user or a pico user
$\alpha_{m,k}$ or $\alpha_{l,k}$	The large-scale fading gain from the MBS or the lth PBS to user k
$\boldsymbol{h}_{m,k}$ or $\boldsymbol{h}_{l,k}$	The small-scale fading gain from the MBS or the lth PBS to user k
$\rho_{m,k}$ or $\rho_{l,k}$	The average downlink receive SNR from the MBS or the lth PBS to user k
γ_k or ρ_k	The instantaneous downlink receive SINR or average downlink receive SNR at user k from its serving BS
$\gamma_{u,k}$ or $\rho_{u,k}$	The instantaneous or average uplink receive SNR at the serving BS from user k

can be written as

$$y_{l_i^{\rm L}} = \alpha_{l,l_i^{\rm L}} \boldsymbol{h}_{l,l_i^{\rm L}}^H \boldsymbol{w}_{l_i^{\rm L}} s_{l_i^{\rm L}} + \sum_{l_j^{\rm L} \in \Phi_l \setminus \{l_i\}} \alpha_{l,l_i^{\rm L}} \boldsymbol{h}_{l,l_i^{\rm L}}^H \boldsymbol{w}_{l_j^{\rm L}} s_{l_j^{\rm L}} + \sum_{b_z^{\rm M} \in \Phi_0} \alpha_{m,l_i^{\rm L}} \boldsymbol{h}_{m,l_i^{\rm L}}^H \boldsymbol{v}_{b_z^{\rm M}} x_{b_z^{\rm M}} + z_{l_i^{\rm L}} \quad (2)$$

where $z_{l_i^{\rm L}}$ is the complex white Gaussian noise with variance σ_l^2 at the receiver.

The main system parameters are listed in Table I.

Assume that each BS equally allocates the total transmit power to its served users, therefore $\mathrm{E}\{|x_{l_n^{\rm M}}|^2\} = \frac{P_{\rm M}}{LK_{\rm M}}$ and $\mathrm{E}\{|s_{l_i^{\rm L}}|^2\} = \frac{P_{\rm L}}{K_{\rm L}}$. Then, the downlink average receive SNRs from the MBS and from the lth PBS to the macro user $l_n^{\rm M}$ are denoted as $\rho_{m,l_n^{\rm M}} = \frac{P_{\rm M}\alpha^2_{m,l_n^{\rm M}}}{LK_{\rm M}\sigma_m^2}$ and $\rho_{l,l_n^{\rm M}} = \frac{P_{\rm L}\alpha^2_{l,l_n^{\rm M}}}{K_{\rm L}\sigma_m^2}$, respectively. The downlink average receive SNRs from the MBS and from the lth PBS to the pico user $l_i^{\rm L}$ are denoted as $\rho_{m,l_i^{\rm L}} = \frac{P_{\rm M}\alpha^2_{m,l_i^{\rm L}}}{LK_{\rm M}\sigma_l^2}$ and $\rho_{l,l_i^{\rm L}} = \frac{P_{\rm L}\alpha^2_{l,l_i^{\rm L}}}{K_{\rm L}\sigma_l^2}$, respectively.

The second terms in (1) and (2) are the multi-user interference (MUI) and the third term is the cross-tier interference, i.e., the ICI. To eliminate the MUI and ICI, we consider CoMP-CB between the MBS and the PBSs using zero forcing beamforming (ZFBF), which is suboptimal but with low complexity [11]. Then, when the MBS serves the $LK_{\rm M}$ macro users, it will avoid generating ICI to the $LK_{\rm L}$ pico users. When each PBS serves the $K_{\rm L}$ pico users, it will null ICI to the adjacent $K_{\rm M}$ macro users. To ensure the feasibility of the multi-cell ZFBF for supporting the maximum number of users, we consider $L(K_{\rm M}+K_{\rm L}) = N_{\rm M}$ and $K_{\rm M}+K_{\rm L} = N_{\rm L}$.

We consider independent and identically distributed (i.i.d.) block Rayleigh fading channels, where the channels remain constant within a block with the duration of one round of uplink feedback and downlink transmission. We assume that the fading channels in the uplink and the downlink are independent since they usually operate at separated frequency bands.

The channel between a user and the MBS is called *macro channel*, and the channel between the user and a PBS is called *pico channel*. For conciseness, we use the notation $k \in \cup_{l=0}^{F} \Phi_l$ to refer to a macro user or a pico user whenever their corresponding expressions can be unified. We use $\boldsymbol{h}_{m,k}$ and $\boldsymbol{h}_{l,k}$ to refer to the *macro* and *pico channel* vectors of user k, and $\bar{\boldsymbol{h}}_{m,k} \triangleq \boldsymbol{h}_{m,k}/\|\boldsymbol{h}_{m,k}\|$ and $\bar{\boldsymbol{h}}_{l,k} \triangleq \boldsymbol{h}_{l,k}/\|\boldsymbol{h}_{l,k}\|$ are the *macro* and *pico CDIs* of user k, respectively.

Assume that each user can obtain perfect channel after downlink channel estimation. The macro and pico CDIs of each user are first quantized through given codebooks with proper sizes. Then, the indices of the quantized CDIs of the user are fed back to its serving BS via uplink transmission.

III. OPTIMAL FEEDBACK STRATEGY

In this section, we study the optimal feedback strategy to maximize the average achievable sum rate of the considered HetNet, where both the CDI quantization distortion and the CDI feedback outage are taken into account. We first present the quantization error model of the CDI, then introduce the uplink transmission outage model for the CDI feedback. Finally, we formulate the problem that jointly optimizes the overall number of feedback bits for each user and the bit allocation for quantizing the macro and pico CDIs.

A. CDI Quantization

Denote the quantized version of the macro and pico CDIs of user k as $\hat{\boldsymbol{h}}_{m,k}$ and $\hat{\boldsymbol{h}}_{l,k}$. Here we take the macro channel as an example to model the CDI quantization error. The relationship between the CDI and its quantized version is [11]

$$\bar{\boldsymbol{h}}_{m,k} = \cos\theta_{m,k} \hat{\boldsymbol{h}}_{m,k} + \sin\theta_{m,k} \boldsymbol{q}_{m,k} \quad (3)$$

where $\boldsymbol{q}_{m,k}$ is the quantization error vector of unit norm and orthogonal to $\hat{\boldsymbol{h}}_{m,k}$ [10], and $\cos\theta_{m,k} = |\bar{\boldsymbol{h}}_{m,k}^H \hat{\boldsymbol{h}}_{m,k}|$ reflects the accuracy of the quantized CDI.

Then the average CDI quantization distortion is defined as

$$\mathrm{E}\{\sin^2\theta_{m,k}\} \triangleq 1 - \mathrm{E}\{|\bar{\boldsymbol{h}}_{m,k}^H \hat{\boldsymbol{h}}_{m,k}|^2\} \quad (4)$$

whose value depends on the employed codebook. Finding an optimal codebook generally requires numerical searching [12]. Moreover, the analytical performance characterization for the optimal codebook is intractable. We therefore consider a quantization cell approximation method used in [10], whose average CDI quantization distortion is

$$\mathrm{E}\{\sin^2\theta_{m,k}\} = \frac{N_{\rm M}-1}{N_{\rm M}} 2^{-\frac{B_{m,k}}{N_{\rm M}-1}} \quad (5)$$

where $B_{m,k}$ is the number of bits for quantizing the macro CDI of user k. It was shown in [10] that the approximation method performs closely to the codebooks optimized for i.i.d. Rayleigh fading channels. It was further observed in [14] that this model of quantization error is valid even when the number of feedback bits is not large.

Similarly, for the pico channel the average CDI quantization distortion is

$$\mathrm{E}\{\sin^2\theta_{l,k}\} = \frac{N_{\rm L}-1}{N_{\rm L}} 2^{-\frac{B_{l,k}}{N_{\rm L}-1}} \quad (6)$$

where $B_{l,k}$ is the number of bits for quantizing the pico CDI of user k.

B. Feedback Outage

The overall number of bits for CDI feedback of user k is B_k, where $B_k = B_{m,k} + B_{l,k}$. Suppose that the available uplink resources for feedback are equally allocated to multiple users, and each user employs the same number of resource blocks, β_S. When $\beta_S > 1$, each block conveys B_k/β_S bits for user k.

Because of channel fading, the instantaneous uplink capacity between user k and its serving BS may not be large enough to feed back these B_k bits. Denote $\gamma_{u,k}$ and $\rho_{u,k}$ respectively as the instantaneous and average uplink receive SNRs at the serving BS of the signals from user k. For conciseness, we express N_k as the number of antennas at the serving BS for user k, where $N_k = N_{\mathrm{M}}$ if user k is a macro user, and $N_k = N_{\mathrm{L}}$ if user k is a pico user. Further considering that the uplink channel is i.i.d. Rayleigh fading, the instantaneous SNR is subject to Gamma distribution, i.e., $\gamma_{u,k} \sim \mathbb{G}(N_k,\ \rho_{u,k})$. Then the feedback outage probability for user k is [18]

$$P(N_k, \nu_k) \triangleq \Pr\{\beta_S \log_2(1+\gamma_{u,k}) < B_k\} = \int_0^{\nu_k} \frac{\gamma_{u,k}^{N_k-1} e^{-\gamma_{u,k}/\rho_{u,k}}}{\rho_{u,k}^{N_k}\Gamma(N_k)} d\gamma_{u,k} \tag{7}$$

where $\nu_k \triangleq 2^{B_k/\beta_S} - 1$ is a SNR threshold, and $\Gamma(\cdot)$ is the Gamma function.

When an outage occurs in the feedback, the overall B_k bits cannot be reliably received by the serving BS. To simplify the analysis, we assume that in this case the BS receives wrong codeword indices for the per-cell CDIs and the resulting CDIs are independent from those obtained from the correctly received CDI quantization. This reflects the worst-case estimation for any well-designed method for index assignment. Under such an assumption, the received macro CDI at the MBS after feedback can be expressed as

$$\check{\boldsymbol{h}}_{m,k} = \begin{cases} \hat{\boldsymbol{h}}_{m,k}, & \text{not in outage} \\ \boldsymbol{e}_{m,k}, & \text{in outage} \end{cases} \tag{8}$$

where $\boldsymbol{e}_{m,k}$ is a random vector. The received pico CDI $\check{\boldsymbol{h}}_{l,k}$ can be represented in the same way.

We can see from (5), (6), and (7) that a large value of the SNR threshold ν_k allows a large number of bits B_k (i.e., a low average CDI quantization distortion), and also leads to a high outage probability for the CDI feedback. It suggests that we should optimize the threshold to trade off the distortion caused by the CDI quantization error and the uplink transmission error. This is equivalent to optimizing the overall number of bits for user k to quantize its macro and pico CDIs.

C. Optimal Feedback Strategy

In the sequel, we jointly optimize the overall number of feedback bits for each user (i.e., the threshold) and the bit allocation to the macro and pico CDIs for the user that maximizes the average sum rate of the HetNet under the constraint of the feedback resource. We will show that the average data rate achieved by each user is independent from other users. Further considering that each user has identical number of resource blocks for the feedback, the problem of maximizing the sum rate of the HetNet is equivalent to the problem of maximizing the achievable rate of each user.

Denote $\mathbf{H}_{m,l^{\mathrm{M}}} = [\check{\boldsymbol{h}}_{m,l_1^{\mathrm{M}}}, \cdots, \check{\boldsymbol{h}}_{m,l_{K_{\mathrm{M}}}^{\mathrm{M}}}]$ as the received channel matrix from the MBS to the K_{M} macro users adjacent to the lth PBS, and $\mathbf{H}_{m,l^{\mathrm{L}}} = [\check{\boldsymbol{h}}_{m,l_1^{\mathrm{L}}}, \cdots, \check{\boldsymbol{h}}_{m,l_{K_{\mathrm{L}}}^{\mathrm{L}}}]$ as the received channel matrix from the MBS to the K_{L} pico users in the lth pico cell. For a macro user l_n^{M}, the MBS computes the multi-cell ZFBF vector $\boldsymbol{v}_{l_n^{\mathrm{M}}}$ by normalizing the $(l-1)K_{\mathrm{M}}+n$th column of $(\mathcal{H}_m\mathcal{H}_m^H)^{-1}\mathcal{H}_m$ after received the quantized CDIs, where $\mathcal{H}_m = [\mathbf{H}_{m,1^{\mathrm{M}}}, \cdots, \mathbf{H}_{m,L^{\mathrm{M}}}, \mathbf{H}_{m,1^{\mathrm{L}}}, \cdots, \mathbf{H}_{m,L^{\mathrm{L}}}]$. Denoting $\gamma_{l_n^{\mathrm{M}}}$ as the instantaneous receive signal to interference-and-noise ratio (SINR) of macro user l_n^{M}, from (1) we obtain the average achievable rate of the macro user as

$$\mathrm{R}_{l_n^{\mathrm{M}}} = \mathrm{E}\{\log_2(1+\gamma_{l_n^{\mathrm{M}}})\} = \mathrm{E}\left\{\log_2\left(1+\frac{\rho_{m,l_n^{\mathrm{M}}} S_{l_n^{\mathrm{M}}}}{1+\rho_{m,l_n^{\mathrm{M}}} I_{l_n^{\mathrm{M}}}^{mu} + \rho_{l,l_n^{\mathrm{M}}} I_{l_n^{\mathrm{M}}}^{co}}\right)\right\} \tag{9}$$

where $S_{l_n^{\mathrm{M}}} = |\boldsymbol{h}_{m,l_n^{\mathrm{M}}}^H \boldsymbol{v}_{l_n^{\mathrm{M}}}|^2$, $I_{l_n^{\mathrm{M}}}^{mu} = \sum_{b_z^{\mathrm{M}} \in \Phi_0 \setminus \{l_n^{\mathrm{M}}\}} |\boldsymbol{h}_{m,l_n^{\mathrm{M}}}^H \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2$ is the power of the MUI, and $I_{l_n^{\mathrm{M}}}^{co} = \sum_{l_i^{\mathrm{L}} \in \Phi_l} |\boldsymbol{h}_{l,m_n^l}^H \boldsymbol{w}_{l_i^{\mathrm{L}}}|^2$ is the power of the ICI from the interfering PBS.

Denote $\mathbf{H}_{l,l^{\mathrm{L}}} = [\check{\boldsymbol{h}}_{l,l_1^{\mathrm{L}}}, \cdots, \check{\boldsymbol{h}}_{l,l_{K_{\mathrm{L}}}^{\mathrm{L}}}]$ as the received channel matrix from the lth PBS to the served K_{L} pico users, and $\mathbf{H}_{l,l^{\mathrm{M}}} = [\check{\boldsymbol{h}}_{l,l_1^{\mathrm{M}}}, \cdots, \check{\boldsymbol{h}}_{l,l_{K_{\mathrm{M}}}^{\mathrm{M}}}]$ as the received channel matrix from the lth PBS to the adjacent K_{M} macro users. For a pico user l_i^{L}, the PBS computes the ZFBF vector $\boldsymbol{w}_{l_i^{\mathrm{L}}}$ by normalizing the ith column of $(\mathcal{H}_l\mathcal{H}_l^H)^{-1}\mathcal{H}_l$ after received the quantized CDIs, where $\mathcal{H}_l = [\mathbf{H}_{l,l^{\mathrm{L}}}, \mathbf{H}_{l,l^{\mathrm{M}}}]$. Denoting $\gamma_{l_i^{\mathrm{L}}}$ as the instantaneous receive SINR of pico user l_i^{L}, from (2) we obtain the average achievable rate of the pico user as

$$\mathrm{R}_{l_i^{\mathrm{L}}} = \mathrm{E}\{\log_2(1+\gamma_{l_i^{\mathrm{L}}})\} = \mathrm{E}\left\{\log_2\left(1+\frac{\rho_{l,l_i^{\mathrm{L}}} S_{l_i^{\mathrm{L}}}}{1+\rho_{l,l_i^{\mathrm{L}}} I_{l_i^{\mathrm{L}}}^{mu} + \rho_{m,l_i^{\mathrm{L}}} I_{l_i^{\mathrm{L}}}^{co}}\right)\right\} \tag{10}$$

where $S_{l_i^{\mathrm{L}}} = |\boldsymbol{h}_{l,l_i^{\mathrm{L}}}^H \boldsymbol{w}_{l_i^{\mathrm{L}}}|^2$, $I_{l_i^{\mathrm{L}}}^{mu} = \sum_{l_j^{\mathrm{L}} \in \Phi_l \setminus \{l_i^{\mathrm{L}}\}} |\boldsymbol{h}_{l,l_i^{\mathrm{L}}}^H \boldsymbol{w}_{l_j^{\mathrm{L}}}|^2$ is the power of the MUI, and $I_{l_i^{\mathrm{L}}}^{co} = \sum_{b_z^{\mathrm{M}} \in \Phi_0} |\boldsymbol{h}_{m,l_i^{\mathrm{L}}}^H \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2$ is the power of the ICI from the MBS.

Considering the impact of the outage in uplink feedback, the average achievable rate for user k can be obtained as follows

$$\mathrm{R}_k = (1 - P(N_k, \nu_k))\mathrm{R}_k^{no} + P(N_k, \nu_k)\mathrm{R}_k^{ou} \tag{11}$$

where

$$\mathrm{R}_k^{no} = \mathrm{E}\Big\{\log_2(1+\gamma_k)\,|\check{\boldsymbol{h}}_k = \hat{\boldsymbol{h}}_k\Big\}$$
$$\mathrm{R}_k^{ou} = \mathrm{E}\Big\{\log_2(1+\gamma_k)\,|\check{\boldsymbol{h}}_k \neq \hat{\boldsymbol{h}}_k\Big\}$$

are respectively the average rate of user k achieved when the outage does not occur and the outage occurs in the CDI feedback, and user k can either be a macro user ($\gamma_k = \gamma_{l_n^{\mathrm{M}}}$) or a pico user($\gamma_k = \gamma_{l_i^{\mathrm{L}}}$). Since a random codeword is employed when an outage occurs as shown in (8), R_k^{ou} is equal to a special case of R_k^{no} with $B_{m,k} = 0$ and $B_{l,k} = 0$.

As shown in Appendix A, when $N_{\mathrm{M}} = L(K_{\mathrm{M}} + K_{\mathrm{L}})$ and $N_{\mathrm{L}} = K_{\mathrm{M}} + K_{\mathrm{L}}$, the average rate achieved by user k without the outage in the CDI feedback is

$$\mathrm{R}_k^{no} = c_0 \int_0^\infty \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma} \Delta_k(B_{m,k}, B_{l,k}) d\gamma \quad (12)$$

where $c_0 = \log_2 e$, for a macro user, $k = l_n^{\mathrm{M}}$, $\rho_k = \rho_{m,l_n^{\mathrm{M}}}$, and

$$\begin{aligned}&\Delta_k(B_{m,k}, B_{l,k}) \\ &= (1+\gamma 2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}})^{-LK_{\mathrm{M}}+1}(1+\frac{\rho_{l,l_n^{\mathrm{M}}}}{\rho_{m,l_n^{\mathrm{M}}}}\gamma 2^{\frac{-B_{l,l_n^{\mathrm{M}}}}{N_{\mathrm{L}}-1}})^{-K_{\mathrm{L}}} \quad (13)\end{aligned}$$

and for a pico user, $k = l_i^{\mathrm{L}}$, $\rho_k = \rho_{l,l_i^{\mathrm{L}}}$, and

$$\begin{aligned}&\Delta_k(B_{m,k}, B_{l,k}) \\ &= (1+\gamma 2^{\frac{-B_{l,l_i^{\mathrm{L}}}}{N_{\mathrm{L}}-1}})^{-K_{\mathrm{L}}+1}(1+\frac{\rho_{m,l_i^{\mathrm{L}}}}{\rho_{l,l_i^{\mathrm{L}}}}\gamma 2^{\frac{-B_{m,l_i^{\mathrm{L}}}}{N_{\mathrm{M}}-1}})^{-LK_{\mathrm{M}}} \quad (14)\end{aligned}$$

When an outage occurs in the CDI feedback, the average rate achieved by user k can be obtained as a special case of R_k^{no} by setting $B_{m,k} = 0$ and $B_{l,k} = 0$, i.e.,

$$\mathrm{R}_k^{ou} = c_0 \int_0^\infty \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma} \Delta_k(0,0) d\gamma \quad (15)$$

which is a constant irrespective of the feedback.

After substituting (12) and (15) into (11), we can obtain the average achievable rate for user k. From (12)-(15), we can see that the average rate achieved by user k only depends on its own channels (i.e., ρ_k) and codebook sizes (i.e., $B_{m,k}, B_{l,k}$), which means that its average rate is independent from that of the other users. This suggests that we can maximize the achievable sum rate of the HetNet by maximizing the individual data rate achieved by each user.

The optimal feedback strategy to maximize the average achievable rate for each user under the feedback resource constraint is given by

$$\begin{aligned}\max_{B_{m,k},\ B_{l,k},\ \nu_k} \ & (1 - P(N_k, \nu_k))\mathrm{R}_k^{no} + P(N_k, \nu_k)\mathrm{R}_k^{ou} \quad (16)\\ \text{s.t.} \ & B_{m,k} + B_{l,k} = \beta_S \log_2(1+\nu_k), \\ & B_{m,k},\ B_{l,k}, \nu_k \geq 0\end{aligned}$$

where the uplink SNR threshold ν_k determines the overall number of bits for the CDI feedback of user k, and the variables $B_{m,k}$ and $B_{l,k}$ decide the bit allocation to quantize the macro and pico CDIs for the user.

The explicit expression of the outage probability in (7) is given in [19], and the explicit expressions of (12) and (15) are given in Lemma 3 in [20], which are all in a complicate form of the sum of multiple terms and not provided here for conciseness. To find the optimal solution of problem (16), we need to search exhaustively over multiple-dimension space, which is of prohibited complexity. The resulting optimal feedback strategy is infeasible for practical us, but can provide an achievable upper bound for the adaptive feedback schemes.

In next section, we will find suboptimal solutions for the original problem, problem (16), with low complexity by maximizing the lower bounds of the achievable rate.

IV. Adaptive Feedback Schemes

In this section, we formulate a new optimization problem, where the objective function of the original optimization problem (16) is replaced by a lower bound of R_k. We will show that the optimal solution of this new problem can be found by using two steps without iterations meanwhile not losing the global optimality of the new problem.

We start by deriving a lower bound of the average rate achieved by user k for the non-outage case of CDI feedback. Then, we optimize the bit allocation for a given threshold. Finally, we provide two solutions to optimize the threshold.

As shown in Appendix B, the lower bound of $\Delta_k(B_{m,k}, B_{l,k})$ is

$$\Delta_k(B_{m,k}, B_{l,k}) \geq (1 + \frac{\gamma}{K}\delta_k(B_{m,k}, B_{l,k}))^{-K} \quad (17)$$

where $K = LK_{\mathrm{M}} + K_{\mathrm{L}} - 1$, and

$$\delta_k(B_{m,k}, B_{l,k}) = \mu_{m,k} 2^{\frac{-B_{m,k}}{N_{\mathrm{M}}-1}} + \mu_{l,k} 2^{\frac{-B_{l,k}}{N_{\mathrm{L}}-1}} \quad (18)$$

For a macro user, $k = l_n^{\mathrm{M}}$, $\mu_{m,k} = LK_{\mathrm{M}} - 1$, $\mu_{l,k} = K_{\mathrm{L}} \frac{\rho_{l,l_n^{\mathrm{M}}}}{\rho_{m,l_n^{\mathrm{M}}}}$. For a pico user, $k = l_i^{\mathrm{L}}$, $\mu_{m,k} = LK_{\mathrm{M}} \frac{\rho_{m,l_i^{\mathrm{L}}}}{\rho_{l,l_i^{\mathrm{L}}}}$, $\mu_{l,k} = K_{\mathrm{L}} - 1$.

Further using (17), (11) and (12), a lower bound of the average achievable rate of user k is obtained as follows

$$\begin{aligned}\mathrm{R}_k \geq & (1 - P(N_k, \nu_k)) c_0 \int_0^\infty \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}(1+\frac{\gamma}{K}\delta_k(B_{m,k}, B_{l,k}))^{-K} d\gamma \\ & + P(N_k, \nu_k)\mathrm{R}_k^{ou} \quad (19)\end{aligned}$$

Now, we can formulate the new optimization problem, where the objective function is the lower bound of R_k in (19) and the constraints remain the same as in problem (16).

In (19), the term R_k^{ou} does not depend on the optimization variables ν_k, $B_{m,k}$ and $B_{l,k}$, as shown in (15). The term $P(N_k, \nu_k)$ only depends on ν_k but not on $B_{m,k}$ and $B_{l,k}$, as shown in (7). The term $\delta_k(B_{m,k}, B_{l,k})$ only depends on $B_{m,k}$ and $B_{l,k}$, as shown in (18). As a result, we can find the solution of the new problem using a two-step optimization. Specifically, for a given threshold ν_k, we can find the optimal values of $B_{m,k}$ and $B_{l,k}$, which will be expressed as explicit functions of ν_k. If the threshold ν_k can be optimized, the bit allocation $B_{m,k}$ and $B_{l,k}$ will also be optimized.

A. Bit Allocation Optimization For a Given Threshold

We first find the solution of the bit allocation in the new optimization problem with a given threshold ν_k. To do this, we only need to minimize the term $\delta_k(B_{m,k}, B_{l,k})$ inside (19), because other terms does not depend on the optimization variables $B_{m,k}$ and $B_{l,k}$. From (18), the resulting problem to optimize the bit allocation can be formulated as follows

$$\begin{aligned}\min_{B_{m,k},\ B_{l,k}} \ & \mu_{m,k} 2^{\frac{-B_{m,k}}{N_{\mathrm{M}}-1}} + \mu_{l,k} 2^{\frac{-B_{l,k}}{N_{\mathrm{L}}-1}} \quad (20)\\ \text{s.t.} \ & B_{m,k} + B_{l,k} = \beta_S \log_2(1+\nu_k), \\ & B_{m,k} \geq 0, B_{l,k} \geq 0\end{aligned}$$

This is a convex optimization problem. First, the constraints are linear and thus convex. Second, the objective function is a weighted sum of exponential functions with positive weights, which are also convex. The solution can be obtained from solving Karush-Kuhun-Tucker (KKT) condition, which turns into a water-filling algorithm with a unique solution as follows

$$B_{m,k} = (N_{\mathrm{M}} - 1)\Big[\log_2 \mu_{m,k} - \log_2(\lambda(N_{\mathrm{M}} - 1))\Big]^{\dagger}$$
$$B_{l,k} = (N_{\mathrm{L}} - 1)\Big[\log_2 \mu_{l,k} - \log_2(\lambda(N_{\mathrm{L}} - 1))\Big]^{\dagger} \quad (21)$$

where λ is chosen such that $B_{m,k} + B_{l,k} = \beta_S \log_2(1 + \nu_k)$.

Note that the bit allocation accommodates the feature of HetNets, i.e., the unequal receive SNRs in macro and pico channels reflected in the parameter $\mu_{l,k}$ and $\mu_{m,k}$, and the non-identical number of antennas at the MBS and PBS N_{M} and N_{L}. Depending on the receive SNRs of the users, the bit allocation results have two possibilities as follows.

1) Case 1 (Non-zero bit allocation): Both the macro and pico CDIs are allocated with non-zero bits for feedback. In this case, the operation $[\cdot]^{\dagger}$ can be removed. By finding λ and then substituting it into (21), we can obtain explicit expressions for the optimal solution of problem (20), which are

$$B_{m,k} = (N_{\mathrm{M}} - 1)\phi_k + \varphi \log_2 \frac{(N_{\mathrm{L}}-1)\mu_{m,k}}{(N_{\mathrm{M}}-1)\mu_{l,k}}$$
$$B_{l,k} = (N_{\mathrm{L}} - 1)\phi_k + \varphi \log_2 \frac{(N_{\mathrm{M}}-1)\mu_{l,k}}{(N_{\mathrm{L}}-1)\mu_{m,k}} \quad (22)$$

where $\phi_k = \frac{\beta_S \log_2(1+\nu_k)}{N_{\mathrm{M}}+N_{\mathrm{L}}-2}$ and $\varphi = \frac{(N_{\mathrm{M}}-1)(N_{\mathrm{L}}-1)}{N_{\mathrm{M}}+N_{\mathrm{L}}-2}$.

2) Case 2 (Zero bit allocation): Only one CDI (either the macro CDI or the pico CDI) of the user is fed back to its serving BS. The allocation results are

$$B_{m,k} = \beta_S \log_2(1 + \nu_k), \qquad B_{l,k} = 0 \quad (23)$$

or

$$B_{m,k} = 0, \qquad B_{l,k} = \beta_S \log_2(1 + \nu_k) \quad (24)$$

In fact, in this case the user prefers Non-CoMP transmission.

Remind that only the users experienced strong interference are considered in this work as shown in the system model, where the users prefer CoMP transmission, we will only study the first case in the sequel. Applying $B_{m,k} > 0$ and $B_{l,k} > 0$ to (22), we obtain the condition that Case 1 is true as follows

$$(1 + \nu_k)^{\frac{-\beta_S}{N_{\mathrm{L}}-1}} < \frac{(N_{\mathrm{L}}-1)\mu_{m,k}}{(N_{\mathrm{M}}-1)\mu_{l,k}} < (1 + \nu_k)^{\frac{\beta_S}{N_{\mathrm{M}}-1}}$$

which depends on the location of user k.

B. Threshold Optimization

Then, we optimize the overall number of bits for the CDI feedback by finding an optimal threshold ν_k.

We provide two solutions of ν_k for the new optimization problem with the optimized bit allocation. One is the optimal solution of the new problem, which is found via one-dimensional searching. The other is in closed form, which is desirable for practical use but with minor performance loss.

1) Solution A: By substituting the results of optimal bit allocation (BA) in (22) into (18), we obtain

$$\delta_k(B_{m,k}, B_{l,k}) = g_k^{no}(1 + \nu_k)^{-\beta_0} \triangleq \delta_k^{BA}(\nu_k) \quad (25)$$

where $g_k^{no} = (N_{\mathrm{M}} + N_{\mathrm{L}} - 2)(\frac{\mu_{m,k}}{N_{\mathrm{M}}-1})^{\frac{N_{\mathrm{M}}-1}{N_{\mathrm{M}}+N_{\mathrm{L}}-2}}(\frac{\mu_{l,k}}{N_{\mathrm{L}}-1})^{\frac{N_{\mathrm{L}}-1}{N_{\mathrm{M}}+N_{\mathrm{L}}-2}}$, and $\beta_0 = \frac{\beta_S}{N_{\mathrm{M}}+N_{\mathrm{L}}-2}$ can be viewed as a feedback resource constraint on each antenna.

By using (25), (17) and (12), the lower bound of the average rate achieved by user k without the outage in feedback after the optimal bit allocation is given by

$$R_{k,lb}^{no,BA}(\nu_k) = c_0 \int_0^{\infty} \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}(1 + \frac{\gamma}{K}\delta_k^{BA}(\nu_k))^{-K} d\gamma \quad (26)$$

Then from (11) the resulting lower bound of the average rate achieved by user k after the optimal bit allocation becomes

$$R_{k,lb1}^{BA}(\nu_k) \triangleq (1-P(N_k,\nu_k))R_{k,lb}^{no,BA}(\nu_k) + P(N_k,\nu_k)\mathrm{R}_k^{ou} \quad (27)$$

The threshold ν_k to maximize $R_{k,lb1}^{BA}$ can be found from the following problem

$$\max_{\nu_k} \quad R_{k,lb1}^{BA}(\nu_k) \quad (28)$$
$$\text{s.t.} \quad \nu_k > 0$$

The optimal solution of this problem, $\nu_{k,o1}$, can be obtained from one-dimensional searching.

Note that $\nu_{k,o1}$ is the globally optimal solution of the new problem defined after (19) after the two-step optimization in problem (20) and problem (28).

2) Solution B: To obtain a closed form solution for the threshold, we further derive a lower bound for $R_{k,lb1}^{BA}(\nu_k)$. By setting $B_{m,k} = 0$ and $B_{l,k} = 0$ in (18), from (17) we obtain

$$\Delta_k(0,0) \geq (1 + \frac{\gamma}{K} g_k^{ou})^{-K}$$

where $g_k^{ou} = \mu_{m,k} + \mu_{l,k}$.

Applying this inequality to (15), the resulting lower bound of the average rate achieved by user k with the outage in feedback is obtained as

$$R_{k,lb}^{ou} = \int_0^{\infty} \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}(1 + \frac{\gamma}{K} g_k^{ou})^{-K} d\gamma \quad (29)$$

By using this lower bound of R_k^{ou}, from (27) we can derive another lower bound of the average rate achieved by user k after the optimal bit allocation as follows

$$R_{k,lb1}^{BA}(\nu_k) \geq (1 - P(N_k,\nu_k))R_{k,lb}^{no,BA}(\nu_k) + P(N_k,\nu_k)\mathrm{R}_{k,lb}^{ou}$$
$$= c_0 \int_0^{\infty} \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}\Big[(1-P(N_k,\nu_k))\Big(1+\frac{\gamma}{K}\delta_k^{BA}(\nu_k)\Big)^{-K} + P(N_k,\nu_k)\Big(1 + \frac{\gamma}{K} g_k^{ou}\Big)^{-K}\Big] d\gamma$$
$$\geq c_0 \int_0^{\infty} \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}\Big[1+\frac{\gamma}{K}\big((1 - P(N_k,\nu_k))\delta_k^{BA}(\nu_k) + P(N_k,\nu_k) g_k^{ou}\big)\Big]^{-K} d\gamma \quad (30)$$
$$\geq c_0 \int_0^{\infty} \frac{e^{\frac{-\gamma}{\rho_k}}}{1+\gamma}\Big[1 + \frac{\gamma}{K}\big(\delta_k^{BA}(\nu_k) + \frac{g_k^{ou}(1+\nu_k)^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}}\big)\Big]^{-K} d\gamma$$
$$\triangleq R_{k,lb2}^{BA}(\nu_k) \quad (31)$$

where in (30) the inequality $(1-a)(1+x)^{-K}+a(1+y)^{-K} \geq [1+(1-a)x+ay]^{-K}$ for $x, y > 0$ and $0 \leq a \leq 1$ are applied, and the last inequality is because $P(N_k, \nu_{l_k}) \geq 0$ and $P(N_k, \nu_k) \leq \frac{(1+\nu_k)^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}}$ as shown in (D.4).

It is worthy to note that the impact of the feedback outage is over-estimated in the lower bound $R_{k,lb2}^{BA}(\nu_k)$ in (31) because the upper bound of the outage probability is employed. When $P(N_k, \nu_k)$ is close to one, $R_{k,lb2}^{BA}(\nu_k)$ may become loose. Fortunately, the case of $P(N_k, \nu_k)$ closing to one rarely happens, because ν_k is optimize towards maximizing the average rate achieved by each user. If the value of $P(N_k, \nu_k)$ is very large, the received CDI will be with very low quality and the resulting achievable rate will be very low. This is just the case we try to avoid through the threshold optimization.

The threshold ν_k to maximize $R_{k,lb2}^{BA}(\nu_k)$ can be found from the following problem

$$\min_{\nu_k} \quad \delta_k^{BA}(\nu_k) + \frac{g_k^{ou}(1+\nu_k)^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}} \qquad \text{s.t.} \quad \nu_k > 0 \tag{32}$$

By setting the derivative of the objective function as zero, we obtain a closed form solution of the threshold as follows

$$\nu_{k,o2} = \left(\frac{g_k^{no}\beta_0\Gamma(N_k)}{g_k^{ou}}\right)^{\frac{1}{N_k+\beta_0}} \rho_{u,k}^{\frac{N_k}{N_k+\beta_0}} - 1 \tag{33}$$

As we will show, the solution in (33) only has a minor performance loss in per-user DoF.

C. Feedback Procedure

In practice, the low complexity adaptive feedback schemes for user k can be implemented as follows.

1) Find the threshold $\nu_k = \nu_{k,o1}$ numerically from (28) or $\nu_k = \nu_{k,o2}$ from (33). Then the number of bits that can be reliably fed back is $\beta_s \log_2(1+\nu_k)$, which is rounded to the nearest integer.
2) Allocate the number of bits for quantizing the macro and pico CDIs with (22) for the given threshold ν_k obtained from the previous step, where $B_{m,k}$ and $B_{l,k}$ are rounded to the nearest integers.

We call the scheme with threshold $\nu_{k,o1}$ and bit allocation in (22) as the adaptive feedback scheme A, and the scheme with threshold $\nu_{k,o2}$ and bit allocation in (22) as the adaptive feedback scheme B. The optimal solution of problem (16) is referred to as the optimize feedback scheme.

V. DoF Analysis of the Proposed Feedback Schemes

In contrast to the limited feedback schemes for multiuser MIMO systems with fixed size codebooks [12], where a ceiling on average rate exists when the SNR increases [11], the proposed schemes allow the overall number of feedback bits and the numbers of bits for macro and pico CDIs of each user adaptive to its uplink and downlink SNRs, i.e., its location. By optimizing the CDI quantization and feedback transmission, the systems with the proposed feedback schemes will no longer be limited by the cross-tier interference.

To show this, we derive the DoF achieved by each user, which can provide useful insight for understanding the potential of the systems in dealing with various interference [21–24] and reflect the per-user rate in high SNR region. The SNR will be high in prevalent cellular systems if the inter-cluster interference can be reduced by interference avoidance techniques [5]. If the inter-cluster interference can not avoided and simply treated as additive noise, the SNR may be low as pointed out in [25]. Nonetheless, the DoF analysis is able to demonstrate that the proposed adaptive feedback strategy can thoroughly avoid the cross-tier interference in the HetNets with imperfect CDI.

As shown in (7) and (11), the average rate achieved by user k depends not only on the downlink average receive SNRs $\rho_{m,k}$ from the MBS and $\rho_{l,k}$ from the pico BS, but also on the uplink average receive SNR $\rho_{u,k}$. To facilitate the analysis of per-user DoF that is defined for high downlink SNRs, we assume that $\rho_{l,k}$ and $\rho_{u,k}$ are related with $\rho_{m,k}$ as follows,

$$\rho_{l,k} = \kappa_{d,k}\rho_{m,k}, \quad \rho_{u,k} = \kappa_{u,k}\rho_{m,k} \tag{34}$$

where $\kappa_{d,k}$ and $\kappa_{u,k}$ are constants reflecting the receive SNR differences in various channels.

Then, the per-user DoF for the macro user ($k = l_n^{\mathrm{M}}$) and the pico user ($k = l_i^{\mathrm{L}}$) are respectively defined as

$$\mathrm{DoF}_{l_n^{\mathrm{M}}} = \lim_{\rho_{m,l_n^{\mathrm{M}}}\to\infty} \frac{\mathrm{R}_{l_n^{\mathrm{M}}}}{\log_2 \rho_{m,l_n^{\mathrm{M}}}}$$
$$\mathrm{DoF}_{l_i^{\mathrm{L}}} = \lim_{\rho_{l,l_i^{\mathrm{L}}}\to\infty} \frac{\mathrm{R}_{l_i^{\mathrm{L}}}}{\log_2 \rho_{l,l_i^{\mathrm{L}}}} = \lim_{\rho_{m,l_i^{\mathrm{L}}}\to\infty} \frac{\mathrm{R}_{l_i^{\mathrm{L}}}}{\log_2 \rho_{m,l_i^{\mathrm{L}}}} \tag{35}$$

where the last quality is by using $\rho_{l,l_i^{\mathrm{L}}} = \kappa_{d,l_i^{\mathrm{L}}}\rho_{m,l_i^{\mathrm{L}}}$.

When perfect CDI is used for ZFBF, it can be readily verified that the per-user DoF equals to one since the interference is completely eliminated. For limited feedback CoMP-CB systems, the per-user DoF of one can not always be achieved as will be shown in the sequel.

We start by characterizing the per-user DoF of the optimal feedback scheme. Then, we derive the per-user DoFs achieved by the two adaptive feedback schemes.

Lemma 1: The DoF achieved by user k with the optimal feedback scheme is upper bounded as follows

$$\mathrm{DoF}_k^O \leq \min\{\beta_0, 1\} \tag{36}$$

Proof: See Appendix C.

From the analysis in Appendix C, we know that this DoF upper bound can be approached by increasing the number of antennas of the serving BS of the user. When $N_k \to \infty$, the feedback channel degrades to a deterministic channel [26] and the outage probability becomes zero. Then, the upper bound is achieved.

Lemma 2: The DoF achieved by user k with the adaptive feedback scheme A is lower bounded as follows

$$\mathrm{DoF}_k^A \geq \min\{\beta_0, 1\} \tag{37}$$

Proof: See Appendix D.

Theorem 1: The DoF achieved by user k with the adaptive feedback scheme A is the same as the DoF achieved by the user with the optimal feedback scheme, i.e.,

$$\mathrm{DoF}_k^A = \mathrm{DoF}_k^O = \min\{\beta_0, 1\} \tag{38}$$

Proof: Since $\mathrm{DoF}_k^A \leq \mathrm{DoF}_k^O$, from Lemmas 1 and 2, we immediately prove the Theorem 1.

The achieved DoF only depends on the per-antenna feedback resource constraint β_0, which is a pre-determined value as shown in (25), but does not depend on the threshold (or equivalently, the uplink and downlink SNRs). Therefore, the Theorem implies that the impact of feedback outage on the per-user DoF will vanish if we can properly select the number of overall feedback bits and allocate the bits for quantizing the macro and pico CDIs.

By setting the achieved per-user DoF as one, we can find from the Theorem that to allow user k not losing per-user DoF from the CDI distortion, the feedback resources should satisfy $\beta_0 = \frac{\beta_S}{N_\mathrm{M}+N_\mathrm{L}-2} = 1$, or equivalently $\beta_S = N_\mathrm{M} + N_\mathrm{L} - 2$. This indicates that by properly designing the number of resource blocks for the CDI feedback of each user, the proposed adaptive feedback scheme A has no DoF loss.

Theorem 2: The DoF achieved by user k with the adaptive feedback scheme B for is lower bounded as follows

$$\mathrm{DoF}_k^B \geq \min\{\frac{1}{\frac{1}{N_k} + \frac{1}{\beta_0}}, 1\} \tag{39}$$

Proof: See Appendix E.

The lower bound of this DoF is close to the upper bound of the DoF in Theorem 1 when N_k is large. For example, for $\beta_0 = 0.5$, when $N_k = 4$ the lower bound becomes $\frac{1}{\frac{1}{N_k}+\frac{1}{\beta_0}} \approx 0.44$, and when $N_k = 8$ the lower bound becomes $\frac{1}{\frac{1}{N_k}+\frac{1}{\beta_0}} \approx 0.47$, both close to the upper bound $\mathrm{DoF}_k^O = 0.5$. This implies that if a minor performance loss is allowed, the closed form solution for the threshold provides a near-optimal performance.

VI. Simulation Results

In this section, we evaluate the adaptive feedback schemes by comparing with the optimal and existing feedback schemes and verify the asymptotical analysis through simulations.

A. Simulation Settings

The considered setting of the HetNet in the simulation is from [3]. Specifically, the transmit powers of the macro and pico BSs are respectively 46 dBm and 30 dBm, the uplink transmit power is 23 dBm at both the macro and pico users, and the receiver noise power is -75 dBm. The large-scale path loss model of the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6 \log_{10}(d_0/10^3)$ and $PL(dB) = 140.7 + 36.7 \log_{10}(d_0/10^3)$, where d_0 is the distance of a user to the BS. The minimum distances of the pico and maco users to the pico and macro BSs are 10 m and 35 m, respectively.

The topology of the HetNet is shown in Fig. 2, where a macro cell covers two pico cells. The macro cell radius is set as 250 m, the pico cell radius is 40 m, and the distance between the MBS and PBS is 210 m. The MBS and the PBSs are respectively equipped with eight and four antennas. Each PBS serves two pico users, and in the vicinity of each PBS there are two macro users. The users are located on the line connecting the MBS and PBS with a distance of d m from the MBS, where the two macro users close to one pico cell have the same distance and the two pico users in one pico cell also have the same distance to the PBS. In this way, we only need to show the results for one of the macro users and one of the pico users. The channel vectors of the users are independent from each other. The channels are subject to i.i.d. flat Rayleigh fading, and all the results are averaged over 10^4 channel realizations.

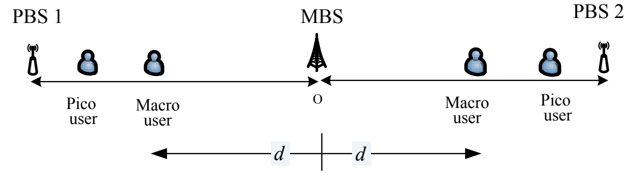

Fig. 2. Topology of the HetNet in the simulation.

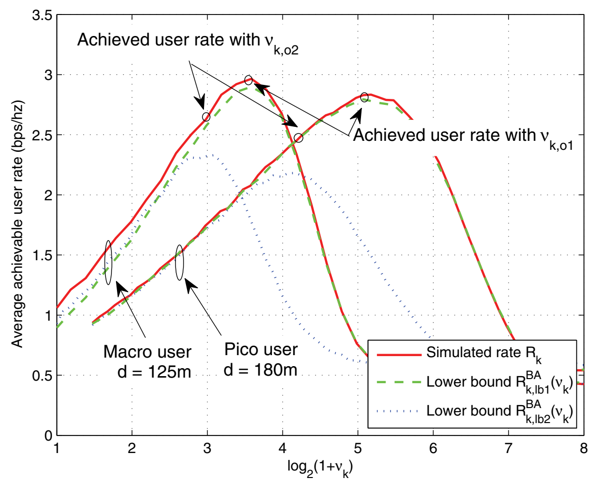

Fig. 3. Tightness of the lower bounds and the tradeoff between the CDI quantization error and the feedback transmission error.

We set the per-user feedback resource constraint as $\beta_S = 12$. Then the overall number of bits for the CDI feedback of user k is equal to $\beta_S \log_2(1 + \nu_k)$. Similar results can be obtained for other parameters. Random vector quantization [11] is applied for the CDI quantization.

Unless otherwise specified, the above-mentioned parameters will be applied in the simulations.

B. Tightness of the Lower Bounds

To evaluate the tightness of the derived lower bounds and to illustrate the tradeoff between the CDI quantization error and the feedback transmission error, we consider the scenarios where the macro users are located at $d = 125$ m, and the pico users are located at $d = 180$ m. Similar results can be observed for the users at the other locations.

In Fig. 3, the simulated average per-user rate R_k computed from (11), the lower bound $R_{k,lb1}^{BA}(\nu_k)$ numerically obtained from (27) and the lower bound $R^{BA}_{k,lb2}(\nu_k)$ from (31) are provided versus different values of ν_k, all with optimized bit allocation. The simulated R_k is obtained with the bit allocation that is found from exhaustive searching the values of R_k for the given values of ν_k. The bit allocation in (22) is used for the two numerical results.

It is shown that for both the macro and pico users, the first lower bound $R^{BA}_{k,lb1}(\nu_k)$ is very close to the simulated average per-user rate R_k. The tightness of the second lower bound $R^{BA}_{k,lb2}(\nu_k)$ depends on the value of the threshold. When ν_k is small where the outage in the feedback is low, the value of $R^{BA}_{k,lb2}(\nu_k)$ is close to the simulated average per-user rate. Otherwise, the value of $R^{BA}_{k,lb2}(\nu_k)$ has a performance gap to the simulated rate R_k. As a result, with the adaptive feedback scheme B, there will be an average rate loss for both the macro and pico users. For the macro user, the average achievable rate using $\nu_{k,o2}$ will be 0.4 bps less than the average achievable rate using $\nu_{k,o1}$.

It is also shown that the maximum average achievable rates of the macro and pico users are neither achieved at very large value of ν_k where the outage probability is high, nor achieved at very small value of ν_k where the CDI quantization is less accurate. The results demonstrate the need for selecting a proper overall number of feedback bits for each user.

C. Average Rate Comparison of Different Feedback Schemes

Now we evaluate the performance of the proposed adaptive feedback schemes by comparing with other feedback schemes. The considered schemes are as follows:

- Perfect CDI: The average user rate is obtained by using perfect CDI at the BSs.
- Optimal threshold ν_k and bit allocation: the results are obtained by exhaustively searching (ES), i.e., the optimal feedback scheme.
- Threshold $\nu_{k,o1}$ and bit allocation: i.e., the adaptive feedback scheme A.
- Threshold $\nu_{k,o2}$ and bit allocation: i.e., the adaptive feedback scheme B.
- Threshold $\nu_{k,o1}$ and equal bit allocation: The overall number of bits is obtained by using $\nu_{k,o1}$ from problem (28). Then, all the bits for each user are equally allocated between the macro and pico CDIs.
- CoMP with fixed size codebook and equal bit allocation: The overall number of bits is fixed and equally allocated to all users, and all the bits for each user are equally allocated between the macro and pico CDIs. This is the conventional CoMP-CB with fixed size codebook. To ensure nearly error-free feedback, one bit per resource block is considered, and thus in total 12 bits are employed for feeding back the CDI of each user.
- NonCoMP with fixed size codebook: As a baseline, we provide the results for a NonCoMP system where the macro and pico BSs serve their users independently and all the 12 bits are allocated to the macro or the pico CDI.

In Fig. 4, we show the average achievable rate of the macro users with different distances to the MBS, where we set $d > 95$ m to avoid zero bit allocation in (21). We can observe substantial performance gain of the proposed feedback

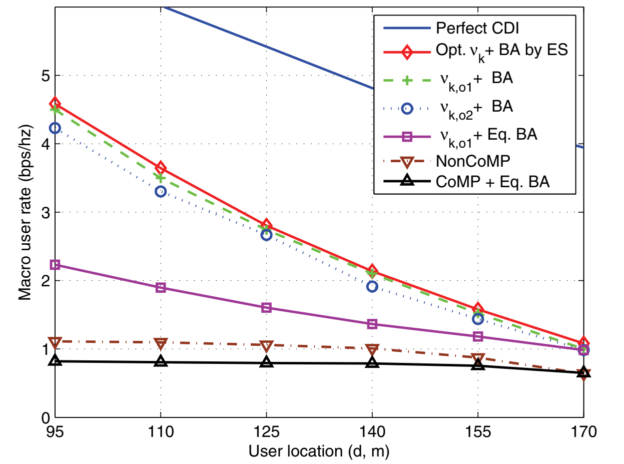

Fig. 4. Average achievable rates of the macro users in different locations.

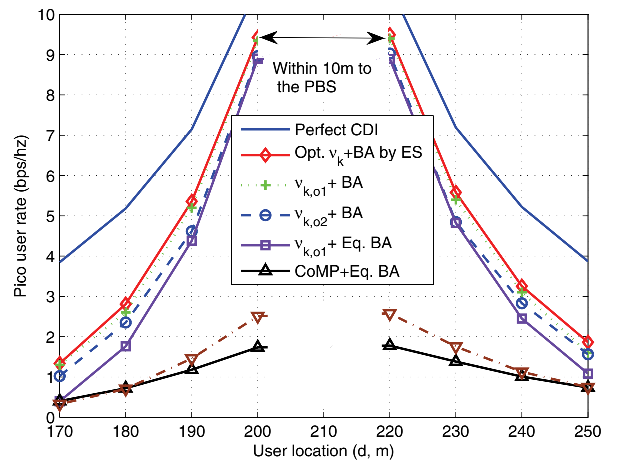

Fig. 5. Average achievable rates of the pico users in different locations.

schemes over the NonCoMP system and the CoMP-CB system with fixed codebook size. With the thresholds of $\nu_{k,o1}$ and $\nu_{k,o2}$, the average rates achieved by the adaptive feedback schemes are only slightly less than that of the optimal feedback scheme. The adaptive feedback scheme A with the optimized bit allocation in (22) outperforms the one with equal bit allocation, which indicates the importance of bit allocation. We can see that in the considered limited feedback system, CoMP-CB is inferior to NonCoMP owing to the quantization and feedback transmission errors. By allowing the adaptive feedback for CDI, the performance of CoMP-CB is largely retrieved.

In Fig. 5, we show the average achievable rate of the pico users with different distances to the PBS. It is shown that by using the same feedback scheme, the rate of a user with $d \geq 220$ m is slightly higher than the one with the same distance to the PBS when $d \leq 200$ m. This is because the residual ICI from the MBS owing to the imperfect CDI decreases with the distance to the MBS. In contrast to the results for the macro users in Fig. 4, with the threshold $\nu_{k,o1}$, we find that equal bit allocation for quantizing the CDIs of macro and pico channels

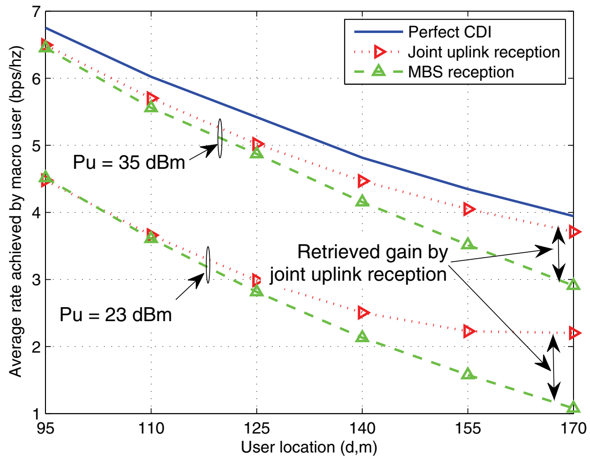

Fig. 6. Average user rate achieved by different uplink transmission schemes.

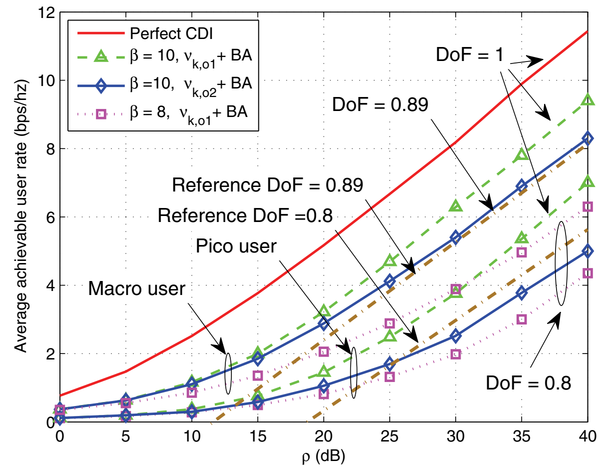

Fig. 7. Average achievable rates of the macro and pico users with different value of SNRs.

performs closely to the optimized bit allocation in (22). This can be explained as follows. For the pico user, the SNR from the PBS is much higher than that from the MBS, such that the pico CDI requires a larger size of codebook. However, the number of antennas at the PBS is less than that of the MBS, such that the pico CDI needs a smaller size of codebook. This two opposite requirements lead to the result that the optimal bit allocation performs almost the same as the equal bit allocation.

D. Impact of Uplink Transmission

As observed in Fig. 4, the adaptive feedback schemes have a larger rate loss for the "cell-edge" macro users, e.g., those located around $d = 170m$. The reason is as follows. The uplink receive SNRs of these macro users at the MBS are low, therefore the received CDIs have a large distortion due to the low feedback capacities. On the other hand, the "cell-edge" macro users experience large ICI from the PBS with the imperfect CDIs, which finally leads to the rate loss.

In Fig. 6, we consider an enhanced adaptive feedback with coherent cooperative reception to illustrate the impact of uplink transmission on the downlink achievable rate. Specifically, the MBS and PBS jointly receive the CDIs of each user with maximum ratio combination (MRC). The optimal value of ν_k is found from maximizing the average per-user rate in (11) by exhaustively searching (the outage probability needs to be modified to accommodate the joint uplink reception) and the bit allocation is obtained from (22) with the optimized ν_k. It is shown that the joint uplink reception for the CDIs can mitigate the performance degradation. This is because when a macro user (in the vicinity of a pico cell) is far from the MBS, although its uplink SNR at the MBS is low, its uplink SNR at the PBS is high. The gap between the rate with perfect CDI and the rate with the joint uplink received CDIs is led by the difference of the downlink and uplink transmit powers. To show this, we increase the uplink transmit power from $P_u = 23$ dBm to 35 dBm, then the received CDIs after the MRC are of high quality and the rate loss from the perfect CDI is minor. This suggests that the performance of the FDD HetNets will be fundamentally limited by the uplink transmit power, considering that we have employed the optimized feedback scheme and the cooperative reception for the CDIs.

E. Validation of the DoF Analysis

We validate the DoF analysis by comparing the analytical and simulation results. Again, the MBS and PBS are respectively equipped with eight and four antennas and each serves two users. Two different per-user feedback resource constraints are compared: $\beta_S = 10$ and $\beta_S = 8$. We set $\rho_{m,k} = \rho$, $\rho_{u,k} = \kappa_{u,k}\rho$ and $\rho_{l,k} = \kappa_{d,k}\rho$, and set $\kappa_{u,k} = 10^{-1}$ and $\kappa_{d,k} = 10^{-0.5}$ for all users in the simulation. The results are similar for other values of $\kappa_{u,k}$ and $\kappa_{d,k}$.

In Fig. 7, the average per-user rates with different values of ρ are plotted. The straight lines with the slops of the DoFs of 0.89 and 0.8 are also shown as a reference. We can see that the user with $\beta_S = 10$ achieves a higher DoF than with $\beta_S = 8$. As indicated by Theorem 1 and shown in the figure, by using the adaptive feedback scheme A, the macro and pico users are all with a DoF of $\beta_0 = 10/(8 + 4 - 2) = 1$ when $\beta_S = 10$ and with a DoF of $\beta_0 = 8/(8 + 4 - 2) = 0.8$ when $\beta_S = 8$. According to Theorem 2, when $\beta_S = 10$, the DoFs of the macro and pico users are $\frac{1}{\frac{1}{8}+\frac{10}{8+4-2}} \approx 0.89$ and $\frac{1}{\frac{1}{4}+\frac{10}{8+4-2}} \approx 0.8$, respectively. The achieved DoF of the macro user is slightly larger than that of the pico user when using the adaptive feedback scheme B, since the MBS has more antennas than the PBS. The simulation results are consistent with the analytical analysis. We can also observe that when the downlink SNR exceeds 30 dB, the DoF can reflect the average rate scaling with the SNR.

VII. Conclusions

In this paper, we studied adaptive feedback strategy for CoMP-CB in heterogeneous networks. We optimized the overall number of bits for the CDI feedback of each user and the bit allocation for quantizing the per-cell CDIs to maximize the average sum rate of the network. This can trade off the CDI quantization error with the feedback transmission error and exploit the different system configurations of macro and pico BSs and the propagation channels of the macro and pico cells. We proposed two adaptive feedback schemes with low complexity, where the threshold determining the overall number of bits and the bit allocation are successively optimized. The per-user DoFs were derived to show that one adaptive feedback scheme has no DoF loss from the optimal solution and the other only has minor DoF loss. The number of resource blocks to achieve a per-user DoF of one was provided. Simulation results demonstrated that the proposed adaptive feedback schemes provide substantial performance gain over existing feedback schemes. The results also suggest that the performance of the FDD heterogeneous networks is limited by the uplink transmit power, even when the optimized feedback scheme and cooperative reception for the CDIs are considered.

Appendix A
Expression of the Average Achievable Per-User Rate

The expectation of $\log_2(1+\gamma_k)$ over the positive random variable γ_k can be derived via integral as [27]

$$\mathrm{E}\{\log_2(1+\gamma_k)\} = c_0 \int_0^\infty \frac{1-\Pr\{\gamma_k \le \gamma\}}{1+\gamma} d\gamma \qquad \text{(A.1)}$$

where $\Pr\{\gamma_k \le \gamma\}$ is the cumulative distribution function (CDF) of the instantaneous SINR γ_k.

When $N_{\mathrm{M}} = L(K_{\mathrm{M}}+K_{\mathrm{L}})$ and $N_{\mathrm{L}} = K_{\mathrm{M}}+K_{\mathrm{L}}$, $\mathcal{H}_m$ and $\mathcal{H}_l$ are square matrices of full rank. From the result in [11], when the ZFBF vectors are obtained from the full rank square matrix $\mathcal{H}_m$, the term $S_{l_n^{\mathrm{M}}}$ in (9) is exponential distributed with unit mean [11]. The same is true for $S_{l_i^{\mathrm{L}}}$ in (10) when the square matrix $\mathcal{H}_l$ is full rank.

To find the CDF of the SINR, we further need to characterize each individual term of MUI and ICI in (9) and (10). To do this, we take the individual interference term $|\boldsymbol{h}_{m,l_n^{\mathrm{M}}}^H \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2$ as an example. When the uplink transmission for feedback is not in outage, $\check{\boldsymbol{h}}_{m,l_n^{\mathrm{M}}} = \hat{\boldsymbol{h}}_{m,l_n^{\mathrm{M}}}$ as shown in (8). Since $\check{\boldsymbol{h}}_{m,l_n^{\mathrm{M}}} \perp \boldsymbol{v}_{b_z^{\mathrm{M}}}$ with ZFBF, from (3) we have

$$|\boldsymbol{h}_{m,l_n^{\mathrm{M}}}^H \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2 = \|\boldsymbol{h}_{m,l_n^{\mathrm{M}}}\|^2 \sin^2\theta_{m,l_n^{\mathrm{M}}} |\boldsymbol{q}_{m,l_n^{\mathrm{M}}} \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2$$

It was proofed in [27] that $X = \|\boldsymbol{h}_{m,l_n^{\mathrm{M}}}\|^2 \sin^2\theta_{m,l_n^{\mathrm{M}}}$ is Gamma distributed as $X \sim \mathbb{G}(N_{\mathrm{M}}-1, 2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}})$, $Y = |\boldsymbol{q}_{m,l_n^{\mathrm{M}}} \boldsymbol{v}_{b_z^{\mathrm{M}}}|^2$ is Beta distributed as $Y \sim \mathbb{B}(1, N_{\mathrm{M}}-2)$, and their product XY is exponential distributed with mean of $2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}}$.

According to Appendix A of [28], the powers of the MUI and ICI in (9) and (10) are subject to Gamma distribution, since each of them are the sum of multiple i.i.d. exponential distributed random variables when no outage occurs in the feedback. To be specific, as shown in [29], the powers of the MUI and the ICI in (9) for macro user l_n^{M} are respectively random variables as follows

$$I_{l_n^{\mathrm{M}}}^{mu} \sim 2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}} \mathbb{G}(LK_{\mathrm{M}}-1, 1)$$
$$I_{l_n^{\mathrm{M}}}^{co} \sim 2^{\frac{-B_{l,l_n^{\mathrm{M}}}}{N_{\mathrm{L}}-1}} \mathbb{G}(K_{\mathrm{L}}, 1) \qquad \text{(A.2)}$$

and the powers of the MUI and ICI in (10) for pico user l_i^{L} are respectively random variables as follows

$$I_{l_i^{\mathrm{L}}}^{mu} \sim 2^{\frac{-B_{l,l_i^{\mathrm{L}}}}{N_{\mathrm{L}}-1}} \mathbb{G}(K_{\mathrm{L}}-1, 1)$$
$$I_{l_i^{\mathrm{L}}}^{co} \sim 2^{\frac{-B_{m,l_i^{\mathrm{L}}}}{N_{\mathrm{M}}-1}} \mathbb{G}(LK_{\mathrm{M}}, 1) \qquad \text{(A.3)}$$

From [30], we know

$$\Pr\Big\{\frac{a_0 X_0}{1+\sum_{i=1}^2 a_i X_i} \le \gamma\Big\} = 1 - \frac{e^{\frac{-\gamma}{a_0}}}{\prod_{i=1}^2 (1+\frac{a_i}{a_0}\gamma)^{N_i}}$$

where X_0 follows exponential distribution with unit mean, $X_1 \sim \mathbb{G}(N_1,1)$, $X_2 \sim \mathbb{G}(N_2,1)$, and a_i, $i=0,1,2$ are positive constants. From this result and by using (A.2) and the distribution of $S_{l_n^{\mathrm{M}}}$, we obtain the CDF of the SINR of macro user l_n^{M} in (9) as

$$\Pr\{\gamma_{l_n^{\mathrm{M}}} \le \gamma\} = 1 - e^{\frac{-\gamma}{\rho_{m,l_n^{\mathrm{M}}}}} \Delta_{l_n^{\mathrm{M}}}(B_{m,l_n^{\mathrm{M}}}, B_{l,l_n^{\mathrm{M}}}) \qquad \text{(A.4)}$$

where

$$\Delta_{l_n^{\mathrm{M}}}(B_{m,l_n^{\mathrm{M}}}, B_{l,l_n^{\mathrm{M}}}) = (1+\gamma 2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}})^{-LK_{\mathrm{M}}+1} (1+\frac{\rho_{l,l_n^{\mathrm{M}}}}{\rho_{m,l_n^{\mathrm{M}}}}\gamma 2^{\frac{-B_{l,l_n^{\mathrm{M}}}}{N_{\mathrm{L}}-1}})^{-K_{\mathrm{L}}}$$

Further using (A.1), we obtain the average rate achieved by macro user l_n^{M} when no outage occurs in the CDI feedback as follows

$$\mathrm{R}_{l_n^{\mathrm{M}}}^{no} = c_0 \int_0^\infty \frac{e^{\frac{-\gamma}{\rho_{m,l_n^{\mathrm{M}}}}}}{1+\gamma} \Delta_{l_n^{\mathrm{M}}}(B_{m,l_n^{\mathrm{M}}}, B_{l,l_n^{\mathrm{M}}}) d\gamma \qquad \text{(A.5)}$$

Similarly, we can derive the average rate achieved by pico user l_i^{L} when no outage occurs in the CDI feedback as follows

$$\mathrm{R}_{l_i^{\mathrm{L}}}^{no} = c_0 \int_0^\infty \frac{e^{\frac{-\gamma}{\rho_{l,l_i^{\mathrm{L}}}}}}{1+\gamma} \Delta_{l_i^{\mathrm{L}}}(B_{m,l_i^{\mathrm{L}}}, B_{l,l_i^{\mathrm{L}}}) d\gamma \qquad \text{(A.6)}$$

where

$$\Delta_{l_i^{\mathrm{L}}}(B_{m,l_i^{\mathrm{L}}}, B_{l,l_i^{\mathrm{L}}}) = (1+\gamma 2^{\frac{-B_{l,l_i^{\mathrm{L}}}}{N_{\mathrm{L}}-1}})^{-K_{\mathrm{L}}+1} (1+\frac{\rho_{m,l_i^{\mathrm{L}}}}{\rho_{l,l_i^{\mathrm{L}}}}\gamma 2^{\frac{-B_{m,l_i^{\mathrm{L}}}}{N_{\mathrm{M}}-1}})^{-LK_{\mathrm{M}}}$$

Appendix B
Lower Bound of $\Delta_k(B_{m,k}, B_{l,k})$

The lower bound of $\Delta_k(B_{m,k}, B_{l,k})$ can be obtained by using the arithmetic-geometry inequality

$$\prod_{i=1}^K (1+x_i) \le (1+\frac{1}{K}\sum_{i=1}^K x_i)^K \qquad \text{(B.1)}$$

where $x_i > 0$.

For macro user l_n^{M}, let

$$x_i = \gamma 2^{\frac{-B_{m,l_n^{\mathrm{M}}}}{N_{\mathrm{M}}-1}}, \text{ for } i = 1, \cdots, LK_{\mathrm{M}}-1$$
$$x_i = \frac{\rho_{l,l_n^{\mathrm{M}}}}{\rho_{m,l_n^{\mathrm{M}}}}\gamma 2^{\frac{-B_{l,l_n^{\mathrm{M}}}}{N_{\mathrm{L}}-1}}, \text{ for } i = LK_{\mathrm{M}}, \cdots, K$$

and then by applying (B.1) to (13), we have

$$\Delta_{l_n^{\rm M}}(B_{m,l_n^{\rm M}},B_{l,l_n^{\rm M}}) \geq (1+\frac{\gamma}{K}\delta_{l_n^{\rm M}}(B_{m,l_n^{\rm M}},B_{l,l_n^{\rm M}}))^{-K} \quad \text{(B.2)}$$

where $\delta_{l_n^{\rm M}}(B_{m,l_n^{\rm M}},B_{l,l_n^{\rm M}}) = (LK_{\rm M}-1)2^{\frac{-B_{m,l_n^{\rm M}}}{N_{\rm M}-1}} + K_{\rm L}\frac{\rho_{l,l_n^{\rm M}}}{\rho_{m,l_n^{\rm M}}}2^{\frac{-B_{l,k}}{N_{\rm L}-1}}$.

For pico user $l_i^{\rm L}$, let

$$x_i = \gamma 2^{\frac{-B_{l,l_i^{\rm L}}}{T_l-1}}, \text{ for } i=1,\cdots,K_{\rm L}-1$$
$$x_i = \frac{\rho_{m,l_i^{\rm L}}}{\rho_{l,l_i^{\rm L}}}\gamma 2^{\frac{-B_{m,l_i^{\rm L}}}{N_{\rm M}-1}}, \text{ for } i=K_{\rm L},\cdots,K. \quad \text{(B.3)}$$

and then by applying (B.1) to (14), we have

$$\Delta_{l_i^{\rm L}}(B_{m,l_i^{\rm L}},B_{l,l_i^{\rm L}}) \geq \left(1+\frac{\gamma}{K}\delta_{l_i^{\rm L}}(B_{m,l_i^{\rm L}},B_{l,l_i^{\rm L}})\right)^{-K} \quad \text{(B.4)}$$

where $\delta_{l_i^{\rm L}}(B_{m,l_i^{\rm L}},B_{l,l_i^{\rm L}}) = LK_{\rm M}\frac{\rho_{m,l_i^{\rm L}}}{\rho_{l,l_i^{\rm L}}}2^{\frac{-B_{m,l_i^{\rm L}}}{N_{\rm M}-1}} + (K_{\rm L}-1)2^{\frac{-B_{l,l_i^{\rm L}}}{N_{\rm L}-1}}$.

APPENDIX C
PROOF OF LEMMA 1

From (9) and (35), for a macro user $k=l_n^{\rm M}$, the per-user DoF can be expressed as

$$\text{DoF}_k = \lim_{\rho_{m,l_n^{\rm M}}\to\infty}\frac{\text{E}\left\{\log_2\left(1+\rho_{m,l_n^{\rm M}}S_{l_n^{\rm M}}+\rho_{m,l_n^{\rm M}}I_{l_n^{\rm M}}^{mu}+\rho_{l,l_n^{\rm M}}I_{l_n^{\rm M}}^{co}\right)\right\}}{\log_2\rho_{m,l_n^{\rm M}}}$$
$$-\lim_{\rho_{m,l_n^{\rm M}}\to\infty}\frac{\text{E}\left\{\log_2(1+\rho_{m,l_n^{\rm M}}I_{l_n^{\rm M}}^{mu}+\rho_{l,l_n^{\rm M}}I_{l_n^{\rm M}}^{co})\right\}}{\log_2\rho_{m,l_n^{\rm M}}} \quad \text{(C.1)}$$
$$=1-\lim_{\rho_{m,l_n^{\rm M}}\to\infty}\frac{\text{E}\left\{\log_2(1+\rho_{m,l_n^{\rm M}}I_{l_n^{\rm M}}^{mu}+\rho_{l,l_n^{\rm M}}I_{l_n^{\rm M}}^{co})\right\}}{\log_2\rho_{m,l_n^{\rm M}}} \quad \text{(C.2)}$$

where the first term in (C.1) equals one, which can be obtained by a simply extension of equation (23) in [11] and hence is not provided for conciseness. Similarly from (10) and (35), for a pico user $k=l_i^{\rm L}$

$$\text{DoF}_k = 1-\lim_{\rho_{m,l_i^{\rm L}}\to\infty}\frac{\text{E}\left\{\log_2(1+\rho_{l,l_i^{\rm L}}I_{l_i^{\rm L}}^{mu}+\rho_{m,l_i^{\rm L}}I_{l_i^{\rm L}}^{co})\right\}}{\log_2\rho_{m,l_i^{\rm L}}} \quad \text{(C.3)}$$

Maximizing the per-user DoF is equivalent to minimizing the second term of (C.2) and (C.3). The upper bound of DoF_k^O achieved by the optimal feedback scheme can be obtained by minimizing the lower bound of the second term in (C.2) and and (C.3).

By using the Jensen's inequality $\text{E}\{\log_2(1+\sum_{i=1}^K a_i x_i)\} \geq \log_2(1+\sum_{i=1}^K a_i e^{\text{E}\{\log x_i\}})$, for a macro user $k=l_n^{\rm M}$, we have

$$\text{E}\left\{\log_2(1+\rho_{m,l_n^{\rm M}}I_{l_n^{\rm M}}^{mu}+\rho_{l,l_n^{\rm M}}I_{l_n^{\rm M}}^{co})\right\}$$
$$\geq \log_2\left(1+\rho_{m,l_n^{\rm M}}e^{\text{E}\{\log I_{l_n^{\rm M}}^{mu}\}}+\rho_{l,l_n^{\rm M}}e^{\text{E}\{\log I_{l_n^{\rm M}}^{co}\}}\right)$$
$$\triangleq \log_2(1+\varepsilon_{l_n^{\rm M}}) \quad \text{(C.4)}$$

and for a pico user $k=l_i^{\rm L}$, we have

$$\text{E}\left\{\log_2(1+\rho_{l,l_i^{\rm L}}I_{l_i^{\rm L}}^{mu}+\rho_{m,l_i^{\rm L}}I_{l_i^{\rm L}}^{co})\right\}$$
$$\geq \log_2\left(1+\rho_{l,l_n^{\rm M}}e^{\text{E}\{\log I_{l_i^{\rm L}}^{mu}\}}+\rho_{m,l_i^{\rm L}}e^{\text{E}\{\log I_{l_i^{\rm L}}^{co}\}}\right)$$
$$\triangleq \log_2(1+\varepsilon_{l_i^{\rm L}}) \quad \text{(C.5)}$$

For conciseness, denote $c_a = \text{E}\{\log X\}$ where $X \sim \mathbb{G}(a,1)$ for $a>1$. Considering that there may be outage in the CDI feedback during the uplink transmission, we have

$$\text{E}\left\{\log I_{l_n^{\rm M}}^{mu}\right\}$$
$$=(1-P(N_{l_n^{\rm M}},\nu_{l_n^{\rm M}}))\text{E}\left\{\log I_{l_n^{\rm M}}^{mu}|\check{\boldsymbol{h}}_{m,l_n^{\rm M}}=\hat{\boldsymbol{h}}_{m,l_n^{\rm M}}\right\}$$
$$+P(N_{l_n^{\rm M}},\nu_{l_k})\text{E}\left\{\log I_{l_n^{\rm M}}^{mu}|\check{\boldsymbol{h}}_{m,l_n^{\rm M}}\neq\hat{\boldsymbol{h}}_{m,l_n^{\rm M}}\right\}$$
$$=(1-P(N_{l_n^{\rm M}},\nu_{l_n^{\rm M}}))(c_{LK_{\rm M}-1}+\log 2^{\frac{-B_{m,l_n^{\rm M}}}{N_{\rm M}-1}})$$
$$+P(N_{l_n^{\rm M}},\nu_{l_k})c_{LK_{\rm M}-1} \quad \text{(C.6)}$$

where (C.6) is obtained from the distribution of $I_{l_n^{\rm M}}^{mu}$ in (A.2) when conditioned on non-outage cases and from treating $B_{m,l_n^{\rm M}}=0$ in (A.2) when conditioned on outage cases.

Then from (C.6), we can derive that

$$e^{\text{E}\left\{\log I_{l_n^{\rm M}}^{mu}\right\}} = e^{c_{LK_{\rm M}-1}}2^{\frac{-B_{m,l_n^{\rm M}}\left(1-P(N_{l_n^{\rm M}},\nu_{l_n^{\rm M}})\right)}{N_{\rm M}-1}} \quad \text{(C.7)}$$

Similarly we can obtain

$$e^{\text{E}\left\{\log I_{l_n^{\rm M},l_i^{\rm L}}^{co}\right\}} = e^{c_{K_{\rm L}}}2^{\frac{-B_{l,l_n^{\rm M}}\left(1-P(N_{l_n^{\rm M}},\nu_{l_n^{\rm M}})\right)}{N_{\rm L}-1}} \quad \text{(C.8)}$$

$$e^{\text{E}\left\{\log I_{l_i^{\rm L},l_j^{\rm L}}^{mu}\right\}} = e^{c_{K_{\rm L}-1}}2^{\frac{-B_{l,l_i^{\rm L}}\left(1-P(t_{l_i^{\rm L}},\nu_{l_i^{\rm L}})\right)}{N_{\rm L}-1}} \quad \text{(C.9)}$$

$$e^{\text{E}\left\{\log I_{l_i^{\rm L},b_z^{\rm M}}^{mu}\right\}} = e^{c_{LK_{\rm M}}}2^{\frac{-B_{m,l_i^{\rm L}}\left(1-P(t_{l_i^{\rm L}},\nu_{l_i^{\rm L}})\right)}{N_{\rm M}-1}} \quad \text{(C.10)}$$

From (C.7) to (C.10), we obtain a unified expression of ε_k for user k in (C.4) and (C.5) as follows

$$\varepsilon_k = \rho_{m,k}\vartheta_{m,k}2^{\frac{-B_{m,k}(1-P(N_k,\nu_k))}{N_{\rm M}-1}} + \rho_{l,k}\vartheta_{l,k}2^{\frac{-B_{l,k}(1-P(N_k,\nu_k))}{N_{\rm L}-1}} \quad \text{(C.11)}$$

where for a macro user $k=l_n^{\rm M}$, $\vartheta_{m,k}=e^{c_{LK_{\rm M}-1}}$ and $\vartheta_{l,k}=e^{c_{K_{\rm L}}}$, and for a pico user $k=l_i^{\rm L}$, $\vartheta_{m,k}=e^{c_{LK_{\rm M}}}$ and $\vartheta_{l,k}=e^{c_{K_{\rm L}-1}}$.

Considering that $\rho_{l,k}=\kappa_{d,k}\rho_{m,k}$, by applying the generalized algorithmic-geometry inequality $p_1x_1+p_2x_2 \geq (p_1+p_2)x_1^{\frac{p_1}{p_1+p_2}}x_2^{\frac{p_2}{p_1+p_2}}$ for $x_1,x_2,p_1,p_2>0$, and by letting

$$x_1 = \frac{\vartheta_{m,k}}{N_{\rm M}-1}2^{-\frac{B_{m,k}(1-P(\nu_k))}{N_{\rm M}-1}}$$
$$x_2 = \frac{\kappa_{d,k}\vartheta_{l,k}}{N_{\rm L}-1}2^{-\frac{B_{l,k}(1-P(\nu_k))}{N_{\rm L}-1}}$$
$$p_1 = N_{\rm M}-1, \quad p_2 = N_{\rm L}-1,$$

from (C.11) we have

$$\varepsilon_k \geq \rho_{m,k}\zeta_k^{BA}2^{-\frac{(B_{m,k}+B_{l,k})(1-P(N_k,\nu_k))}{N_{\rm M}+N_{\rm L}-2}}$$
$$=\rho_{m,k}\zeta_k^{BA}(1+\nu_k)^{-\beta_0(1-P(N_k,\nu_k))} \triangleq \varepsilon_k^{BA}(\nu_k) \quad \text{(C.12)}$$

where $\zeta_k^{BA} = (N_{\rm M}+N_{\rm L}-2)\left(\frac{\vartheta_{m,k}}{N_{\rm M}-1}\right)^{\frac{N_{\rm M}-1}{N_{\rm M}+N_{\rm L}-2}}\left(\frac{\kappa_{d,k}\vartheta_{l,k}}{N_{\rm L}-1}\right)^{\frac{N_{\rm L}-1}{N_{\rm M}+N_{\rm L}-2}}$ and the equality of (C.12) is from $B_{m,k}+B_{l,k} = \beta_S \log_2(1+\nu_k)$, and $\beta_0 = \beta_S/(N_{\rm M}+N_{\rm L}-2)$.

Since ζ_k^{BA} and β_0 are positive constants, minimizing $\varepsilon_k^{BA}(\nu_k)$ is equivalent to maximizing $(1+\nu_k)^{1-P(N_k,\nu_k)}$, which is further equivalent to maximizing the term $(1-P(N_k,\nu_k))\log(1+\nu_k)$.

Considering (7), we can derive that

$$\begin{aligned} &\mathrm{E}\{\log(1+\gamma_{u,k})\} \\ &= \mathrm{E}\{\log(1+\gamma_{u,k})|\gamma_{u,k}\geq\nu_k\}\left(1-P(N_k,\nu_k)\right) \\ &\quad + \mathrm{E}\{\log(1+\gamma_{u,k})|\gamma_{u,k}<\nu_k\}P(N_k,\nu_k) \\ &\geq \mathrm{E}\{\log(1+\gamma_{u,k})|\gamma_{u,k}\geq\nu_k\}\left(1-P(N_k,\nu_k)\right) \quad (C.13) \\ &\geq \log(1+\nu_k\}\left(1-P(N_k,\nu_k)\right) \quad (C.14) \end{aligned}$$

where the last inequality is by replacing the uplink instantaneous SNR $\gamma_{u,k}$ by the threshold ν_k, which is not a random variable and is conditioned by $\gamma_{u,k}\geq\nu_k$.

From the Jensen's inequality, we know that

$$\mathrm{E}\{\log(1+\gamma_{u,k})\} \leq \log(1+\mathrm{E}\{\gamma_{u,k}\}) \quad (C.15)$$

where $\mathrm{E}\{\gamma_{u,k}\} = N_k\rho_{u,k}$ is from the statistics of $\gamma_{u,k}$ discussed before (7).

Then from (C.14) and (C.15) we have

$$\log(1+\nu_k\}\left(1-P(N_k,\nu_k)\right) \leq \log(1+N_k\rho_{u,k}) \quad (C.16)$$

The equalities in (C.13)-(C.16) will hold only when the number of antennas at the serving BS is sufficiently large such that the uplink fading channel degrades to a deterministic channel [26]. This is because when the uplink channel becomes deterministic, $\gamma_k = N_k\rho_{u,k}$ is no longer a random variable and by setting $\nu_k = N_k\rho_{u,k}$, the outage probability is zero.

From (C.12) and (C.16), we can derive that

$$\begin{aligned} \varepsilon_k^{BA}(\nu_k) &= \rho_{m,k}\zeta_k^{BA}e^{-\beta_0\left(1-P(N_k,\nu_{l_k})\right)\log(1+\nu_{l_k})} \\ &\geq \rho_{m,k}\zeta_k^{BA}e^{-\beta_0\log(1+N_k\rho_{u,k})} \\ &= \rho_{m,k}\zeta_k^{BA}(1+N_k\kappa_{u,k}\rho_{m,k})^{-\beta_0} \quad (C.17) \end{aligned}$$

where $\rho_{u,k} = \kappa_{u,k}\rho_{m,k}$ is applied in the last equality.

Further considering (C.2), (C.3), (C.4) and (C.5), we have

$$\begin{aligned} \mathrm{DoF}_k^O &\leq 1 - \lim_{\rho_{m,k}\to\infty}\frac{\log_2\left(1+\varepsilon_k\right)}{\log_2\rho_{m,k}} \\ &\leq 1 - \lim_{\rho_{m,k}\to\infty}\frac{\log_2(1+\varepsilon_k^{BA}(\nu_k))}{\log_2\rho_{m,k}} \quad (C.18) \end{aligned}$$

where (C.18) is due to (C.12).

When $\beta_0 \geq 1$, (C.17) does not increase with $\rho_{m,k}$ and is bounded, since

$$\begin{aligned} &\lim_{\rho_{m,k}\to\infty}\rho_{m,k}(1+N_k\kappa_{u,k}\rho_{m,k})^{-\beta_0} \\ &\leq \lim_{\rho_{m,k}\to\infty}(N_k\kappa_{u,k})^{-\beta_0}\rho_{m,k}^{1-\beta_0} \leq (N_k\kappa_{u,k})^{-\beta_0}. \end{aligned}$$

Using this fact and from (C.17), we have

$$\lim_{\rho_{m,k}\to\infty}\frac{\log_2(1+\varepsilon_k^{BA}(\nu_k))}{\log_2\rho_{m,k}} = 0 \quad (C.19)$$

and consequently from (C.18) we obtain

$$\mathrm{DoF}_k^O \leq 1 \quad (C.20)$$

When $\rho_{m,k} > 1$, we have

$$\begin{aligned} &\rho_{m,k}(1+N_k\kappa_{u,k}\rho_{m,k})^{-\beta_0} \\ &\geq \rho_{m,k}(\rho_{m,k}+N_k\kappa_{u,k}\rho_{m,k})^{-\beta_0} = (1+N_k\kappa_{u,k})^{-\beta_0}\rho_{m,k}^{1-\beta_0}. \end{aligned}$$

It means when $\beta_0 < 1$, (C.17) is unbounded as $\rho_{m,k}$ approaches infinity. Using this fact we can safely ignore "1" inside the $\log(\cdot)$ function in (C.18) and obtain

$$\begin{aligned} \mathrm{DoF}_k^O &\leq 1 - \lim_{\rho_{m,k}\to\infty}\frac{\log_2\varepsilon_k^{BA}(\nu_k)}{\log_2\rho_{m,k}} \quad (C.21) \\ &= \beta_0 \quad (C.22) \end{aligned}$$

where (C.21) is by (C.22) is from using (C.17) by ignoring the constant irrespective of $\rho_{m,k}$.

Therefore, from (C.20) and (C.22) we immediately prove the Lemma for both cases, i.e., $\mathrm{DoF}_k^O \leq \min\{\beta_0, 1\}$.

Appendix D
Proof of Lemma 2

The lower bound of DoF_k^A achieved by the adaptive feedback scheme A can be derived from

$$\begin{aligned} &\mathrm{R}_{k,lb1}^{BA}(\nu_{k,o1}) \\ &= (1-P(N_k,\nu_{k,o1}))\mathrm{R}_{k,lb}^{no,BA}(\nu_{k,o1}) + P(N_k,\nu_{k,o1})\mathrm{R}_k^{ou} \\ &\geq (1-P(N_k,\nu_k))\mathrm{R}_{k,lb}^{no,BA}(\nu_k) \quad (D.1) \end{aligned}$$

where ν_k is an arbitrary threshold that may not be the optimal solution of problem (28), considering that $P(N_k,\nu_k)\mathrm{R}_k^{ou} \geq 0$.

Because $(1+\frac{x}{K})^{-K} \geq e^{-x}$ for $x>0$, from (26) we have

$$\mathrm{R}_{k,lb}^{no,BA}(\nu_k) \geq c_0\int_0^\infty \frac{e^{-\gamma(\frac{1}{\rho_k}+\delta_k^{BA}(\nu_k))}}{1+\gamma}d\gamma$$

where $c_0 = \log_2 e$.

Applying this inequality and (25) into (D.1), we can obtain a lower bound of $\mathrm{R}_{k,lb1}^{BA}(\nu_{k,o1})$ as follows

$$\mathrm{R}_{k,lb1}^{BA}(\nu_{k,o1}) \geq (1-P(N_k,\nu_k))c_0\int_0^\infty \frac{e^{-\gamma(\frac{1}{\rho_k}+g_k^{no}(1+\nu_k)^{-\beta_0})}}{1+\gamma}d\gamma$$

Considering the following inequality

$$c_0\int_0^\infty \frac{e^{-\frac{x}{a}}}{1+x}dx = \int_0^\infty \log_2(1+ax)e^{-x}dx \geq \log_2(1+ac_1) \quad (D.2)$$

where the first equality is obtained from integral by parts, and $c_1 = e^{\int_0^\infty e^{-x}\log x dx} > 0$, we further obtain that

$$\begin{aligned} &\mathrm{R}_{k,lb1}^{BA}(\nu_{k,o1}) \\ &\geq (1-P(N_k,\nu_k))\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_k)^{-\beta_0}}\right) \quad (D.3) \end{aligned}$$

To obtain a lower bound of DoF_k^A, we can find the DoF achieved by the right hand side of (D.3). To find an achievable DoF, we can choose a value of ν_k to ensure $P(N_k,\nu_k)\to 0$ as $\rho_{u,k}\to\infty$. It is worthy to note that there are many choices to set ν_k for this purpose, which may lead to different lower bounds of DoF_k^A. By applying $e^{-\gamma_{u,k}/\rho_{u,k}} < 1$ in (7), we obtain

$$P(N_k,\nu_k) \le \int_0^{\nu_k} \frac{\gamma_{u,k}^{N_k-1}}{\rho_{u,k}^{N_k}\Gamma(N_k)} d\gamma_{u,k} = \frac{\nu_k^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}} \le \frac{(1+\nu_k)^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}} \quad \text{(D.4)}$$

We select a threshold as $\nu_k = \nu_{k,s} \triangleq \frac{\rho_{u,k}}{\log \rho_{u,k}} - 1$, which ensures

$$\lim_{\rho_{u,k}\to\infty} P(N_k,\nu_{k,s}) \le \lim_{\rho_{m,k}\to\infty} \frac{(\log \rho_{u,k})^{-N_k}}{\Gamma(N_k+1)} = 0 \quad \text{(D.5)}$$

where $\rho_{u,k} = \kappa_{u,k}\rho_{m,k}$ in (34) is applied.

By using (D.3) and (D.5), we have

$$\begin{aligned}\text{DoF}_k^A &= \lim_{\rho_{m,k}\to\infty} \frac{\text{R}_{k,lb1}^{BA}(\nu_{k,o1})}{\log_2 \rho_{m,k}} \\ &\ge \lim_{\rho_{m,k}\to\infty} \frac{(1-P(N_k,\nu_{k,s}))\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}}\right)}{\log_2 \rho_{m,k}} \\ &= \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}}\right)}{\log_2 \rho_{m,k}} \quad \text{(D.6)}\end{aligned}$$

For a macro user, by using $\rho_k = \rho_{m,k}$ defined in (12) and $\rho_{u,k} = \kappa_{u,k}\rho_{m,k}$, the term inside the log function of (D.6) is

$$\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}} = \frac{c_1\rho_{m,k}}{1+\frac{\rho_{m,k} g_k^{no}\rho_{u,k}^{-\beta_0}}{(\log \rho_{u,k})^{-\beta_0}}} = \frac{c_1\rho_{m,k}}{1+\frac{\kappa_{u,k}^{-\beta_0} g_k^{no}\rho_{m,k}^{1-\beta_0}}{(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}}}. \quad \text{(D.7)}$$

For a pico user, by using $\rho_k = \rho_{l,k}$ defined in (12) and $\rho_{l,k} = \kappa_{d,k}\rho_{m,k}$ in (34), the term inside the log function of (D.6) is

$$\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}} = \frac{c_1\kappa_{l,k}\rho_{m,k}}{1+\frac{\kappa_{l,k}\rho_{m,k} g_k^{no}\rho_{u,k}^{-\beta_0}}{(\log \rho_{u,k})^{-\beta_0}}} = \frac{c_1\kappa_{l,k}\rho_{m,k}}{1+\frac{\kappa_{l,k}\kappa_{u,k}^{-\beta_0} g_k^{no}\rho_{m,k}^{1-\beta_0}}{(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}}} \quad \text{(D.8)}$$

In the case of $\beta_0 \le 1$, the term $\frac{\rho_{m,k}^{1-\beta_0}}{(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}}$ in the denominator of both (D.7) and (D.8) approaches infinity as $\rho_{m,k}$ increases. Using this fact we can safely ignore "1" in both (D.7) and (D.8) to derive a unified expression of (D.6) for both the macro and pico users as follows

$$\begin{aligned}&\lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}}\right)}{\log_2 \rho_{m,k}} \\ &= \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}\rho_{m,k}^{\beta_0}}{\kappa_{u,k}^{-\beta_0} g_k^{no}}\right)}{\log_2 \rho_{m,k}} \\ &\ge \lim_{\rho_{m,k}\to\infty} \frac{\beta_0 \log \rho_{m,k} + \log\left(\frac{c_1(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}}{\kappa_{u,k}^{-\beta_0} g_k^{no}}\right)}{\log_2 \rho_{m,k}} \\ &= \beta_0 \quad \text{(D.9)}\end{aligned}$$

In the case of $\beta_0 > 1$, the term $\frac{\rho_{m,k}^{1-\beta_0}}{(\log(\kappa_{u,k}\rho_{m,k}))^{-\beta_0}}$ approaches zero as $\rho_{m,k}$ increases. Then we can derive a unified expression of (D.6) for both the macro and pico users as follows

$$\lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}}\right)}{\log_2 \rho_{m,k}} = \lim_{\rho_{m,k}\to\infty} \frac{\log(1+c_1\rho_k)}{\log_2 \rho_{m,k}} = 1. \quad \text{(D.10)}$$

Combining (D.9) and (D.10), we obtain

$$\lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1 g_k}{1+g_k g_k^{no}(1+\nu_{k,s})^{-\beta_0}}\right)}{\log_2 \rho_{m,k}} = \min\{\beta_0, 1\} \quad \text{(D.11)}$$

By using (D.6), and (D.11), we have

$$\text{DoF}_k^A \ge \min\{\beta_0, 1\} \quad \text{(D.12)}$$

Appendix E
Proof of Theorem 2

The lower bound of DoF_k^B achieved by the adaptive feedback scheme B can be derived from $R_{k,lb2}^{BA}(\nu_{k,o2})$. Applying $(1+\frac{x}{K})^{-K} \ge e^{-x}$ for $x>0$ into (31), we have

$$\begin{aligned}R_{k,lb2}^{BA}(\nu_{k,o2}) &\ge c_0 \int_0^\infty \frac{e^{-\gamma(\frac{1}{\rho_k}+\delta^u(\nu_{k,o2}))}}{1+\gamma} d\gamma \\ &\ge \log_2\left(1+\frac{c_1\rho_k}{1+\rho_k\delta^u(\nu_{k,o2})}\right) \quad \text{(E.1)}\end{aligned}$$

where $\delta^u(\nu_{k,o2}) \triangleq \delta_k^{BA}(\nu_k) + \frac{g_k^{ou}(1+\nu_k)^{N_k}}{\Gamma(N_k+1)\rho_{u,k}^{N_k}}$ is the objection function in (32), the last inequality is from (D.2).

By substituting $\nu_{k,o2}$ in (33) into $\delta^u(\nu_{k,o2})$, we obtain

$$\delta^u(\nu_{k,o2}) = \rho_k^{BA}\left(1+\frac{\beta_0}{N_k}\right)\left(\frac{\rho_k^{BA}\beta_0\Gamma(N_k)}{g_k^{ou}}\right)^{-\frac{\beta_0}{N_k+\beta_0}} \rho_{u,k}^{\frac{-N_k\beta_0}{N_k+\beta_0}} \quad \text{(E.2)}$$

Recalling that $\rho_{u,k} = \kappa_{u,k}\rho_{m,k}$, for a macro user $\rho_k = \rho_{m,k}$, and for a pico user $\rho_k = \rho_{l,k} = \kappa_{d,k}\rho_{m,k}$, then in the case of $\frac{N_k\beta_0}{N_k+\beta_0} \le 1$, $\rho_k\delta^u(\nu_{k,o2})$ is unbounded as $\rho_{m,k}$ increases. Using this fact we can safely ignore "1" in the denominator of (E.1) and obtain

$$\begin{aligned}\text{DoF}_k^B &= \lim_{\rho_{m,k}\to\infty} \frac{\text{R}_{k,lb2}^{BA}(\nu_{k,o2})}{\log_2 \rho_{m,k}} \\ &\ge \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k\delta^u(\nu_{k,o2})}\right)}{\log_2 \rho_{m,k}} = \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1}{\delta^u(\nu_{k,o2})}\right)}{\log_2 \rho_{m,k}} \\ &\ge \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(\frac{c_1}{\delta^u(\nu_{k,o2})}\right)}{\log_2 \rho_{m,k}} = \frac{N_k\beta_0}{N_k+\beta_0} \quad \text{(E.3)}\end{aligned}$$

where the last equality is from (E.2).

In the case of $\frac{N_k\beta_0}{N_k+\beta_0} \ge 1$, $\rho_k\delta^u(\nu_{k,o2})$ does not increase with $\rho_{m,k}$ and is bounded, and thus we have

$$\begin{aligned}\text{DoF}_k^B &= \lim_{\rho_{m,k}\to\infty} \frac{\text{R}_{k,lb2}^{BA}(\nu_{k,o2})}{\log_2 \rho_{m,k}} \\ &\ge \lim_{\rho_{m,k}\to\infty} \frac{\log_2\left(1+\frac{c_1\rho_k}{1+\rho_k\delta^u(\nu_{k,o2})}\right)}{\log_2 \rho_{m,k}} \\ &= \lim_{\rho_{m,k}\to\infty} \frac{\log_2(1+c_1\rho_k)}{\log_2 \rho_{m,k}} = 1 \quad \text{(E.4)}\end{aligned}$$

Combining (E.1), (E.3), and (E.4), we complete the proof for Theorem 2.

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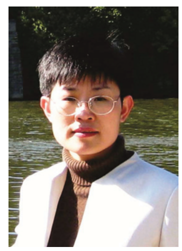

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