Spatial Spectrum Holes for Cognitive Radio with Directional Transmission[††]

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***Abstract*— In this paper, we propose a *cognitive radio* (CR) transmission scheme, which enables secondary users to coexist with primary users by exploiting *spatial spectrum holes* (SSHs) through directional antennas or antenna arrays with beamforming. To ensure reliable CR links and avoid interference to primary users, some CR users may act as relays . We investigate successful communication probability of CR users when this scheme is applied. We further demonstrate that the spectrum efficiency can be greatly improved by multiplexing CR links with directional transmission.**

I. INTRODUCTION

Cognitive radio (CR), a very promising technology for future wireless communications, can enable much higher spectrum efficiency by means of dynamic and opportunistic spectral access. The term *cognitive radio* was first proposed in late 1990s [1] and a comprehensive overview has been provided in [2]. The basic concept of CR is to allow unlicensed CR users, also called secondary users, to use licensed bands as long as they will not cause interference to licensed users, also called primary users. Therefore, secondary users must identify and use the spectrum bands that are not being used by primary users. In practice, the available spectrum bands for secondary users vary with time and location. The region of location-time-frequency available for a secondary user is called a *spectrum hole* (SH). Generally, CR system can coexist with a primary system by working through SHs.

Apparently, SH is the basic resource for CR users. A report of the *Federal Communication Commission* (FCC) [3] shows that with a very large probability there are always some licensed spectrum bands that are not used by primary users in a certain location or time. So far, lots of work has been done to increase the spectrum efficiency by allowing CR users to exploit these unused bands. Spectrum sensing [4]-[8] seeks SH in time and location so that secondary users can utilize it. While most of existing work on spectrum sensing focuses on detecting SH in time and location, the possibility that a secondary system coexists with a primary system at the same location, time, and frequency has not been well investigated.

Motivated by smart antenna and *spatial division multiple access* (SDMA) technologies [9], we will introduce the concept of *spatial spectrum holes* (SSHs), which can be utilized by CR users with directional transmission ability. There have been some contributions considering the utilization of multiple antennas in CR. Beanforming in CR has been introduced in [10] and [11], where only point-to-point CR links were considered. The scheme proposed in this paper is investigated from the perspective of the whole CR network. By using directional transmit antennas at CR users and relay assisted communication technologies, our proposed scheme enables a primary system and a secondary system to coexist without interfering each other. The scheme is especially suitable where only a small number of primary users are active most of the time.

The rest of this paper is organized as follows. In Section II, we formulate the problem and introduce the concept of SSHs. We present a relay assisted secondary transmission scheme with directional antennas in Section III. Then we analyze the performance of the proposed scheme in Section IV and present the numerical results in Section V, respectively. Finally, we conclude the paper in Section VI.

II. PROBLEM FORMULATION

Generally, whether or not a spectrum hole exists at certain time and location can be described as the presence or absence of the primary signal according to the observed signal

$$y(t) = \begin{cases} n(t), & \mathcal{H}_0, \\ hs(t) + n(t), & \mathcal{H}_1, \end{cases} \tag{1}$$

where $s(t)$ is the primary signal, h denotes the channel coefficient between the *primary transmitter* (PT) and the CR user, and $n(t)$ represents *additive white Gaussian noise* (AWGN). Hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the absence and presence of the primary signal, respectively.

A. Temporal Spectrum Hole and Geographic Spectrum Hole

Conventionally, there are two scenarios involved in CR's communication. The first one is that the CR user can access a spectrum when the primary user is not using it temporarily, which means that both the CR and primary users can be

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deployed in the same spectrum and area but at different time slots. In this case, $\mathcal{H}_0$ and $\mathcal{H}_1$ in (1) represent that the primary user turns on and off, respectively, and the secondary transmission is realized by utilizing the silent time slots of the primary users, which is called *temporal spectrum holes* (TSHs) in this paper.

The second scenario for CR communication is that the CR user can access the spectrum when the CR and primary users are in different geographic areas since path-loss and shadowing of wireless channels separate them and make it possible for both to work at the same time without interfering each other. In this case, $\mathcal{H}_0$ and $\mathcal{H}_1$ in (1) represent whether or not the CR and primary users can be separated by path-loss, respectively. We call this secondary communication opportunity as *geograhpic spectrum holes* (GSHs).

B. Spatial Spectrum Hole

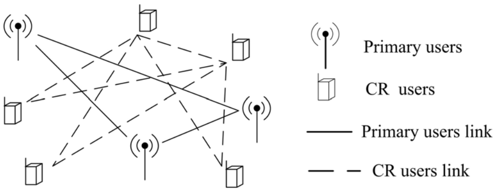

Fig. 1. Spatial Spectrum Holes

Usually, it is hard for the CR and primary users to work at the same time and location. However, Figure 1 shows a schematic representation of the coexistence of the primary and secondary systems at the same time and location. If each CR and primary user is with only one omni-directional antenna and works at the same frequency, it is impossible for CR users to coexist with primary users at the same time with acceptable interference. However, if CR users are with directional transmission ability, which can be implemented by a directional antenna or by transmit antenna arrays with beamforming, they will be able to coexist with primary users as shown by the dashed line links in the figure. The secondary communication opportunity in this case is called *spatial spectrum holes* (SSHs) in this paper since we are using different spatial domains that primary and CR links are in.

III. SECONDARY COMMUNICATION WITH DIRECTIONAL TRANSMISSION IN CR NETWORK

As discussed in the previous section, CR users can exploit SSHs through directional transmission. Figure 2 illustrates how the SSHs can be utilized for secondary communication in a CR network. As shown in the figure, two primary users are communicating with each other and a number of CR users are operating in the same area and spectrum band. Each CR user is with directional transmission and omin-directional receiving ability. For simplicity, we assume the direction of transmission of each CR user is random.

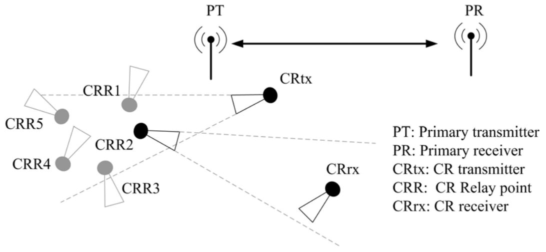

Fig. 2. System Diagram

When the *CR transmitter* (CRtx) intends to send messages to the *CR receiver* (CRrx), it first senses the spectrum by its directional antenna. If the sensing result is idle, the spectrum can be used by the CRtx; otherwise, it is referred as CR system's outage and the CRtx needs to vary its direction of its transmission randomly. If the sensing result of the CRtx is idle and the CRrx is within the coverage of the CRtx, the direct link from the CRtx to CRrx can be created; otherwise, relay stations must be used. The candidate relay points must satisfy the following two requirements:

- The coverage of its directional transmission is clear, that is, the sensing result of the relay point is idle so as not to cause interference to the primary users.
- Its directional transmission faces the CRrx, or faces another candidate relay point if multi-hop relay is assumed.

In Figure 2, the *CR relay point 2* (CRR2) satisfies the requirements. Therefore, a two-hop relay link can be established between the CRtx, CRR2, and CRrx.

All above discussion is to share or multiplex the spatial resource among the primary and CR users. Obviously, multiple CR users can also multiplex available SSHs, as shown in Figure 1. To facilitate analysis, we divide CR links into different classes according to their priorities. The link in Class I has higher priority than that in Class II and it only needs to avoid interfering the primary users. In contrast, the link in Class II has to avoid interfering both primary users and the CR users in Class I. Apparently, there will be fewer SSHs for Class II than for Class I. Similarly, the secondary transmission of the CR users in Class III, Class IV, and so on, can be realized. Thus, multiple CR links can work simultaneously, which dramatically increases the spectrum efficiency.

Apparently, CR users will undergo interference from the primary users and hence appropriate interference avoidance or cancellation techniques, such as dirty-paper-coding [12], need to be used. Meanwhile, dedicated control channels are needed to effectively organize all the nodes in the CR network. While a lot of issues need to be investigated regarding the proposed scheme, we will focus on the probability of successfully establishing CR links and the multiplexing of the proposed scheme in the rest of this paper.

IV. PERFORMANCE ANALYSIS

In this section, the instantaneous *successful communication probability* (SCP) from the CRtx to CRrx will be analyzed first. Then, the SCP within multiple consecutive time slots will be investigated. Also, the multiplexing of the proposed scheme in a CR network will be considered. For simplicity, ideal spectrum sensing and data transmission are assumed throughout this section.

A. Successful Communication Probability

In this subsection, we only focus on CR transmitters with directional antennas even through the discussion here can be easily applied to CR transmitters with beanforming. Denote the directional antenna aperture as θ degrees and the normalized antenna aperture as $\gamma = \frac{\theta}{360}$. Suppose there are L primary users and N CR users in a certain area and the locations of them are independent and random with uniform distribution on each direction. Therefore, L and N actually represent the density of primary and CR users in the considered area.

1) Direct Transmission: The probability that the sensing result is idle for the CRtx is given by

$$P_{CRtx,idle} = (1-\gamma)^L. \tag{2}$$

At the same time, the probability that the CRrx falls in the coverage of the CRtx is γ. Therefore, the probability of one-hop direct communication between the CRtx and CRrx is given by

$$P_1 = \gamma(1-\gamma)^L. \tag{3}$$

2) Two-hop Transmission: When two-hop communication between the CRtx and CRrx is considered, a relay point needs to be selected to connect them. Suppose there are n CR users in the coverage of the CRtx. Obviously, n is a random variable with binomial distribution and its *probability mass function* (PMF) is given by

$$f(n) = \binom{N}{n}\gamma^n(1-\gamma)^{N-n},\ 0 \le n \le N, \tag{4}$$

where $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ and ! denotes factorial operation. For each of the n CR users in the coverage of the CRtx, according to the two requirements for CR relay candidate, the probability that it qualifies as a relay point is $\gamma(1-\gamma)^L$. Therefore, given n, the probability that there exists at least one relay candidate point is given by

$$P_{relay,n} = 1 - \left[1-\gamma(1-\gamma)^L\right]^n. \tag{5}$$

Define $A = 1-\gamma(1-\gamma)^L$, then the mean value of $P_{relay,n}$ can be obtained by

$$P_{relay} = \mathbb{E}\{P_{relay,n}\} = 1 - \mathbb{E}\{A^n\} = 1 - \mathbb{E}\{e^{n\ln A}\}, \tag{6}$$

where $\mathbb{E}(\cdot)$ represents expectation operation. Recall that the *moment-generating function* of the binomial random variable, n, is given by [13]

$$M_n(t) = \mathbb{E}(e^{nt}) = (1-\gamma+\gamma e^t)^N. \tag{7}$$

According to (6) and (7), we obtain

$$P_{relay} = \mathbb{E}\{P_{relay,n}\} = 1-(1-\gamma+\gamma A)^N. \tag{8}$$

Therefore, the average SCP of two-hop relay link can be obtained by

$$P_2 = P_{CRtx,idle}\cdot P_{relay} = (1-\gamma)^L[1-(1-\gamma+\gamma A)^N]. \tag{9}$$

Although it is theoretically feasible to realized secondary transmission between the CRtx and CRrx through multiple hops, we are concerned with the overhead and complexity issues in the proposed scheme and only consider direct transmission and two-hop transmission with the help of one single relay point in this paper.

Since P_1 and P_2 are different possibilities, they are independent. Thus, the SCP of a maximum of two-hop relay is given by

$$\begin{aligned} Q_2 &= \Pr\left[\bigcup_{m=1}^{2} P_m\right] = 1 - \prod_{m=1}^{2}(1-P_m) \\ &= 1-[1-\gamma(1-\gamma)^L] \\ &\times \{1-(1-\gamma)^L[1-(1-\gamma+\gamma A)^N]\}. \end{aligned} \tag{10}$$

where $\bigcup$ represents the union operator and $\Pr(a)$ denotes the probability of a.

B. SCP in Multiple Consecutive Time Slots

In the previous subsection, we assumed that the directions of the transmission of CR users were fixed and investigated the instantaneous SCP within one single time slot. In this subsection, we will investigate the SCP between the CRtx and CRrx in multiple consecutive time slots, which may caused by CR and primary user's movements or the different bean vectors in different time slots. To facilitate analysis, we assume the direction of the CR transmission varies independently from slot to slot, which may be caused by the CR user's random movements or by different beanpattern vectors at different time slots. Define v as the time slot index, P as the SCP in each time slot, then the overall SCP within consecutive V time slots is given by

$$P_V = \Pr\left[\bigcup_{v=1}^{V} P\right] = 1-(1-P)^V. \tag{11}$$

Apparently, a higher SCP can be achieved by letting CR users attempt to communicate in more time slots.

C. Spectrum Multiplexing among CR Users

Assume there is a primary network and a secondary network coexisting in the same area using the same frequency with bandwidth, W, in a particular time period, T. Therefore, the time-bandwidth product is $\mathbb{M} = TW$, which represents the available *time-frequency resource* (TFR). Suppose there are L' primary users and N' CR users operating in the considered area. If only one CR link coexists with the primary links, we call it the link in Class I. For the link in Class I, the SCP of m_1-hop can be calculated from (3) or (9) as $P_{m_1}^{(1)}(L = L', N = N')$, where $1 \le m_1 \le 2$ and the upper index of P represents the class.

For the link in Class II, the link in Class I should be seen as an equivalent primary link to avoid interference. Therefore, the corresponding SCP of m_2-hop link in Class II can be expressed as

$$P_{m_2}^{(2)} = \Pr\left[\bigcup_{m_1=1}^{M} P_{m_2}^{(2)}(L' = L + m_1, n = N' - m_1)\right], \quad (12)$$

where $1 \leq m_2 \leq M$ and $M = 2$. Similarly, the SCP of m_k-hop link in Class k is given in (13) and the TFR of the link in Class k with a maximum of M hops is given by

$$\mathbb{M}_k = \mathbb{M} \cdot \Pr\left[\bigcup_{m_k=1}^{M} P_{m_k}^{(k)}\right] = \mathbb{M} Q_M^{(k)}, \quad (14)$$

where $Q_M^{(k)} = 1 - \prod_{k=1}^{K}(1 - P_{m_k}^{(k)})$. Therefore, the overall TFR for CR network is

$$\mathbb{M}_{CR} = \sum_{k=1}^{K} \mathbb{M}_k = \mathbb{M} \sum_{k=1}^{K} Q_M^{(k)}, \quad (15)$$

where K is the number of active secondary links. Then, the equivalent reuse ratio of the TFR for the CR network can be expressed as

$$\eta = \frac{\mathbb{M}_{CR}}{\mathbb{M}} = \sum_{k=1}^{K} Q_M^{(k)}, \quad (16)$$

which is determined by the density of the primary and CR users, the CR's directional antenna aperture, and the maximum number of hops, etc.

V. NUMERICAL RESULTS

In this section, we deliver the numerical results to show the performance of the proposed CR transmission scheme. We assume that there is a primary network and a secondary network deployed in the same area. We further assume that the number of CR users is much larger than that of primary users, i.e. $N \gg L$, which may happen when the CR users want to coexist with primary users with shorter communication range comparing to primary user's.

Figure 3 illustrates the CR's SCP, Q_2, versus normalized antenna aperture γ for different N's, when $L = 2$ and $M = 2$, i.e., there are two primary user communicating with each other in the area and the secondary link can consist of a maximum of two hops. Figure 3 indicates that the maximum Q_2 increases with the number of CR users, which is reasonable since more CR users means a more sufficient exploitation of the SSHs. Also, it can be observed that the larger the number of CR users is, the narrower the optimal CR directional antenna aperture is. This is because a large number of CR users means that the CRtx can easily find relay candidates and the interference from the primary users will be the bottleneck for SCP. In this case, a narrow antenna aperture can avoid the interference from the primary users. On the other hand, when the number of CR users is small, lacking of relay point will be the bottleneck and in this case a large antenna aperture can help the CR transmitter find relay points easily.

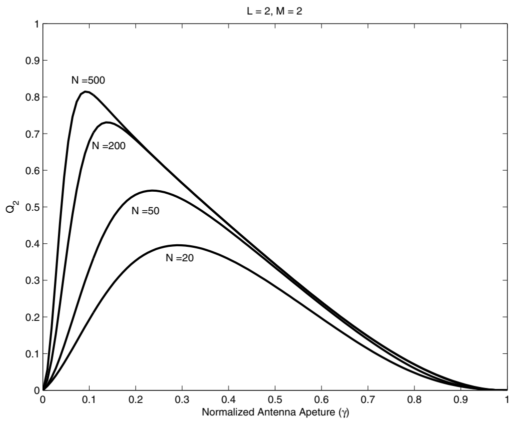

Fig. 3. SCP versus γ for Different Ds

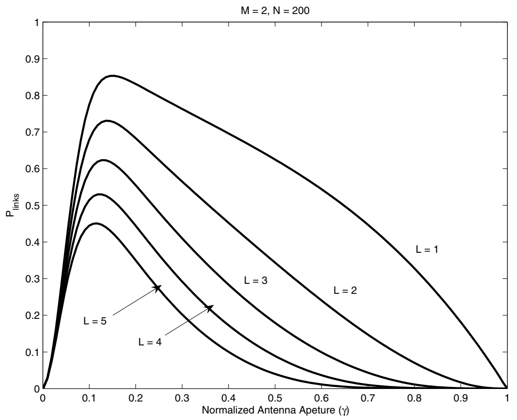

Fig. 4. SCP versus γ for Different Ls

Figure 4 shows the SCP versus γ curves for different L's, when $M = 2$ and $N = 200$. Figure 4 indicates that both the maximum Q_2 and the corresponding optimal antenna aperture go down as the number of primary user, L, increases. This is because, as L increase, the chance of SSHs decreases and the antenna aperture needs to be decreased so as not to cause interference to the primary users.

Figure 5 shows the SCP versus γ for different M's when $L = 2$ and $N = 200$. It can be observed from Figure 5 that Q_M dramatically increases as M goes from 1 to 2, indicating the utilization of relay techniques greatly improves the system performance.

Figure 6 demonstrates how the SCP increases with the number of time slots. Here we assume the CR user's antenna directions vary independently from slot to slot. It can be observed from Figure 6 that the SCP goes up dramatically as time goes by, indicating the successful second communication can be guaranteed if only the CRtx attempts to launch the connection continually.

Figure 7 shows the SCP versus γ when $L = 2$, $M = 2$, and $N = 50$. The solid lines in the figure are SCP curves of different classes and the dash line is the summation of them, which represents the overall SCP in the CR network. Figure 7 indicates that the SCP goes down as the Class number k increases, which is reasonable since a larger k means a lower priority and more equivalent primary users to avoid

$$P_{m_k}^{(k)} = \Pr\left[\bigcup_{m_{k-1}=1}^{M} \ldots \bigcup_{m_1=1}^{M} P_{m_k}^{k}\left(L = L' + m_{k-1} + \ldots + m_1, N = N' - m_{k-1} - \ldots - m_1\right)\right]. \tag{13}$$

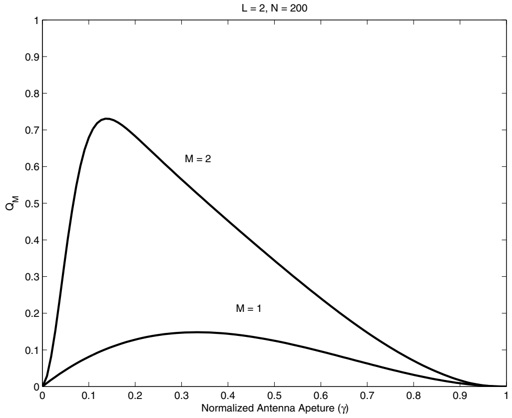

Fig. 5. SCP versus γ for Different Ms

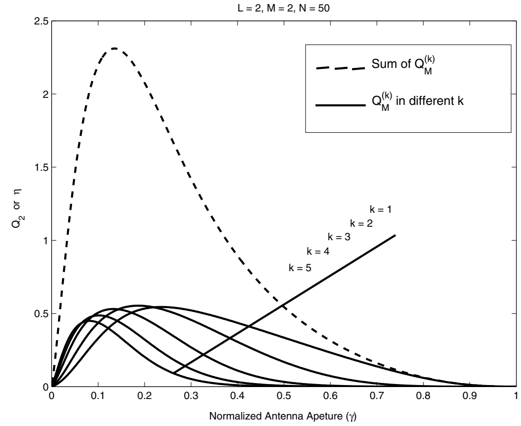

Fig. 7. SCP versus γ in Different classes

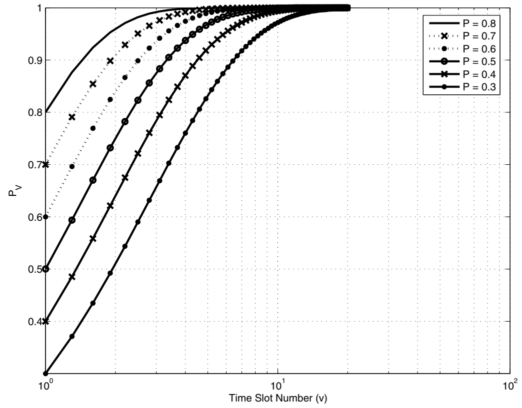

Fig. 6. SCP versus time slot number

interfering. From the dashed line, it can be observed that the proposed scheme can greatly increase the spectrum efficiency. Figure 7 also indicates that the optimal γ is different for different classes. Therefore, an appropriate antenna aperture shall be selected according to the CR user's density in the CR network and the quality-of-service requirement. For instance, the antenna aperture that maximizes η can be chosen to optimize the overall network performance.

VI. CONCLUSION AND FUTURE RESEARCH

In this paper, we have discussed the benefits by exploiting directional transmission ability of CR in its networks. We have investigated *spatial spectrum holes* by relay assisted CR secondary directional transmission. The successful communication probability and multiplexing ability of this scheme has been analyzed. The numerical results have shown that the reliable communication can be achieved if only the density of CR users is sufficiently high or enough time slots are available. Moreover, the spectrum efficiency of the CR network can be dramatically increased by multiplexing multiple CR links. In our future work, we will consider the impact of imperfect spectrum sensing and data transmission on the exploitation of SSHs.

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